The Impact of Imperfect Credibility in a Transition to Price Stability*

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Abstract

In this paper we study the impact of a temporary lack of credibility in a transition to price stability. We quantify the effects of a period of disinflation on temporary output losses, and the impact of the lack of credibility on the optimal speed of disinflation. We demonstrate that the "disinflationary booms" found by Ball (1994) and Ireland (1997) may or may not disappear in an environment with imperfect credibility, depending on the speed of learning relative to the speed of disinflation. Finally we enquire whether the speed of the Volcker disinflation was excessive or not.

JEL Classification: E20; E32; F32; F41.
Keywords: Price stability; imperfect credibility; optimal speed of disinflation.

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1. Introduction

In this paper we study the effects of a disinflationary monetary policy when policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price-level. The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices. Recent research has revealed much about the effects of monetary contraction in such an environment.

For our purpose, three broad results stand out from this work. First, in the periods following a contraction in the money stock, real output is likely to fall below its (now altered) long-run equilibrium level. Second, a gradual disinflation may actually result in output, after its initial decline, rising above its new steady-state level, and remaining so for some time. And finally, it is optimal to end high inflations quickly, low inflations gradually, and maintain inflation at or near zero, thereafter. The key papers that develop these results are due to Ball (1994), Ireland (1997), King and Wollman (1999) and Khan, King and Wollman (2002). Important precursors to the analytical foundations of these results are contained in Danziger (1988), Benabou and Konieczny (1994) and Lucas and Stokey (1983), while the contributions of Sargent (1982) and Gordon (1982), as emphasized by Ireland, provide an important focus on policy implications of differential speeds of disinflation.

The theoretical papers just mentioned, and many others besides, assume perfect foresight (or rational expectations). For some purposes this assumption is obviously appropriate: what other assumption makes sense when one wishes to analyze an economic model in, or in a close neighborhood of, an unchanging steady state? However, the assumption of perfect foresight may be less attractive when one wishes to analyze the effects of 'large' changes in policy. For one thing, the steady state of the model may well be changing. In addition, policymakers may not enjoy complete credibility. In this paper we examine the effects of a disinflationary monetary policy when policymakers initially do not enjoy complete credibility, and where the steady state of the economy is changing. We model the monetary policymakers as doggedly pursuing the goal of price stability in the face of this imperfect, but improving, credibility.

Two important recent contributions address some of the issues we do. The first is Ball (1995). He demonstrates that if credibility is sufficiently low, a period of disinflation may lead to expected output losses. In his model agents harbour a nagging suspicion that the authorities will renege and give up on the path of disinflation. He models agents' scepticism as a constant conditional probability of reneging. This may be a somewhat rigid way of modelling the evolution of agents' priors. On the one hand, as the disinflation proceeds it is plausible that agents accord increasing weight to the announced path for the money supply. On the other hand, perhaps as the disinflation proceeds and the extent of nominal rigidity in the economy optimally rises, the authorities may be more likely to renege (to exploit a flattening of the Phillips curve). *Ex ante* both of these cases seem intuitively plausible, and so we propose an 'expectations updating rule' that nests these alternatives. The distinction between these two cases can be important, as we demonstrate below. In addition, Ball (1995) leaves to one side the issue of the optimal speed of disinflation under imperfect credibility, a topic we take up here.

The second related paper is by Ireland (1995). He also finds that higher output losses are the price of imperfect credibility during a period of disinflation. However, the attainment of price stability is desirable (i.e., welfare enhancing) in general, except when the loss of seigniorage is replaced in the low inflation state by a rise in other distortionary taxes. Again, his modelling of the expectations formation process misses the effects to which we have just referred. In addition, we examine the issue of a lack of credibility in a more complex, but now standard, supply-side with a continuum of monopolistically competitive producers. This set up leads to some computational complexities related to the optimal choice of prices by firms who not only have to forecast future demand and cost conditions, but also have to forecast their covariances. This may be why these other authors employ somewhat simpler supply-sides in their set-ups. We also extend Ireland's (1997) calculation of the optimal speed of disinflation to the case of imperfect credibility, and enquire whether or not imperfect credibility materially impacts on the optimal speed of disinflation, as compared to the situation under perfect foresight. This is a question of first-order policy importance but which, to our knowledge, has not been addressed hitherto in the class of models employed here, and which is proving popular for policy-oriented analyses.

1.1. Outline of the Paper

In the next section we outline our model and discuss its salient features. In section 3 we display some benchmark results that demonstrate the three key points we mentioned above. In section 4 we propose our expectations updating rule. In section 5 we analyze the impact of imperfect credibility during a period of disinflation. In section 6 we use our model to analyse the Volcker disinflation. Given the actual course of disinflation during this period, we back out the implied speed of learning of agents in the economy (by matching the output gap of our model to a measure of the actual US output gap during this period). Given the implied speed of learning, we then compare the speed of the Volcker disinflation with the optimal speed of disinflation. In Section 7 we discuss our results and offer some thoughts on areas for future research.

2. The Model

2.1. The Representative Agent

Our basic framework extends the perfect foresight model of Ireland (1997). Its component parts are now familiar in the literature and so we can develop the key equations somewhat briskly. The economy consist of many identical consumers. Each period a representative agent makes plans for consumption and leisure/labour such that (expected) present discounted value of utility is maximised. This measure of utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\alpha} - 1}{1-\alpha} - \gamma N_t \right\} \qquad \alpha, \gamma > 0, \tag{2.1}$$

and is separable in consumption, C_t, and labour supply, N_t. $\beta \in (0,1)$ is a discount factor. Following Dixit and Stiglitz (1977), C_t is defined over a continuum of goods,

$$C_t = \left[\int_0^1 c_t(i)^{\frac{b-1}{b}} di \right]^{\frac{b}{b-1}} \qquad b > 0, \tag{2.2}$$

where $c_t(i)$ denotes, in equilibrium, the number of units of each good i from firm i that the representative agent consumes. b is the price elasticity of demand. $p_t(i)$ is the nominal price at which firm i must sell output on demand during time t. The Dixit-Stiglitz aggregate price level, P_t, at time t is given by:

$$P_t = \left[\int_0^1 p_t(i)^{1-b} di \right]^{\frac{1}{1-b}}. \tag{2.3}$$

Let N_t be given by:

$$N_t = \int_0^1 n_t(i)\, di,$$

where $n_t(i)$ denotes the quantity of labour supplied by the household to each firm i, at the nominal wage W_t, during each period. This assumption means that households effectively supply a portion of labour to *all* firms. The reason why we need such an assumption (and the one below regarding the representative agent's share portfolio) is to ensure that the marginal utility of wealth equalizes across agents.

