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Dissertation

Macroeconomic Effects of the Population Ageing Phenomenon

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Abstract

In the next decades, Romania will experience demographic changes that are likely to have a significant impact on its economic development. Using an overlapping generations model in the Auerbach-Kotlikoff tradition, I attempt to assess the effects of the ageing population phenomenon. The driving force of the model is the population evolution which is imposed as exogenous and is based on projections from Eurostat. The study is not designed to provide precise forecasts but rather a set of economic trends which are likely to be induced by the demographic transition. Simulations covering the next 100 years were conducted under two alternative scenarios, depending on the method of balancing the social security budget. First, I assume the contribution rate adjusts in order to cover the growing financing needs. Second, the contribution rate is constant and the pension to wage ratio (and indirectly the pension) adjusts. The possibility of increasing the retirement age was also considered. The results confirm that the population ageing has a greater impact in the first scenario.
1 Introduction

In the last decades, the world’s countries experienced persistent demographic shifts. According to current projections significant changes in the features of the populations will continue well into the future. For some of the countries, the changes imply an increase of the share of older individuals in total population, which characterizes the population ageing phenomenon.

The main causes of population ageing are considered to be the decline in fertility rates and the increase in life expectancy, which contribute to a constantly rising median age. These two factors are also associated with diminishing population growth rates, with negative values for some countries.

The process is affecting large parts of the world, but its timing and magnitude differ substantially. According to United Nations Population Division\textsuperscript{1}, the ratio of the population aged 65 years or over to the population aged 15-64 for Europe will increase from 24 in 2010 to 47 in 2050. For the Eastern Europe region\textsuperscript{2} the ratio will go from 19 to 43, while for Romania it will reach 49, from a current value of 21. The magnitude of the ageing population process for Romania is similar in a projection provided by a different institution. Figure 1 illustrates the future evolution of the ratio of population aged 61 years or over to the population aged 21 or over, according to Europop2008 projection from Eurostat.

The interest in the demographic transition as the topic of this paper is easily justified on one hand by the extent of the phenomenon for Romania, and on the other hand by standard economic theory consequences of ageing on economic development. The most obvious is the direct effect on the labor force, since intensification of ageing diminishes the active population. The labor force shrinkage is also determined (perhaps even to a higher degree) by the evolution of total population, which is projected to drop by more than

\textsuperscript{1}World Population Prospects: The 2008 Revision

\textsuperscript{2}Belarus, Bulgaria, Czech Republic, Hungary, Poland, Republic of Moldova, Romania, Russian Federation, Slovakia, Ukraine
20 percent by 2060\(^3\).

Figure 1: Different age groups evolution in the Europop2008 population projection

Börsch-Supan et al. (2006) suggest the analysis of the ageing effects from two perspectives, namely a macroeconomic and a microeconomic point of view. Accordingly, from the macroeconomic perspective, the capital to labor ratio will rise, leading to higher marginal productivity of labor and lower marginal productivity of capital. Thus a rise in wages is implied, simultaneously with a decline in interest rates.

The microeconomic perspective is concerned with agents’ saving and consumption patterns over the life period and is based heavily on the life-cycle theory. Individuals are assumed to accumulate assets while economically active, (especially with a higher savings rate for middle-aged persons) and

\(^3\)Source: Europop2008
afterwards to dissave, or decumulate their wealth in the post-retirement period. By aggregating the individual patterns, this view suggests that countries with a high share of population aged close to retirement will own an abundant capital stock, but this will decline as the demographic transition progresses and the share of individuals at the ending stages of life increases.

The effects of ageing on the fiscal sector are of great importance. A smaller share of active individuals determines falling fiscal revenues, while the government’s health related expenditures will grow with the number of older persons. But the main challenge of the ageing process in the fiscal area is related to the social security system. The total amount of public pension expenditure will rise while the revenues from active individuals will decrease. For this reason pay-as-you-go (PAYG) pension systems are being reformed, by gradually increasing the contribution rate for the private pension system.

Another measure proposed for mitigating the problem of the pension system is the gradual increase in the retirement age. For Romania it was suggested that this should reach 65 years in 2030\(^4\). This appears necessary especially because the ratio of retired to active agents will accelerate around 2030\(^5\). This is a particular demographic feature of Romania, due to the fact that the generations born immediately after 1968 are significantly more numerous than the previous ones. The baby boom was generated by the communist regime through legislative change, namely a decree from 1967 that banned abortion and contraception in order to stimulate population growth.

In order to quantify some of the effects of population ageing on the Romanian economy in the presence of different fiscal policies, I employ an overlapping generations (OLG) general equilibrium model in the Auerbach-Kotlikoff tradition. Although initially developed for assessing the impact of fiscal policy changes, this methodology has proven tractable for the study of demographic transition and became a workhorse in this field.

