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Value at Risk Models - an Approach to Measuring Bank Foreign Exchange Exposure
-dissertation paper-

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Bucharest, June 2000

Abstract

Following the 1993 Amendment to the European Commission's Capital Adequacy Directive, and the 1998 implementation of a similar approach in the US, banks are currently required to hold capital against market risk. There are now created the opportunities for banks to use their internal value at risk models, as opposed to the standard regulatory formulae, as a basis for setting capital charges. A wider use of VaR models also in emerging market economies could be an important step towards greater stability in the banking sector and further development of solid financial markets. In a number of emerging markets, banks are already allowed to use VaR techniques in order to assess and manage their exposure to adverse changes in the market conditions.

As prospects for accession to the EU increase, and with the launching of the euro, CEE transition economies bank risk management practices and policies are likely to face new pressures for further mutation. New members of the EU will be expected to adopt the aquis communitaire, and this will include the common framework for treating risk and the system of capital requirements. Although the supervision authority does not indicate a specific approach to be used, the penalties associated with internal model failures in accurately forecasting the distribution of future losses raise the issue of model selection.

This paper examines the empirical performance of several value at risk estimation techniques employed to model bank foreign exchange exposure perceived from a banking regulation perspective. We compare the performances of each model through a simulation methodology for a random portfolio containing spot position in five currencies against ROL, daily adjusted over a sample period of 22 months. Performance assessment is based on a range of tests that address the relative size and variability of VaR estimates, accuracy features from a backtesting perspective and nevertheless efficiency in setting capital charges.

We found an important dispersion between different models VaR estimates, but no model was identified as being insufficiently conservative in its risk measurement. The paper concludes that, although the tests are not precise enough to allow a categorical discrimination between models, they do provide useful diagnostic information for evaluating model performances.

Introduction

'Weaknesses in the banking system of a country, whether developing or developed, can threaten financial stability both within that country and internationally' the Basle Committee Core Principles. In the recent years we have witnessed an unprecedented surge in the usage of risk management practices, with the VaR based risk management emerging as the industry standard by choice or by regulation.

The need to improve the strength of financial systems has attracted growing international concern. Numerous official bodies have recently been examining ways to strengthen financial stability throughout the world and notably in the emerging market economies. In the "Recommendations for Public Disclosure of Trading and Derivatives Activities of Banks and Securities Firms", the Basle Committee on Banking Supervision and the Technical Committee of the International Organization of Securities Commissions 'consider transparency of banks and securities firms activities to be a key element of an effectively supervised financial system'. Public statements made by the G-7 Heads of State and Finance Ministers recognize that improved transparency of institutions' financial conditions, performance, business activities, risk profile and risk management practices, facilitates effective market discipline by promoting safety and soundness in individual institutions and financial system as a hole.

The increase in the relative importance of trading risk in bank portfolios has obliged regulators to reconsider the system of capital requirements agreed in the 1988 Basle Capital Accord. The common framework for treating risk designed in 1988 aimed to limit the credit risk, ignoring some important features related to trading risk and off-balance sheet positions. The European Commission's Capital Adequacy Directive (1993) established EU minimum requirements for the trading books of banks and securities firms. They proposed a system comprising two alternative ways of calculating trading books capital: the 'standardized' and the 'alternative' model. The standardized approach is a set of rules that assign risk charges to specific instruments and specify how these charges are to be aggregated into an overall market risk capital requirement. The internal models approach determines market risk capital charges on potential loss estimates generated by banks' internal risk measurement models.

Following the 1993 Amendment to the European Commission's Capital Adequacy Directive, and the 1998 implementation of a similar approach by the Federal Reserves, banks are currently required to hold capital against market risk defined as the risk that changes in the market conditions (prices and volatilities) would adversely affect the portfolio value of a bank. There are now created the opportunities for banks to use their internal value at risk models, as opposed to the standard regulatory formulae, as basis for setting capital charges. But VaR concept is also widely applied beyond the regulatory compliance, in determining trading limits and capital allocation decisions.

In a widely used definition, value at risk measures the potential loss on a portfolio over a specified period that will not be exceeded with a given probability. A VaR measure is dependent on two parameters: the holding period and the significance level. Current recommendations of the Basle Committee are that 1% VaR measures, calculated over a holding period of 10 working days are used to calculate capital charges.

A formal definition of VaR may be written as:

$$\Pr ob(\Delta P < -VAR) = \alpha$$

where ΔP denotes the portfolio change during the holding period and α represents the significance level.

If we let $N(\Delta P)$ represent the cumulative probability distribution function of portfolio returns, then

$VaR = N^{-1}(\alpha)$ where $N^{-1}(\alpha)$ denotes the inverse cumulative distribution function.

The standards for in-the-house model construction imply that banks must calculate the distribution of their losses over a ten-day holding period using a panel of historical data of at least twelve months and must yield capital requirements sufficient to cover losses on 99% of occasions. As a check on the accuracy of models, under the proposed alternative Basle approach, the supervisors will carry out back testing, the comparison of actual trading results with model generated risk measures. The Basle Committee proposed that the capital requirements should be equivalent to the higher of the current VaR estimate and the average VaR estimate over the previous 60 days multiplied by three.

According to the Basle committee recommendations, market risk capital charges, denoted by MRC_t are determined as follows:

$$MRC_t = \max[VaR_t(10,1); M * \frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}(10,1)]$$

where $VaR_t(10,1)$ is the current VaR_t estimate over a ten days holding period with a 0.01 significance level and M represents the multiplier indicated by the supervision authority to reflect the quality of risk management estimation models and practices for each bank in the system.

Strengthening the financial system's ability to evaluate and manage market risk has been usually identified as a precondition for further market integration. This entails improving the internal risk management of individual financial institutions on the basis of VaR models, in order to assess their balance sheet vulnerability with respect to changes in asset prices such as exchange rates, interest rates or equity prices. As a result, the risk of international illiquidity should be reduced. A reason for the excessive short-term foreign borrowing in many of the emerging economies could be the result of an inadequate risk management. In a number of emerging market countries important steps have already been taken in this regard and in some of them banks are required to use VaR techniques in order to assess and manage their exposure to adverse changes in the market conditions. A wider use of VaR models in emerging market economies could be an important step towards greater stability in the banking sector and further development of solid financial markets.

In the transition process, the usual elements of a well-functioning regulatory/supervisory system: 'adequate accounting and disclosure requirements, adequate capital standards, prompt corrective action, careful monitoring of the institution's risk management procedures and monitoring of financial institutions compliance with the regulations' Mishkin 1999 are considered a prerequisite toward further integration. As prospects for accession of CEE transition economies to the EU increase, and with the launching of the euro, bank risk management practices and policies are likely to face new pressures for further mutation. New members of the EU will be expected to adopt the aquis communitaire, and this will include the common framework for treating risk and the system of capital requirements.

The existent academic literature has been mostly concerned with measuring VaR (Duffie and Pan 1997, Linsmeier and Pearson 1996, Danielsson and De Vries 1997, Jackson, Maude and Perraudin 1997), with evaluating properties of VaR and other risk measures (Artzner, Delbaen, Eber and Heath 1998, Cvitanic and Karatzas 1998, Wang 1998), or with the evaluation VaR models from different perspectives (Hendricks 1996, Christoffersen 1997, Lopez 1997, Engel and Gizycki 1999). Regulation and banking aspects of VaR literatures have been emphasized by Kupiec and O'Brien (1997), Schachter (1997) and also Galai (1999) in the proceedings of the International Conference on Risk Management and Regulation in Banking 1997. Comprehensive overviews of VaR philosophy, instruments and techniques can be found in P. Jorion (1997) Value at Risk: The New Benchmark for Controlling Market Risk, K. Dowd (1998) Beyond VaR: the New Science of Risk Management or C. Alexander (editor), J. Hull (1998-1999) Risk Management and Analysis. Ahn, Boudoukh, Richardson and Whitelaw 1998, Luciano 1998, Basak and Shapiro 1998 consider the economic agents' wish to limit the VaR of their market exposure and optimal portfolio policies. A macro risk management tool has recently been developed by Blejer and Schumacher (1998), employing the same methodology as used in VaR models for individual financial institutions but focusing on the country's balance sheet vulnerability as opposed to a regime's sustainability.

Our paper examines the empirical performance of several value at risk estimation techniques employed to model foreign exchange exposure from a banking regulation perspective. We compare the performances of each model through a simulation methodology for a random portfolio containing spot position in five currencies against ROL, daily adjusted over a sample period of 22 months. Performance assessment is based on a range of tests that address the relative size and variability of VaR estimates, accuracy features from a backtesting perspective and nevertheless efficiency in setting capital charges.

We found support in our attempt to model VaR for measuring foreign exchange bank exposure in Hendricks (1996), Mahoney (1996), Lopez (1997), Danielsson and De Vries (1997 and 1998), Jackson, Maude and Perraudin (1997), Kupiec and O'Brien (1997), Butler and Schachter (1996 and 1997), Klaasen (1998), Engel and Gizycki (1999) and Alexander (1997 and 2000).

Since the introduction of the simplest VaR models, a little over ten years ago, the range of techniques used to obtain VaR estimates has expanded both in number and in complexity. The original VaR model uses classical multivariate statistics. Returns are assumed to follow a multivariate normal distribution, the paradigm until the late 1960s. Great advances have been made in multivariate statistics since then, and some of the developments in market risk models can be seen as stages in a catching-up process.

An example is the use of nonparametric estimates of probability distributions. Rather than starting with an assumed distribution, characterized by a few parameters like mean and variance, nonparametric estimates start with a sample of data and estimate a distribution from it. Since the distribution is based closely on the sample, a nonparametric model of returns may give more accurate estimates of market risk than a parametric one. A drawback of nonparametric methods is that they can fit the data too closely, so that noise as well as useful information is worked into the estimated distribution, but given the likely benefits of nonparametric methods, for example in estimating the tails of a distribution, the use of such methods in risk management will increase.

To summarize, a wide range of approaches has been developed to calculate VaR. The variance-covariance and Monte Carlo approaches require explicit assumptions to be made about the statistical distribution underlying movements in market prices (the normal distribution being most commonly used), while the historical simulation and extreme value estimation methods make no such assumption. In this study, we will refer to the following VaR estimation techniques:

- The variance-covariance technique assumes that the market returns have a joint-normal distribution. The fixed-weight approach assumes that return covariances and variances are constant over the period of estimation; exponential smoothing moving average method takes into account the potential for the variance-covariance matrix to vary through time by placing more weight on the most recent observations (JP Morgan and Reuters 1996);

- GARCH models, first introduced by Engel (1982) and generalized by Bollerslev (1986), are designed to describe volatility as a time varying process in high frequency data. GARCH allow for both autoregressive and moving average behaviour in variances and covariances and capture the volatility clustering effects. Constant correlation GARCH and orthogonal GARCH techniques represent variants of GARCH used in modeling portfolio returns.

- Kernel estimation uses non-parametric methods of weighting the historical data in estimating the variance-covariance matrix.

- Historical simulation uses past movements in market prices to compute a hypothetical distribution of returns.

- Antithetic historical simulation takes into account the trend behaviour of asset prices by augmenting the original data series with the negative of the profits and losses used in standard historical simulation

- Exponential historical simulation exploits the non-parametric nature of historical simulation while imposing an exponential-weighting scheme on the historical data.

- Monte Carlo simulation using normally distributed returns estimates the variance-covariance matrix using a fixed-weight variance-covariance method and estimated returns are drawn random.

- Extreme-value estimation focuses attention on estimation of a distribution’s tail.

'It is too early to judge how successful the various methods nonlinear modeling will eventually be. The less constrained a method is by prior information, the greater its potential for matching complex market structures. Pure black-box methods like neural networks are extreme cases of this kind. There is a trend towards letting the data produce models rather than basing models on assumptions' Trends in Risk Modeling, Chris Deas 1998. Another trend is implied by the fact that larger amounts of data about the market are needed to derive the information provided otherwise by assumptions. Econometric studies also reveal aspects of market structure, which should be built into pricing and risk management models.

But the use of VaR models in risk management systems should not be perceived as a global panacea. There are important shortcomings of different VaR approaches:

- VaR estimates are based on historical data and to the extent that the past may not be a good predictor of the future, VaR measure may underpredict or overpredict risk;

- VaR provides no indication of the magnitude of losses that may occur if adverse market movements are larger than predicted by the chosen confidence level. To address this problem, stress testing is developed together with VaR (specification of stress scenarios and assessment of their impact on portfolio value);

- The potential for aggregating exposures in a wide array of industry and markets is both a strength and a weakness of VaR approach. The aggregating procedure may hide imbalances between exposures from different risk sources.

The remainder of the paper is organized as follows: the first section presents the data and the simulation methodology, the second section describes the panel of models used in estimating VaR together with the estimation results, in section three is developed a model performance analysis and, finally, section four concludes.

Section 1 Data and simulation methodology

Our study employs 17th value at risk estimation approaches to model bank FX exposure: classical variance-covariance method (equally weighted moving average), five classes of exponentially weighted moving average models (with lambda coefficient values: 0.9, 0.92, 0.94, 0.96, 0.98), three types of GARCH models - constant correlation GARCH (1,1), constant correlation GARCHFIT and orthogonal GARCH, three approaches of the historical simulation method: the classical one, the exponentially weighted historical simulation and antithetic historical simulation, structured Monte Carlo simulation, kernel estimation and three types of tail estimation (with the assumed cumulative probability percentage in the distribution's tail: 5%, 10% and 15%). All the models mentioned above will be presented in the Section 2, together with the results obtained.

The data consists of daily exchange rates (reference exchange rates communicated by the National Bank of Romania) against the ROL for the following five currencies: USD, AUS, FRF, DEM and GBP. The historical sample covers the period: June 18th, 1997 – April 6th 2000.

The performances of all value at risk estimation models are determined over the sample period through a simulation methodology for a portfolio daily adjusted and containing spot position in five currencies. The portfolio is considered constant over the estimation period - 24 hours.

The performance analysis consists in several steps:

a) Selection of the daily random portfolio over the sample period, by drawing the positions in each currency from a normal independent distribution with the following configuration: USD, DEM and FRF daily positions ~ 50000*N(1,0.65), GBP positions ~30000*N(1,0.65) and AUS positions ~100000*N(1,0.65). Even if it may appear to be unrealistic that bank FX portfolios are driven by a random process, we find support for our approach in Hendricks (1996) and Mahoney (1996) in the sense that VaR measures are scale independent.

b) Estimation of the daily value at risk for the portfolio selected in the first step, using each of the 17th approaches for each observation in the sample period starting with June 18th 1998. For each estimation, we use a rolling window containing the 259 observations that precede the date for which the estimation is made.

c) Calculation (ex post) of daily outcomes from the portfolio, over the estimation sample period, as benchmark in backtesting procedures.

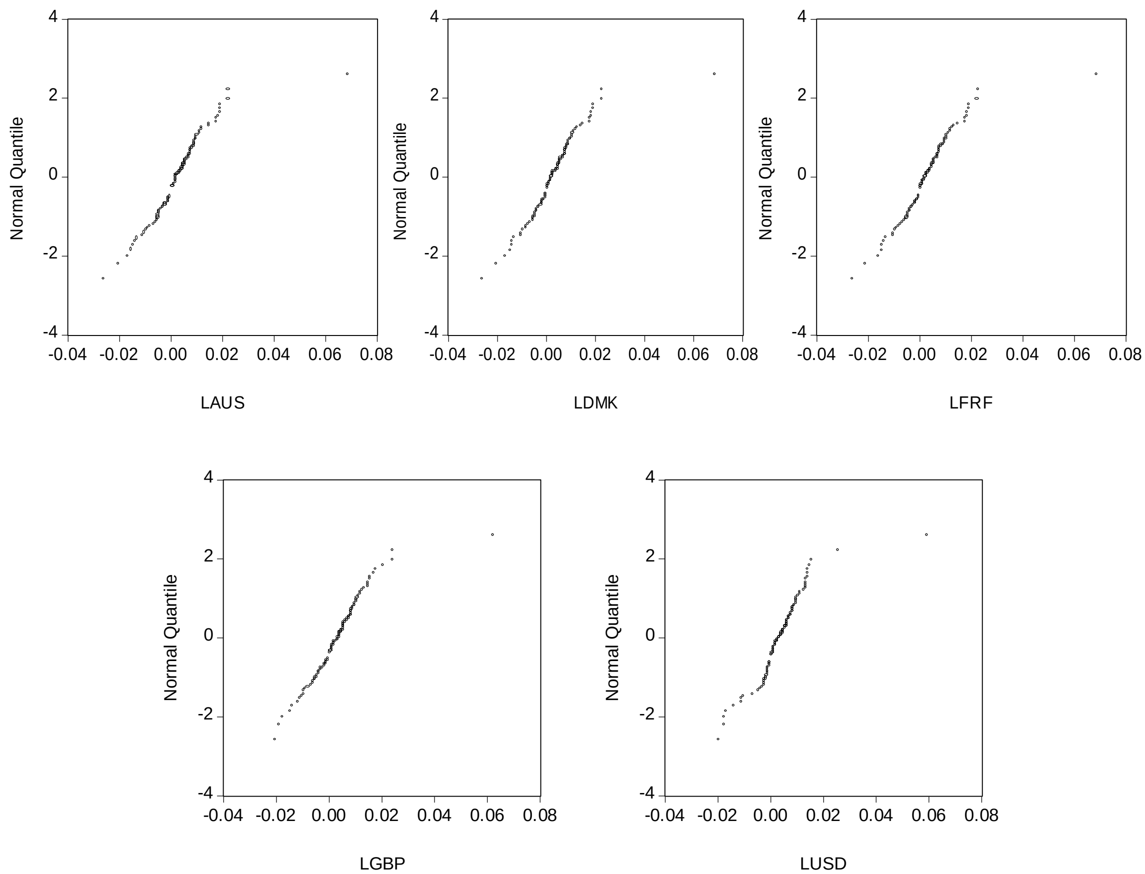
d) Assessing the performance of each value at risk approach, by comparing the estimations obtained in step two from different approaches with the daily portfolio outcomes through statistical and operational instruments.