Each period, the representative agent faces a budget constraint of the following sort:

$$\int_0^1 \left[Q_t\left(i\right)s_{t-1}\left(i\right)+\Phi_t\left(i\right)+W_t n_t\left(i\right)\right]di \geq \int_0^1 \left[p_t\left(i\right)c_t\left(i\right)+Q_t\left(i\right)s_t\left(i\right)\right]di. \quad (2.4)$$

Here $Q_t\left(i\right)$ denotes the nominal price of a share in firm i, s_t denotes the quantity of shares, $\Phi_t\left(i\right)di = D_t(i)s_t(i)$, where $D_t(i)$ is the dividend associated with a unit share, and $\int_0^1 p_t\left(i\right)c_t\left(i\right)di = P_t C_t$ denotes total nominal expenditure. We assume that for $t = 0$, $s_{-1}(i) = 1$, for all $i \in [0,1]$. In effect, then, we are assuming that each household owns an equal share of all the firms. The constraint (2.4) says that each period (and, under uncertainty, in each state of nature) income (financial plus labour) can be worth no less then the value of expenditure (on non-durable consumption plus financial investment). The household problem, then, is to choose $c_t(i), n_t(i), s_t(i)$ and total consumption, C_t, such as to maximize (2.1) subject to the sequence of constraints (2.4), and the relevant initial and transversality conditions. Optimal household behaviour is described by the requirement that household consumption spending must be optimally allocated across differentiated goods at each point in time (i.e., the optimal $c_t(i)$). It can be shown that the Dixit-Stiglitz preference relation requires that purchases of each good i satisfies:

$$c_t(i) = C_t\left(\frac{p_t(i)}{P_t}\right)^{-b}. \quad (2.5)$$

As in Ireland (1997) it will simplify things somewhat if we let aggregate nominal magnitudes be determined in equilibrium by a quantity-type relation:

$$M_t = \int_0^1 P_t\,(i)\,c_t\,(i)\,di = P_t C_t. \tag{2.6}$$

An interior optimum for the agent's problem will include (2.4) with equality, (2.5) for all i, (2.6) and the following conditions:

$$C_t^{-\alpha} = \lambda_t P_t; \tag{2.7}$$

$$\gamma = \lambda_t W_t. \tag{2.8}$$

And for all i

$$Q_t(i) = D_t(i) + \beta E_t(\lambda_{t+1}/\lambda_t)Q_{t+1}(i), \tag{2.9}$$

where λ_t is an unknown multiplier associated with (2.4).

2.2. The Corporate Sector

There is a continuum of firms indexed by i over the unit interval, each of them producing a different, perishable consumption good. So, goods may also be indexed by $i \in [0,1]$, where firm i produces good i.

Each firm i sells shares, at the beginning of each period t, at the nominal price $Q_t(i)$, and pays, at the end of the period, the nominal dividend $D_t(i)$. The representative household trades the number of shares that it owns, $s_t(i)$, in each of the firms at the end of each period t. Under market clearing, $s_t(i) = 1, \forall i \in [0,1]$, in each period. Firms are able to change prices each period, subject to a fixed cost. As a consequence, in equilibrium firms will not necessarily be willing to change prices in each period. The criterion for the price-setting decision at time t is to maximise the return to shareholders.

At time t we assume that firms are divided into two categories, such that firms from the first category can freely change their prices, $p_{1,t}(i)$, while the firms belonging to the second must sell output at the same price set a period before, $p_{2,t}(i) = p_{2,t-1}(i)$, unless they pay the fixed cost $k > 0$, measured in terms of labour. We may think of this cost as being associated with information collection and decision making. At time $t+1$, the roles are reversed and the first set of firms keep prices unchanged, $p_{1,t+1}(i) = p_{1,t}(i)$ unless they are willing to pay the fixed cost k, while the second set of firms can freely set new prices.

The model assumes, then, that firms are constantly re-evaluating their pricing strategy, weighing the benefits of holding prices fixed versus the alternative of changing prices and incurring the fixed penalty. However at moment t the firms belonging to the set of firms that can freely change price are able to choose between two strategies, depending on whether the inflation rate is moderate or high. At moderate rates of inflation, or in the face of gradual changes in the monetary stance, they are more likely to keep their prices constant for two periods and hence avoid the cost k (single price strategy). On the other hand, in the case of a high inflation, or in the face of sharp changes in the monetary stance, firms are more likely to choose a new price and pay the cost k (two price strategy).

We assume a simple linear production technology $y_t(i) = l_t(i)$, where $y_t(i)$ and $l_t(i)$ are output of firm i and the labour used to produce it, respectively. Let us denote aggregate output as Y_t, then equilibrium profits at time t for firm i are given by,

$$D_t(i) = [p_t(i) - W_t] \left(\frac{p_t(i)}{M_t} \right)^{-b} C_t - I_t(i) W_t(i) k. \tag{2.10}$$

While, in equilibrium, the units of labour supplied to each firm at nominal wage W_t are given by:

$$n_t(i) = Y_t^{1-b} \left(\frac{p_t(i)}{M_t} \right)^{-b} + I_t(i) W_t(i) k,$$

where

$$I_t(i) = \begin{cases} 1, & \text{if the firm pays the cost of price adjustment } k \text{ at moment } t; \\ 0, & \text{if the firm does not pays the cost } k \text{ at moment } t. \end{cases}$$

2.3. Single price strategy

Under this strategy we may think of firm i choosing $p_t(i)$ so as to maximize the following expression:

$$\Pi_t(i) = D_t(i) + \beta E_t (\lambda_{t+1}/\lambda_t) D_{t+1}(i), \tag{2.11}$$

which follows from (2.9), and implies that prices are set to maximize market value. We then set $\gamma = 1$, substitute (2.7), (2.8), the quantity equation and goods market equilibrium into (2.10). It then follows that the price for firm i that will be used for two consecutive time periods is:

$$p_t(i) = \frac{b}{b-1} \frac{M_t^b Y_t^{b-1} + \beta E_t M_{t+1}^b Y_{t+1}^{b-1}}{M_t^{b-1} Y_t^{b-\alpha} + \beta E_t M_{t+1}^{b-1} Y_{t+1}^{b-\alpha}}. \tag{2.12}$$

This equation is familiar from the New Keynesian economics. It basically says that the optimal price will be a function of current and future anticipated demand and costs conditions, and where in steady state price will be a fixed mark-up over marginal costs. As is familiar in models of monopolistic competition based on Dixit-Stiglitz preferences, the markup is constant and determined by the elasticity of demand (that is, tied down via the preference side of the model), the lower the elasticity, the higher the mark-up.

2.4. Two price strategy

In this case the firm chooses the price $p_t(i)$ to maximise profits in each period

$$\Pi_t(i) = D_t(i). \tag{2.13}$$

The optimising price in this case is given by:

$$p_t(i) = \frac{b}{b-1}\frac{M_t}{Y_t^{1-\alpha}}. \tag{2.14}$$

Here we see that prices are a mark-up as before; now it is only current period demand and cost conditions that are relevant.

3. Model Calibration and Benchmark Results Under Perfect Foresight

The calibration of the model follows Ireland (1997). We set $\alpha = 0.1$ so that the intertemporal elasticity of substitution follows Ball, Mankiw and Romer (1988). As in Rotemberg and Woodford (1992) $b = 6$, corresponding to a benchmark value of 1.2 for the steady-state markup. Following Ball and Mankiw (1994), each interval of time in the model corresponds to a period of six months, determining the choice of $\beta = 0.97$, consistent with an annual discount rate of 5 percent. k, the inflation rate at which the rigidity of individual goods prices vanishes, (i.e., firms switch from the single price strategy to the two price strategy) is set at 0.1075. Finally, we assume $\gamma = 1$.