\(^4\)This measure is stipulated in the current proposed law for pension reform
\(^5\)This is observable in figure 1
The paper draws heavily on Miles (1999), Börsch-Supan et al. (2006), Krueger and Ludwig (2007) and Babecký and Dybczak (2009) more recently, with the emphasis being placed on the case of Romania. Results from the conducted simulations, which have different assumptions about the retirement age and the way in which the social security budget is balanced, are presented over the 2000-2100 horizon. As Babecký and Dybczak (2009) also point out, this type of study is not designed to provide a precise forecast of the macroeconomic development, but rather a set of economic trends that are likely to be induced by the demographic changes.

The remainder of this paper is organized as follows. Section 2 tries to give a description and evaluation of the current knowledge on the study of ageing population using the Auerbach-Kotlikoff OLG model. The theoretical model is described in section 3. Section 4 contains the description of the data used and details of the calibration. The main results and implications for the present analysis are summarised in section 5. Finally, section 6 gives the conclusions.

2 Literature review

The overlapping generations (OLG) framework was originally proposed by Samuelson (1958) and later expanded by Diamond (1965). The seminal work of Auerbach and Kotlikoff (1987) has brought forward a model that is used to study a wide range of economic issues.

When connecting their work with the existing studies, Krueger and Ludwig (2007) identify three strands of literature. The first one refers to assessments of the transition path of the economy after a policy change by making use of large-scale OLG models. In this line of work Altig et al. (2001) analyzed the economic and welfare consequences resulting from fiscal alternatives to U.S. federal income tax. They are able to assess the gains for different income-groups of agents in the five alternatives considered and also quantify the effects of these fiscal shifts on the long-run output.
The second strand of literature used the Auerbach-Kotlikoff model in a multi-country framework. This includes among others Börsch-Supan et al. (2006), Domeij and Flodén (2006) and Krueger and Ludwig (2007). In particular the focus was on world interest rates and capital flows, as the ageing phenomenon affects world regions with different timing.

The third strand (on which this paper also draws) concerns the macroeconomic effects of the ageing population in closed economy frameworks. Nardi et al. (2001) focus on the concept of general equilibrium models in the study of pension reform under demographic ageing. By relying on previous empirical studies, they warn about the calculations outside a general equilibrium framework when quantifying the tax increase needed to finance pensions because of the ageing process; the postponement of the retirement age and the taxation of benefits are suggested in order to alleviate the tax increase.

By putting in use the Auerbach-Kotlikoff framework, Miles (1999) studied the impact of population ageing on the United Kingdom and Europe as a whole. The simulations were conducted for three cases, depending on the form of balancing the social security budget. First a constant replacement rate (pension to wage ratio) is assumed. Then the contribution rate is fixed and finally the replacement rate is considered declining gradually to zero by 2040. The last case corresponds to phasing out the public pension system and the welfare consequences for the young and old generations are discussed. The author concludes that because only the future generations will benefit from such a policy change, the majority alive now don’t have the incentive to support a government who would undertake such a reform.

The author also tries to explain the puzzle of the saving rate for the older generations. The life-cycle theory predicts negative saving rates for the retired agents, while micro survey data contradicts it. He argues that the pension receipts should not be considered income, but a depletion of assets accumulated during active life. This implies smaller saving rates and is consistent with the life-cycle theory. The simulation results are used to
support this hypothesis.

Babecký and Dybczak (2009) provides an assessment for the impact of the demographic ageing on the Czech economy. The source is valuable because it concerns an emerging country with a similar profile to that of Romania. They also follow the approach of two scenarios for balancing the social security budget (constant contribution rate versus constant replacement ratio) and also study the effects of retirement age modification.

A different element in their analysis is the open economy assumption, where they impose as exogenous the interest rate evolution. More specifically, they assume perfect capital mobility which implies that the domestic interest rate is equal to the one on international markets; the results of Miles (1999) for Europe are considered as the external interest rate. The results obtained are consistent with the insights of Miles (1999) and advocate for a constant contribution rate fiscal policy because the impact of population ageing is smaller as compared to the scenario of constant replacement ratio.

Catalan et al. (2008) should also be mentioned in this direction of study, as they focus on the effects of world interest rate evolution on Cyprus as a small open economy. Again they consider the interest rate evolution as exogenous.

3 The model

In this section I present the theoretical foundation of the model employed, the behavior of its sectors and the equilibrium relationships along with a description of the numerical implementation.

Among the advantages of OLG models in studying the ageing population process, Miles (1999) mentions the possibility of undertaking policy simulations. These models have also the property of being theory based and allow the researcher to assess the results of a wide range of individual behavior patterns by varying the parametrisation of the model. However it must be noted that the sensitivity of the model to parameters results in uncertainty
in the simulations, since the values of some parameters are difficult to assess.

From the beginning it should be noted that the emphasis of the model on the long term trends is confirmed by its specification of variables in real terms, without considering monetary developments\(^6\). The sectors of this closed model are the households, the firm and the government, with their details described below.