Statistical features of the financial series of exchange rates against ROL (first difference in logs):

1. **Tests for normality assumption**: all five series provide evidence of non-normality with excess kurtosis (higher probabilities for extreme events than indicated in the normal distribution – fat tails) and positive skweness, therefor, more than 50% probability of exchange rate depreciation. The empiric quantiles graph (Graph 1.1) indicates, for each serie, deviations below the normal line for smaller quantiles and over the normal line for higher quantiles. The normality null hypothesis is strongly rejected by Jarque-Berra normality test. The statistic features of the exchange rate series are as follows:

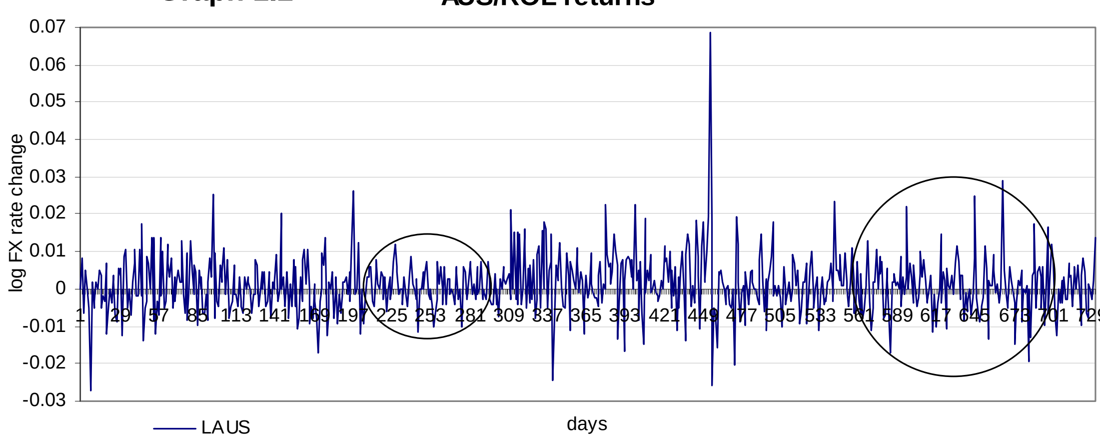
	LAUS	LDMK	LFRF	LGBP	LUSD
Mean	0.001149	0.001148	0.001155	0.001324	0.001359
Median	0.000750	0.000734	0.000736	0.000734	0.000818
Maximum	0.068813	0.068822	0.068907	0.062027	0.059451
Minimum	-0.027352	-0.027692	-0.027871	-0.030756	-0.024543
Std. Dev.	0.007591	0.007580	0.007628	0.006746	0.004673
Skewness	1.120261	1.144260	1.072458	0.978115	2.578952
Kurtosis	12.20665	12.32719	12.19776	13.20850	40.30094
Jarque-Bera	2734.621	2809.290	2716.866	3290.730	43188.81
Probability	0.000000	0.000000	0.000000	0.000000	0.000000

Graph 1.1: empiric quantiles for exchange rates returns



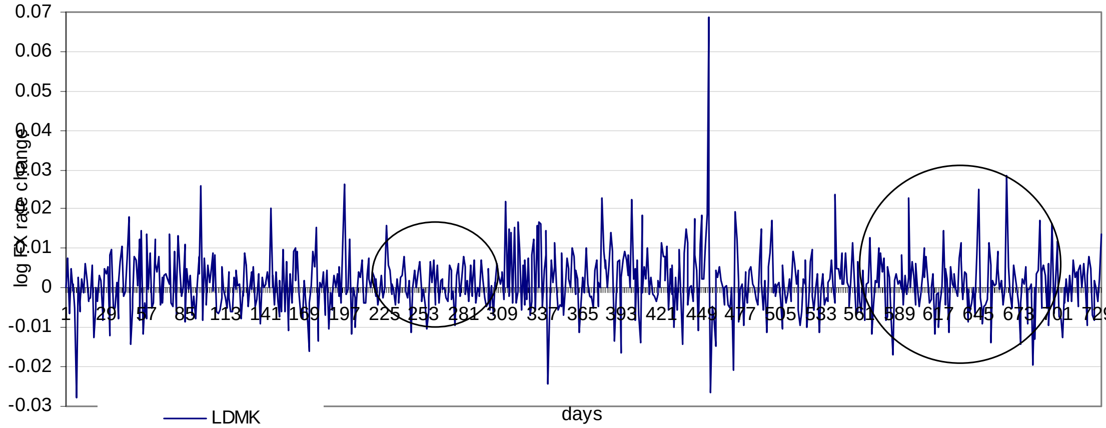
2. **Homoscedasticity assumption**: a simple visual inspection of the series indicates a volatility clustering process (volatility mean reverts), the alternance between high and low volatility periods and even more, a possible correlation between the variance processes for the five series, which justifies a multivariate approach for the portfolio value at risk modelling. The returns (first difference in logs)of all the exchange rates employed are plotted in Graphs 1.2 to 1.6. Periods of high and low volatility are marked distinctively on the graphs.

Graph 1.2 **AUS/ROL returns**



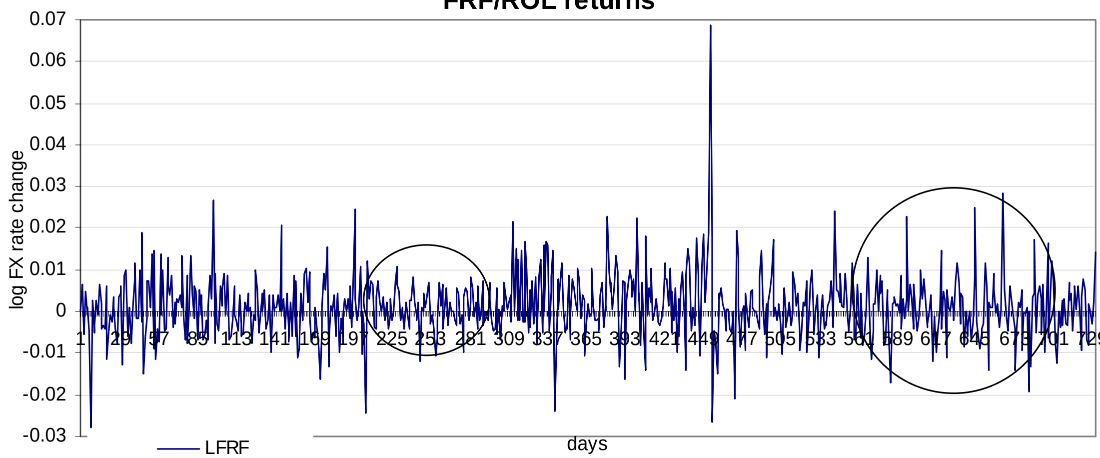
Graph 1.3

DEM/ROL returns



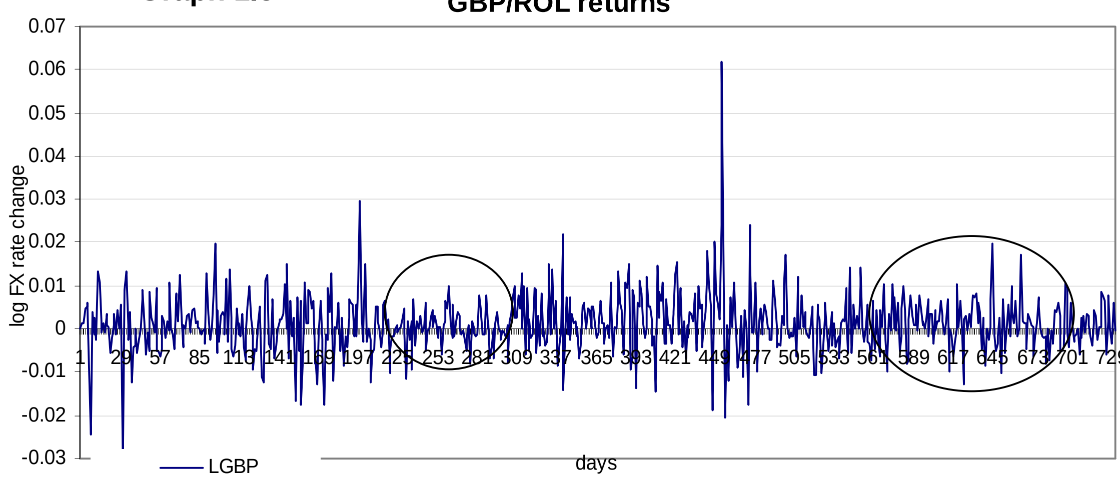
Graph 1.4

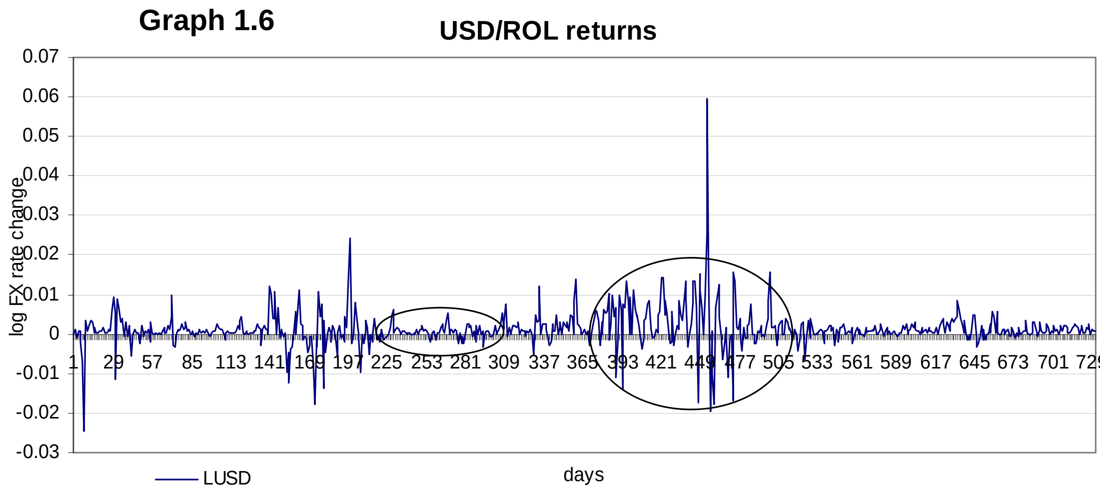
FRF/ROL returns



Graph 1.5

GBP/ROL returns





3. **Stationarity assumption**: the unit root null hypothesis for the first difference in logs time series of exchange rates is rejected by ADF unit root test for the 1% critical level, as follows:

ADF TEST STATISTIC (level, 2 lagged differences) on LAUS

ADF Test Statistic	-16.29343	1% Critical Value*	-3.4419
		5% Critical Value	-2.8659
		10% Critical Value	-2.5691

*MacKinnon critical values for rejection of hypothesis of a unit root.

ADF TEST STATISTIC (level, 2 lagged differences) on LDEM

ADF Test Statistic	-16.33812	1% Critical Value*	-3.4419
		5% Critical Value	-2.8659
		10% Critical Value	-2.5691

*MacKinnon critical values for rejection of hypothesis of a unit root.

ADF TEST STATISTIC (level, 2 lagged differences) on LFRF

ADF Test Statistic	-16.23796	1% Critical Value*	-3.4419
		5% Critical Value	-2.8659
		10% Critical Value	-2.5691

*MacKinnon critical values for rejection of hypothesis of a unit root.

ADF TEST STATISTIC (level, 2 lagged differences) on LGBP

ADF Test Statistic	-16.63797	1% Critical Value*	-3.4419
		5% Critical Value	-2.8659
		10% Critical Value	-2.5691

*MacKinnon critical values for rejection of hypothesis of a unit root.

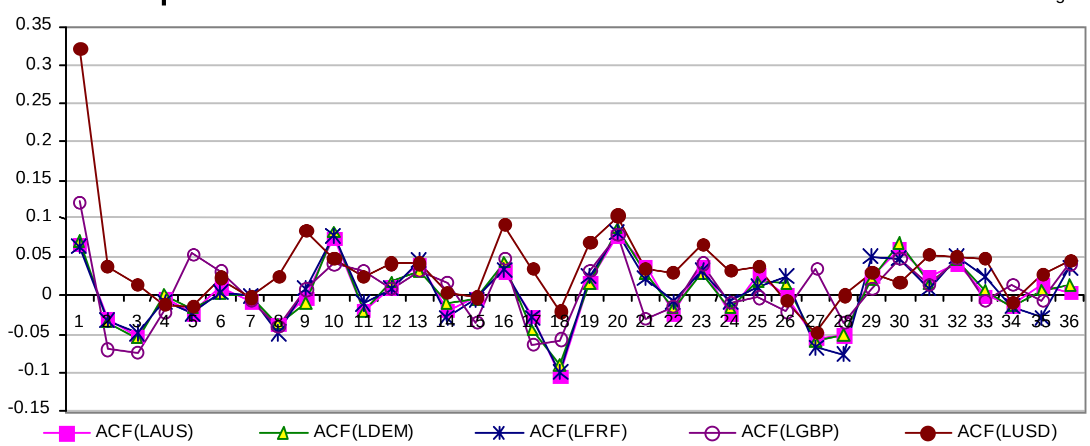
ADF TEST STATISTIC (level, 2 lagged differences) on LUSD

ADF Test Statistic	-13.91237	1% Critical Value*	-3.4419
		5% Critical Value	-2.8659
		10% Critical Value	-2.5691

*MacKinnon critical values for rejection of hypothesis of a unit root.

4. **Serial independence assumption**: the autocorrelation coefficients for 36 lags lie between –0.1 and 0.1, with one major exception: the autocorrelation coefficient of the LUSD serie for the first lag is 0.321. The autocorrelation coefficients are presented in Graph 1.7. The hypothesis of a common process for at least three of the exchange rates, AUS, FRF and DEM is confirmed also by the structure of the autocorrelation coefficients.

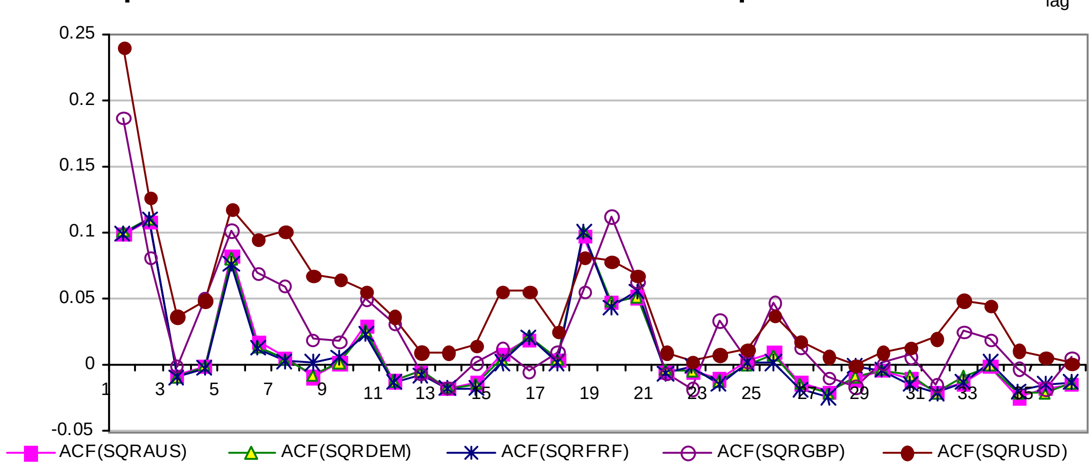
Graph 1.7: Autocorrelation coefficients



For confirming the low evidence of autocorrelation, we also use Box-Liung Q statistic. For the LAUS, LFRF and LDEM, no autocorrelation coefficient is significantly different form zero at the 5% confidence level. For LGBP and LUSD series, all the coefficients are statistically significant, till the 34th lag for LGB and till the 36th lag for LUSD.

Autocorrelation analysis must be extended also to the squared log changes in the exchange rate series, in the aim of identifying a possible ARCH process. The serial autocorrelation coefficients for the squared log changes are statistically significant for the first 30 lags at the 5% confidence level, and, at least for the first lag, are over 0.1 (Graph 1.8). Based on the strong positive autocorrelation for the first lags, combined with excess kurtosis, we may conclude that there are indications for a conditional variance process for the log changes in the series of exchange rates. This finding is rather a common result for high frequency exchange rates data.

Graph 1.8: Autocorrelation coefficients for squared returns



Section 2 Value at risk estimation models and results

In this section we will discuss the methods used to estimate value at risk, together with the estimation results. Each model estimation was implemented in Eviews 3.1 through the programes presented in Appendix 3.

Variance-covariance (or moving average) models presume a normal distribution for market returns and serial independence. All moving average models estimate the unconditional variance of the returns time series based on the restrictive assumption of constant volatility. In fact, moving average models are not forecasting but estimation models: the current volatility estimation is also the volatility forecast, whatever the time horizon. The normality assumption simplifies the estimation procedure by allowing all percentiles to be known multiples of standard deviation, and by reducing the number of distribution parameters to be evaluated to only two: mean and standard deviation. The serial independence implies that returns from two different moments are not correlated and, then, the rule of square root of time rule can be applied.

Define R_t to be the return matrix at moment t, and Σ_t the variance-covariance matrix of R_t. The return on a portfolio containing spot FX positions can be expressed as a linear combination of the returns on individual positions. Portfolio sensitivity to movements in a specific risk factor is defined as the change in portfolio value following a change of 1% in the risk factor. The sensitivities vector (with a number of elements equal to the number of risk factors) is denoted by δ.

