Inflation contracts, targets and strategic incentives for delegation in international monetary policy games*

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ABSTRACT

In this paper we study delegation by inflation targets and contracts as a mechanism of implementing the cooperative optimum in international monetary policy games. First, we prove that state-contingent inflation targets and contracts are equivalent in this framework and show how they can be designed to implement the collusive outcome. Then we study the strategic incentives governments have to delegate with the optimal contracts and targets. Regarding the game as a two-stage one we solve for the Subgame Perfect Nash Equilibrium in the delegation game in terms of contracts. We do this in two models, one with policy spillovers and one with policy spillovers and an inflation bias. We find that the perfect equilibrium is different, more specifically inefficient, when compared to the optimal contracts. Only for *state-independent contracts* will the solutions be similar but this only eliminates the inflation bias without affecting shock stabilisation.

Our results suggest that for the cooperative (and ex-ante commitment) optimum to be implemented, cooperation between countries or some form of coordination from a supranational authority is needed. However, this implies that the delegation solution merely relocates the problem from the policy rules choosing to the delegation stage. Implementation of the collusive outcome is thus not non-cooperative, as argued by Persson and Tabellini (1995, 1996). Nevertheless, domestic commitment to own agents and coordination at the delegation stage seem more plausible than binding agreements between countries over the policy outcomes in real world situations.

Content:

1. Introduction

In this paper we study delegation by inflation targets and contracts as an (allegedly[1]) non-cooperative solution to implement the cooperative outcome in international monetary policy games. Persson and Tabellini (1995, 1996) show how delegation by inflation contracts can achieve the same solution as a cooperative (and ex-ante commitment) regime and argue that this can be viewed as a non-cooperative mechanism of implementing the cooperative outcome. However, their result is hard to reconcile with the idea of implementing the collusive outcome *non-cooperatively*. We show that when playing Nash at both stages (delegation and policy rules), the governments *do not have the right incentives* to delegate by the *optimal contracts* (or targets). For the cooperative equilibrium to be implemented *cooperation* (or *coordination* from a supra-national principal) at the delegation stage is needed. This is an implicit assumption in Persson and Tabellini and their solution just relocates the problem from the *monetary policy stage* (whereby authorities choose policy rules) to the *monetary policy regimes* choosing stage (where governments choose to delegate to a central bank by contracts or targets). This is somehow reminiscent of McCallum's (1985) critique regarding solution to dynamic inconsistency in a domestic policy context. However, the problem is different in the international policy games context where there are different (or additional) incentives and we try to briefly overview it below.

While in the literature on strategic interaction in monetary policy it has been argued since the early work of Hamada (1976) that cooperation is Pareto efficient it is also well known that enforcing the cooperative outcome is unlikely. Three main motives are usually listed for this (Persson and

[1] Please find qualification below

Tabellini 1995). The first is that cooperation is counterproductive in the absence of domestic commitment with respect to the private sector (the 'Rogoff'-1985b-problem). Secondly, there are issues related to uncertainty regarding initial positions, loss functions or parameters of the model. We do not deal with these issues here and an exhaustive exposition of them can be found in Ghosh and Masson (1994). The third problem concerns the countries' individual incentives to deviate form the cooperative equilibrium.

Two main solutions to this last problem have been recently proposed. Firstly, it has been argued by Canzoneri and Henderson (1991, chs. 4-5) or Ghosh and Mason (1994, ch.8) that if the game between policymakers is repeated over time reputational mechanisms relax these incentives. The main idea, consisting of the application of the Folk Theorem of repeated games, is well known, as well as its major drawbacks (e.g., the lack of predictive power due to multiple equilibria), hence we do not deal with it here.

The second mechanism to decentralise the cooperative outcome refers to institutional design and has been recently proposed by Persson and Tabellini (1995, 1996). The solution consists mainly in delegating monetary policy to an independent Central Bank and imposing a linear inflation contract and is reminiscent of the microeconomic literature on contracts and principal-agent relations. Such applications already exist for solutions to the dynamic inconsistency problem in the domestic policy context by Walsh (1995) and Persson and Tabellini (1993).

The idea is an application of the Folk Theorem in delegation games of Fershtman, Judd and Kalai (hereinafter FJK, 1991). They conclude that 'cooperation outcomes emerge as equilibria in the game with delegation if the principal is fully committed to the contract with the agent and the contracts are fully observed' (p. 553) and implementation can be done by target compensation functions.

In this paper we try to bring new insights to the literature on institutional design in two directions. Firstly, based on the ideas of Svensson (1997) in a closed economy context, we show that delegation to a Central Bank with a non-zero inflation target can achieve the same (second-best) outcome as delegation by a contract.