### 3.1 The households

The agents (or individuals) enter the model at age 21 with no initial wealth endowment and die after \(T + T^R\) periods, without leaving any bequest. All the agents with the same age form a generation (or cohort). Therefore, the total population consists of \(T + T^R\) unequal coexisting generations, of age \(21, 22, \ldots, 21 + T + T^R\). Each household in the economy contains a single agent\(^8\). In the first \(T\) periods the agent works and afterwards he retires for the next \(T^R\) years. The household born in period \(t\) maximizes:

\[
\sum_{s=1}^{T+T^R} \beta^{s-1} U(c^s_{t+s-1}, l^s_{t+s-1})
\]

where the future utility from consumption and leisure is discounted by the subjective factor \(\beta\). According to the adopted notation convention for the variables, the subscript indicates time and the superscript denotes the age of the generation. The duration of a period is normalized to 1 with the time endowment being dedicated to labor \((n^s_{t+s-1})\) and leisure \((l^s_{t+s-1})\). Therefore:

\[
n^s_{t+s-1} + l^s_{t+s-1} = 1
\]

The active agent obtains income from work and from lending capital, leading to the following budget constraint:

\[
k^{s+1}_{t+s} = (1 + r_{t+s-1})k^s_{t+s-1} + (1 - \tau_{t+s-1})w_{t+s-1}\xi^s_{t+s-1}n^s_{t+s-1} - c^s_{t+s-1}
\]

\(^6\)This is consistent with the long run monetary neutrality

\(^7\)The model doesn’t take into account agents younger than 21

\(^8\)I will use these notions interchangeably
The capital $k_{t+s}^{s+1}$ also represents the household’s accumulated wealth, so the notions are interchangeably used. The next period’s wealth results after subtracting consumption from the wage and current wealth income. The effective labor supply of the household is $\xi_s n_{t+s}^s$, and is obtained by multiplying the labor effort $n_{t+s}^s$ with the age-specific efficiency $\xi_s$. It seems natural to assume that the skills and abilities of the representative agent increase during the first part of the working period, reaching a peak and then stagnating or even slowly decaying. To incorporate this stylized fact, I introduce the age-productivity profile $\xi_s$. The income from wage is also adjusted with the taxation rate $\tau_{t+s-1}$ which represents the percentage social security contribution collected by the government.

During the $T^R$ years after retirement the agent receives pension $b$ giving rise to a different budget constraint:

$$k_{t+s}^{s+1} = (1 + r_{t+s-1})k_{t+s-1}^s + b_{t+s-1} - c_{t+s-1}^s \quad (4)$$

The pension amount is independent of wage income or time worked during active life.

### 3.2 The firm

The models assumes a single firm that rents capital and labor from households to produce a single homogenous good according to a Cobb-Douglas production function with labor-augmenting technological progress:

$$Y_t = (A_t N_t)^{1-\alpha} K_t^\alpha \quad (5)$$

For the technological progress the assumption of constant deterministic growth is imposed. Thus, $A_t$ grows in each period with $g_A$. Taking into account the capital depreciation rate $\delta$, firms maximize profits given by:

$$\pi_t = Y_t - r_t K_t - w_t L_t - \delta K_t \quad (6)$$
3.3 The government

The governmental sector is limited to the social security system, being responsible for collecting contributions from the employed households and distributing them (in the form of pensions) to the retired population. Every period, the government maintains its budget balanced:

\[ \tau_t w_t N_t = \sum_{s=T+1}^{T+TR} b_t P_t^s \]  

(7)

where \( P_t^s \) is the \( s \)-year old population. In order to ensure the budgetary balance, the government can adjust either the revenue side, through the social security contribution rate, or the expenditure side, through the value of the pension paid. In the second case, the actual instrument is the replacement ratio, which is a fraction between the pension and the average net wage earning:

\[ \zeta = \frac{b_t}{(1 - \tau_t) w_t \bar{n}_t} \]  

(8)

where the average effective labor supply \( \bar{n} \) is given by:

\[ \bar{n}_t = \sum_{s=1}^{T} n_t^s \xi_s \frac{P_t^s}{\sum_{s=1}^{T} P_t^s} \]  

(9)

3.4 Equilibrium

3.4.1 Competitive equilibrium

In the competitive equilibrium the households are on their optimal path (maximize their future discounted utility), the firm maximizes its profit, the government has a balanced budget, and the labor, capital and goods markets clear.

The household’s optimization problem consist of maximizing discounted sum of future utilities (1) subject to the budget constraints (3) and (4); this can be solved using the method of Langrange multipliers resulting in the
following first order conditions (FOCs):

\[ U_l(c^s_{t+s-1}, l^s_{t+s-1}) = (1 - \tau_t) w_t U_c(c^s_{t+s-1}, l^s_{t+s-1}) \]  \hspace{1cm} (10)

\[ U_c(c^s_{t+s-1}, l^s_{t+s-1}) = \beta (1 + r_t) U_c(c^{s+1}_{t+s}, l^{s+1}_{t+s}) \]  \hspace{1cm} (11)

where \( U_l \) and \( U_c \) denote the marginal utility of leisure and consumption. The first of the two conditions states that the marginal rate of substitution between consumption and leisure is equal to the net wage. This FOC concerns only active agents, since they face the labor-leisure trade-off. The second FOC is the well-known Euler equation that relates intertemporal marginal utility of consumption and affects the decisions of both working and retired agents.