For $\Delta P \sim N(0, \delta\Sigma\delta)$, $VaR = -Z(\alpha)\sqrt{\delta'\Sigma\delta}$, where $Z(\alpha) = N^{-1}(\alpha)$

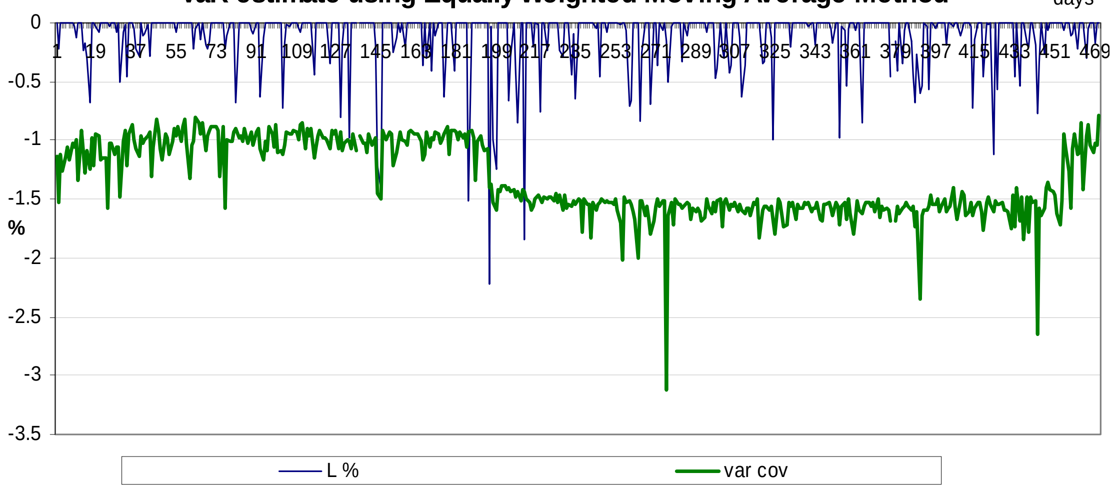
Once a distribution of possible profits and losses has been specified, standard properties of normal distribution are used to determine the loss that will not be exceeded (1-alfa) percent of the time, i.e. value at risk. The model calibration consists in estimating the parameters of the variance-covariance matrix, which can be done by several methods.

The **equally weighted moving average method** calculates a given portfolio variance using a fixed amount of historical data. Each element from the variance-covariance portfolio matrix is estimated as follows: the variance for each risk factor is computed as an equally weighted average of squared returns and the covariance between each two risk factors is evaluated as an equally weighted average of cross products of returns.

$$\hat{\Sigma}_{t+1} = \frac{1}{T}\sum_{i=0}^{T-1} R_{t-i} R'_{t-i}$$

The main drawback on the equally weighted moving average method is represented by the fact that stress events cause 'ghost features in volatility' (Alexander 1998). An extreme event will keep volatility estimates high for a period equal to the time length of historical date used in estimation, although the underlying volatility has long ago returned to normal levels. Equally weighted moving average method does account for the phenomenon of 'pressure relieving' implied by an extreme event. The VaR estimates obtained are plotted in Graph 2.1.

Graph 2.1 **VaR estimate using Equally Weighted Moving Average Method**



Exponentially weighted moving average method places more weight on more recent observations and this procedure eliminates the ghost features in volatility. Because the weights decline exponentially, the most recent observations receive more weight than the earlier ones. The exponentially weight is done by using a smoothing constant, lambda, as a decay factor which determines the rate at which the weights on past observations decay as they move further into the past. The larger the value of the decay factor, the more weight is placed on past observation and the smoother the series becomes.

For the log returns is hypothesized the following random walk model:

$$p_t = \mu + p_{t-1} + \sigma_t \varepsilon_t, \varepsilon_t \sim N(0,1)$$

The variance-covariance matrix is given by:

$$\hat{\Sigma}_{t+1} = (1-\lambda)\sum_{i=0}^{T-1} \lambda^{i+1} R_{t-i} R'_{t-i} \qquad 0 < \lambda < 1$$

This can be rewritten as

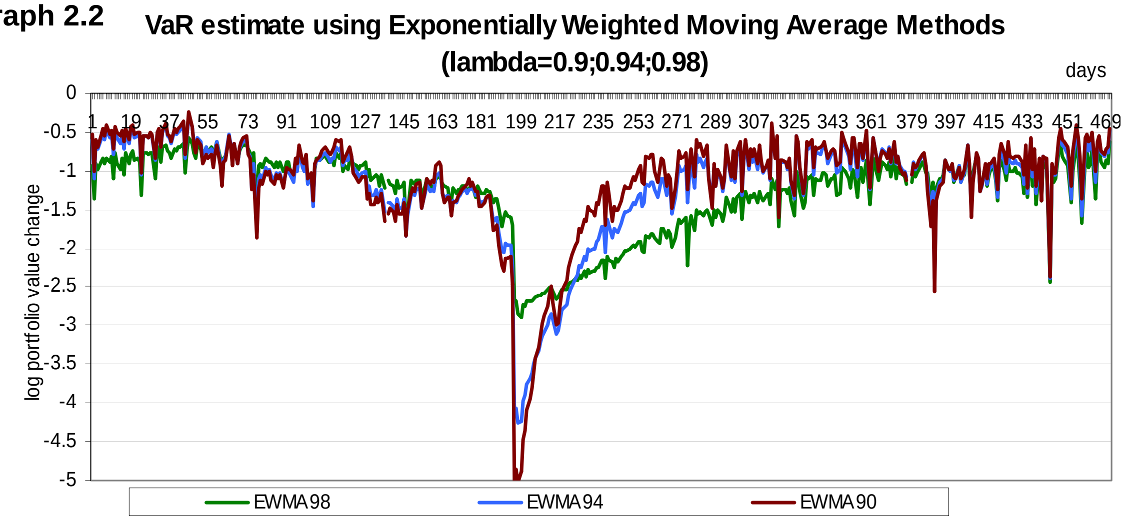
$$\hat{\Sigma}_{t+1} = \lambda \hat{\Sigma}_t + c R_t * R'_t$$

which implies a first order autoregressive structure for the variance/covariance matrix that is a form of IGARCH model without constant term. In this expression, $(1-\lambda)$ represents the volatility speed of reaction to market events, and the coefficient of lagged variance, λ, determines the persistence in volatility.

Exponentially weighted moving average is a quick and easy method that captures the volatility clustering. Current estimates respond quickly to changing market conditions, but the volatility and correlation forecast are still constant.

Another problem with exponentially weighted moving average method is that there is no optimal way to choose the smoothing parameter. JP Morgan recommend for lambda a value of 0.94, while usually a smaller persistence is obtained from empirical estimations. In order to diminish the problem related to choosing one or another value for the parameter lambda, this study applies the exponentially weighted moving average method with 5 different values for lambda: 0.9, 0.92, 0.94, 0.96 and 0.98. The estimation results for three values of the underlying parameter lambda are presented in Graph 2.2.

Graph 2.2



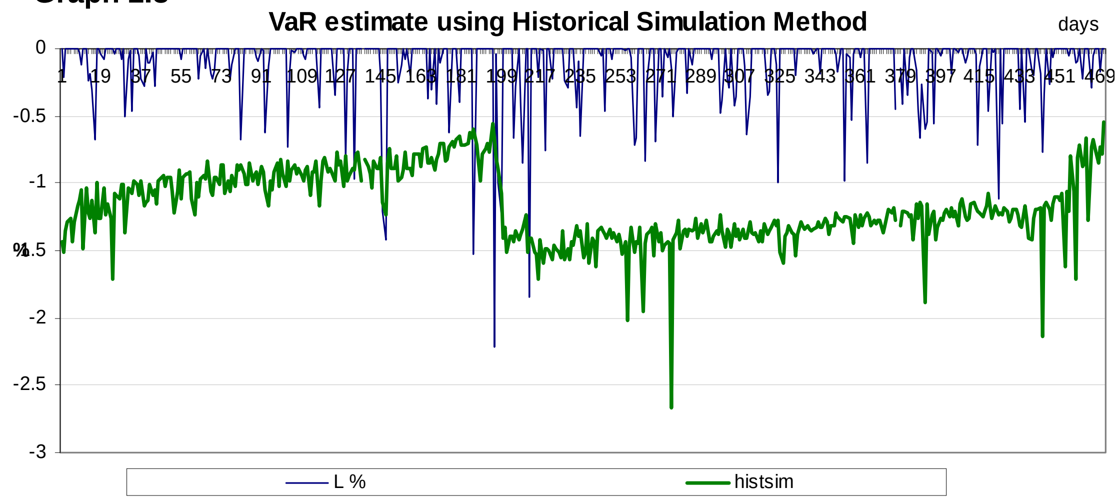
Historical simulation is the simplest non-parametric method of value at risk estimation, strongly advocated by Hendricks (1996); it makes no assumptions about the properties of the empirical returns distribution. Historical simulation uses past movements in the risk factors to compute a hypothetical distribution of daily returns on the current portfolio.

The distribution of profits and losses is constructed by subjecting the current portfolio structure to actual changes in market factors experienced in the last T observations. T sets of hypothetical values for each risk factor are constructed using their current values and the changes experienced in the last T days. Using these hypothetical risk factor values, T hypothetical portfolio outcomes are computed and the distribution of profits and losses determined.

The value at risk for the current portfolio is set equal to the percentile of the hypothetical P&L distribution associated with the required level of confidence. For 99% coverage estimation over a rolling window of 259 past observations, value at risk is set equal to the 3rd largest loss observed in the hypothetical outcome distribution. The estimation results are plotted in Graph 2.3.

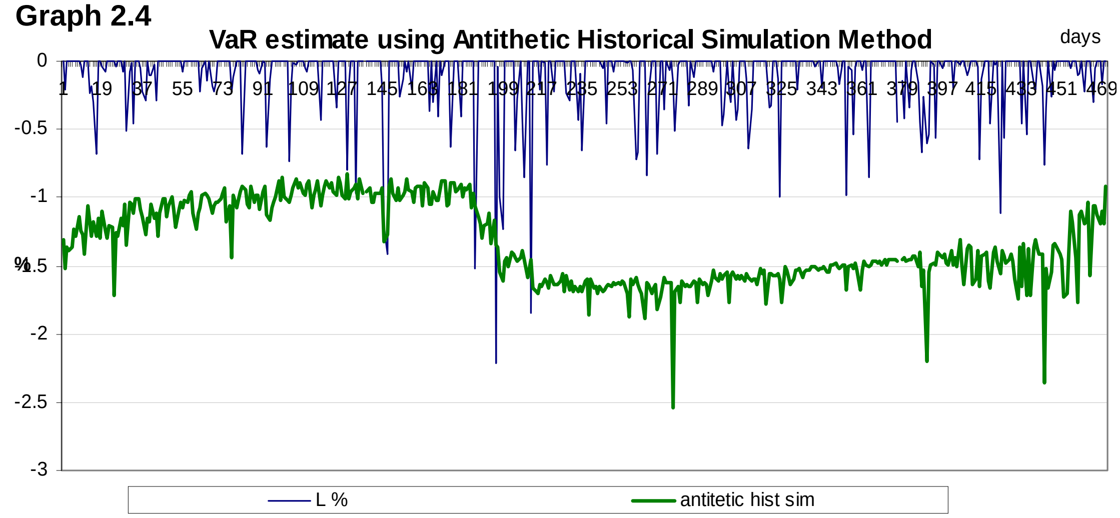
The main shortcoming on this method is the fact that extreme percentiles are difficult to estimate accurately without employig a large sample of data. But a large sample of data is not consistent with non-stationarity and induces the problem of ghost features in volatility, because all past observations receive an equal weight.

Graph 2.3



Another issue related to historical simulation is the trending behaviour often exhibited by financial series. If the data sample is not large enough, the possible outcomes generate by a trend change are not taken into account. Adressing this problem, Holton (1998) suggests as for imposing symmetry to the distribution to double the sample size by taking for each observed outcome it's negative value and augmenting the original sample. This approach is known as **antithetic historical simulation**. The value at risk estimation results from this approach are presented in Graph 2.4.

Graph 2.4

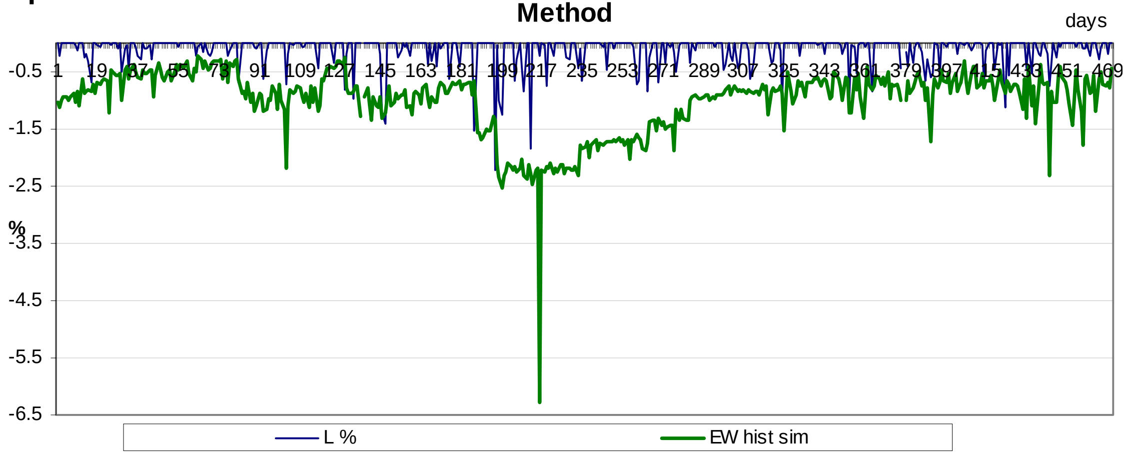


Boudoukh (1998) proposes another hybrid approach that eliminates the distortion caused by the sample size and non-stationarity in the return distribution, exponential historical simulation. This approach imposes a weighting scheme on data, whilst exploiting the benefits from the non-parametric nature of historical simulation.

$$w_{t-k+1} = \frac{\lambda^{k-1}(1-\lambda)}{(1-\lambda^{T})}$$

Each observation from the past is associated with a weight according to its distance from the current observation. The returns are then ascendingly ordonated. The weights associated to each observation provide the probability density function of the hypothetical outcome distribution. Value at risk for a given confidence level is set equal to the first realization in the distribution for which the cumulative distribution function (i.e. the sum of all preceding ordonated realization weights) reaches the given confidence level. For the parameter lambda, we used a value of 0.97. The results are plotted in Graph 2.5.

Graph 2.5 **VaR estimate using Exponentially Weighted Historical Simulation Method**



GARCH models

As we documented in the first section of this paper, exchange rate volatility is a time varying process in high frequency data and periods of high volatility tend to cluster. To capture this, many authors employed ARCH models, first introduced by Engel (1982) and generalized by Bollerslev (1986). GARCH allow for both autoregressive and moving average behaviour in variances and covariances and capture the volatility clustering effects.

The GARCH regression model contains two equations: one for the expected returns (assumed to be very simple) and another for the returns variance. In the GARCH (p,q) model proposed by Bollerslev (1986), the conditional variance takes the form of:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

The parameters are estimated by constrained maximum likelihood. In the simples GARCH(1,1), vanilla GARCH, the parameter α determines the volatility speed of reaction to market events, while β determines the volatility persistence in estimates. GARCH volatility forecasts converge to the unconditional variance $\omega/(1-(\alpha+\gamma))$ only if $\alpha+\beta<1$. On financial markets, the speed of reaction is usually documented to be below 0.25 and the volatility persistence over 0.7. The constant term in the variance equation indicates the expected level of convergence for long term GARCH forecast.

The next step in estimating a GARCH model is the specification of the mean equation, which is usually a random walk model, ARMA(2,1) for daily observations, or AR(2).

The advantages of GARCH models consist in providing a convergent volatility term structure and in estimation of optimal parameter. The main limitations regard estimation problem and difficulties in achieving the convergence for multivariate models. In order to eliminate such drawbacks, several methods have been proposed: constant correlation univariate modeling, risk factor orthogonalization, Integrated-GARCH, Exponential-GARCH, Asymmetric-GARCH.

A common approach to estimate a portfolio variance is the **constant correlation GARCH** model proposed by Bollerslev (1990). This model estimates each diagonal element of the variance-covariance matrix using a univariate GARCH model. The constant correlation assumption allows the off-diagonal elements to be computed from the variances under the assumption that the risk factor correlation in time is invariant. The off-diagonal elements are estimated as follows:

$$\hat{\sigma}^2_{ij,t+1} = \rho_{ij}\sigma_{i,t+1}\sigma_{j,t+1}$$

In modeling the variance process followed by the exchange rates against ROL we found to be the appropiate speciffication an AR(1) process for the mean equation of GBP and USD, and a constant equation for the other three currencies mean equations. In all mean equation it appeard to be statistically significant a dummy variable for the observation 454, when ROL suffered a drastic depreciation of 6% against USD.

For the variance equation, based on previous insights, we tried to estimate first a vanilla GARCH, which proved not to be satisfactory for all the exchange rates. It appears, as suggested in the first section of this paper by the pattern in the ACF of the squared log changes, that two slightly different processes derive the exchange rates: the first for AUS, FRF and DEM, and the second for GBP and USD. For AUS, FRF and DEM the appropriate specification is ARCH(1), while for GBP and USD, the process seems to be a GARCH(2,1). Let this second approach, which uses different specification forms in variance equation, be denoted as GARCHFIT.

For standard errors estimation we used the heteroscedsticity consistent estimator proposed by Bollerslev and Woodridge (1989), that minimizes the problem of innovations non-normality. The GARCH parameter estimation algorithm used is Marquandt with initial coefficient values estimated with OLS/TLS. The results are presented in the Appendix 2.

For the first three currencies in the portfolio (AUS, DEM and FRF), the variance process exhibits low persistence in volatility estimates together with very unstable coefficient estimates obtained from the rolling window of 259 historical observation. The coefficient instability may partially be due to the weak specification of the variance equation.

The variance equations for GBP and USD, better specified, indicate high persistence in volatility estimates. Such high persistence is usually explained either by the frequency of financial data observations, or by a regime switching volatility process. Lamoureux si Lastrapes (1990) show that structural changes in the variance process can induce overestimation of persistence in GARCH models, by changing the level of unconditional variance. For example, if variance is low and constant over a period of time, and high but constant another period of time, such persistence of high and low homoscedastic volatility **periods** cannot be discriminated in a clasical GARCH model from a individual **event** volatility persistence.