Secondly, the Folk Theorem in delegation games indeed states that the cooperative outcomes can emerge as equilibria in the game with delegation. This seems plausible in the European Union context concerning the policy arrangements between the ins and the outs of the EMU (studied by Persson and Tabellini 1996) where the European authority can act as an international principal. However, there is nothing to insure that in the absence of a benevolent international principal – which seems more plausible in a more general policy cooperation exercise - the contracts (or targets) by which the governments will choose to delegate will be the optimal (cooperative) ones. To study this problem we evaluate the contracting incentives of the individual governments when they play Nash (non-cooperatively). We will thus try to compare the subgame perfect equilibrium penalties and targets with the ones that the international planner would like to design to implement the cooperative optimum.

We do this in two different models, similar to the reduced forms in Canzoneri and Henderson (1991) or Rogoff (1985). In the first one, an adaptation of Dolado, Griffiths and Padilla (1994) there is no domestic credibility problem, there are policy spillovers between countries and cooperation is ex post Pareto optimal. The second one is a version of Persson and Tabellini (1995, 1996) and it has, apart from policy spillovers through the real exchange rate, a domestic inflation bias. This bias, as pointed by Rogoff (1985b), makes cooperation not being ex post Pareto optimal in the absence of commitment with respect to the private sector.

In both models we find that the subgame perfect equilibrium contracts (and targets) are different from the ones that implement the cooperative, respectively cooperative and commitment, optimum. Thus, when playing Nash, the countries wouldn't have the right incentives to delegate monetary policy by those contracts or targets that implement the cooperative optimum. For the collusive outcome to be actually implemented, cooperation or coordination should be enforced at the delegation stage. Thus, the Persson and Tabellini solution moves the problem one step backward in the timing of the game: cooperation (or coordination) should take place when countries choose the policy regimes and not when fixing policy rules. One may question the ability of governments to cooperate at this early stage given their inability to do it in the monetary policy game. However, this type of arrangement may seem more plausible if one thinks about an international principal trying to coordinate the policy regimes and not the policy rules. Moreover, commitment with respect to agents seems more plausible than commitment with respect to the other country as there are instances in which the latter reduces the countries' welfare.

In the rest of the paper we will proceed as follows: chapter 2 presents a brief review of the literature and chapter 3 shows the equivalence of contracts and non-zero inflation targets. In chapter 4 and 5 we study the contracting incentives - first in the model with policy spillovers, then we add a domestic inflation bias. Chapter 6 concludes, while some of the derivations and proofs can be found in the appendices.

Chapter 2.

Review of literature

In what follows we will use the term *policy cooperation* to refer to the situation in which countries jointly optimise an aggregate measure of their welfare. By *policy coordination* we will mean the situation in which countries decide on choosing one among a multiplicity of Nash equilibria (so welfare maximisation is done individually, non-cooperatively). We will thus follow the terminology of Canzoneri and Henderson (1991), which is slightly different from other uses of the terms in the literature (e.g. Ghosh and Masson, 1994, Nolan and Schaling, 1996).

Crucial to gains from cooperation is the existence of policy spillovers, thus the transmission of domestic policies through linkages like trade flows, capital movements and the exchange rate. A theoretical exposition as well as an empirical survey of the international transmission of policies may be found in Ghosh and Masson (1994) and does not constitute an objective of this paper.

2.1 Monetary policy spillovers and cooperation

The modern literature on macroeconomic policy cooperation and coordination can be traced to Cooper (1969). He argued that, given the interdependence of economies a lack of policy coordination is costly as it makes national objectives more difficult to attain. If governments assigned their policy instruments to respond to both domestic and foreign targets the world economy would return to equilibrium after a shock more quickly than under individual optimisation. However, his model had little to say about the way spillovers affect different economies and optimal policy responses.

Hamada (1976) provides the first analytical framework to analyse the policy cooperation using game-theoretical tools, giving definitions of the cooperative and coordination regimes.

Throughout the literature, no matter which is the initial model for the world economy, externalities appear due to the presence in each policymaker's loss function of the money supply of the other country. Generally, non-cooperative behaviour in the presence of externalities (either positive or negative) as a result of a shock leads to non-Pareto optimal outcomes. The nature of the bias (contractionary or expansionary) depends on the sign of externalities (negative or positive) or the nature of the shocks that make the policies be strategic complements or substitutes. There is however a special case studied by Canzoneri and Gray (1985) in which non-cooperation is Pareto optimal, that is following a supply shock in a symmetric model in which one of the policymakers acting as a fixed-exchange-rate leader. Additionally, a form of Stackelberg leadership can be shown to be welfare improving when compared to Nash playing (as studied by Canzoneri and Henderson, 1991). There are however dissatisfactions related to the Stackelberg (and the fixed exchange rate) equilibrium in one shot games. First, it requires commitment by the leader and it is not clear why policymakers would commit. Moreover, there is no clear answer concerning the positions of the two players (i.e. who will be the leader and who the follower). These results, however, are derived in one-shot games, where it is common to find that cooperation is optimal.