Solving the firm’s optimization problem implies a factor market equilibrium in which factors are paid with their marginal product. The wage and interest rate that ensure the firm is on its maximizing path are given by:

\[ r_t = \frac{\partial Y_t}{\partial K_t} - \delta = \alpha N_t^{1-\alpha} K_t^{\alpha-1} - \delta \]  \hspace{1cm} (12)

\[ w_t = \frac{\partial Y_t}{\partial N_t} = (1 - \alpha) N_t^{-\alpha} K_t^\alpha \]  \hspace{1cm} (13)

The labor market clears so that the aggregate effective labor \( N_t \) that enters the production function is equal to the sum of individual labor supply \( n^s_t \) adjusted with the efficiency \( \xi^s \) and also taking into account the size of the generation \( P^s_t \). The condition can be written as:

\[ N_t = \sum_{s=1}^{T} n^s_t \xi^s P^s_t \]  \hspace{1cm} (14)

The capital market clearing implies that aggregate capital stock is given by the sum of individual wealths:

\[ K_t = \sum_{s=1}^{T} k^s_t P^s_t \]  \hspace{1cm} (15)

\[ Y_t = C_t + I_t = C_t + K_{t+1} - (1 - \delta) K_t \]  \hspace{1cm} (16)
Equation (16) states the goods market clearing condition, where aggregate consumption is given by:

\[ C_t = \sum_{s=1}^{T} c_t^s P_t^s \]

### 3.4.2 Balanced growth path

The balanced growth path is defined as an equilibrium in which variables grow at constant, but possibly different rates. The specification of certain growth rates is based on some "stylized facts" of growth. It was observed that the average growth rates of output, consumption, capital stock over long periods of time are very close. This property is known as "balanced growth". It was also observed that the real wage displays a rising trend similar to that of output while the interest rate doesn’t exhibit a long run trend.

In order for the neoclassical framework to be consistent with these stylized facts and to ensure the existence of the balanced growth path, the model specification must fulfill some conditions. King et al. (1988) among others show that the technological progress must enter the production function in a labor-augmenting fashion, the utility function must have constant elasticity of intertemporal substitution and the marginal rate of substitution between consumption and leisure needs to be homogeneous of degree one in consumption.

Consistent with the stylized facts of growth, in the balanced growth path, output per capita, aggregate capital per capita and aggregate consumption per capita grow at \( g_A \). Also individual variables (the wage, the individual wealth, the pension and individual consumption) grow at the growth rate of technological progress.

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9Gali (2005)

10See also Heer and Maussner (2009), p. 67
3.5 The stationary transformation

Usually, it is the evolution of variables without the influence of an exogenous growth factor, such as the technological progress, that is of interest. Therefore, it is convenient to express the model in terms of stationary variables. The stationarization of the model implies removing the deterministic time trend induced by the technological progress. Also aggregate variables will be expressed in per capita terms. Therefore the aggregate variables are divided by $A_tP_t$ or $P_t$, where $P_t$ is total population in period $t$:

$$\tilde{Y}_t = \frac{Y_t}{A_tP_t}, \tilde{C}_t = \frac{C_t}{A_tP_t}, \tilde{K}_t = \frac{K_t}{A_tP_t}, \tilde{N}_t = \frac{N_t}{P_t}$$

and the individual variables are divided by $A_t$:

$$\tilde{c}_t = \frac{c_t}{A_t}, \tilde{k}_t = \frac{K_t}{A_t}, \tilde{w}_t = \frac{w_t}{A_t}, \tilde{b}_t = \frac{b_t}{A_t}$$

It is important to note that the stationary-transformed variables represent deviations from original variables’ long term trend. The balanced growth path can now be expressed as a steady state in de-trended variables\(^{11}\). The production function (5) becomes:

$$\tilde{Y}_t = \tilde{N}_t^{1-\alpha} \tilde{K}_t^\alpha$$

(17)

Plugging relation (8) into (7) and using the above notations, the government’s balanced budget condition becomes:

$$\tau_t \tilde{w}_t \tilde{N}_t = \tilde{b}_t \sum_{s=T+1}^{T+TR} \frac{P_t^s}{P_t} = \zeta(1 - \tau_t) \tilde{w}_t \tilde{N}_t \sum_{s=T+1}^{T+TR} \frac{P_t^s}{P_t}$$

(18)

Also, the budget constraints (3) and (4) will be written in the following way:

$$\tilde{k}_{t+s}^{s+1}(1 + g_A) = (1 + r_{t+s-1})\tilde{k}_{t+s-1}^s + (1 - \tau_{t+s-1})\tilde{w}_{t+s-1}\xi^n_{t+s-1} - \tilde{c}_{t+s-1}^s$$