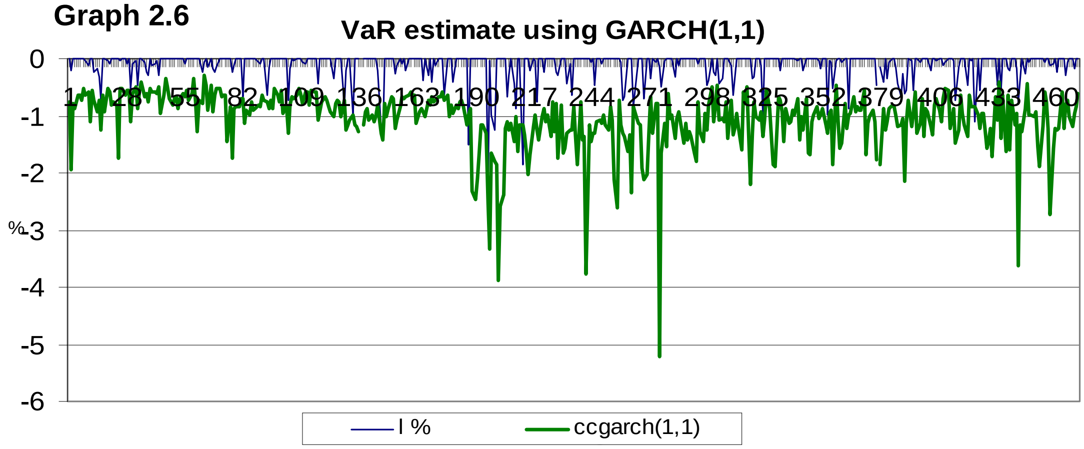
Structural changes in the variance process are usually originated in changes of economic policy or institutional reforms, therefor regime switching volatility models are appealing instruments to be used in transition countries, where economic and institutional challenges are likely to have an impact over the structural behaviour of the markets.

One approach in modelling a regime switching volatility is the use of dummy variables to identify the deterministic changes in the variance process and to separate periods with different volatility regimes. If the dummy variables are found to be significant, the persistence in volatility estimates is expected to be reduced. A usefull diagnosis tool to identify the presence of a regime switching volatility is the GARCH parameter estimates instability. The graphs of the parameters estimates for all GARCH models are presented in the Appendix.

The solution chosen to reduce the persistence in volatility estimates for USD/ROL exchange rate (first difference in logs) and to allow forecast convergence, consists in introducing two dummy variables in the variance equation in order to separate between two different volatility regimes. The first dummy variable, DUMMY1 points out the transition from a low volatility period to a high volatility period caused by the Russian crises –August 1998- that determined massive foreign capital withdrawals from emerging economies markets and implicitly affected the liquidity of the FX markets. The second dummy variable, DUMMY2 marks the beginning of a new tranquil period in the FX market – June 1999 – initiated by the success of the National Bank of Romania in avoiding a potential default in the service of the foreign debt and another stand-by agreement with the IMF. DUMMY1 is also used to replace the point variable DUMMY in the mean equation.

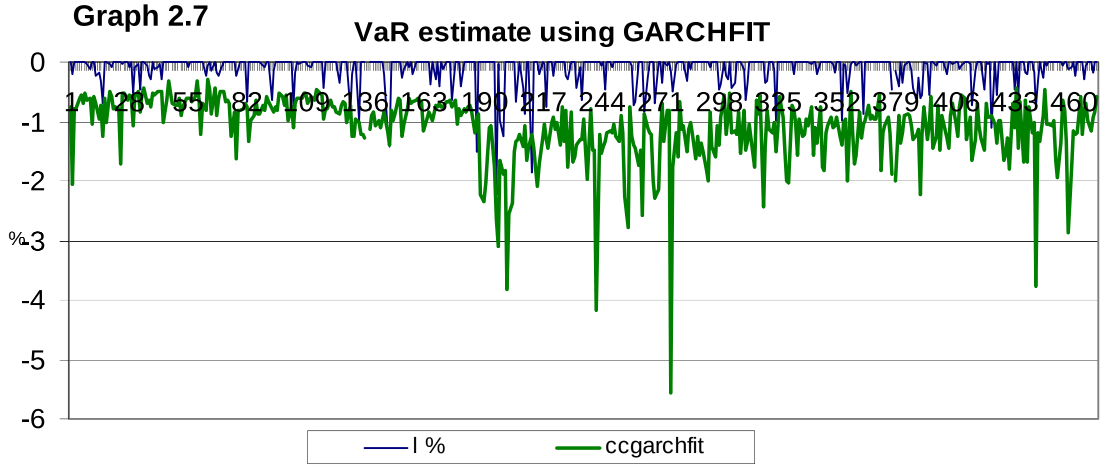
For the first model estimated, vanilla GARCH, both dummy variables are statistically significant and the associated coefficients have the expected sign: DUMMY1 is positively related to the level of variance, while DUMMY2 is negatively related to the variance. The persistence of volatility estimates is reduced below unit, allowing the model forecast to converge to the steady state unconditional variance. The GARCH(1,1) estimation results are plotted in Graph 2.6.

Graph 2.6



In GARCH(2,1) model, the coefficients of the two dummy variables are no longer statistically different form zero, at a confidence level of 10%, but they still manage to reduce the persistence of volatility estimates below unit. The GARCHFIT estimation results are plotted in Graph 2.7.

Graph 2.7



Another approach to model portfolio variance using univariate GARCH estimation- **orthogonal GARCH** - was first introduced by Engle, Ng and Rotschild (1990) and further developed by Alexander and Chibumba (1998). This approach uses the principal component analysis for the risk factor orthogonalization. The variance-covariance matrix of the innitial risk factors is obtained from the variances of the orthogonalized factors.

The first step in estimating VaR with orthogonal GARCH consists in determining the principal components and risk factor orthogonalization. Because the principal components are orthogonalised, the covariances modelling is no longer neccesary and the number of parameters to be estimated is substantially reduced. The orthogonalization procedure allows the properties of the whole innitial variance-covariance matrix to be deduced from a univariate volatility estimation.

Let R denote the observed return matrix (T*k). W refers to the eigenvectors matrix of R'R. the ortogonal principal components are P_1, P_2, P_k , with

$[P_1\ P_2 \ldots P_k]=RW$

A change in the risk factor I can be written as a linear combination of the principal components, the weight being given by the I-th eigenvector.

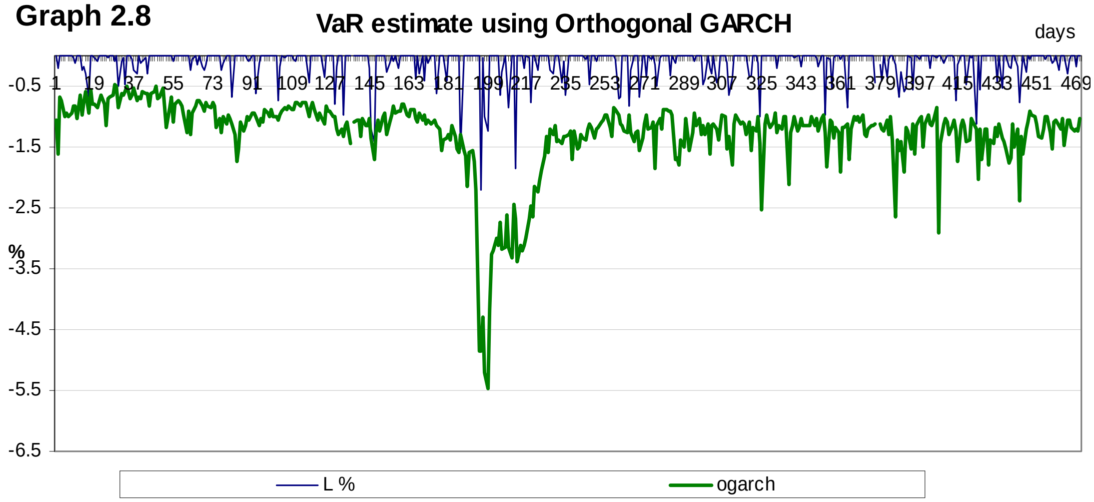
$R=PW' \Rightarrow R_i = w_{i1}P_1 + w_{i2}P_2 + \ldots + w_{ik}P_k$

The variance-covariance matrix is then given by:

$$\hat{\Sigma}_t = W \operatorname{var}(P) W'$$

For modeling the variances of the principal components, we used an univariate constant-correlation GARCH(1,1) specification. The problems associated with the estimation of constant correlation GARCH, high persistence, different specification forms among currencies, and parameter instability are not also the case for the orthogonalized factors. The persistence is low, the model is better specified and ensures the same mean and variance equation form for all the currencies and, most important, the estimates for the parameters are stable over time. The estimation results are presented in Graph 2.8.

Graph 2.8



Structured Monte Carlo simulation

For generating a sequence of random variables with zero mean and a variance-covariance matrix that replicates the observed matrix at a certain moment, a specific form of Monte Carlo simulation is used. The structured Monte Carlo simmulation provides a set of possible scenarios for the value of the portfolio.

The first step in the structured Monte Carlo simulation implies a Cholesky decomposition of a given variance-covariance matrix.

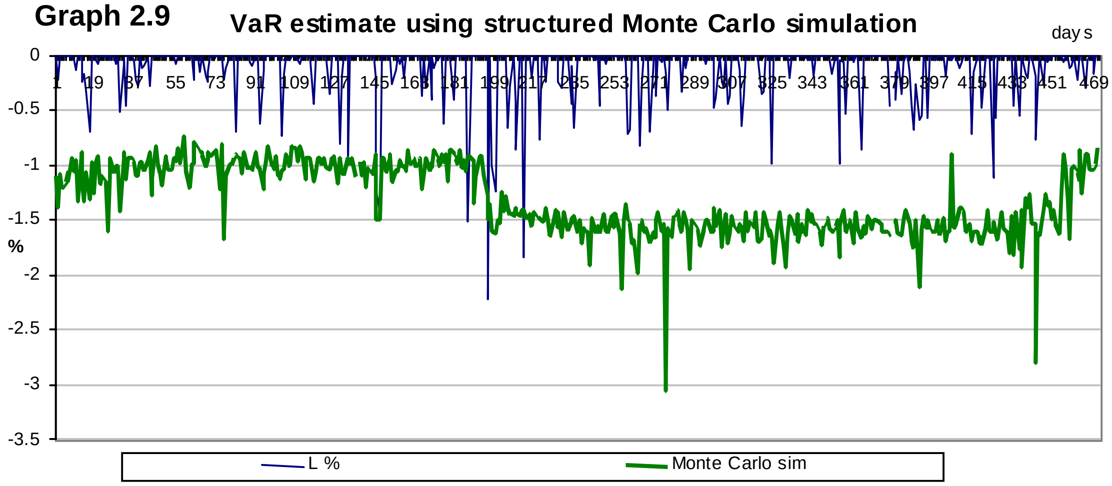
$\sum_T = AA'$, A and A’ are triangular matrix

The neat step consistes in generating a n*1 vector, denoted by Z, of random independent variables drawn from the standard distribution. Let Y=AZ, where Y elements have unit variance and are correlated according to the given variance-covariance matrix.

This method provides individual random elements of the possible profits and losses vector that are consistent with the given correlation between the market factors. The simulation is repeated thousands of times in order to generate a representative distribution of possible outcome.

The hypothetical profits and losses are sorted ascendingly, and the value at risk is set equal to the percentile of the distribution associated with the given confidence level. The estimation results are presented in Graph 2.9.

Graph 2.9 **VaR estimate using structured Monte Carlo simulation**



Gaussian Kernel density estimation model represents a combination of historical simulation and normal kernel estimation provided by Butler and Schachter (1996) aiming to improve the precision of VaR non-parametric estimate based on historical observations. The density of the return on a portfolio is estimated using a non-parametric method called Gaussian kernel, which can be seen as a simple generalization of a histogram obtained by smoothing the data with a normal continuous shape.

Gaussian kernel estimation is obtained form an estimated portfolio return distribution that is continuous and differentiable. Thus, the kernel density estimation produces a non-parametric estimate of the continuous probability distribution function of portfolio returns.

The Gaussian kernel density function, $\hat{f}(x)$, attaches a normal pdf to each data point. It's important to mention that the use of a normal kernel estimation does not imply parametrization of final estimate, smoothing could be done with any continuous shape. The smoothing is done by centering each pdf on the data point with a bandwidth (st dev suggested by Silverman 1986) equal to $0.9\sigma n^{-0.2}$, where σ is the standard deviation of the data estimated from the available observations.

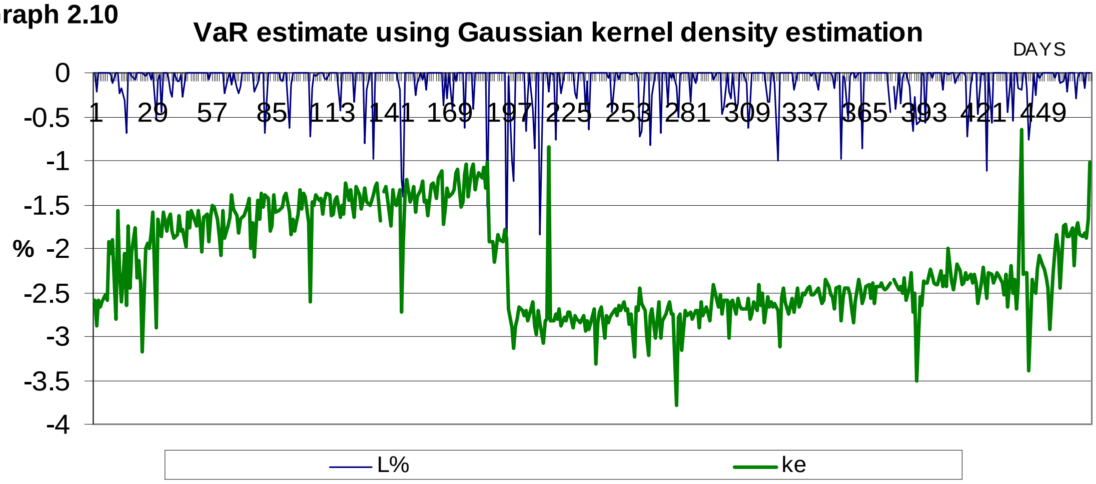
The Gaussian kernel estimate of the probability density function of the portfolio returns is given by:

$$\hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{0.9\sigma n^{-0.2}\sqrt{2\pi}} \exp(-\frac{1}{2}(\frac{x - X_i}{0.9\sigma n^{-0.2}})^2)$$

The kernel density estimation method is based on the hypothetical profits and losses discrete distribution constructed in the historical simulation. Another ascendingly sorted larger (1000 observations) data series is generated in order to represent the basis of estimation for the pdf. The bandwidth is computed as sugested by Silverman: 0.9*259^(-0.2). For each data point in the newly generated series, the pdf is computed as the average of pdf estimates for the normal function parameters: mean=each observation from the hypothetical profits and losses discrete distribution and standard deviation=bandwidth.

The cumulative distribution function for each data point is obtained by summing all discrete pdf values that preced the current observation. The value at risk estimate is set equal to the percentile of the cdf associated with the given confidence level. The estimation results are shown in Graph 2.10.

Graph 2.10



Extreme return (tail) estimation

Vaue at risk analysis highly depends on the accuracy of extreme returns estimation. As a basic rule, the properties of the tail return distribution are significantly different from those of the process generating them. Fat tails phenomenon is frequently identified in financial time series analysis.

Extreme value technique focuses attention on the estimation of the distribution's tails. These techniques use the larges or the smalles realizations of the data series to estimate the tail index, which is a measure of the tail thickness. The estimation method is semi-parametric, combining non-parametric historical simulation with parametric estimation of distribution's tails.

The simplicity of extreme value estimation derives from the fact that extreme value distribution belongs to one of just three possible distribution famillies, regardless the original return distribution. Particularly for financial series, if the distribution is fat tailed, the familly to which belongs is:

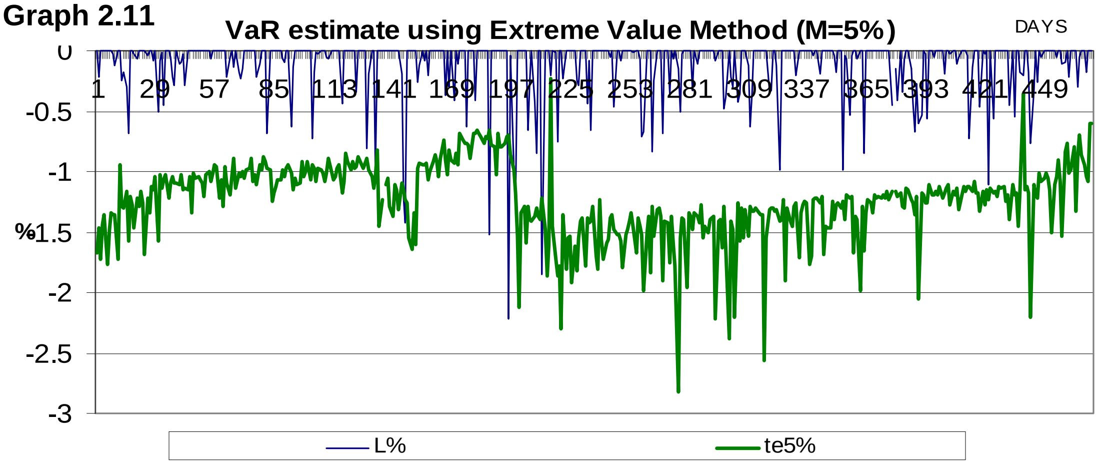
$f(x)=\exp(-x^{l})$, if $x>=0$ and zero otherwise.

The only parameter that needs to be estimated is the tail index. The first step in estimating the tail index implies sorting ascendingly the hypothetical profits and losses distribution constructed as in the historical distribution. The maximum likelihood estimator of the tail index (Hill 1976) is given by: $\frac{1}{\gamma} = \frac{1}{M-1}\sum_{k=1}^{M-1}\log(X_k / X_M)$ where M the distribution rank at which the tail starts.