2.2 Challenges for the optimality of monetary policy cooperation

The ex post Pareto optimality of policy cooperation has been challenged by various researchers. One of the most prominent critiques is that of Roggoff

(1985b), who showed that policy cooperation between countries may be counterproductive if there are domestic credibility problems. He augments a two-country model with a Barro-Gordon (1983) dynamic inconsistency problem and shows that the cooperative outcome reduces the welfare of the countries compared to the non-cooperative one when commitment with respect to the private sector is infeasible. Miller and Salmon (1985) obtain a similar result for a dynamic version of the model, providing also a numerical example.

Oudiz and Sachs (1985) report another such situation, in which there is no inflation bias but policymakers would like to be able to commit with respect to the private sector concerning the future exchange rate path. Their result is that commitment with respect to the private sector may be counterproductive when commitment between the two policymakers is impossible.

A third 'paradoxical' result is obtained by Canzoneri and Henderson (1991) in a three-country model, in which cooperation between two countries (Germany and France, in their example) and Nash playing with respect to the third (USA) is worse than the Nash-Nash equilibrium (no cooperation at all).

All the three above results can be interpreted in terms of coalitions: coalitions among a subset of players may be counterproductive if commitment with respect to the other players is unfeasible[2]. Thus, the above results appear as just an example of a more general result in game theory.

Kohler (1999) has extended the study of coalitions to an n-country monetary policy game. Her main conclusion is that in an n-country set-up, the size of a stable coalition will be less than n countries due to free-riding incentives.

These incentives refer to the opportunity an outsider has to export inflation due to the discipline of the union. A large union cannot be sustained due to the too high discipline it would impose. A second best solution is found to consist of the co-existence of several smaller coalitions.

Jensen (1997) has recently challenged Rogoff's (1985b) result showing that if one of his assumptions is changed, i.e. if wage-setters are considered to be *non-atomistic and inflation-averse*, monetary policy cooperation may not be counterproductive.

Moreover, Canzoneri and Henderson (1991) make another observation regarding this type of results. They argue that whether cooperation can be counterproductive depends on the commitment technology used to form a coalition. If this technology does not allow for commitment with respect to third parties, examples of counterproductive cooperation are easily obtained. However, these are incomplete until the commitment technology is fully specified. If the technology allows commitment with respect to third parties, the coalition may act as a Stackelberg leader and cooperation can never be counterproductive as the coalition could always choose to play the old Nash policies. The authors see these examples as arguments for the absence of some coalitions in practice, as policymakers would not employ a technology that only serves to lower their welfare.

2.3 Mechanisms to implement the cooperative outcome

The cooperative outcome can be achieved through either a commitment technology or a non-cooperative game mechanism. Assuming commitments are unfeasible, the literature focused on studying the latter mechanisms.

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[2] This result is obtained in other economic contexts as well, e.g. in Alesina and Tabellini (199) where commitment between the monetary and fiscal authority is counterproductive due to lack of commitment to the private sector.

One of the means to enforce cooperation concerns *reputation and trigger mechanisms* in a repeated game, considering that policymakers interact repeatedly over time. A second class of solutions on which we will focus further on consists of implementation by delegation mechanisms or institutional design.

The first stream is extensively discussed in Canzoneri and Henderson (1991, chs. 4-5) and the arguments rely on the folk theorem in repeated games. If the game is repeated, the incentives to cheat are diminished since the governments realise that if they cheat they will not be trusted again. Thus, there appears a trade-off between the short-term utility gain and the present value of the welfare loss during the (arguably finite-horizon) punishment period. If the instantaneous utility gain is not too high, the discount rate not too high and the punishment period sufficiently long, cooperation may be sustained by rational governments with no external mechanisms. Alternatively, if the money supplies are not observed but the inflation rates are, governments could agree on a *trigger strategy*, the punishment period being triggered whenever a deviation of the inflation rate in either of the countries is observed, where the choice of the appropriate trigger level is essential.

There are some severe drawbacks of this type of arguments. First of all, it lacks predictive power due to multiple equilibria (Persson and Tabellini 1995). Secondly, for a trigger strategy to work, the policymakers' horizon should be either infinite or the final period should be unknown (Canzoneri and Henderson, 1991). Another requirement for the trigger strategy to work is that the threat of triggering the non-cooperative equilibrium in case of cheating be credible. Thus, a commitment mechanism is required to ensure the non-cooperative regime will indeed be triggered when one of the policymakers has cheated, since cooperation would still be optimal.

The second mechanism to enforce the cooperative outcome is based on institutional design and was pioneered by Persson and Tabellini (1995, 1996). The institutions could be either domestic or international. Fixed exchange rate arrangements, like the Bretton Woods agreement or the EMS for example, could be regarded as such arrangements. The institutional solution to the problem may be interpreted as strategic delegation by contracts implementing the desirable outcome, in a manner resembling the microeconomic literature on contracts and principal-agent problems.