(19)

$$\tilde{k}_{t+s}(1 + g_A) = (1 + r_{t+s-1})\tilde{k}_{t+s-1} + \tilde{b}_{t+s-1} - \tilde{c}_{t+s-1}^s$$

(20)

\(^{11}\)As Barro and Sala-i Martin (2003) (p. 34) note, the concept of balanced growth path refers to the state in which all variables grow at constant rates while the concept of steady state is used for the particular case when the growth rate is zero.
while the market clearing condition (16) becomes:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{K}_{t+1}(1 + g_A)\frac{P_{t+1}}{P_t} - (1 - \delta)\tilde{K}_t$$

(21)

Using the same notations, the first order conditions for the household’s optimization problem are transformed in the following way:

$$U_l(\tilde{c}_t^*, l_t^*) = (1 - \tau_t)\tilde{w}_t U_c(\tilde{c}_t^*, l_t^*)$$

(22)

$$U_c(\tilde{c}_t^*, l_t^*)(1 + g_A) = \beta(1 + r_t)U_c(\tilde{c}_t^{*+1}, l_t^{*+1})$$

(23)

while the prices of the production factors are given by:

$$r_t = \frac{\partial \tilde{Y}_t}{\partial \tilde{K}_t} - \delta = \alpha \tilde{N}_t^{1-\alpha} \tilde{K}_t^{\alpha-1} - \delta$$

(24)

$$\tilde{w}_t = \frac{\partial \tilde{Y}_t}{\partial \tilde{N}_t} = (1 - \alpha)\tilde{N}_t^{-\alpha} \tilde{K}_t^\alpha$$

(25)

3.6 Numerical implementation

In their seminal work Auerbach and Kotlikoff (1987) tried to assess the reaction of the economy following a fiscal shock. They assumed the economy was in a steady state when the shock occurred and that it moved to a new steady state afterwards. The period of interest is between the two steady states and is referred to as transition period because the economy moves from one steady state to another. In our case, the shock whose effects are studied is of a demographic nature.

The implementation requires historical and projected demographic data for the transition period for every age group between 21 and $T + T^R$; the detailed description of the data is given in the next section. The numerical procedure starts by providing initial guesses for the trajectories of the aggregate capital stock per capita and aggregate labor supply per capita for the selected period. These trajectories allow me to compute (from equations (24) and (25)) time series for the interest rate and wage. For a given value of the replacement ratio, I am also able to calculate the time series for
the contribution rate $\tau_t$ by making use of the balanced budget condition (18). Consequently, as the sequences of wages, interest rates and contribution rates that every generation will face are known, by solving the system of FOCs I can obtain the optimal individual labor supply, consumption and wealth for every generation. After computing these optimal individual variables, I aggregate them across generations and compare the aggregate capital stock and aggregate labor supply with the initial guesses. If they are far from the starting values, I update the initial guess and repeat the procedure.

A special attention needs to be given to the optimal sequences of wealth, labor supply, and consumption for the generations that are born before the beginning of the transition period or die after the end of the transition period. This is because when aggregating the capital stock and labor supply we need the optimal allocations of these cohorts.

If the economy is in a steady state at the beginning of the transition period, then the optimal allocations of the agents alive at this moment are known. Analogously, for the agents alive after the end of the transition period, their optimal allocations correspond to the values of the aggregate variables in the new steady state. The first period of the new steady state is chosen so that the economy has enough time to converge, in our case I chose 2149. But the assumption of an initial steady state in 2010 is not very realistic. I overcome this inconvenience by taking Krueger and Ludwig (2007), among others approach. I assume an initial steady state farther in the past, in 1950, so that in 2010 all the agents alive are born after 1950\footnote{I will assume the agents enter the model at age 21 and live to a maximum age of 80; the details are presented in section 4}. This way the allocations in 2010 are not affected by the decisions made before 1950.

Heer and Maussner (2009) describe the procedure in detail. They also provide a set of GAUSS codes for this type of models, which I used after modifying and adapting them for the features of my application.
4 Data and calibration

4.1 Data

The demographic evolution is taken exogenous and can be considered the driving force of the model. Specifically, the demographic transition influences the model through the evolution of the ratio of the population aged $21 + T + 1$ years or over to the population aged 21 or over (which is the total population in the model). This ratio, which is also the share of retirees in total population, affects the model outcomes through the balance between government revenues and expenditures\textsuperscript{13}.

I collected historical demographic data for Romania for the period 1968-2009 from Eurostat. In order to backcast the series to 1950, I used data from the United Nations which provides values for every five years (1950, 1955 . . . ) and interpolated them. From the data obtained after interpolation, I calculate the growth rates for every generation between 1950 and 1967 and apply them backward from the last available data from Eurostat, recursively generating values until 1950.