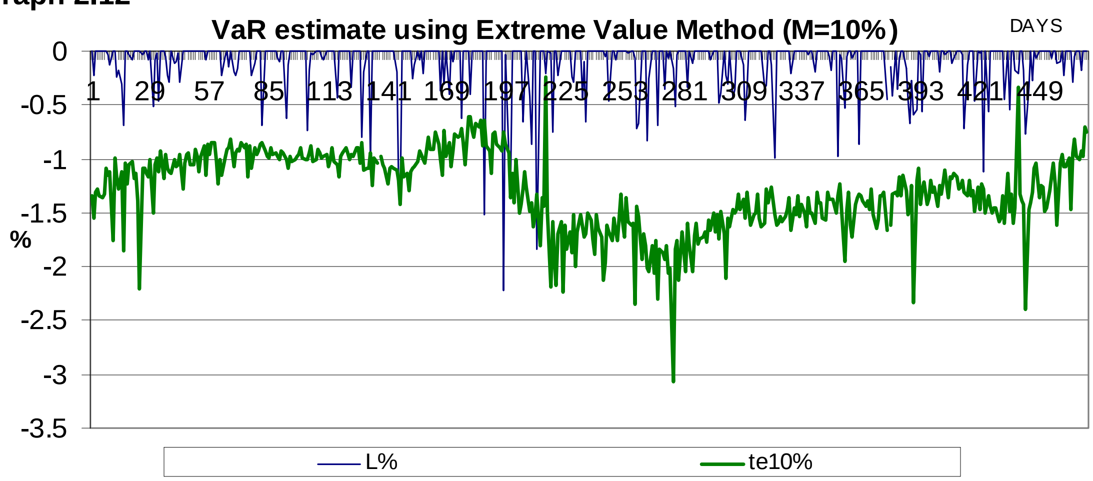
In our analysis we employed three values for the tail cumulative distribution function: 5%, 10% and 15%. The estimation results are shown in Graphs 2.11 to 2.13.

For M and ϒ known, the extreme quantile estimate is given by: $\hat{x}_p = X_{M+1}\left(\frac{M}{Tp}\right)^{\frac{1}{\gamma}}$

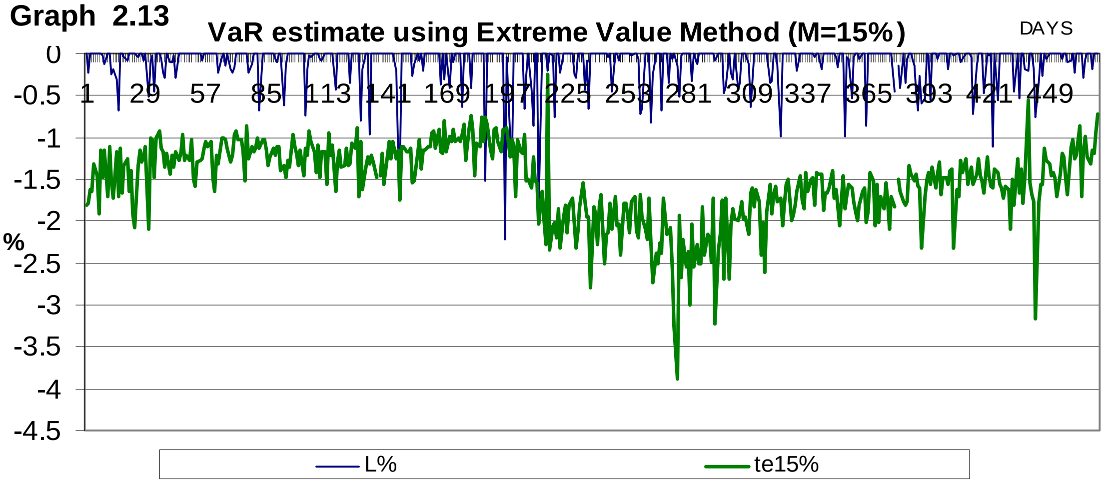
Graph 2.11



Graph 2.12

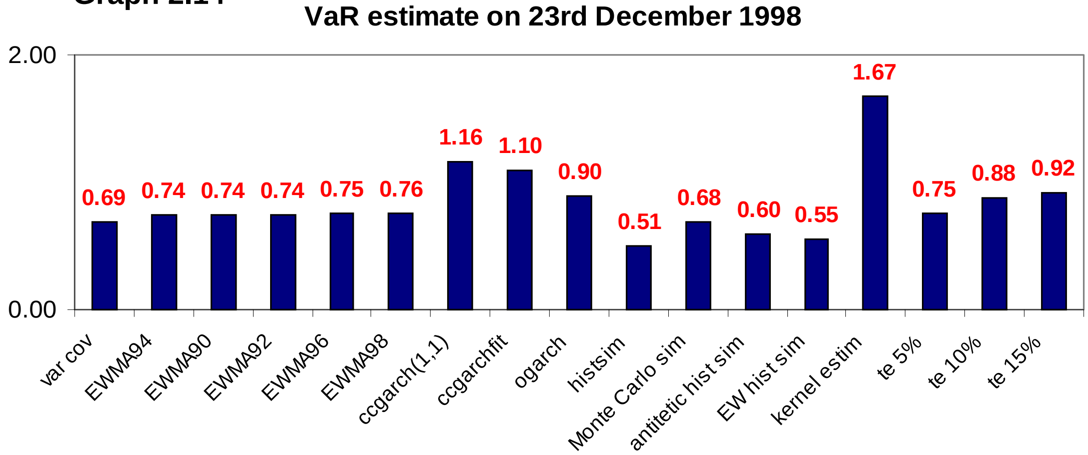


Graph 2.13

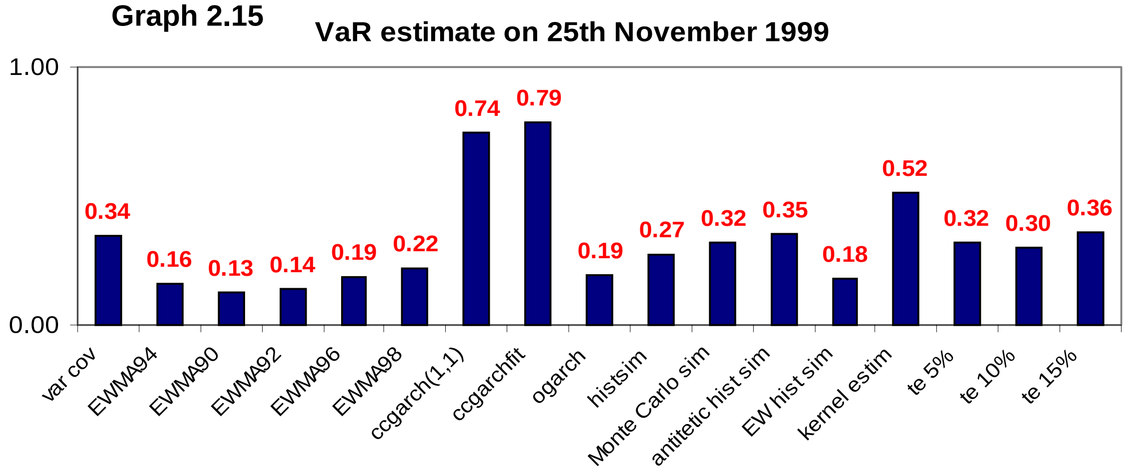


Two of 731 simulated portfolios are defined by small net positions generated from a compensation between large long and short positions. For these two portolios, value at risk estimates are quite large, some models identifying possible losses larger than the net portfolio investment. The VaR estimates for these two moments are presented in Graph 2.14 and 2.15.

Graph 2.14



Graph 2.15



Section 3 Estimation performance analysis

Although the supervision authority does not indicate a specific approach to be used in VaR estimation, the penalties associated with internal model failures in accurately forecasting the distribution of future losses raise the issue of model selection and performance criteria.

Performance assessment is based in our study on a range of tests that address the relative size and variability of VaR estimates, accuracy features from a backtesting perspective and nevertheless efficiency in setting capital charges. Being aware that the tests employed are not relevant enough to allow a categorical discrimination between models, they do provide useful diagnostic information for evaluating model performances. The results for each performance criterion are presented in Appendix 1.

1. **Mean relative bias:** measures the relative size and average conservatism of different VaR estimation approaches (Hendricks 1996). This performance criterion points out the extent to which different VaR techniques produce risk estimates of similar average size. Based on the average size of different VaR estimates, it allows further assessments about the degree of conservatism of individual techniques.

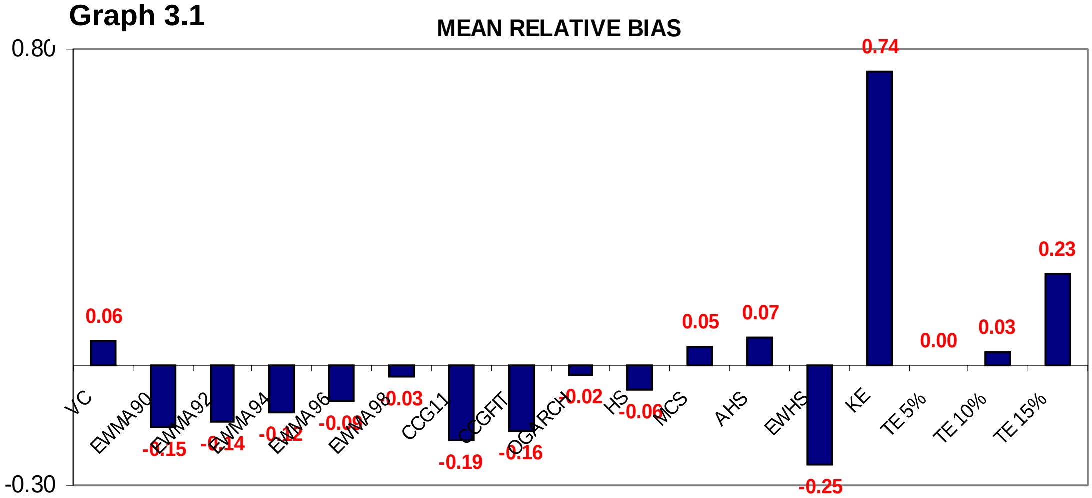
Given N models and T value at risk estimates from each model, mean relative bias of I model is expressed as the average of daily differences, in percentage terms, between the estimates produced by model I and the average of all models estimates:

$$MRB_i = \frac{1}{T}\sum_{t=1}^{T}\frac{VaR_{it} - \overline{VaR}_t}{\overline{VaR}_t}$$

Given the MRB numbers for all models, it appears to be quite a large dispersion between estimates provided by different VaR approaches. The spread is mostly induced by three models: kernel estimation, tail estimation 15% and exponentially weighted historical simulation. Without these three notable exceptions, all other model's MRB tend to lay between –0.2/+0.07. The MRB values are plotted in Graph 3.1.

As suggested by MRB, the model that produces the most conservative estimates of value at risk is Gaussian kernel estimation, its risk estimates being, on average, with 70% higher than all models average. The next most conservative model is tail estimation with M=15%, which overestimates the average of all models with 23 %. Models that provide estimates constantly below the average of all models are exponentially weighted historical simulation (-25%) and exponentially weighted moving average models. Models that produce the estimates closest to the average level are tail estimation for M=5 and 15%, orthogonal GARCH(1,1), EWMA (lambda=0.98), historical simulation and structured Monte Carlo simulation.

Graph 3.1



Evaluating the performances of different underlying parameters indicates, in the case of exponentially weighted moving averages, that the degree of estimates conservatism is positivelyy related to lambda coefficient, which is also true for M coefficient in tail estimation procedures. Analysing tail estimation MRB numbers, it appears to be quite a large spread yielded by M values: if tail estimation 5% replicates almost perfectly the average of all models risk measures, tail estimation 15% overstates the same average with more than 20%. If the distance of the estimate from the average would be a parameter selection criteria, the most accurate estimates are given by lambda=0.98 and M=5 or 10%.

Considering the evidences of non-normality presented in the first section, we would expect models heavily based on the normality assumption to underpredict the estimation provided by other models. This is the case for the models like equally and exponentially weighted moving averages, GARCH models that tend to underestimate the average of all model estimates more than models like tail estimation or kernel estimation. The exception is provided by the simulation approaches: exponentially weighted historical and structured Monte Carlo. The first produces estimates below the average while the second, which relies heavily on normality assumptions, seems to be more conservative.

2. **Root mean square relative bias** examines the degree to which the risk measures tend to vary around the average risk measure for a given observation. This statistic acts like a standard deviation measure.

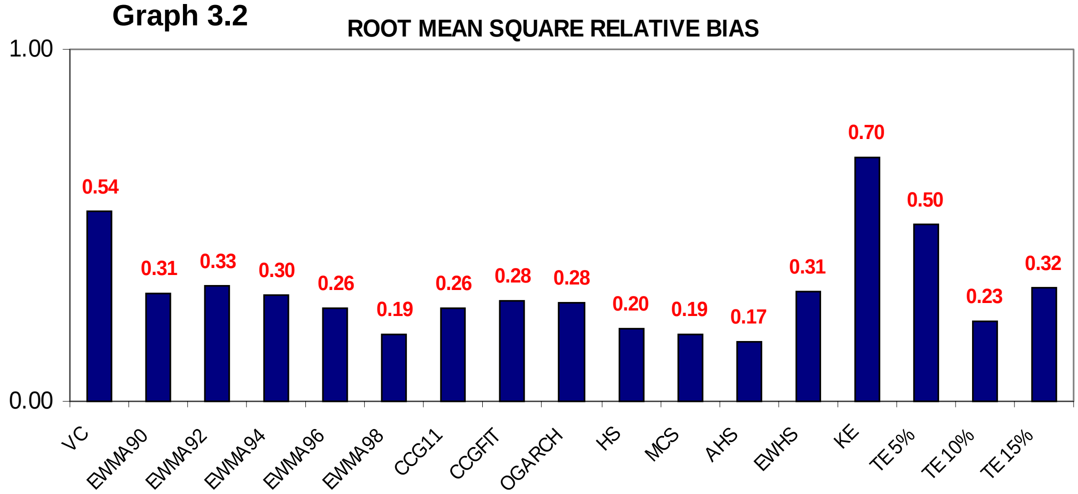
RMSRB emphasises the fact that for any given moment, dispersion between the risk estimates produced by different models is likely to occur.

Root mean square relative bias is computes as:

$$RMSRB_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{VaR_{it} - \overline{VaR}_t}{\overline{VaR}_t}\right)^2}$$

RMSRB statistic captures two different effects: the extent to which a given model estimate systematically differs from all models average and the intrinsic variability of the model estimate. The RMSRB numbers are shown in Graph 3.2.

Graph 3.2



For the majority of the models, RMSRB tends to lay between 0.2 and 0.3. Models that provide the least variable estimates are simple and antithetic historical simulation, structured Monte Carlo simulation and EWMA(0.98). All exponentially weighted moving averages produce less variable estimates. The highest variability is associated with kernel estimation technique, followed by tail estimation (M=5%) and equally weighted moving average. From the parameter selection perspective lowest variability is exhibited by EWMA with lambda=0.98 and tail estimation with M=10%.

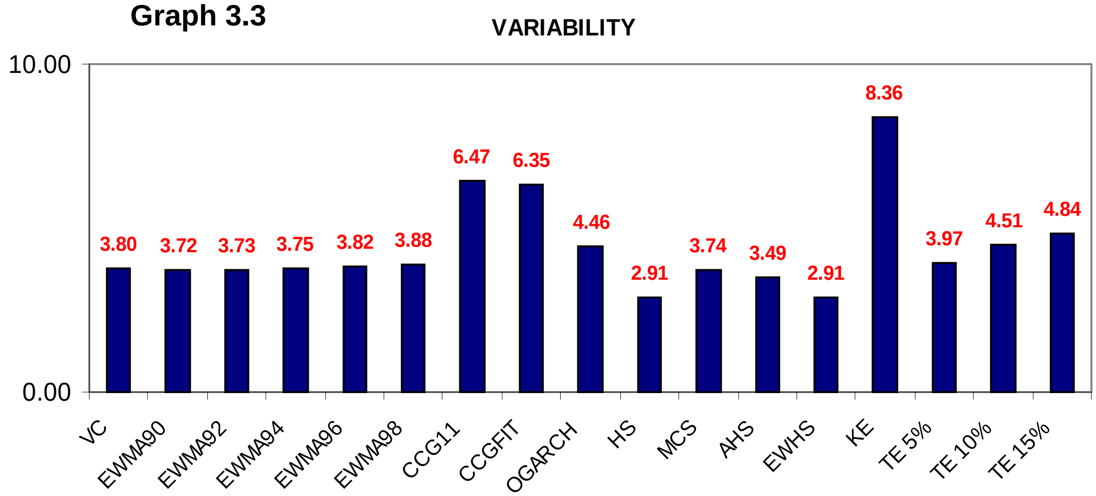
By comparing the two statistics, MRB and RMSRB, we can explain the highest values of RMSRB exhibited by kernel estimation and tail estimation 15% by their high degree of conservatism. This in not the case for equally weighted moving average and tail estimation 5%, models that precisely replicate the average of all models estimate, thus their variance cannot be attributed to a systematically difference from the all model average. In order to separate the two effects mentioned, it is necessary to compute another measure of model estimate variance, which is called variability.

3. **Variability** assumes a zero mean for all models average, but it has the disadvantage of not being scale independent.

Variability is computed as:

$$v_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(VaR_{it})^2}$$

Graph 3.3



Even if variability is no longer scale independent, we can assess that the largest degree of variability is still associated with kernel estimation results. Also over the average variability is produced by constant correlation GARCH models. The lowest estimate variability is generated by the historical simulation technique. Variability results are plotted in Graph 3.3.