The main idea starts from the result of Fershtman, Judd and Kalai (1991), stated in a Folk Theorem for delegation games. They show that the cooperative outcome can be implemented in a decentralised manner in the game with delegation, subject to the condition that each principal is fully committed to its agent and once signed, contracts become public information. Implementation can be done by target compensation functions[3].

Persson and Tabellini (1995) apply this result to show how *domestic* institutions can be designed by delegating monetary policy to achieve the cooperative optimum and how various existing policy arrangements can be reinterpreted in this framework. Moreover, they show how *international* institutions (e.g., multilateral pegs or monetary unions) can be designed to achieve the same cooperative optimum. Even in the presence of domestic credibility problems (the 'Rogoff problem') they show that optimal contracts

[3] Polo and Tedeschi (1994) derive a more general folk theorem for delegation games, showing that 'all the individually rational allocations are implementable as subgame perfect equilibrium outcomes with differentiable contracts'

can be designed to implement the cooperative and ex-ante commitment optimum.

Persson and Tabellini (1996) apply these general results to a more specific problem, i.e. optimal monetary policy arrangements for the countries inside and outside EMU (where all the countries are however members of the EU). After deriving the same result concerning the design of optimal contracts to eliminate both incentives (inflation bias and competitive devaluation due to spillovers) they appreciate that contracts can be difficult to implement. Therefore, they move to analysing simpler policy regimes, consisting of inflation targeting, monetary targeting and an EMS-like regime in which the outside countries peg their exchange rates to the Euro. They conclude that the desirable policy arrangement consists of a symmetric regime in which all the countries choose to adopt a *zero inflation target*, solution that '*approximates* an optimal policy of international cooperation' (PT 1995), thus it still achieves a third-best equilibrium.

One of the things we show in this paper is that a *non-zero inflation target* can implement exactly the optimal policy with cooperation and ex-ante commitment, being equivalent to the linear contracts. However, the optimal targets are shown to be *state-contingent*.

We then move to analysing the incentives that governments have to delegate in the first place, i.e. we analyse the subgame perfect equilibrium contracts and targets of the game with delegation. We obtain that these are usually different from the optimal contracts and targets an international social planner would like to implement, questioning the implementation of cooperative policies by *purely noncooperative* mechanisms. In the Persson and Tabellini framework, there is an implicit assumption about the ability of governments to commit and cooperate at the first stage, when delegating to the monetary authority. We feel this is not consistent with the search for non-cooperative means to implement cooperative outcomes.

To the best of our knowledge, this type of exercise has not been yet pursued in the literature. The only related paper is the one by Dolado, Griffiths and Padilla (1994), in which they do analyse the subgame perfect equilibrium of a similar delegation game. They show that governments have the right incentives to delegate to a conservative ('Rogoff ', 1985a) banker, such delegation occurring not only as a solution to dynamic inconsistency but also due to monetary spillovers. They also show that governments have the incentives to distort the true output-inflation preferences even when monetary policies are co-ordinated by a supranational authority. However, they do not show how delegation can literally be used as a decentralised way of achieving the cooperative solution. Moreover, their results are different depending on the nature of the spillovers as discussed in chapter 4. In the same model, we derive the optimal contracts and targets that implement the cooperative solution and compare them to the subgame perfect equilibrium. In both cases (with and without a domestic inflation bias) we find that the cooperative outcome cannot be implemented in a purely non-cooperative manner.

2.4 A brief survey of the literature on dynamic inconsistency

Since our results in chapters 3 and 5 make use of the time inconsistency literature and the solutions proposed here for the problem at hand have their origins in it, some notes on this are also in order. The conventional wisdom in the field suggests, starting from Kydland and Prescott (1977) and

Barro and Gordon (1983) that the combination of discretionary monetary policy and a short-run benefit from surprise inflation (due, e.g. to distortions in the labour market that make the natural rate of unemployment inefficiently high) leads to a *fourth-best equilibrium*. In this equilibrium there is an inflation bias relative to the second-best equilibrium resulting from commitment to an optimal rule (where, following Svensson, 1997, the first-best equilibrium would imply elimination of the distortions that make the natural rate of unemployment inefficiently high). Several improvements of the fourth-best equilibrium have been proposed in the literature.

Rogoff (1985a) suggested delegation to an inflation-averse or weight-conservative central banker (putting more weight on inflation stabilisation than society does). His solution, however, although reduces the inflation bias affects the stabilisation of shocks and leads to higher than optimal employment/output variability leading to a *third-best equilibrium*. The same type of equilibrium is achieved by simple rules with escape clauses of the type studied by Lohmann (1992), where discretionary behaviour is allowed for large shocks.