Eurostat also provides the Europop2008 population projection which goes as far as 2061. By analyzing the projection, it is noticeable that the ratio of the population aged 61 years or over to the population aged 21 or over appears to stabilize at the end of the period (see figure 1). Given this evolution I assumed the share of individuals aged 61 years or over in total population remains constant at 48%, the projected value corresponding to year 2061, for the next 88 years, until 2149. To transpose this assumption in the detailed data, I set the sizes of all cohorts for the period 2062-2149 equal to the sizes of cohorts in 2061 (in other words the population structure remains fixed after 2061)\textsuperscript{14}. Thus, the final data set contains historical data or forecasts

\textsuperscript{13}See for example equation (18)

\textsuperscript{14}This assumption is not too distorting, since the model is influenced only by the share
4.2 Calibration

Given the lack of studies for Romania, the parameters of the model had to be calibrated with values documented in the international literature. This is a serious drawback especially because the model is very sensitive to some parameters, which leads to uncertainty in the simulations’ outcomes. In most cases, the values used by Heer and Maussner (2009) were kept, with replacement made when possible.

The maximum age was set to 80 years, in accordance to other studies like Babecký and Dybczak (2009), Miles (1999) and also because of the data availability. However, it should be noted that agents older than 80 years are also taken into consideration, since the demographic projection includes them\textsuperscript{15}. Retirement age in the baseline scenario is considered 60 years old, consistent with current legislation\textsuperscript{16}. Thus, since agents enter the model at 21 years old, in the baseline simulations $T$ is set to 40 and $T^R$ to 20. Following Heer and Maussner (2009), utility takes a CRRA form:

$$U(c, l) = \left(\frac{cl^\gamma}{1-\eta}\right)^{1-\eta} - 1$$

(26)

The parameter $\gamma$ governs the preference for leisure or labor and is set to 2, while the of relative risk-aversion $\eta$ is also considered equal to 2.

The subjective discount factor $\beta$ indicates the preference of current utility over future utility and is set to 0.99, in accordance with the literature. The contribution rate $\tau$ is set to 0.105, consistent with the current legislation. The capital share $\alpha$ in the production function is considered 0.3, while the depreciation rate $\delta$ is set to 0.06, consistent with the real business cycle literature. Following Babecký and Dybczak (2009), a value of 2% per year is of retirees in total population, and the baseline scenario assumes 60 as the retirement age

\textsuperscript{15}The last age group in the Europop2008 projection consists of individuals aged 80 years or over

\textsuperscript{16}At present, the retirement age is above 60 for men, and below for women
associated to $g_A$. For the simulations in which the contribution rate adjusts in order to insure a balanced budget, the replacement ratio is set to 0.5, which is approximately the ratio between the average pension and the net average wage. In the absence of data for Romania, the age productivity profile is assumed to be quadratic following Miles (1999) and to be given by $\xi_s = 0.03s + 0.0006s^2$. As shown in figure 2 it is hump-shaped and implies that maximum efficiency is reached at around 50 years.

![Figure 2: Age specific efficiency](image)

5 Results

This section presents the details of the conducted simulations and discusses the results. As mentioned in subsection 3.3, the budgetary balance can be ensured with the help of two instruments: the contribution rate $\tau$ or the replacement ratio $\zeta$ (which in turn affects the value of the pension). I will consider the extreme cases in which only one of the two instruments adjusts in order to balance the social security budget, while the other is assumed constant. This approach has also been adopted by Börsch-Supan et al. (2006)
and Babecký and Dybczak (2009). They suggest that the scenario in which the replacement ratio is held constant bears strong resemblance to the old-fashioned pay-as-you-go public pension system, in which the efforts to finance the pensions become more and more pronounced.

Because of the rising ratio of retired to employees, the scenario with constant contribution rate is expected to generate a decline in the pension, which will provide an incentive for higher private savings. The gradual decumulation of these private savings after retirement can be viewed as a private pension income. Given that the agents are aware of the future decline in the public pension, the situation has similarities with the transition to a partially funded pension system, with a second or third pillar, just as it is happening in Romania.

The conducted simulations try to quantify also the results of modifying the retirement age, which is 60 years in the baseline scenario. The current pension reform legislation stipulates the gradual increase in the retirement age so that it reaches 65 years in 2030. Therefore I assumed a scenario in which the retirement age increases with a year at every four years starting with 2010\textsuperscript{17}.