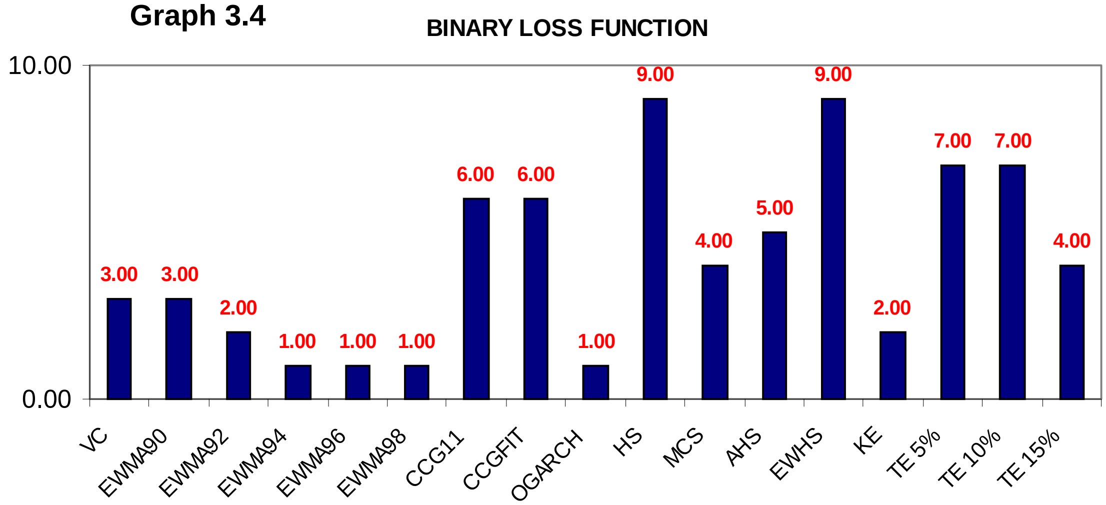
4. **Binary loss function:** as measure of accuracy (conservatism at the right time) counts the number of model estimate failures (moments when the risk measure estimate is lower than the actual loss on the portfolio). Binary loss functions are the instrument used by the supervision authorities in backtesting procedures aiming to assess the quality of internal models. In a binary loss function, all the exceptions, regardless their

dimension, receive equal weights - 1, while the moments when the actual losses are smaller than the risk estimates, the weight is 0.

$$L_{I,t+1}= \begin{cases} 1 \text{ f } L_{I,t+1}<VaR_{I,t} \\ 0 \text{ if } L_{I,t+1}>=VaR_{I,t} \end{cases}$$

Based on backtesting results over a given period (usually 1-year), the models are classified in three categories: the green zone (maximum number of exceptions/year 4), the yellow zone (maximum number of exceptions/year 9) and the red zone (more than 10 exceptions/year). The multiplication factor, as basis in calculating capital charges, is to be set within a range of three to four, depending upon the supervisor's assessment of the bank's risk management practices.

Graph 3.4



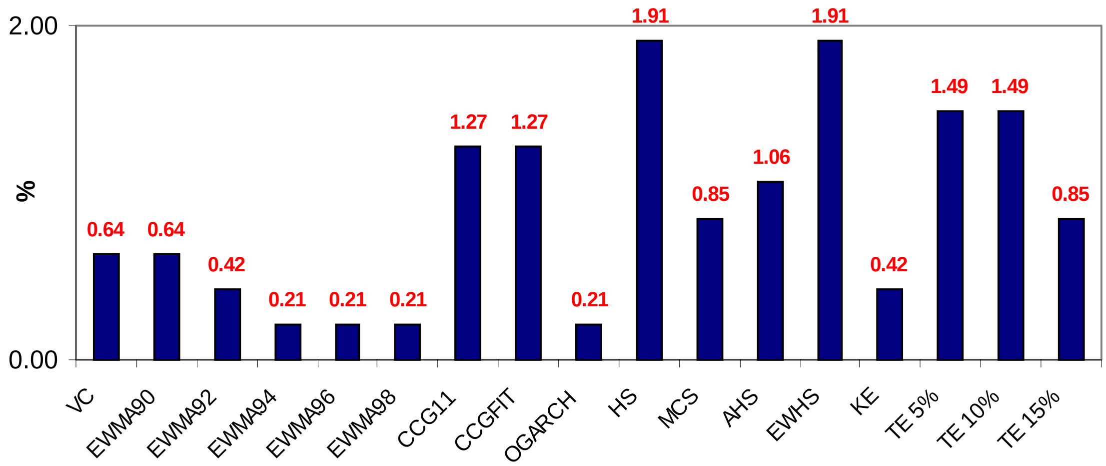
Given the total number of exceptions recorded in two years, the majority of the models are classified in the green zone: equally and exponentially weighted moving averages, constant correlation and orthogonal GARCH models, structured Monte Carlo simulation, antithetic historical simulation, kernel and tail estimation. EWMA, orthogonal GARCH and kernel estimation models have generated an exceptionally low number of estimation failures (less than 1 failure per year). The results of backtesting are shown in Graph 3.4.

Only two models are allotted to the yellow zone, both based on historical simulation: simple and exponentially weighted.

None of the models falls into the red penalising zone.

Binary loss functions also allow assessing if a given model attains the confidence level for which it was designed. If one model provides the desired coverage, in our case 99%, the failure percentage should be below 1%. The confidence level attained indicator is plotted in Graph 3.5.

Graph 3.5



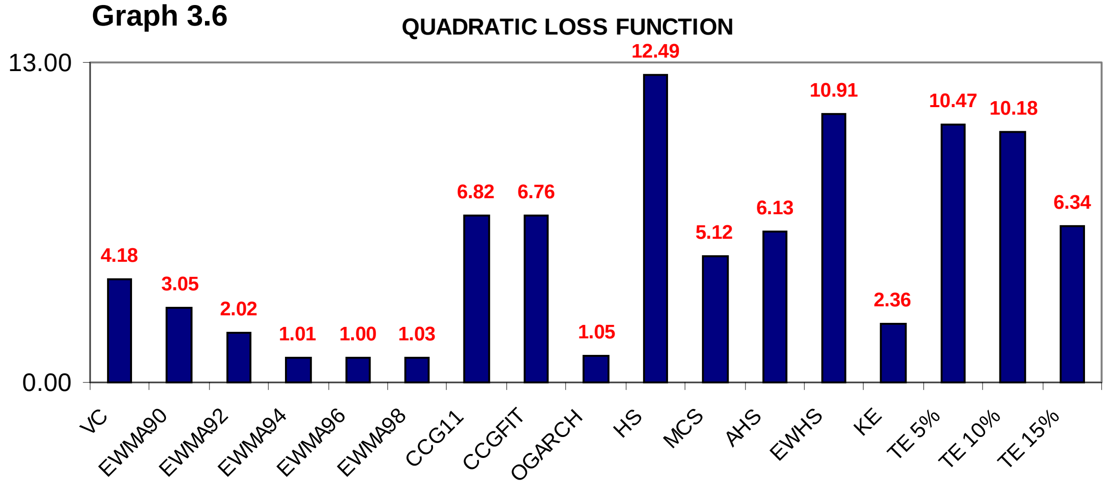
The models that provide the assumed confidence level are the equally and exponentially weighted moving average, orthogonal GARCH, structured Monte Carlo, kernel estimation and tail estimation for M=15%. The antithetic historical simulation is also very close to 1% failure percentage.

As a measure of accuracy, the confidence level attained indicates as conservative the techniques based on moving averages, tail estimation 15%, structured Monte Carlo and kernel estimation. Models with lower performances are based on historical simulation approaches. Nevertheless, all models are consistent and acceptably adequate from a backtesting regulatory perspective.

5. **Quadratic loss function** accounts, besides the number of model's exceptions, also their magnitude, being a better instrument in judging the accuracy degree of an estimate then the binary loss function. Moreover, large failures are penalised also by the quadratic form of the loss function. The quadratic loss function is defined as:

$$L_{I,t+1}= \begin{cases} 1+(L_{I,t+1} - VaRI_{,t})^2 \text{ if } L_{I,t+1}<VaR_{I,t} \\ 0 \text{ if } L_{I,t+1}>=VaR_{I,t} \end{cases}$$

Graph 3.6



The quadratic loss function values for all the models are plotted in Graph 3.6.

Quadratic loss function brings additional information in the aim of discriminating between models that provide the same coverage level. From the models that provide the expected coverage level, the most accurate estimations are produced by the exponentially weighted moving average with lambda parameter equal to 0.96, followed by EWMA 0.94 and 0.98 and orthogonal GARCH. The exceptions magnitude penalises most the risk measures given by tail estimation 15% followed by structured Monte Carlo simulation.

Regarding the models that don't provide the desired confidence level, discrimination between historical simulation approaches: simple and exponentially weighted is allowed by the quadratic loss function results. Exponentially weighted method is less penalised by the magnitude of failures than the simple approach. As a parameter selection criterion, quadratic loss function indicates as more appropriate the percentage of 10 in tail estimation rather then 5, ranking suggested also by RMSRB statistic.

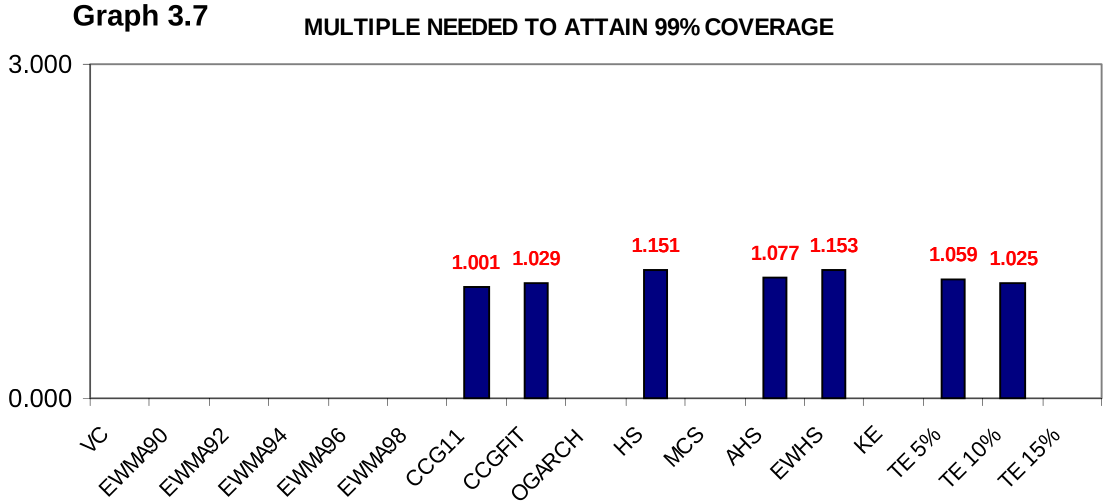
6. **Multiple needed to attain desired coverage**: in order to enforce the comparison between the uncovered loss magnitude induced by various models, a useful tool is the multiple needed to attain coverage, which measures the dimension of adjustments necessary to each model in order to attain the full coverage at the desired confidence level. For each model is computed ex-post the multiple needed to attain desired coverage – 99%, X_I as follows:

$F_I=T_I(1\text{-alfa})$ where $F_I=$ sum 1 if $L_{I,t+1}<X_I VaR_{I,t}$

0 if $L_{I,t+1}>=X_I VaR_{I,t}$

The adjustments necessary for models that are not consistent with the assumed confidence level suggest multiples very close to unit. The largest multiple is required by the historical simulation approaches, leading to a possible indication of inadequacy of the historical return distribution with the current one. The multiples are shown in Graph 3.7.

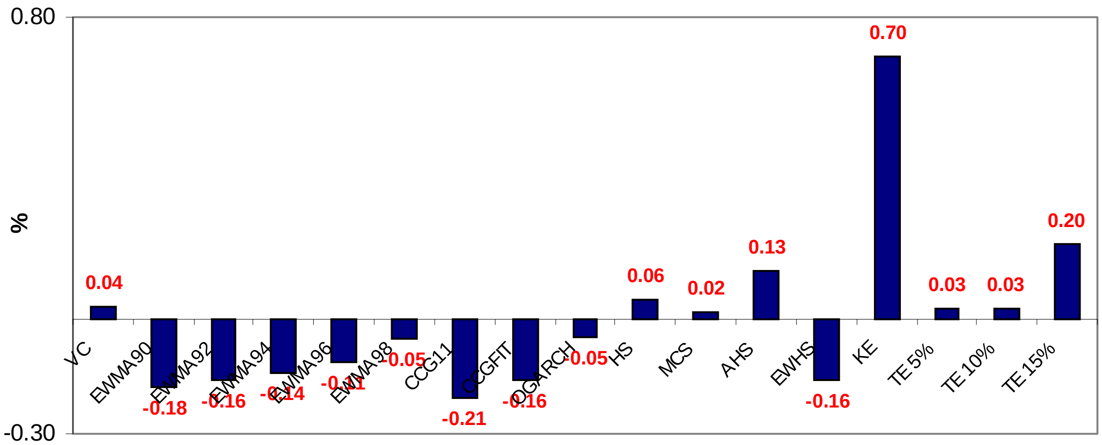
Graph 3.7



Considering again the normality assumptions, it appears to be relevant a comparison between the coefficients indicated by the normal distribution for the transition from a given confidence level to another one, and the coefficients needed by the empirical distribution to realise the same transition. The multiplier needed by historical simulation approaches is larger than the coefficient indicated by the normal distribution. For example, the transition form a 98.34 to 99% coverage in the case of antithetic historical simulation estimates requires a multiplier of 1.013 under normality assumptions, while the actual coefficient necessary equals 1.077. However, the situation is not the same for GARCH models and tail estimation techniques, where the actual transition coefficients are smaller than indicated by the normal distribution.

7. **Mean relative scaled bias**: after scaling all models to ensure a full 99% coverage percentage, it might be relevant to revert to the mean relative bias analysis. The results are presented in Graph 3.8.

Graph 3.8 **MEAN RELATIVE SCALED BIAS**



The initial ranking is still valid, after scaling the estimates with the multiples associated. The estimates closest to all model average are produced by structured Monte Carlo simulation, tail estimation 5 and 10%, orthogonal GARCH and equally weighted moving average models. Scaling also allows the reduction of initial spread induced by different values for the underlying parameter in tail estimation technique.

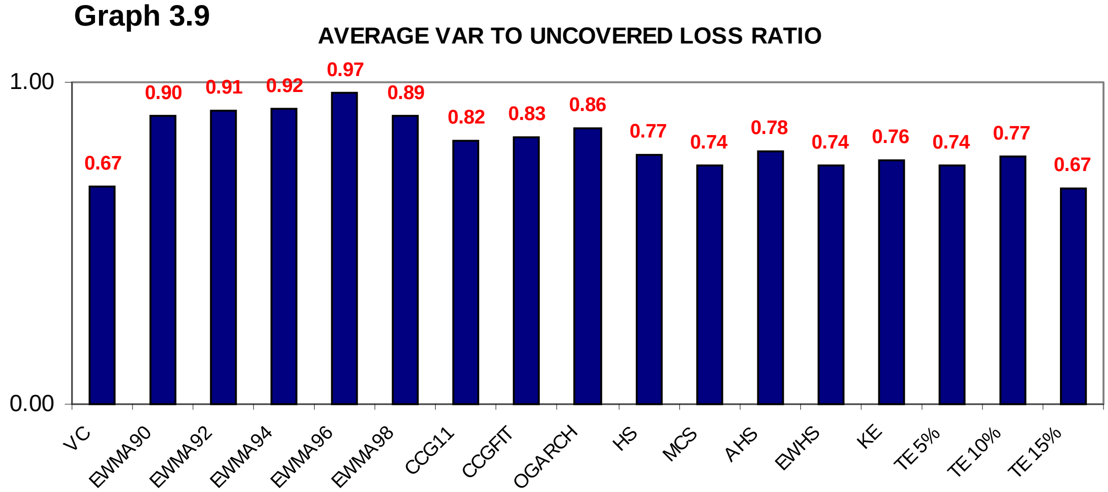
A finding of considerable practical significance, in fact the idea behind MRSB, is to identify the model that, while still offering full 99% coverage, produces the lowest average value at risk estimates. From this point of view, the recommended models are constant correlation GARCH, exponentially weighted moving average and exponentially weighted historical simulation.

8. **Average VaR to uncovered loss ratio**: measures the average size of the loss not covered by the risk estimation.

$$AM_i = \frac{1}{K}\sum_{t=1}^{K}\frac{VaR_{it}}{L_{t+1}}$$

A given VaR number does not provide information about the magnitude of possible losses that occur within the given confidence level, in our case 1%. Nevertheless, this information is essential in risk management. The size of extreme losses represent an useful instrument in assessing whether VaR estimates are able to capture the excess kurtosis of the empirical return distribution. That is, the larger the average VaR to uncovered loss ratio, the more satisfactory the VaR estimate is. The estimated ratios are reported in Graph 3.9.

Graph 3.9



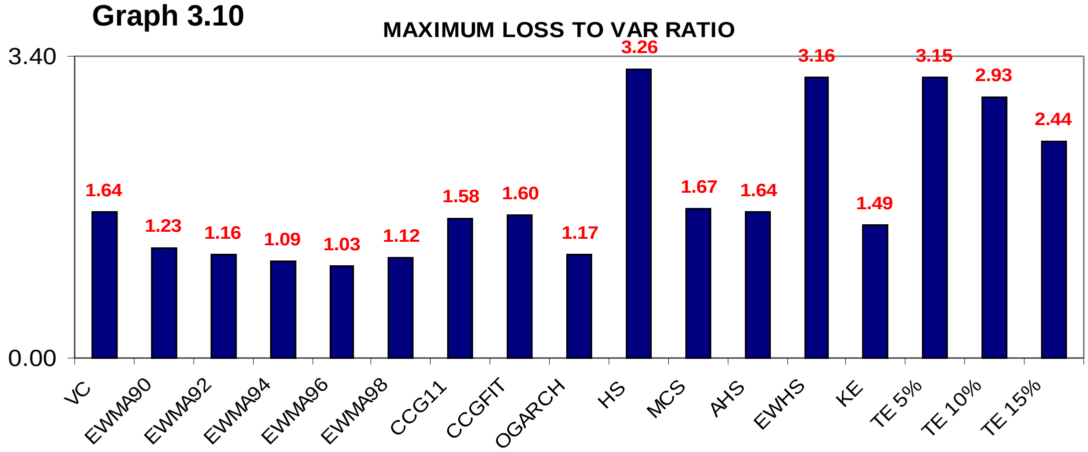
The most accurate estimations of uncovered losses are provided by exponentially weighted moving average models, starting from lambda=0.96, for which average VaR estimates cover 97% of the loss. Loss coverage larger than 70% are given by GARCH models, historical simulation approaches, structured Monte Carlo simulation, kernel estimation and tail estimation. The only models which fail to provide a medium coverage of 70% are equally weighted moving average and tail estimation for M=15%. The fact that models with larger number of exceptions provide better coverage derives from the averaging effect induced by the computation procedure used. To assess the impact of extreme events, we also analyse the maximum loss to VaR ratio.

9. **Maximum loss to VaR ratio**: measures the thickness of return distribution tail. In a theoretical distribution this ratio tends to infinity, emphasising the fact that value at risk numbers does not provide a maximum loss limit. These measures are reported in Graph 3.10.