The second best equilibrium can be achieved by an optimal central bank contract as suggested by Walsh (1995) or Persson and Tabellini (1993). The contract is linear and consists of delegating to a Central Bank that has a loss function equal to that of the society plus a linear inflation penalty. Although simple, the contract is difficult to implement in practice. First, as shown by Goodhart and Vinals (1994), it implies monetary rewards for the governor when inflation is low, which may generate political tensions if this is associated with a high unemployment. Secondly, as argued by Svensson (1997) the loss function is expressed in utils, whereas the linear penalty is expressed in monetary units, so the marginal penalty should incorporate somehow the preferences of the Central Banker. Thirdly, there is the question about the ability and incentives of the government to monitor the Central Bank since an increased inflation would only mean in the short run lower unemployment (Obstfeld and Rogoff, 1996). Other, more general critiques shall be mentioned later.

Svensson (1997) proposed a more simple solution that achieves the second-best equilibrium. It consists of delegating to an instrument-independent (as opposed to goal-independent, in the terminology of Debelle and Fisher, 1994) Central Bank with a non-zero inflation target, different from that of the society. In a static context, the target is shown to be equivalent to the linear contract.

Blinder (1997) has made a more fundamental critique, arguing that in fact the time inconsistency problem does not exist since Central Banks only aim for the natural rate of output. However, Walsh (1998) shows that such a situation would lead to counterintuitive conclusions regarding the ability of monetary policy to influence output. This problem is also discussed at length in Svensson (1995).

McCallum (1995) has also criticised the time-consistency literature, showing that it merely relocates the problem from the Central Bank's level to the principal's level, for example the government (let alone that the government is itself an agent of the society). The principal will always have the incentives to change the delegation arrangement after inflation expectations are formed. For example, in Svensson's (1997) model the inflation target could be changed by the government once expectations have been determined.

Finally, we mention that the solutions modify if there is persistence in output or other real variable, as shown by Lockwood (1997), Lockwood and Phillipopoulos (1994) or Svensson (1997). An autoregressive term in one of the real variables introduces a state-contingent inflation bias and makes inflation variability too high and output variability too low. For the state-contingent inflation bias and the non-optimal variability to be removed, state-contingent inflation contracts are needed, as shown by Lockwood, Miller and Zhang (1995).

A state-contingent inflation target eliminates the state-contingent inflation bias but leaves inflation variability too high and output variability too low, as shown by Svensson (1997). He shows that augmenting the state-contingent inflation target with a 'Rogoff-conservative' central banker solves the problem.

Beetsma and Jensen (1998) argue, based on McCallum's (1995) critique that the state-dependent nature of such delegation schemes undermines their credibility and show that the optimal rule can nevertheless be attained through state-independent delegation. More specifically, the second best solution is obtained when the central bank is required to make an appropriate trade-off between achieving a constant nominal income growth target and attaining the socially optimal (constant) inflation and output targets.

Chapter 3.

Delegation by inflation targets to achieve the cooperative and commitment optimum. Equivalence of contracts and targets.

In this chapter we follow the ideas of Svensson (1997) that delegation by an inflation target eliminates the inflation bias. We extend this to show that in a two-country model as the one in Persson and Tabellini (1996) non-zero inflation targets can be used to implement exactly the second-best optimum corresponding to cooperation and ex-ante commitment. This is in opposition to Persson and Tabellini (1996), who show that *zero* inflation targets *approximate* the second best optimum, which in fact means that it leads to a *third-best equilibrium*. We see nothing to constrain the inflation targets to take only a value of zero. On the contrary, in practice zero targets are the exception rather than the rule, due to different motives (seigniorage, competitiveness, measurement errors, etc.).

3.1. The model

The model we use to derive our result is the same as in Persson and Tabellini (1996), i.e. a parameterised version of the general model in Persson and Tabellini (1995). It puts together the two building blocks on policy cooperation and dynamic inconsistency surveyed in chapter 2. It is a two-country model (each country being specialised in the production of one good) with policy spillovers and incentives on the side of each country to engage in competitive devaluations. Moreover, there is a domestic inflation bias in each country. Monetary policy is neutral in the long run but is used to stabilise the economy in the short run. Although the model may seem postulated adhoc, it is closely related to the more complete models in Canzoneri and Henderson (1991) or Rogoff (1985b).

All the variables are defined as rates of change and we preserve the notation in Persson and Tabellini (1996), the change in the log of the real exchange rate being given by:

$$z = s + q - q^* \qquad (3.1)$$

where s represents nominal depreciation of domestic currency. We will let letters without an asterisk denote variables in the domestic country and letters with asterisk the ones in the foreign country (as opposed to Persson and Tabellini who study countries outside and inside the monetary union). Thus, q and q* represent producer price inflation. In the domestic country, CPI-inflation and PPI inflation are given by

$$p = q + \beta z \qquad (3.2)$$

$$q = m + v \qquad (3.3)$$

where β is the share of foreign goods in the domestic country's consumption basket, *m* is the rate of money growth and v is a demand or velocity shock. The natural rate of output growth is normalised to zero and output growth *x* is given by the expectations-augmented Phillips curve:

$$x = \gamma(q - q^e) - \varepsilon \qquad (3.4)$$

where γ is a parameter, ε an adverse supply shock and qe the rationally expected value of q. The equilibrium z is dependent on the relative supply of foreign goods in relation to its relative demand and thus it satisfies (provided the relative demand is increasing in z):

$$z = \delta(x - x^*) + \phi \qquad (3.5)$$

where δ is the inverse elasticity of the outside good and ϕ is a speculative shock. The structural shocks v, ϕ and ε are assumed to be independently distributed with an expected value of zero.