We study the effect of a monetary policy that brings money growth to zero over some horizon. This was the approach adopted in Ireland (1997), following Ball (1994). Specifically, at period 0, the authorities make a surprise announcement about the path for the money supply, $\left\{M_s^A\right\}_{s=0}^{T}$, such that by time period T inflation will be zero. The superscript A indicates the 'announced' level of the money supply. This announced path for the money supply, in turn, implies a gradual decrease in the growth rate of the money supply. Let θ_t denote the growth rate of the money stock at time t. We study, then, processes for the money growth rate of the following sort:

$$\theta_t = \theta_{t-1} - \frac{\theta - 1}{T}, \tag{3.1}$$

for any value of t from 0 to $T-1$,where θ_{-1} is equal to the initial rate of inflation, and where $\theta_{t\,>\,T} = 1$. So, a horizon of time $T = 1$ entails immediate disinflation, while for $T > 1$ the policymakers engineer a more gradual path towards price stability.

Figure 3.1 shows the effect of an immediate disinflation on output when the initial inflation rate is 3% (the dashed line) and when the initial rate is 200% (the solid line, which is coincident with the x-axis). We see that at relatively low rates of inflation, this is quite costly as firms follow a single-price strategy. The 'hump-shaped' response is due to the fact that the first set of firms to set new prices increase their price; past inflation has eroded their relative real price and now they face a relatively large increase in demand for their products (since the firms that don't re-price have relatively high prices and hence relatively low demand). At higher rates of initial inflation, firms re-price every period (two-price strategy) and hence disinflation can proceed with no relative-price distortion.

Figures 3.2 and 3.3 show the effects of a gradual disinflation from 3% and 200% respectively. After the initial drop in output, a gradual disinflation leads to a boom in output–a relatively prolonged period of above steady state output. Agents set prices for two periods, and because inflation will be lower in the future, they set lower prices today, causing a boom. At high initial rates of inflation, the loss in output in the initial periods can be substantial. The problem is that with gradual disinflation from high rates of inflation, firms do not initially change their prices.

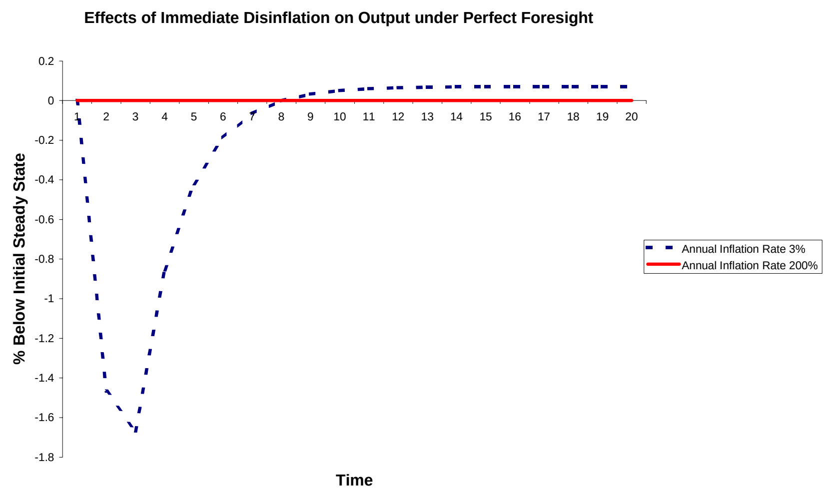


Figure 3.1:

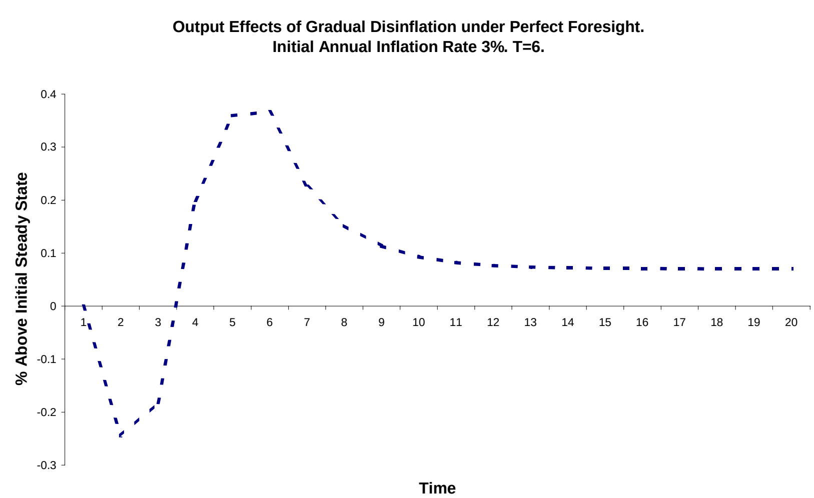


Figure 3.2:

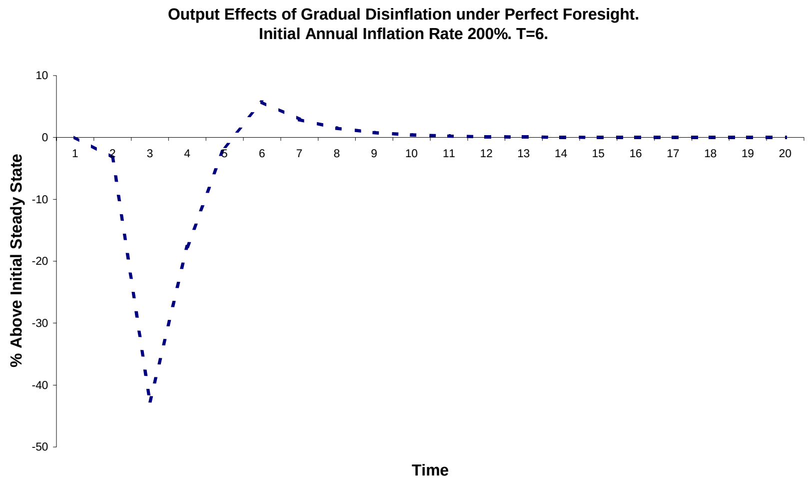


Figure 3.3:

Finally, figure 3.4 shows the optimal speed of disinflation for initial inflation rates of between 1% and 20%. From various initial levels of inflation we calculate the level utility associated with different speeds of disinflation. During big inflations firms are more likely to follow a two price strategy and hence under perfect foresight disinflation is costless. On the other hand, at relatively low rates firms are more likely to follow a single-price strategy and rapid disinflation is more likely to be costly. It turns out that the optimal equilibrium strategy for each initial inflation reported is the single price strategy (although two price strategies were employed along a number of sub-optimal disinflation paths). At very high rates of inflation (not reported) the two price strategy is optimal. We return to what determines the optimal speed of disinflation below.

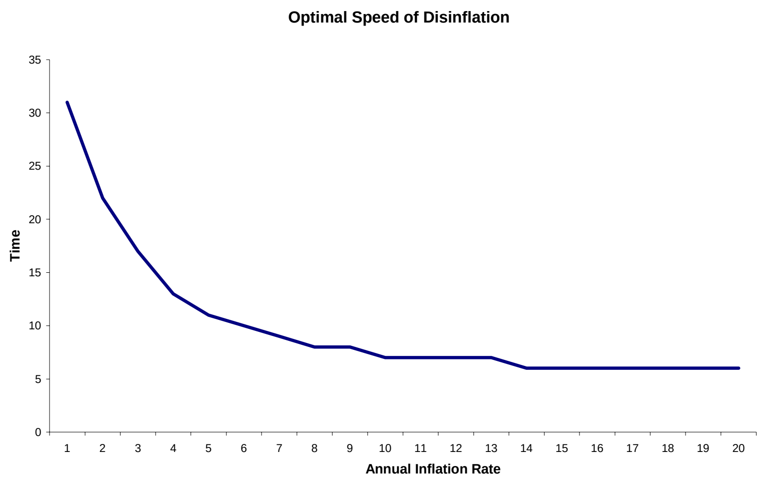


Figure 3.4:

4. Imperfect Credibility

In this section we consider what might happen when credibility is imperfect, but nevertheless improving through time. We run variants of the above experiments in an environment where the probability mass characterising agents' subjective expectations is shifting through time onto the central bank's announced money supply path. Introducing uncertainty into our framework results in some computational complexity which an appendix discusses[1]. Again the policy employed is to lower money growth linearly to zero over some time horizon, $T \geq 1$. To retain computational manageability, we assume that agents perceive of only two possible outcomes. One outcome is the monetary authority's announced path for the money supply. The other outcome is a reversion to an alternative, more inflationary, path for the money supply. There are two obvious choices for this

[1] This appendix is available on Nolan's website: http://www.st-andrews.ac.uk/~cn14/home.htm

alternative path: First, agents perceive the authorities as reverting to the previous steady state inflation rate. Second, alternatively, they fear the government will 'run out of steam' such that at time t (for $0 < t < T$) the growth rate of the money stock will be equal to the growth rate between $t-1$ and t. Algebraically, we can characterize these alternate expectations as follows:

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{-1}M_{t+j-1} + (1-\rho_{t+j})M^A_{t+j}; \tag{4.1}$$

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{t-1}M_{t+j-1} + (1-\rho_{t+j})M^A_{t+j}. \tag{4.2}$$

We will assume that the authorities stick to the announced path of disinflation, so in practice (4.1) and (4.2) may be rewritten as

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{-1}M^A_{t+j-1} + (1-\rho_{t+j})M^A_{t+j};$$
$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{t-1}M^A_{t+j-1} + (1-\rho_{t+j})M^A_{t+j}.$$

In characterizing $\{\rho_s\}_{s=0}^{T+J}$ we need to decide on ρ_0, a measure of the initial level of credibility, the time it takes until $\rho_{T+J} = 0$, for $J \geq 0$, and the path of ρ_s in the transition between these extrema. One option is simply to let ρ_s converge linearly to zero in the following way:

$$\rho_t = \rho_{t-1} - \alpha\frac{\rho_0}{N} \quad t \geq 1, \tag{4.3}$$

where N is the period of the disinflation (measured in half-years) and $\alpha \in (0,1)$. α captures the time it takes for agents to believe completely the central bank's announcements–i.e., for a perfect foresight equilibrium to obtain. However, there may be more plausible characterizations[2]. The following function is useful for capturing such paths:

[2] This linear path for ρ leads to results intermediate between those which we label below as 'concave' and 'convex'. The results showing this are available on request.

$$\rho_t = (-1)^{\delta} k(a^2 - (t - \delta a)^2)^{\frac{1}{2}} + \delta \rho_0, \tag{4.4}$$

where $\delta = 0$ or 1[3]. In the event that $\delta = 0$ it can be shown that ρ_t follows the simple recursive process:

$$\rho_t/k = \left\{ \left(\rho_{t-1}/k\right)^2 + (1 - 2t) \right\}^{\frac{1}{2}}. \tag{4.5}$$

On the other hand, if $\delta = 1$ then we have that

$$\rho_t/k = \left\{ \left(\left(\rho_{t-1} - \rho_0\right)/k \right)^2 + [1 - 2(t - a)] \right\}^{\frac{1}{2}} + \rho_0/k. \tag{4.6}$$

Given ρ_0, (4.5) plots the path $\{\rho_s\}_{s=0}^{T}$ as a concave function. This captures the intuitive idea that agents may be reluctant to update their priors initially. However, as time goes by and the central bank sticks to its announced money supply targets, they increasingly come to believe the announced target path. We shall refer to this case as *concave (expectations) updating*[4]. On the other hand, (4.6) reflects a population, although happy to accept that the monetary authority dislikes the current relatively high rate of inflation, nevertheless worries that as the slope of the short-run Phillips curve flattens the monetary authority may be tempted to renege. The importance of the exploitability of the Phillips curve has been emphasized by Ball, Mankiw and Romer (1988) and is a crucial factor in high inflation equilibria in games of the Barro and Gordon (1983) sort.[5] We refer to this as *convex (expectations) updating.*

[3] It can be shown that $a = N$ and $k = N/\rho_0$.

[4] That is, ρ plots as a concave function of time on the $x-y$ plane, where the $x-$axis measures time.

[5] Intermediate cases are possible to imagine, such as a truncated bell-shaped path for ρ. This would capture a situation in which agents initially place little weight on the authority's announcements, as in (4.5). However, after some time (characterised by an inflexion in the path of ρ), agents once again become more sceptical, as in (4.6). We ignore these alternate paths.

We still have two difficult questions to answer. First, what is a reasonable value for ρ_0, and at what point T do we have that $\rho_{s \geq T} = 0, \forall s$; how credible is the authority's announcement at date zero, and how long does it take for agents to 'arrive' at perfect foresight? We know of no studies that we can easily draw on to parameterize functions (4.5) and (4.6), so our approach has been to analyze the outcome of various thought experiments under many different parameterizations and to present the results we believe to be robust. For now we assume that $\rho_0 = 1$,[6] and that $\rho_T = 0$ after three years[7]. In Section 6 we shall attempt to parameterize our learning rules using the experience of the US in the early 1980s.

5. The Effect of Imperfect Credibility

5.1. Concave Expectations Updating Under (4.1)

In all of the charts that follow the dashed line is the perfect foresight case, and the solid line is the imperfect credibility case. Figure 5.1 compares the path of output under perfect foresight and concave expectations updating, given an initial inflation rate of 3%.

The contraction in output is more pronounced and more protracted under imperfect credibility. And even though by period 6 agents in both economies have the same information, the effects of imperfect credibility remain for some time due to the overlapping nature of price setting.

One of the potentially counterfactual implications of the perfect foresight case, as emphasized by Ball (1994, 1995), is the implication of the 'disinflationary boom': i.e., the tendency for output to rise above its new steady state level under

[6] We experimented with a number of different initial values for ρ_0. The results were virtually unchanged for values of ρ_0 as low as 0.5.

[7] If ρ takes a longer time to reach zero, output obviously also takes a longer time to reach its new steady state level. Our assumption implies agents finally believe completely the anouncement when, and only when, price stability is actually achieved.