The historical simulation exhibits the largest distance between risk estimate and maximum actual loss. Following backtesting procedure results, this model would have had been penalised with a multiplier coefficient of 3.4, it follows that the maximum loss would have had been covered by capital charges. A similar ratio is to be found also in the case of exponentially weighted historical simulation model. This is not also the case for the third historical simulation approach, antithetic historical simulation. We can derive from here the observation that simple historical simulation approaches that rely heavily on the assumption of distribution stationarity are less adequate in modelling an exchange rate process with such strong trending behaviour.

The model majority suggests a maximum loss to VaR ratio around 2, which seems to indicate that the minimum of 3 multiplier imposed by supervisors, is excessive. The contraargument is given by tail estimation 5% model, which is classified in the first category from a supervision perspective and receives a multiplier of 3, but the extreme loss is 3.15 times larger than the risk estimate.

Graph 3.10

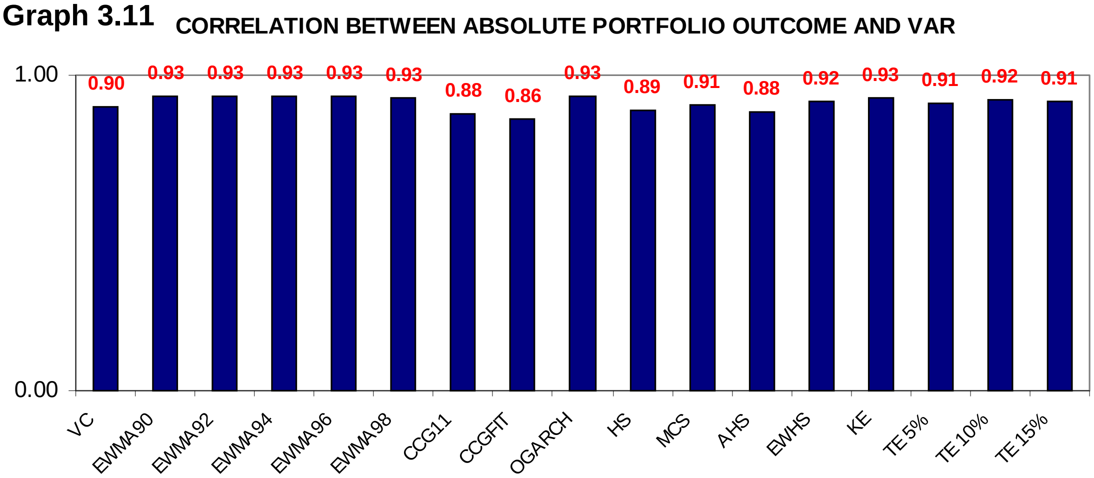


The most accurate models from the perspective of extreme events are exponentially weighted moving averages and orthogonal GARCH.

10. **Correlation between absolute portfolio results and VaR estimate**: reveals how well are correlated the actual changes in portfolio value with changes in risk estimates. The results are presented in Graph 3.11.

For all the models, correlation coefficients with the actual absolute changes in the portfolio value are extremely high, but not conclusive enough considering the daily adjustment of the portfolio structure and value. The high correlation could reflect mostly the change in structure and value of the portfolio, not in the risk exposure. In a similar study, Hendricks (1996) obtained for portfolios constant in structure over the sample period correlations around 0.4 or less.

Graph 3.11



Section 4 Conclusions

Based on the results revealed by the individual measures of estimation performance outlined in the previous section, we can drive the features that stand out for each model employed to estimate value at risk.

- **Equally weighted moving average model**: produces estimates close to all model average, but significantly variable; the failure percentage is less then the confidence level assumed, the model being classified in the green zone from a supervision perspective; however, the average VaR to uncovered loss ratio is one of the lowest in the sample, only 0.67; after scaling all models to attain 1% confidence level, the estimates provided by the equally weighted moving average model continue to overstate by little all model average. Based on this, we may conclude that equally weighted moving average model ensures relative conservative estimates of value at risk, the trade off between an easy to implement algorithm and the estimate accuracy being reflected in the high variability of results and in the low coverage percentage.

- **Exponentially weighted moving average models**: tend to produce estimates below all model average with the associated spread positively related to the value of the underlying parameter lambda; results variability is moderate; the number of exceptions is the lowest in the models sample (2 for EWMA90 and 1 for the others per two years), and also the failures exhibit low magnitude. The coverage of extreme losses with VaR estimates is over 90%. Moreover, finding of considerable practical importance, the coverage level actually attained, 99.58% is achieved with the most important underestimation of all model average estimates. The coefficients of correlation between absolute portfolio results and VaR estimate are also the largest in the sample. In order to choose the appropriate smoothing parameter, we have to decide which selection criterion should be employed. If we were interested in the model that provides the lowest VaR numbers, while is still consistent with the confidence level assumed, we would set lambda equal to 0.9. The least variable estimations and the highest extreme loss coverage is provided by lambda=0.96 or 0.98. A comparison with the equally weighted moving average model reveals advantages of exponentially weighting in the area of estimate accuracy and variability. From an operational perspective, exponentially weighted moving average method appears to be preferable.

- **Constant correlation GARCH models**: produce estimates significantly below all models average with a reasonably high variability; do not attain desired confidence level (6 failures/two years), but still qualify to the green supervision model category. The multipliers needed to attain desired coverage are very close to unit: 1.001 and 1.02. By scaling to attain desired coverage, provide the lowest VaR estimates in the sample, this being an important advantage in setting capital charges. The average value at risk to uncovered loss ratio is over 0.8, supporting the feature of an efficient conservatism degree. The maximum loss to VaR ratio is close to 1.6, being fully covered by the common multiplier of 3. A comparison between the two types of constant correlation GARCH models, GARCH(1,1) and GARCHFIT, allows the conclusion that GARCHFIT performs better in some regards: MRB, variability, quadratic loss function and the average VaR to uncovered loss ratio. Based on these features, we may conclude that constant correlation GARCH models exhibit a degree of conservatism large enough to attain the desired confidence level and also to provide the lowest estimates of VaR numbers and an accurate cover of the extreme losses.

Although the number of exceptions provided is higher than the average, it is still below the green category limit and the constant correlation GARCH method is not penalised by additional capital charges. Also, the difference between the risk estimates and the uncovered losses are reasonably small. Beyond the difficulties in implementing and correctly specifying the models for the variance process, the results obtained justify the recommendation of these models in value at risk estimation.

- **Orthogonal GARCH model**: provides risk estimates very close to all models average, inducing a smaller variability compared to constant correlation GARCH models. The number of model exceptions is the lowest in the sample: 1 estimation failure in almost two years, ensuring a coverage of 99.79%, larger than assumed. Also a relative large figure reflects the proportion of VaR estimates on uncovered losses-86%. Although the coverage percentage is larger than assumed, orthogonal GARCH estimates are slightly below the average of all models scaled to 99% coverage estimates. Another remarkable result concerns the maximum loss to VaR ratio, which is only 1.17. Better performances obtained by the orthogonal GARCH model, regarding variability and accuracy, come as no surprise, considering the superior specification of the processes governing the orthogonal risk factor variances and the relaxation of the constant correlation assumption. If better statistical and operational features as accuracy and conservatism recommend orthogonal GARCH over constant correlation approaches, lower VaR numbers generated by constant correlation GARCH may justify a trade-off between accuracy and higher capital charges.

- **Historical simulation**: examining mean relative bias measures, leads to the conclusion that historical simulation tends to slightly overestimate all models average, while its estimates variability is low. The number of exceptions is considerably higher than the average (9 failures for the entire estimation period), providing a lower coverage than assumed, only 97.9%, but the multiplier needed to attain 99% coverage is still close to unit. After scaling, the estimates continue to be over all model average. The main shortcoming on this approach is the large distance between VaR estimates and extreme losses. The maximum loss is 3.26 times higher than the corresponding VaR estimate. The average VaR to uncovered loss ratio is 0.77. Studying the number of model failures and their magnitude we can assess that historical simulation produces the weakest performances from the sample. Although from a supervision perspective historical simulation is an acceptable model, it is recommendable the attempt to improve its performances by adjusting the historical database.

- **Exponentially weighted historical simulation**: the results obtained by using an exponentially smoothing method improve in some areas the performances of the simple historical simulation approach. First, MRB indicates a substantial underestimation of all models average (-26%), underestimation favourable under the circumstances of 99% coverage. But the actual coverage percentage is identical to the one offered by the simple historical simulation – 97.9%. After scaling with a multiplier of 1.156, this model continues to underestimate significantly all models average. The main disadvantage of the method, large distance between VaR estimate and extreme losses is not overcame by exponentially smoothing the data. Considering the performances slightly improved and the advantage of reducing the capital charges with lower VaR estimates, exponentially smoothing the data is fully justified in a historical simulation approach.

- **Antithetic historical simulation**: by doubling the data sample in the aim of eliminating the trend exhibited by the exchange rates, substantial improvement of the historical simulation approach performance is achieved. The estimates provided by this method slightly overstate all models average, indicating a superior degree of conservatism. The variability exhibited is the lowest in the sample. Trend elimination reduces the number of model failures from 9 to 5 and provides a coverage very close to the one assumed – 98.94%. The multiplier needed for scaling to 99% coverage is 1.077. Average VaR to uncovered loss ratio is 74%, which places this model close to all models average. The main shortcoming of the historical simulation, the disproportion between extreme losses and corresponding risk estimates is fully eliminated, the maximum loss to VaR ratio being less than 2. A comparison between all historical simulation approaches indicates as clear winner antithetic historical simulation, method that exhibits the closest performances to the ones provided by the best models in the sample.

- **Structured Monte Carlo simulation** produces estimates slightly over all models average with low variability. The small number of model failures, 4, allows for a coverage larger than assumed – 99.05%. After scaling all estimates to attain 99% coverage, structured Monte Carlo underestimates slightly all models estimates. The ratios between uncovered losses and VaR estimates lay in the range provided by the other models and the maximum loss exceeds the VaR estimate with 67%. Based on these performance measures, we may conclude that structured Monte Carlo simulation ensures moderate conservative estimates in the green model category, without significant drawbacks or advantages over other approaches.

- **Gaussian kernel density estimation**: the small number of model failures suggests its conservative characteristic and induces a significantly high overestimation of all models average (+74%). Compared to other models that produce a similar number of exceptions, such high degree of conservatism is not fully justified. Even after scaling all models to attain desired confidence level, kernel estimation continues to substantially overstate all models average. The high variability associated is explained mainly by its systematical distance from all models average. The surprise comes from the fact that, while producing the largest average VaR estimates, kernel estimation VaR numbers cover only 74% of the extent of extreme losses. It appears that the conservative characteristic of this model does not manage to justify the limited accuracy and high variability of its estimates.

- **Tail estimation**: in assessing the performances of tail estimation approaches, the impact of the underlying parameter M is essential. Based on MRB numbers, the average size of VaR estimates is lower but close to all model average in the case of M=5 and 10%, and substantially higher (+23%) in the case of M=15%. It follows that the degree of model conservatism is positively related to the size of parameter M. High variability can be explained just in the case of M=15% by its systematical distance from all models average. The number of failures qualifies tail estimation in the first model category. Consequently, the number of model exceptions, between 7 and 4, is negatively related to the degree of conservatism. The large magnitude of failures comes as a surprise in the case of M=15%, a high conservative model, suggesting possible accuracy problems. The multipliers needed to attain desired coverage are all close to unit: 1.059 and 1.025. After scaling, all tail estimation models produce estimates higher than all models average. The average VaR to uncovered losses ratio indicates a value lower than expected in the case of M=15%, just 67%. Moreover, the maximum loss exceeds VaR estimates with percentages between 215 and 144. The magnitude of model failures represents the main drawback on these approaches. For M=15%, the loss exceeds also the capital charges based on the common multiplier of

3. The apparent conservatism of the 15% tail estimation fails to capture the real magnitude of the extreme loss. If we should select the appropriate value for M, the cumulative percentage of the distribution tail limit, we would have to choose between 10 and 15%. 10%, even if it is less conservative and quite variable, brings the advantage of lower capital charges and higher extreme losses coverage (77%). A percentage closer to 15 is justified in a conservative perspective, which ignores the lower extreme losses coverage (67%).

As a final consideration, our work intended to highlight some individual features of several value at risk models, when applied to measuring foreign exchange exposure of a portfolio denominated in ROL. Even if no model was identified as being insufficiently conservative in its risk measurements, it was obvious that some adjustments need to be made in order to achieve a higher degree of accuracy and efficiency. The limitations mentioned above regard notably the features exhibited by the processes driving exchange rates against ROL and impact the use of historical data and the specification of conditional variance. Being aware of the limited relevance of these results for the banking regulation purposes, we trust in their further refinement in a future work.

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Appendix 1

Model	MRB[1]	MRSB[2]	RMSRB[3]	VAR[4]	BLF[5]	QLF[6]	MOC[7]	AUL/V[8]	ML/V[9]	COR[10]
VC	0.06	0.04	0.54	3.80	3.00	4.18		0.67	1.64	0.90
EWMA 90	-0.15	-0.18	0.31	3.72	3.00	3.05		0.90	1.23	0.93
EWMA 92	-0.14	-0.16	0.33	3.73	2.00	2.02		0.91	1.16	0.93
EWMA 94	-0.12	-0.14	0.30	3.75	1.00	1.01		0.92	1.09	0.93
EWMA 98	-0.09	-0.11	0.26	3.82	1.00	1.00		0.97	1.03	0.93
EWMA 96	-0.03	-0.05	0.19	3.88	1.00	1.03		0.89	1.12	0.93
CCG11	-0.19	-0.21	0.26	6.47	6.00	6.82	1.00	0.82	1.58	0.88
CCGFIT	-0.16	-0.16	0.28	6.35	6.00	6.76	1.02	0.83	1.60	0.86
OGARCH	-0.02	-0.05	0.28	4.46	1.00	1.05		0.86	1.17	0.93
HS	-0.06	0.06	0.20	2.91	9.00	12.4	1.15	0.77	3.26	0.89
MCS	0.05	0.02	0.19	3.74	4.00	5.12		0.74	1.67	0.91
AHS	0.07	0.13	0.17	3.49	5.00	6.13	1.07	0.78	1.64	0.88
EWHS	-0.25	-0.16	0.31	2.91	9.00	10.9	1.15	0.74	3.16	0.92
KE	0.74	0.70	0.70	8.36	2.00	2.36		0.76	1.49	0.93
TE 5%	0.00	0.03	0.50	3.97	7.00	10.4	1.05	0.74	3.15	0.91
TE 10%	0.03	0.03	0.23	4.51	7.00	10.1	1.02	0.77	2.93	0.92
TE 15%	0.23	0.20	0.32	4.84	4.00	6.34		0.67	2.44	0.91

[1] mean relative bias
[2] mean relative scaled bias
[3] root square relative mean bias
[4] variability
[5] binary loss function
[6] quadratic loss function
[7] multiple to obtain coverage
[8] average uncovered loss to VaR ratio
[9] maximum loss to VaR ratio
[10] correlation

Appendix 2

Estimation results for GARCH(1,1) model

Dependent Variable: LAUS
Method: ML - ARCH
Date: 06/04/00 Time: 20:40
Sample(adjusted): 6/18/1997 4/05/2000
Included observations: 731 after adjusting endpoints
Convergence achieved after 14 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001020	0.000264	3.862225	0.0001
DUMMY	0.067545	0.000362	186.6211	0.0000
		Variance Equation		
C	1.52E-05	2.31E-05	0.657511	0.5109
ARCH(1)	0.028768	0.032931	0.873594	0.3823
GARCH(1)	0.675475	0.457073	1.477828	0.1395
R-squared	0.108977	Mean dependent var		0.001149
Adjusted R-squared	0.104067	S.D. dependent var		0.007591
S.E. of regression	0.007185	Akaike info criterion		-7.028965
Sum squared resid	0.037477	Schwarz criterion		-6.997540
Log likelihood	2574.087	F-statistic		22.19835
Durbin-Watson stat	1.971437	Prob(F-statistic)		0.000000

Dependent Variable: LDMK
Convergence achieved after 18 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001028	0.000264	3.894503	0.0001
DUMMY	0.067558	0.000354	191.0144	0.0000
		Variance Equation		
C	1.75E-05	2.87E-05	0.609572	0.5421
ARCH(1)	0.026896	0.033933	0.792624	0.4280
GARCH(1)	0.630773	0.565301	1.115818	0.2645
R-squared	0.109320	Mean dependent var		0.001148
Adjusted R-squared	0.104412	S.D. dependent var		0.007580
S.E. of regression	0.007174	Akaike info criterion		-7.031830
Sum squared resid	0.037360	Schwarz criterion		-7.000405
Log likelihood	2575.134	F-statistic		22.27678
Durbin-Watson stat	1.961230	Prob(F-statistic)		0.000000

Dependent Variable: LFRF
Convergence achieved after 18 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001034	0.000266	3.894075	0.0001
DUMMY	0.067653	0.000344	196.8855	0.0000
		Variance Equation		
C	1.74E-05	2.98E-05	0.582510	0.5602
ARCH(1)	0.025923	0.032981	0.786015	0.4319
GARCH(1)	0.639320	0.581049	1.100285	0.2712
R-squared	0.108215	Mean dependent var		0.001155

Adjusted R-squared	0.103301	S.D. dependent var	0.007628
S.E. of regression	0.007223	Akaike info criterion	-7.018361
Sum squared resid	0.037875	Schwarz criterion	-6.986935
Log likelihood	2570.211	F-statistic	22.02430
Durbin-Watson stat	1.972509	Prob(F-statistic)	0.000000