The policy instrument is m and it is chosen by the Central Bank with the preferences described by the loss function:

$$L = \frac{1}{2}\left[p^2 + \lambda(x-\theta)^2 + \mu(z-\xi)^2\right] \qquad (3.6)$$

where λ and μ are positive weights and θ and ξ are stochastic policy targets for employment and the real exchange rates. Assuming $E(\theta)>0$ creates a systematic inflation bias, whereas $E(\xi)>0$ generates incentives to engage in competitive devaluations. Shocks to θ would capture the difference between the target and the natural rate and shocks to ξ variations in the clout of the export industry, lobbying for higher profitability through a weaker exchange rate.

The foreign country is modelled in the same way, only *z* enters with an opposite sign in both the CPI inflation and loss function. The structural parameters are equal across countries but differences in targets are allowed as well as different variances of shocks and arbitrary covariances of pairs of these. The timing of the game is as follows:

(i) policy targets $\tau=(\theta,\theta^*,\xi,\xi^*)$ are revealed, (ii) private expectations (q^e, q^{*e}) are formed, (iii) structural shocks $\omega=(\varepsilon,\varepsilon^*, v, v^*,\phi)$ are revealed, (iv) policies (m, m^*) are simultaneously set, (v) macroeconomic outcomes are realised.

We first reproduce Persson and Tabellini's (1996) result regarding contracts and then derive our result regarding targets.

3.2. Optimal policy

To get an idea as to what contracts or targets would be implemented, a benchmark for the analysis is needed. It consists of the hypothetical case in which the two authorities decide to (a) *cooperate* before stage (i) and (b) to *commit ex-ante* to a pair of state-contingent policy rules m and m*. They will thus minimise the joint losses $E(L+L^*)$ subject to $q^e=E_\tau(q)$, $q^{e*}=E_\tau(q^*)$, *i.e.* expectations about inflation are formed rationally as conditional (on targets) expected values of PPI inflations. Thus, authorities internalise the externalities of their actions on both the other country and inflation expectations of the private sector.

Lemma 3.1

In the cooperative and ex-ante commitment optimum the monetary authorities choose the money supplies m by the following policy rules

$$\begin{aligned} p(\tau,\omega)+\lambda\gamma x(\tau,\omega)+2\mu\gamma\delta z(\tau,\omega)+\beta\delta\gamma\left(p(\tau,\omega)-p^*(\tau,\omega)\right)=0 \\ p^*(\tau,\omega)+\lambda\gamma x^*(\tau,\omega)-2\mu\gamma\delta z(\tau,\omega)+\beta\delta\gamma\left(p^*(\tau,\omega)-p(\tau,\omega)\right)=0 \end{aligned} \quad (3.7)$$

Proof *– Please find Appendix 1.*

At the cooperative and commitment optimum, there is a trade-off between the direct effects of policy on domestic variables (first two terms) and effects on domestic and foreign losses induced directly (third term) through the exchange rate or indirectly through inflation (last term). Persson and Tabellini also derive the optimal rule by solving the model and the foreign first order condition but we do not derive this here as it is not necessary for our purpose. Also, due to ex ante commitment, targets do not appear in the first order condition since they are observable and real variables are neutral to expected policy in the model.

3.3 Non-cooperative and discretionary equilibrium

Supposing, more realistically, that countries cannot commit ex-ante to either each other to cooperate or to the private sector, the *fourth best equilibrium* is obtained (to preserve the terminology in chapter two). Each central bank chooses its policy ex post to minimise L with respect to m, taking m^*, $E_\tau(q)$ and $E_\tau(q^*)$ as given.