Intuitively, if the replacement ratio is fixed and the number of retired persons grows as the active population drops, an increase in the contribution rate is expected in order for the budget to be balanced. Analogously, in the case of a constant contribution rate, the government revenues will decline with the labor force resulting in a smaller benefit for the retired. This intuition is confirmed by the expressions for the contribution rate $\tau_t$ and for the replacement ratio $\zeta_t$, which result after some manipulation of the government’s budget balancing condition (18):

$$
\tau_t = \frac{\zeta \rho_t}{\zeta \rho_t + 1 - \rho_t}
$$

\textsuperscript{17}In the legislation the increase is very smooth, because it uses the month as time unit, while the period in the model is equal to one year, which leads to small jumps in the results.
\[ \zeta_t = \frac{\tau}{1 - \frac{1 - \rho_t}{\rho_t}} \]

(28)

where \( \rho_t \) denotes the ratio of retired agents to total population:

\[ \rho_t = \frac{\sum_{s=T+1}^{T+T_R} P_t^s}{\sum_{s=1}^{T+T_R} P_t^s} \]

From equations (27) and (28) it is easily verifiable that an increase in \( \rho_t \) determines an increase in \( \tau_t \) and a decrease in \( \zeta_t \) respectively, which confirms our intuition.

Next the results of the conducted simulations are presented. The figures on the left side of the page correspond to the scenario in which the replacement ratio is held constant at the value of 50% while on the right side of the page there are the figures for the constant contribution rate scenario. The dashed line illustrates the outcomes for the policy in which the retirement age gradually increased up to 65 years; the solid line is for a constant retirement age of 60 years. The first figures describe the individual behaviour for an agent that belongs to the generation that enters the labor force in 2010, and then the aggregate variables’ path is depicted for the 2000-2100 horizon.

Because the model has been transformed to a stationary form, when interpreting the results we must bear in mind that the variables (except for the interest rate and the individual labor supply) are productivity de-trended and that the aggregate variables are also expressed in per capita terms. The productivity de-trended variables can be thought of as deviations from their deterministic trend.

Figures 3 and 4 depict the optimal capital holdings for an agent that belongs to the generation that enters the workforce in 2010. Consistent with the life cycle theory, the household accumulates wealth during the working period in order to finance expenditures after retirement. The ability to capture this behavior of the household is one of the advantages of the model.
As it can be seen, the two policies for balancing the budget provide similar results, although in the case of constant contribution rate the agent is encouraged to save more because he expects a smaller pension benefit after retirement.

Figure 3: Age wealth profile for 2010 labor force entrant - constant replacement ratio

Figure 4: Age wealth profile for 2010 labor force entrant - constant contribution rate

Extending the retirement age (solid line) has somewhat different impact in the two scenarios: the individual saves more in the constant replacement scenario and less in the constant contribution rate scenario. For this latter scenario, figures 8 and 6 provide a completer picture: the individual works harder to accumulate more in order to compensate the total income drop through a higher capital income. The agent works harder in the fixed retirement age case as compared to increased retirement age case also because he will also have to rely on pension for a longer period.

The agent’s wealth is negative in the first periods of life, because he is forced to borrow as he earns a low wage (as unskilled) and has no capital income. This finding was reported also by Miles (1999).

When analyzing the optimal individual labor supply (fraction of time dedicated to work) for the 2010 workforce entrant, shown in figures 5 and 6, it is noticed that the agent chooses to work less when the relative benefit during retirement is constant, which is also because taxation is expected to be higher in this scenario. Even if he saves less and his capital income is
smaller than in the constant contribution scenario, his total income after retirement is still higher due to the generous pension. The pattern of the individual labor supply follows the evolution of the age-efficiency profile, i.e. agents work more when they are the most productive.

Due to the decrease of the share of working age individuals, the aggregate employment (shown in figures 9 and 10) is constantly falling in line with the demographic transition. Consistent with the figures 5 and 6, the policy of constant contribution rate gives rise to a higher aggregate supply of labor.
per capita. Extending the retirement age appears to have only a scale effect upon labor supply on the aggregate level, as the lines seem translated upward. The retirement of the baby boom cohorts is easily observed around 2030 as afterwards the decline in the labor supply is significantly faster; this shock is not eliminated by increasing the retirement age, but only postponed.

As a result of the fall in the share of the cohorts with the highest capital holdings in favor of older ones with lower levels of wealth (more than 70 years old) the productivity de-trended aggregate capital stock per capita (figures 11 and 12) will experience a sharp decline after 2030-2040. In the fixed replacement ratio scenario, the fall is accelerated by the rising contribution rate which diminished the net wages.

Also, the aggregate capital stock in the case of constant retirement age (dotted line) is higher than in the case of extending retirement age in the fixed replacement ratio scenario and lower in the fixed contribution rate scenario. This finding is consistent with the individual behavior (wealth profiles for the 2010 labor force entrant) and has been documented also by Babecký and Dybczak (2009).

The evolution of the interest rate and wage is determined by the trajectory of the capital to labor ratio, according to equations (24) and (25). For both scenarios, the aggregate labor supply per capita decreases in the first part of time horizon and then stabilizes, while the de-trended aggregate capital stock...
stock per capita increases until 2030-2040 and then experiences a decline. Consequently, the interest rate falls in both policy set-ups in the first part of the selected period, while afterwards rebounds in one scenario and stabilizes in the other. In the constant replacement ratio case, the decline in the capital stock in the second part of the period combined with the steady evolution in the labor supply leads to the capital to labor ratio to diminish, resulting in a rising interest rate. The interest rate’s projected evolution is depicted in figures 13 and 14.