Dependent Variable: LGBP
Convergence achieved after 8 iterations

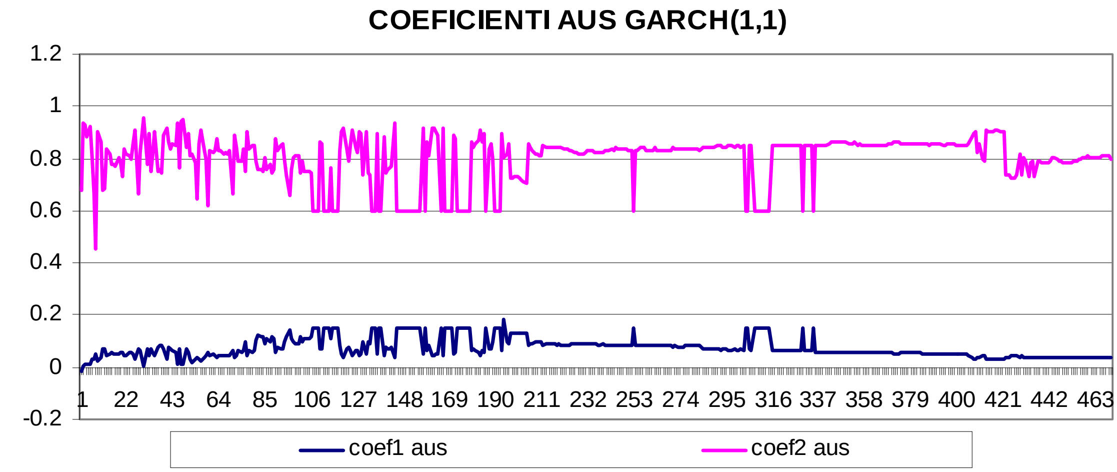
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000934	0.000235	3.974587	0.0001
DUMMY	0.058356	0.001223	47.69628	0.0000
LGBP(-1)	0.090312	0.048946	1.845135	0.0650
Variance Equation				
C	9.51E-06	6.21E-06	1.532468	0.1254
ARCH(1)	0.146054	0.060537	2.412639	0.0158
GARCH(1)	0.622436	0.188918	3.294738	0.0010

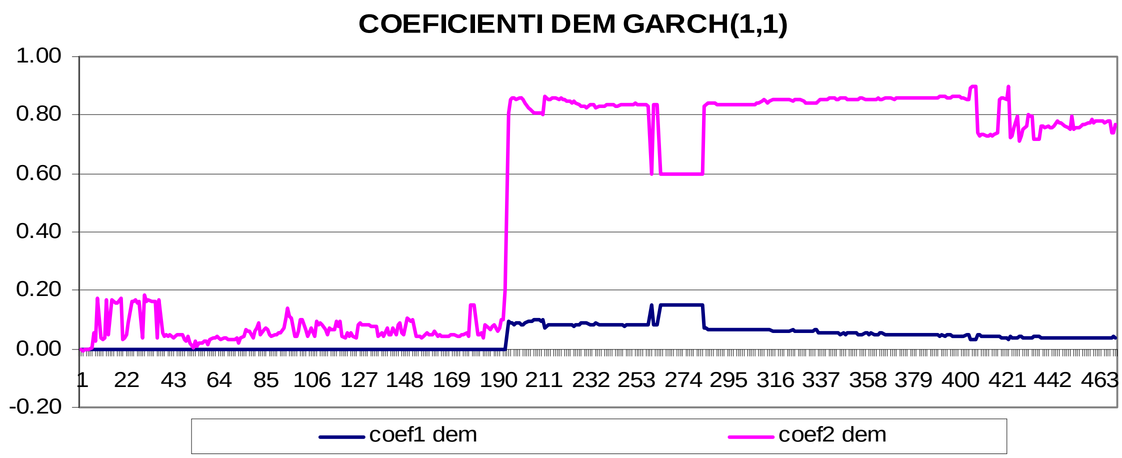
R-squared	0.116990	Mean dependent var	0.001326
Adjusted R-squared	0.110892	S.D. dependent var	0.006750
S.E. of regression	0.006365	Akaike info criterion	-7.315527
Sum squared resid	0.029330	Schwarz criterion	-7.277776
Log likelihood	2676.167	F-statistic	19.18450
Durbin-Watson stat	2.036751	Prob(F-statistic)	0.000000

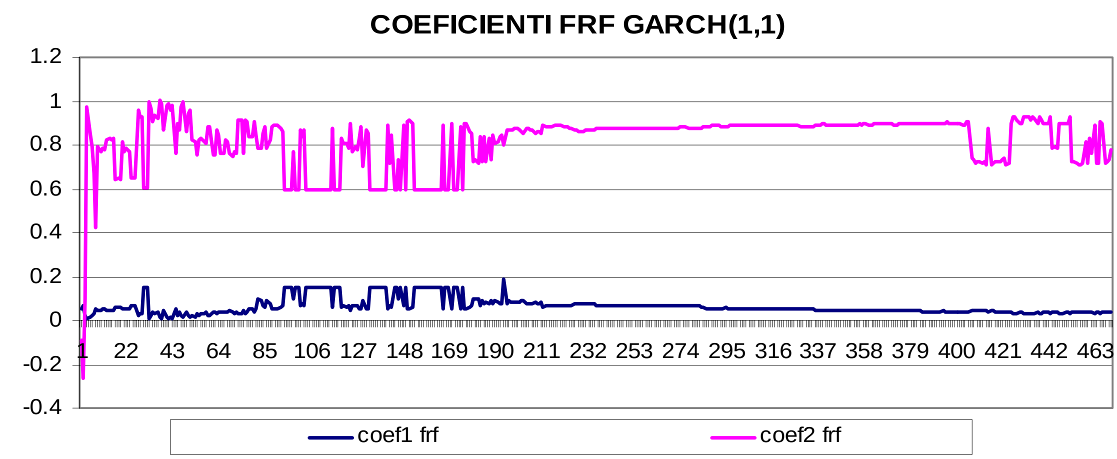
Dependent Variable: LUSD
Convergence achieved after 8 iterations
Bollerslev-Wooldrige robust standard errors & covariance

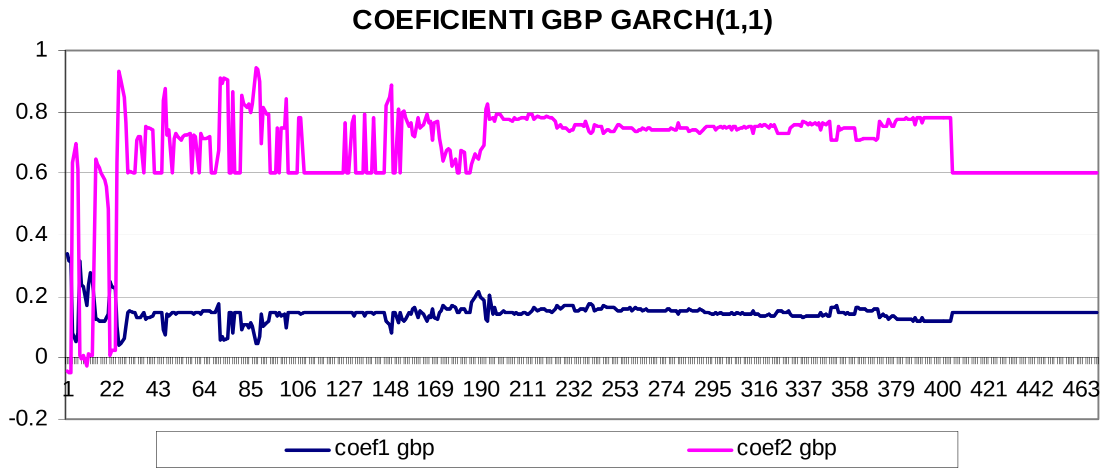
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000630	8.61E-05	7.324102	0.0000
DUMMY1	0.000503	0.000268	1.880018	0.0601
LUSD(-1)	0.286190	0.056350	5.078778	0.0000
Variance Equation				
C	1.27E-06	4.99E-07	2.548857	0.0108
ARCH(1)	0.184868	0.101349	1.824075	0.0681
GARCH(1)	0.656671	0.101895	6.444589	0.0000
DUMMY1	1.64E-06	8.26E-07	1.983528	0.0473
DUMMY2	-1.01E-06	4.58E-07	-2.197328	0.0280

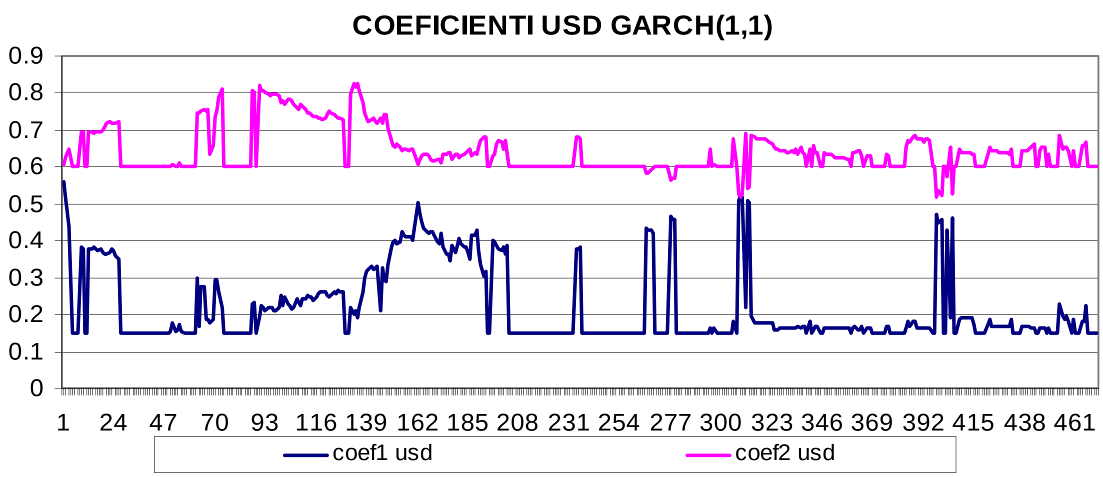
R-squared	0.110387	Mean dependent var	0.001361
Adjusted R-squared	0.101762	S.D. dependent var	0.004676
S.E. of regression	0.004432	Akaike info criterion	-8.906432
Sum squared resid	0.014181	Schwarz criterion	-8.856098
Log likelihood	3258.848	F-statistic	12.79839
Durbin-Watson stat	1.905494	Prob(F-statistic)	0.000000


COEFICIENTI AUS GARCH(1,1)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
1 22 43 64 85 106 127 148 169 190 211 232 253 274 295 316 337 358 379 400 421 442 463
coef1 aus
coef2 aus


COEFICIENTI DEM GARCH(1,1)
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
1 22 43 64 85 106 127 148 169 190 211 232 253 274 295 316 337 358 379 400 421 442 463
coef1 dem
coef2 dem


COEFICIENTI FRF GARCH(1,1)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
1 22 43 64 85 106 127 148 169 190 211 232 253 274 295 316 337 358 379 400 421 442 463
coef1 frf
coef2 frf


COEFICIENTI GBP GARCH(1,1)
1
0.8
0.6
0.4
0.2
0
-0.2
1 22 43 64 85 106 127 148 169 190 211 232 253 274 295 316 337 358 379 400 421 442 463
coef1 gbp
coef2 gbp


COEFICIENTI USD GARCH(1,1)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 24 47 70 93 116 139 162 185 208 231 254 277 300 323 346 369 392 415 438 461
coef1 usd
coef2 usd

Estimation results for GARCHFIT model

Dependent Variable: LAUS
Method: ML - ARCH
Date: 06/04/00 Time: 20:47
Sample(adjusted): 6/18/1997 4/05/2000
Included observations: 731 after adjusting endpoints
Convergence achieved after 22 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001047	0.000264	3.962384	0.0001
DUMMY	0.067028	0.000816	82.15867	0.0000
	Variance Equation			
C	4.90E-05	3.86E-06	12.69471	0.0000
ARCH(1)	0.047290	0.047240	1.001053	0.3168

R-squared	0.108986	Mean dependent var	0.001149
Adjusted R-squared	0.105309	S.D. dependent var	0.007591
S.E. of regression	0.007180	Akaike info criterion	-7.030917
Sum squared resid	0.037477	Schwarz criterion	-7.005777
Log likelihood	2573.800	F-statistic	29.64147
Durbin-Watson stat	1.970532	Prob(F-statistic)	0.000000

Dependent Variable: LDMK
Convergence achieved after 12 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001042	0.000263	3.969909	0.0001
DUMMY	0.068905	0.001234	55.85552	0.0000
	Variance Equation			
C	4.82E-05	3.91E-06	12.33715	0.0000
ARCH(1)	0.060742	0.049022	1.239087	0.2153

R-squared	0.109301	Mean dependent var	0.001148
Adjusted R-squared	0.105625	S.D. dependent var	0.007580
S.E. of regression	0.007169	Akaike info criterion	-7.035009
Sum squared resid	0.037361	Schwarz criterion	-7.009869
Log likelihood	2575.296	F-statistic	29.73757
Durbin-Watson stat	1.963748	Prob(F-statistic)	0.000000

Dependent Variable: LFRF
Convergence achieved after 11 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001054	0.000265	3.983014	0.0001
DUMMY	0.066413	0.001560	42.58234	0.0000
	Variance Equation			
C	4.90E-05	4.05E-06	12.08990	0.0000
ARCH(1)	0.058246	0.047016	1.238873	0.2154

R-squared	0.108180	Mean dependent var	0.001155
Adjusted R-squared	0.104500	S.D. dependent var	0.007628
S.E. of regression	0.007218	Akaike info criterion	-7.020923
Sum squared resid	0.037876	Schwarz criterion	-6.995782
Log likelihood	2570.147	F-statistic	29.39555

Durbin-Watson stat	1.970281	Prob(F-statistic)	0.000000

Dependent Variable: LGBP
Convergence achieved after 10 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000934	0.000226	4.137939	0.0000
DUMMY	0.064424	0.005524	11.66361	0.0000
LGBP(-1)	0.085790	0.050078	1.713135	0.0867
	Variance Equation			
C	1.26E-06	1.05E-06	1.195921	0.2317
ARCH(1)	0.198961	0.070697	2.814276	0.0049
ARCH(2)	-0.156538	0.080263	-1.950302	0.0511
GARCH(1)	0.926779	0.052425	17.67818	0.0000

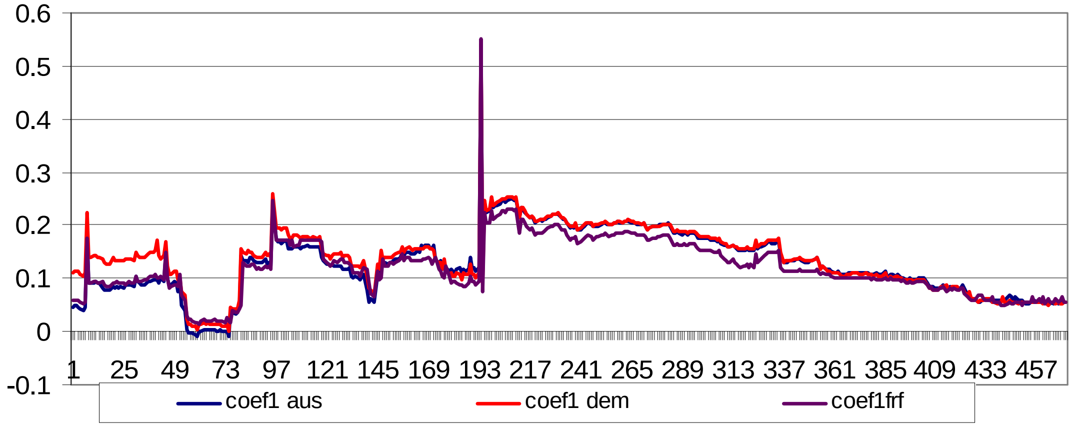
R-squared	0.116118	Mean dependent var	0.001326
Adjusted R-squared	0.108783	S.D. dependent var	0.006750
S.E. of regression	0.006372	Akaike info criterion	-7.323880
Sum squared resid	0.029358	Schwarz criterion	-7.279837
Log likelihood	2680.216	F-statistic	15.83044
Durbin-Watson stat	2.042195	Prob(F-statistic)	0.000000

Dependent Variable: LUSD
Convergence achieved after 20 iterations

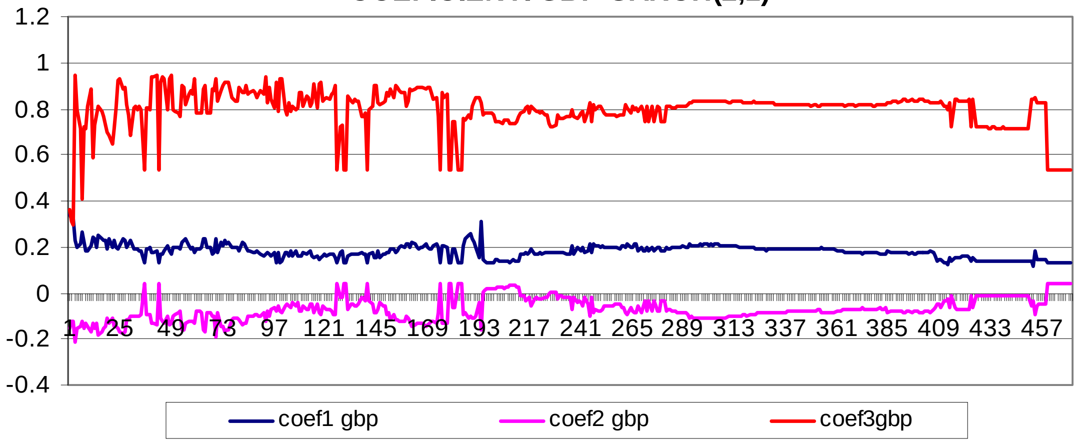
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000701	8.60E-05	8.150644	0.0000
DUMMY1	0.000418	0.000236	1.770946	0.0766
LUSD(-1)	0.208926	0.063714	3.279119	0.0010
	Variance Equation			
C	2.89E-07	1.94E-07	1.484958	0.1376
ARCH(1)	0.520702	0.132052	3.943157	0.0001
ARCH(2)	-0.363939	0.124018	-2.934562	0.0033
GARCH(1)	0.831980	0.050730	16.40027	0.0000
DUMMY1	5.19E-07	3.98E-07	1.303037	0.1926
DUMMY2	-2.14E-07	1.76E-07	-1.218116	0.2232

R-squared	0.097606	Mean dependent var	0.001361
Adjusted R-squared	0.087593	S.D. dependent var	0.004676
S.E. of regression	0.004467	Akaike info criterion	-8.988290
Sum squared resid	0.014385	Schwarz criterion	-8.931664
Log likelihood	3289.726	F-statistic	9.748220
Durbin-Watson stat	1.753373	Prob(F-statistic)	0.000000

COEFICIENTI ARCH(1)



COEFICIENTI GBP GARCH(2,1)



COEFICIENTI USD GARCH(2,1)