Lemma 3.2

In the non-cooperateive and discretionary equilibrium the monetary authorities choose the m's to fulfil:

$$\begin{aligned}
&p(\tau,\omega)+\lambda\gamma x(\tau,\omega)+2\mu\gamma\delta z(\tau,\omega)+\beta\delta\gamma(p(\tau,\omega)-p^*(\tau,\omega))= \\
&\lambda\gamma\theta+\mu\gamma\delta\xi+\mu\gamma\delta z-\beta\delta\gamma p^* \\
&p^*(\tau,\omega)+\lambda\gamma x^*(\tau,\omega)-2\mu\gamma\delta z(\tau,\omega)+\beta\delta\gamma(p^*(\tau,\omega)-p(\tau,\omega))= \\
&\lambda\gamma\theta^*+\mu\gamma\delta\xi^*-\mu\gamma\delta z-\beta\delta\gamma p
\end{aligned} \tag{3.8}$$

Proof *– please find Appendix 1.*

The RHS reflects the 'incentive constraints' (PT, 1996) existent in the non-cooperative discretionary equilibrium. First, at the domestic level, there is an inflation bias (first term) due to the 'credibility' (ex post optimality) constraint, i.e. to the central bank ignoring the effect of policy on expectation formation. Second, due to the 'individual rationality' constraint, the spillover effects on the foreign country are ignored and a permanent competitive depreciation bias appears (second term). The two 'constraints' make the targets enter the first order condition in this case. Also due to non-cooperation (individual rationality constraint) the stabilisation of shocks is distorted. The home country ignores that it exports inflation abroad if it appreciates its real exchange rate in response to a shock, externality that might be either positive or negative depending on the nature of the shock.

3.4. Optimal inflation contracts

We follow Persson and Tabellini (1996) and assume that there is an international principal (the European Union, in their case, for countries inside and outside EMU) who imposes performance contracts on both central banks. Delegation by linear inflation contracts can achieve the second-best equilibrium where the folk theorem for delegation games of Fershtman et al. (1991) is used. We will argue with this point in the next chapter.

For the moment we assume that such contracts can be imposed and they are linear penalties for not achieving the target (equal to zero in PT 1996 by assumption), for example for the home country:

$$T(p) = tp(\tau,\omega) \qquad (3.9)$$

Now the central bank minimises the sum $L+T$ with respect to m in a discretionary non-cooperative fashion. It is straightforward to notice that the only difference from the first order condition (3.8) will be the introduction of a new term on the left hand side equal to $(1+\beta\delta\gamma)t$. Comparing (3.8) and the new term with the optimal first order condition we can state Proposition 3.1.

Proposition 3.1

There exists a unique pair of state-contingent contracts (t,t) that implement the cooperative and ex-ante commitment with the marginal penalties given by:*

$$\begin{aligned} t(\tau,\omega) &= \frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma\theta + \mu\gamma\delta\xi + \mu\gamma\delta z(\tau,\omega) - \beta\delta\gamma p^*(\tau,\omega)\right) \\ t^*(\tau,\omega) &= \frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma\theta^* + \mu\gamma\delta\xi^* - \mu\gamma\delta z(\tau,\omega) - \beta\delta\gamma p(\tau,\omega)\right) \end{aligned} \qquad (3.10),$$

where z and p are evaluated at the ex-ante cooperative optimum.*

The first thing to note is that contracts are **state contingent**. This result is usually obtained in a dynamic context (e.g. by Lockwood, 1997 or Svensson, 1997) but not in static models. Thus, it seems to be a result of the two-country nature of the mode since the state-dependence comes from the policy spillovers. To see this more clearly, one may solve for the *z* and *p** at the cooperative optimum in terms of the shocks. This is straightforward but tedious and does not make any difference for the purpose at hand since the contract can be interpreted based on (3.10). We will however pursue this type of exercise in chapter 5.

The intuitive interpretation of the marginal penalty is as follows: the first two terms help eliminate the systematic biases (the inflation bias and the competitive depreciation bias). The other two help correcting the *stabilisation bias* resulting from the failure to internalise the externalities imposed on the foreign country either directly through the exchange rate or indirectly through CPI inflation. The marginal penalty for the foreign country t* can be interpreted in a similar way.

For reasons discussed briefly in chapter two and extensively in McCallum (1995) and Beetsma and Jensen (1998) such state-contingent penalties may be difficult to enact. Thus, Persson and Tabellini (1996) derive state-independent contracts under this constraint, which, given the linearity of the model, are given by the expected values of (3.10):

$$\begin{aligned} t &= \frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma E(\theta) + \mu\gamma\delta E(\xi)\right) \\ t^* &= \frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma E(\theta^*) + \mu\gamma\delta E(\xi^*)\right) \end{aligned} \qquad (3.11)$$

These state-independent contracts eliminate the systematic biases with the cost of a sub-optimal response to shocks, achieving a *third-best equilibrium*.

3.5. Optimal inflation targets

Observing that even simple linear penalties may be difficult to enact, Persson and Tabellini move to discussing and comparing different policy regimes, including symmetric zero inflation targets, monetary targets and an EMS-lie regime (exchange rate pegging). All these arrangements result in (Pareto-ranked) *third-best equilibria*, the best among them being shown to be a generalised system of zero-inflation targets.