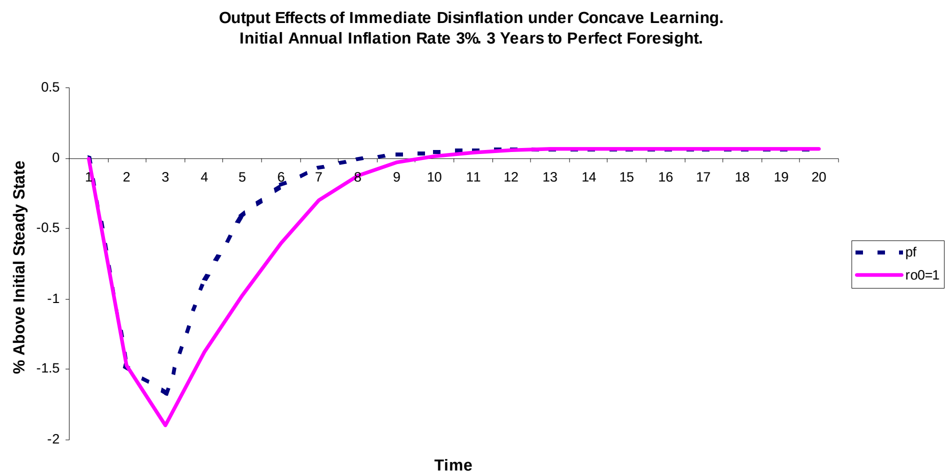
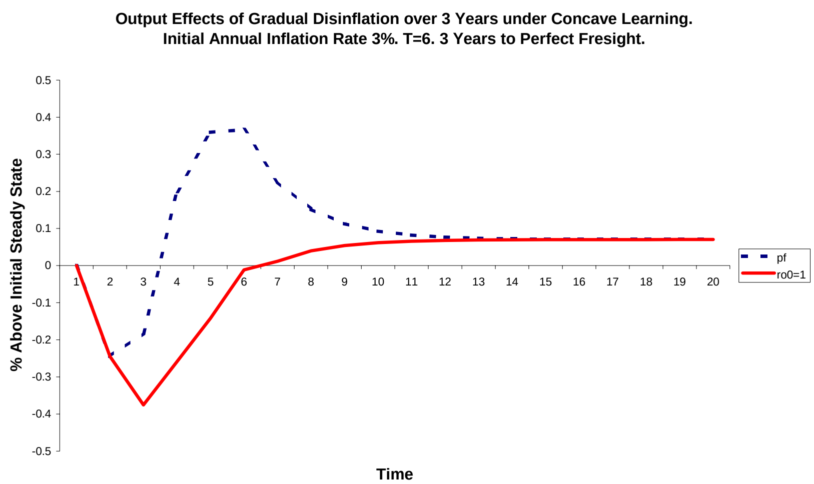


Figure 5.1:

a gradual disinflation as agents anticipate lower future price-levels.

Figure 5.2 shows that under concave updating this effect may vanish as output falls more sharply and does not rise above its new steady state value along the transition path. Agents only gradually come to realise that the price-level is to grow at a zero rate–a realization that is all the more tardy because of the gradualness of the disinflationary process itself. For very high initial inflation rates, the fall in output following an immediate disinflation is catastrophic as figure 5.3 demonstrates and it is also of a similar order of magnitude under a more gradual disinflation, as figure 5.4 shows.

Given the extra cost imposed by imperfect credibility, what is the quantitative impact on the optimal speed of disinflation? Figure 5.5 reveals that a good 'rule-of-thumb' is that disinflations from initial rates between 2%-11% should take an extra year, as compared with the perfect foresight case. In contrast for inflation rates above 12% and less than or equal to 1% the optimal speed of disinflation is



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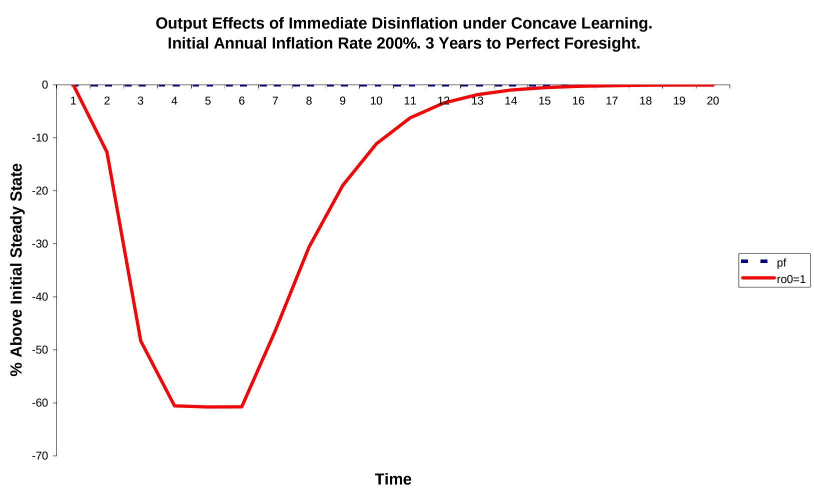

Figure 5.3:

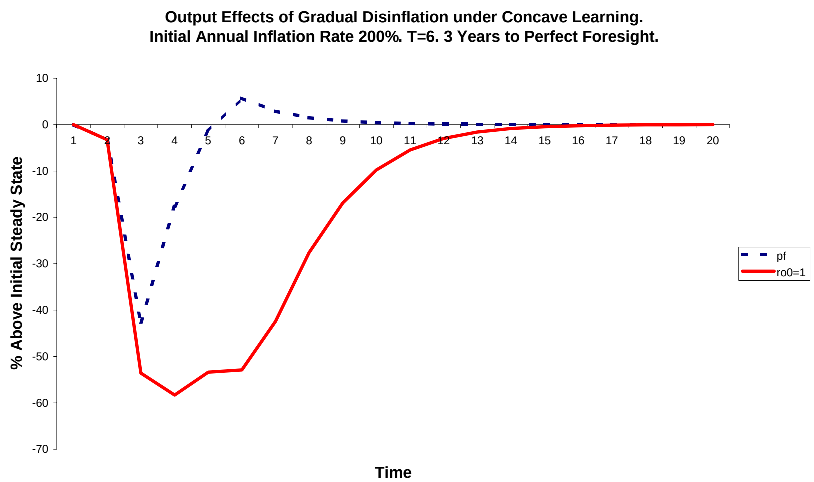

Figure 5.4:

indistinguishable from the perfect foresight case.

The key reason that gradual disinflations are attractive is that, with some price stickiness and under perfect foresight, they often imply prolonged periods of above trend output and consumption. However, as the initial inflation rate rises the contraction in output in the early periods of the disinflation is more pronounced, increasingly offsetting the utility gain from the subsequent boom– the optimal speed of disinflation rises.

Under imperfect credibility, the initial contraction in output is more severe for any initial inflation rate than is the case under perfect foresight. Furthermore, the utility gain from the disinflationary boom may be absent. It turns out that a more gradual period of disinflation is optimal up until an initial inflation rate of around 12%. For initial inflation rates greater than 12% the optimal speed of disinflation is the same as under perfect foresight.

In short, therefore, under perfect foresight gradual disinflations are primarily

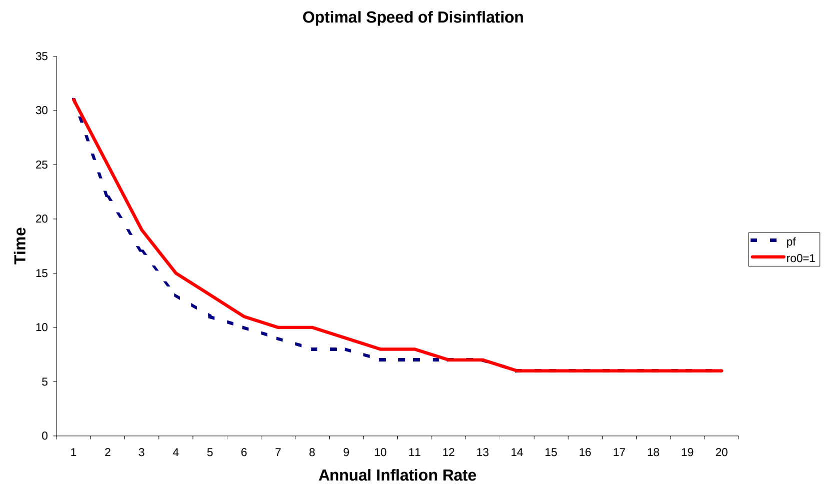

Figure 5.5:

about reaping the utility from output gains following an initial contraction in activity, while under imperfect credibility a primary concern is avoiding over-sharp contractions in activity in the early period of the disinflation.