Since the wage rate depends inversely to the capital to labor ratio relative to the interest rate, the evolution of the factor prices $\tilde{w}$ and $r$ are symmetrical, with the productivity de-trended wage depicted in figures 15 and 16.
The evolutions of the contribution rate $\tau$ and replacement ratio $\zeta$, shown in figures 17 and 18, are consistent with the demographic transition and the development of the above discussed variables. It is important to note the advantages of extending the retirement age, as the contribution rate is smaller and the replacement ratio is above the values for the fixed retirement age case. The figures also show how the the baby boom generations put pressure on the pension system when they reach the retirement age.

In both budget balancing alternatives, the productivity de-trended pension is expected to decline. In the constant replacement ratio scenario, the decrease is induced by the smaller wage and by the drop in working population. However, the decline is less pronounced than in the constant contribution rate scenario. In both cases, extending the working life period alleviates the problem of smaller income after retirement.

Overall, the magnitude of the impact from increasing the retirement age is different in the two alternative scenarios. When considering adjusting the contribution rate to balance the budget, extending the retirement age has a small impact, especially on the individual variables.

Therefore, in advocating for the constant contribution rate scenario, which corresponds to a transition toward a partially funded multi-pillar pension system, the advantage of better response to varying the retirement age can be taken into account. However, the main strengths of this scenario are given
by the fact that it encourages labor and saving; the evolutions of the aggregate capital stock labor indicate that also the production will be higher in this scenario as compared to the alternative with constant replacement ratio. Having in mind that this approach maintains a constant taxation level, while the other guarantees a higher social security benefit through rising the contribution rate, it could be interpreted that this alternative is closer to a liberal right-wing economic view; analogously, the approach where the replacement ratio is constant would correspond to a socialist left-wing economic view.
6 Conclusions

In the coming decades Romania will experience a significant population ageing process. Using an overlapping generations model in the spirit of Auerbach and Kotlikoff (1987), this paper attempted to assess some of the implications of the demographic change for the Romanian economy.

Following an approach common in the literature, the simulations conducted assumed two extreme cases of balancing the social security budget in the face of a growing number of retirees: either by raising the contribution rate in order to maintain the relative benefit after retirement, either by ensuring a constant contribution rate at the cost of smaller pension income. Also the extension of the retirement age, as proposed in the current legislation for pension system reform, was taken into account. In both alternatives for balancing the budget, the rising share of retirees in total population significantly affects the simulation outcomes. The demographic evolution which is considered exogenous and based on the population projection from Eurostat, is the driving force of the model.

The individual behavior findings confirm that agents accumulate wealth during active life in order to dissave after retirement, a pattern also suggested by the life-cycle theory. In the constant replacement ratio scenario, the relative size of the benefit after retirement is guaranteed, which stimulates a low labor supply (given also the high taxation); together with the increase of the retired generations as share in total population, this leads to a fall in the aggregate labor supply per capita. The aggregate (productivity detrended) capital stock also experiences a decline after 2030-2040, when the older cohorts (with low wealth) rise as a share of total population and the contribution rate determines lower net wages. Extending the retirement age appears to alleviate only marginally and to postpone for a few years shortly the fall in the pension and the rise in the contribution rate. All the results are significantly affected by the retirement of the baby boom generations around 2030, when the ratio of retired to active individuals accelerates.

Maintaining a fixed contribution rate stimulates agents to work and save
more, as they are aware of the low level of pension income after retirement. As a result the aggregate employment and capital stock decline less than in the fixed replacement ratio scenario. As dictated by the evolution of the capital to labor ratio, the interest rate declines and the wage increases in a symmetrical manner. Similar to Babecký and Dybczak (2009) finding, I can conclude that the impact of ageing is more pronounced when the government guarantees a relative benefit after retirement than in the constant contribution rate scenario. This idea is in accordance with the proposed pension system reforms through shifting toward funded multi-pillar systems.¹⁸

The main criticism of the Auerbach-Kotlikoff model is related to the strong assumption of the closed economy framework (in the base version of the model). The solutions to overcome this are the adoption of a multi-country framework (which is not suited my case), as in Krueger and Ludwig (2007) for instance, or, when studying a single economy, exogenously imposing the interest rate future development, as in Babecký and Dybczak (2009) or Catalan et al. (2008). The problem with the second solution is that providing an exogenous path for the interest rate could be an assumption strong as the closed economy framework hypothesis.

Another important analysis we have abstracted from in this paper refers to the study of welfare consequences of the two main scenarios taken into consideration. Assessing which generations benefit more from a policy change is crucial in understanding the trade-off faced when designing and implementing a pension system reform.

¹⁸The idea of a system with a publicly managed mandatory pillar, a private managed mandatory pillar and a private managed voluntary pillar was proposed in the influential Averting the Old Age Crisis Policy Research Report from The World Bank
References