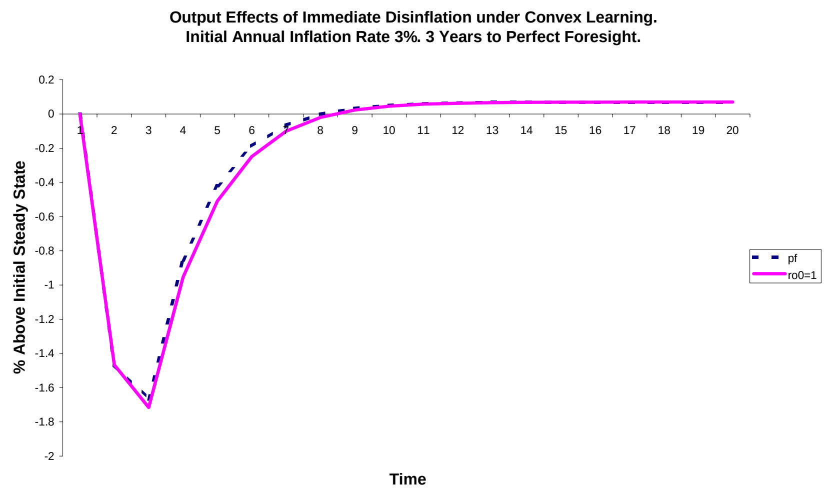

Figure 5.6:

5.2. Convex Expectations Updating Under (4.1)

Many of the same qualitative results found under concave updating are present with convex updating. However, as is apparent from Figures 5.6-5.9, the outturns look closer to the case of perfect foresight compared with concave updating. The reason for this is that the convex path of ρ means that agents avoid some of the more costly mistakes early on in the disinflation that may occur under concave updating.

Figure 5.6 shows that the drop in output under immediate disinflation leads to a drop in output more severe than, but close to, that under perfect foresight.

This tendency for agents to believe the authorities when they announce decreases in the rate of growth of money also permits disinflationary booms to occur, as figure 5.7 shows, for 'moderate' rates of inflation whilst such booms are absent for higher initial rates of inflation, as figure 5.8 demonstrates.

The optimal speed of disinflation, figure 5.9, is closer to the case of perfect

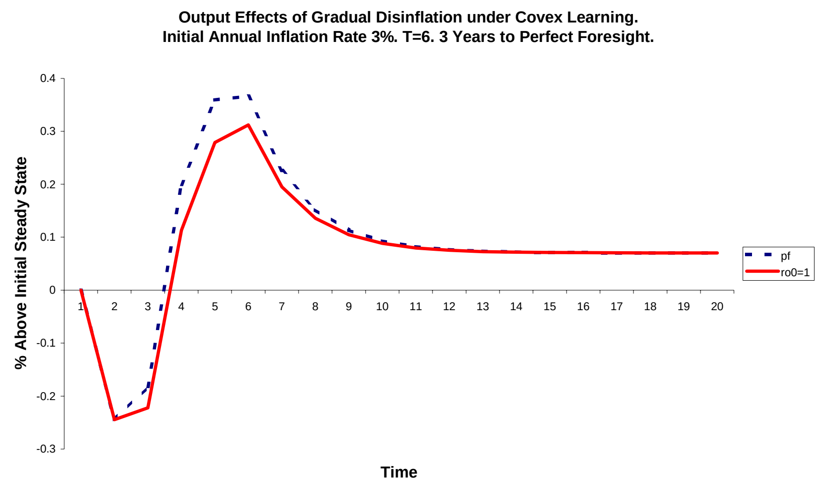

Figure 5.7:

foresight than under concave expectations updating.

5.3. Concave and Convex Expectations Updating Under (4.2)

Under (4.2) agents expect the disinflation policy to stall such that there is some probability that next period's inflation will equal this period's. We found that because the initial expectational errors are smaller than under (4.1), the results tend to be closer to those found under perfect foresight; deflationary booms are more likely to occur under (4.2). As before, concave updating tends to make these booms disappear,[8]although this is now less likely to occur than under (4.1). For example, Figure 5.10 compares the effects of gradual disinflation under perfect foresight (dashed line) with concave learning under (4.1), the bottom line; and with concave learning under (4.2), the middle line.

[8]We do not present the results in detail, but they are, in chart form, available upon request.

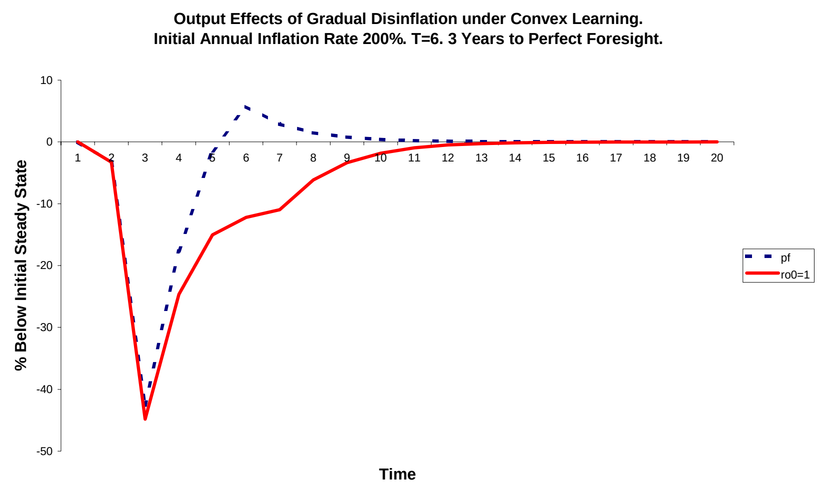

Figure 5.8:

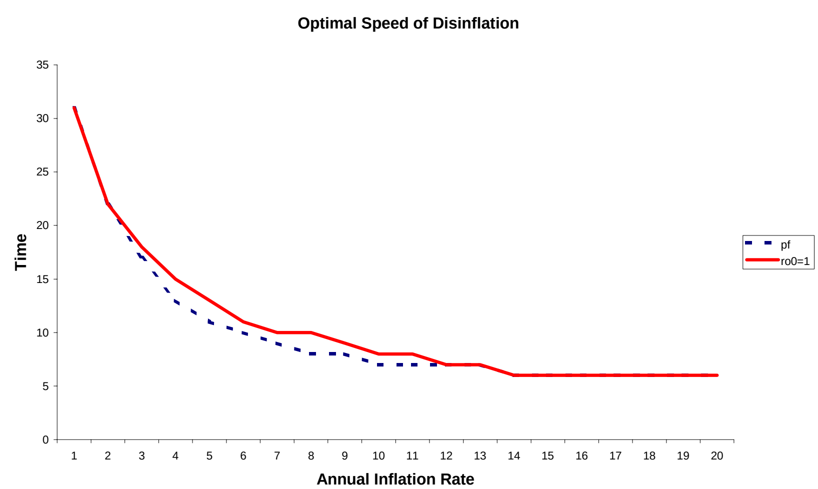

Figure 5.9:

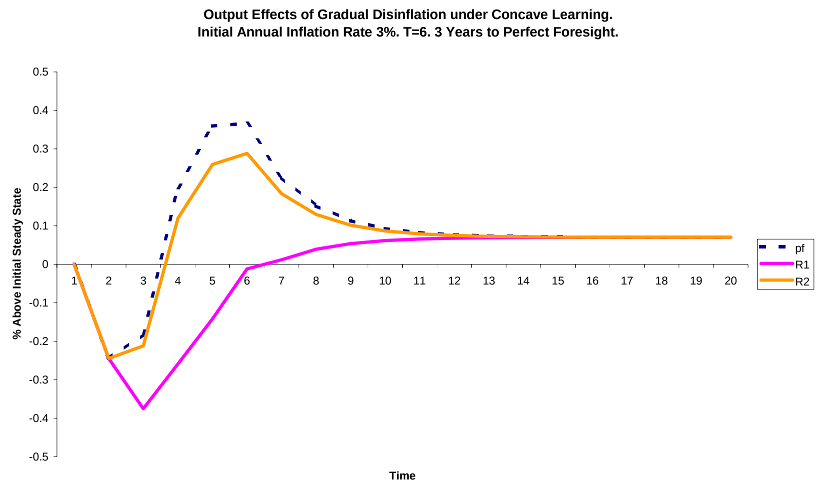

Figure 5.10:

6. The Volcker Disinflation and the Optimal Speed of Disinflation[9]

In sections 4 and 5 we adopted a number of different expectations updating rules. In this section we attempt to use the experience of the US in the early 1980s to look at, for this period at least, what might be an empirically plausible version of the expectations updating rule.

Between late 1979 and 1985 inflation in the US fell, broadly speaking, from 10% to 4%, and has subsequently fallen further. This reduction in the inflation rate was costly, as measures of the output gap from this period indicate. In this section, we take a first pass at the issue: might the Volcker disinflation have been too rapid? To do this we assume that the initial steady state inflation was 10% (around the highest level that actual US inflation reached before coming down),

[9] We would like to thank an anonymous referee for suggesting the analysis that we undertake in this section.

and that Volcker intended to reduce inflation to 2% (close to the average inflation rate since 1984). We use the currency component of M1 from 1979 until 1984 as the path for the money stock during the disinflation. Given this path and assuming perfect foresight we calculate the flexible price level of output of our model. We then back out a learning process such that the output gap in our model tracks the actual US output gap. Given that learning process we can then calculate the optimal speed of disinflation.

The dashed line in figure 6.1 is the US output gap as measured by the OECD. The solid line is the output gap of our model. To generate such a sharp contraction in output we had to adopt a somewhat extreme form of concave learning, where agents to not believe the 'announced' money supply growth rates for almost the whole course of the disinflation (5 years), and then in the course of year six, believe entirely the announced path. And even then we cannot quite generate the sharpness of the contraction indicated in the OECD data; although both measures of the output gap bottom out in 1982, the model ouput gap at the trough is 1% less than the OECD estimate.

The optimal speed of disinflation from an initial rate of inflation of 10% to 2% under perfect foresight is three and half years. In contrast the optimal speed under the learning process implied above is seven years. Our calculations suggest that even had the disinflation taken longer than it actually did the output losses would have been only marginally lower.

Clearly one needs to be cautious in an exercise such as this as we are squeezing real world data into a highly simplified framework, but we think these results are indicative. It appears that once inflation has risen substantially, imperfect credibility makes sizeable output losses in the transition to price stability highly likely, even when the speed of disinflation is 'optimal'. In a related analysis Erceg and Levin (2003) come to some similar conclusions. They examine the

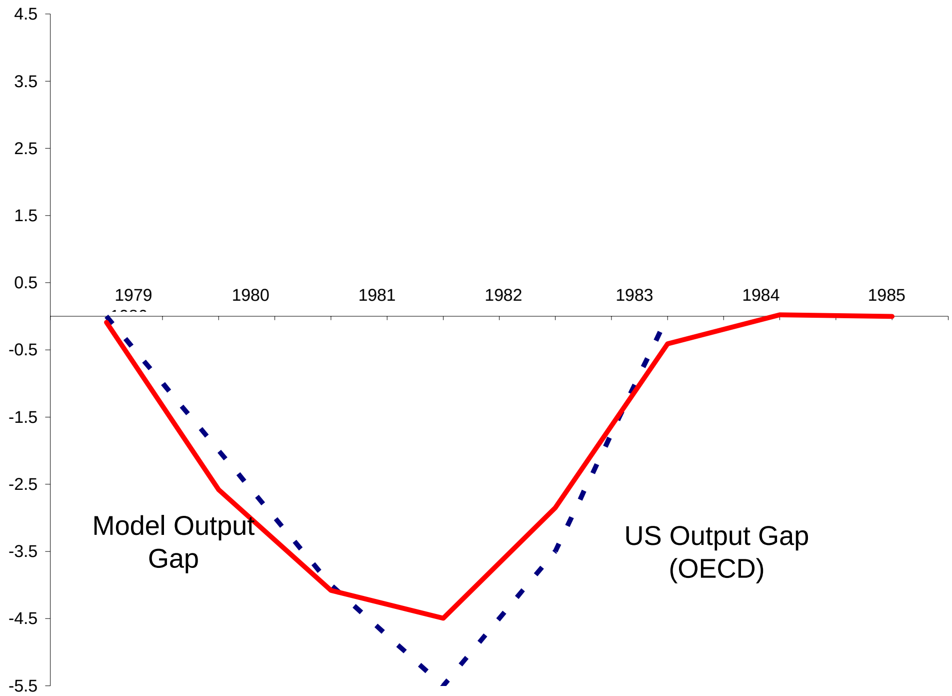

Figure 6.1:

Volcker disinflation under the assumption that agents use optimal filtering to disentangle persistent changes in the inflation target from temporary shocks to the monetary policy rule. Unlike in the current set up, in their model the steady state is unchanged throughout this period. However, leaving aside this important difference[10], they also find as we do that sizeable output losses may be the price of imperfect credibility.

[10]The difference may be important for the sorts of reasons discussed in Albanesi, Chari and Christiano (2003).

7. Discussion and Conclusions

We think two results stand out from our investigation. First, we find that imperfect credibility and price stickiness are jointly neither necessary nor sufficient for monetary contractions to cause lengthy recessions, at least for the relatively modest initial levels of inflation under scrutiny here. In this sense our results conflict with Ball (1995).

Second, in our model imperfect credibility need not be an overriding concern to the policymaker initiating a period of disinflation; under imperfect credibility the optimal speed of disinflation is quite similar to the case of perfect foresight. That said, under concave expectations updating initially sharp falls in output were a concern for the policymaker. Under convex updating, and perfect foresight, the optimal speed of disinflation was driven to a large extent by the protracted period of above steady state output.

What is central to both of these results is the size of the expectational errors early on the regime shift relative to the degree of price stickiness. For a given level of imperfect credibility longer nominal contract length will generally imply sharper recessions following a monetary contraction. Under convex updating these initial errors are relatively small while under concave updating they are somewhat larger.

Future research might investigate three issues. First, how large does the initial fall in output have to be before it is optimal for the policymaker to renege on the announcement to attain price stability. We accommodated this issue by focussing on situations where the initial rate of inflation was relatively modest. For countries with very high initial inflation rates, however, reneging might well be optimal, as Figure 5.3 strongly suggests.

A second issue concerns the interaction between monetary and fiscal policy.

We side-stepped this issue altogether by ignoring distortionary taxation of real activity. Again, so long as initial inflation, and hence seigniorage, is relatively modest the fiscal implications of disinflation are probably limited. As initial inflation rises, the public sector budget constraint is likely to play an increasing role in the welfare calculus.

A final issue concerns how quickly agents actually take to learn about regime shifts. In this paper we picked what we thought were sensible scenarios. In truth, we know relatively little about such issues, although in section 6 we make a tentative first step in this direction as do Erceg and Levin (2003). Given its importance to our results, this is an important topic for future research.

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8. Appendix: Solving the model

In this appendix we set out our approach to solving the model. We do this in two steps. The first step entails mapping the non-linear stochastic problem into a form more convenient to work with numerically. The second step then constructs the equilibrium solution to that simpler problem. The solution to the model under perfect foresight requires only the second of these steps and hence we do not spell this out in detail.

8.1. Step 1

In what follows let x_t denote the vector of endogenous variables excluding the control variables; s_t denotes the vector of control variables; θ denotes the parameters of the model; $E_t(y|\Omega)$ denotes the expectation of y conditional on the information set Ω at time t. We need to solve a stochastic non-linear expectations model of the following general form:

$$fE_t\left(x_{t+1}, x_t, s_{t+1}, s_t; \theta|\Omega\right) = 0, \tag{8.1}$$

where $f : R^{d_x+d_x} \times R^{d_s+d_s} \times R^{d_\theta} \to R^{d_x}$ is the equilibrium function, and d_x, and d_s are the dimensionalities of the x, s, and θ spaces respectively. In what follows we shall generally denote the conditional expectation using the shorthand, $E_t(y)$.

The expectation operator is given by a discrete distribution that takes on value v_{t+j} with probability w_j as follows:

$$E_t\left[f(v_t)\right] \simeq \sum_{j=0}^{k} w_j f_j(v_{t+j}). \tag{8.2}$$

In the current set up $k = 1$. So applying (8.2), the expected money supply is calculated as follows:

$$E\left[s_{t+1}\right]=\left[z(s_{t+1})\right]=\sum_{j=0}^{1}w_j z_j(s_{t+j})=w_0 z_0(s_t)+w_1 z_1(s_{t+1}),$$

where the weights reflect agents' perceptions as to the likelihood of the possible outturns: $w_0 = \rho_0$ is the probability that money supply $(z_0(M_t) = \theta_{-1}M_t)$ will revert to its previous steady state growth rate, while with probability $w_1 = (1-\rho_0)$ the money supply is expected to follow its announced path: $\left(z_1(M_{t+1}) = M_{t+1}^A\right), \rho_t \in [0,1],\ t = 0,1,2,...$ The path of ρ_t through time is determined by (4.4) in the main text. This means that we may re-write equation (8.1) as follows:

$$f\left(E_t\left[x_{t+1}\right], x_t, E_t\left[s_{t+1}\right], s_t, Z_{t+1}; \theta\right)=0,$$

where $Z_{t+1} \in R^{d_x}$ is the expectation variable Z_{t+1} given by:

$$Z_{t+1}=E_t\left[h(x_{t+1}, s_{t+1})\right], \tag{8.3}$$

and where the expectation function $h : R^{d_x} \times R^{d_s} \rightarrow R^{d_x}$ takes the form:

$$h_i(x_{t+1}, s_{t+1})=x_{t+1}^a s_{t+1}^b. \tag{8.4}$$

The expectation variable Z_{t+1} :

$$Z_{t+1}=E_t\left[h_i\left(x_{t+1}^a s_{t+1}^b\right)\right],$$

may be written as:

$$Z_{t+1}=E_t\left[x_{t+1}^a\right]+E_t\left[s_{t+1}^b\right]+cov\left(x_{t+1}^a, s_{t+1}^b\right)$$

where we have used the fact that

$$E\left[XY\right]=E\left[X\right]E\left[Y\right]+cov\left(X,Y\right).$$

The covariance may be calculated as

$$cov\left(x^a_{t+1}, s^b_{t+1}\right) = E\left[z\left(\left(x^a_{t+1} - E\left[x^a_{t+1}\right]\right)\left(s^b_{t+1} - E\left[s^b_{t+1}\right]\right)\right)\right].$$

Consequently, we may write,

$$cov\left(x^a_{t+1}, s^b_{t+1}\right) = \sum_{j=0}^{1} w_j z_j\left(\left(x^a_{t+1-j} - E\left[x^a_{t+1}\right]\right)\left(s^b_{t+1-j} - E\left[s^b_{t+1}\right]\right)\right). \tag{8.5}$$

Further, we note that $E\left[x^a_{t+1}\right] = \sum_{j=0}^{1} w_j z_j\left(x^a_{t+1-j}\right)$,and $E\left[s^b_{t+1}\right] = \sum_{j=0}^{1} w_j z_j\left(s^b_{t+1-j}\right)$,where w_j, are probabilities associated to the distribution function $z_j : R^n \rightarrow R^n$ which in this case is the identity function, for all $j = \overline{0,1}$.

As a result, system (8.2) may be re-written as

$$E\left[x_{t+1}\right] = f\left(E\left[x_{t+1}\right], E\left[x^a_{t+1}\right], x_t, E\left[x^b_{t+1}\right], E\left[s_{t+1}\right], s_t, cov\left(x^a_{t+1}, s^b_{t+1}\right); \theta\right). \tag{8.6}$$

8.2. Step 2

In order to solve (8.6), we apply an iterative Jacobi method. For the case of perfect foresight, the expectations variables are redundant as are the covariance terms; otherwise the solution algorithm is identical across the problems. Starting from an initial guess $E\left[x_{t+1}\right]^0 = x$, each iteration $q, q = \overline{1,Q}$ generates a new vector of values $E\left[x_{t+1}\right]^q$ based on the previous iteration. Convergence is achieved when

$$\max_i \left|E\left[x_{it+1}\right]^q - E\left[x_{it+1}\right]^{q-1}\right| < \varepsilon,$$

where $i = \overline{1,202}$ is the index of an element in the vector $E\left[x_{t+1}\right]$ and ε is chosen precision. The initial guess is given by the previous steady state for all the variables except the covariance term. To find $cov\left[s_{t+1}, x_{t+1}\right]$ the model is first solved assuming that there is no correlation between the control and endogenous variables s_t and x_t. We do this under the assumption of perfect foresight (corresponding to $w_0 = 0$ and $w_1 = 1$) and then assuming a complete lack of credibility ($w_0 = 1$ and $w_1 = 0$). Given this information we may construct an initialization for the covariance term. The expected value of the price $E\left[x_{t+1}^a\right]$ is calculated as described in step 1 given that *at the first iteration* , $z_0(x_t^a)$ obtained under the first assumption and $z_1(x_{t+1}^a)$ under the second one.

After the settings for the initial point are realized, the iterative process starts and continues until convergence of the following mapping is reached.

$$E\left[x_{t+1}^{\max}\right]^q = \phi f\left(E\left[x_{t+1}^{\max}\right]^{q-1}, x_t, E\left[x_{t+1}^b\right], E\left[s_{t+1}\right], s_t, cov\left(x_{t+1}^a, s_{t+1}^b\right)^{q-1}; \theta\right) + (1-\phi)\, E\left[x_{t+1}^{\max}\right]^{q-1}$$

During this process, at each new iteration, we find the profit maximising price and calculate the new covariance terms, based on the outcome from the previous iteration.

The scalar $\phi \in (0, 1)$ is a relaxation parameter which reduces the change between the iterations in order to speed the process of convergence.