As argued before, we see no reason in assuming that the inflation target should be zero. On the contrary, we see this as the exception rather than the rule and show that non-zero inflation targets can be designed to achieve the *second-best equilibrium (cooperation and ex ante commitment).*

Suppose again that the international principal can impose delegation to an instrument-independent central bank whose loss function is modified by the introduction of a non-zero inflation target as follows:

$$\begin{aligned} L^{cb} &= \frac{1}{2}\left[\left(p-\hat{p}\right)^2+\lambda(x-\theta)^2+\mu(z-\xi)^2\right] \\ L^{cb}* &= \frac{1}{2}\left[\left(p-\hat{p}*\right)^2+\lambda(x-\theta*)^2+\mu(z-\xi*)^2\right] \end{aligned} \qquad (3.12)$$

Suppose further that minimisation of the loss function is done by each central bank in a discretionary and non-cooperative manner, taking expectations and the other country's policy as given.

The first order condition for the home country, rearranged for the ease of comparison with the optimal rule, would be:

$$\begin{aligned} &p(\tau,\omega)+\lambda\gamma x(\tau,\omega)+2\mu\gamma\delta z(\tau,\omega)+\beta\delta\gamma(p(\tau,\omega)-p*(\tau,\omega))-(1+\beta\delta\gamma)\hat{p}(\tau,\omega)= \\ &\lambda\gamma\theta+\mu\gamma\delta\xi+\mu\gamma\delta(\tau,\omega)-\beta\delta\gamma p*(\tau,\omega) \end{aligned} \qquad (3.13)$$

It is clear from (3.13) and the optimal rule (3.7) or the contract case that Proposition 3.2 holds.

Proposition 3.2

There exists an unique pair of state-contingent optimal inflation targets, i.e. implementing the cooperative and ex-ante commitment optimum, targets given by:

$$\begin{aligned} \hat{p}(\tau,\omega) &= -\frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma\theta + \mu\gamma\delta\xi + \mu\gamma\delta z(\tau,\omega) - \beta\delta\gamma p^*(\tau,\omega)\right) \\ \hat{p}^*(\tau,\omega) &= -\frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma\theta^* + \mu\gamma\delta\xi^* - \mu\gamma\delta z(\tau,\omega) - \beta\delta\gamma p(\tau,\omega)\right) \end{aligned} \tag{3.14}$$

As (3.14) shows, in this model a state-contingent target is perfectly equivalent to a state-contingent contract. It eliminates both systematic biases (the inflation and competitive depreciation biases – first two terms) and stabilisation biases resulting from non-internalising the policy externalities. It is also to be noted that the optimal target is equal to the negative of the marginal penalty in a contract.

Thus, the following will hold:

$$\begin{aligned} \hat{p} &= -t \\ \hat{p}^* &= -t^* \end{aligned} \tag{3.15}$$

for both the state-contingent and the state-independent targets and contracts.

Due to the same arguments as before one can study the state-independent inflation targets, and the result will be perfectly equivalent to the one in the case of contracts, i.e. elimination of systematic biases but sub-optimal response to shocks. Thus the constant targets in this case will simply be given by the negative of (3.11).

The equivalence of the two arrangements can be alternatively shown as follows, starting from the loss function under inflation targeting:

$$L^{cb} = \frac{1}{2}\left[\left(p - \hat{p}\right)^2 + \lambda(x-\theta)^2 + \mu(z-\xi)^2\right] = \frac{1}{2}\left[\left(p^2 - 2p\hat{p} + \hat{p}^2\right) + \lambda(x-\theta)^2 + \mu(z-\xi)^2\right] =$$

$$= L - p\hat{p} + \frac{1}{2}\hat{p}^2$$

The loss function differs from the one without delegation by a linear term in inflation and a constant. Choosing the target equal to the contract but of opposite sign achieves the same outcome.

This equivalence result is similar to Svensson (1997) who finds that in a static closed-economy context the two are equivalent. However, it is different in that in Svensson's paper, state-contingent inflation contracts and targets (resulting there due to output persistence) are no longer equivalent.

The result is different from the one in Persson and Tabellini (1996) in that in our case appropriately chosen, non-zero and state-contingent inflation targets can implement the second-best equilibrium with cooperation and commitment. However the equivalence carries to the problem of the incentives the governments would have to delegate with exactly these targets. We will deal with this issue in the next two chapters.

Finally, in the framework of the folk theorem for delegation games of Fershtman et al. (1991) one can regard our result as a mean to implement the cooperative (and commitment, in our case) optimum through delegation, subject to qualification in Chapters 4 and 5. Their result states that implementation can be done by target compensation functions and Persson and Tabellini (1995) show that in this framework linear contracts can achieve the same solution. We obtain that implementation can be done also by delegating to an agent with a *distorted utility function* in that it does not share the same objectives as the principal. This conforms the idea of Fershtman et al. (1991) that '*sending an agent can be equivalent to credibly* *reporting distorted utility function*' (p. 551), provided the agent's utility function is public information (which is true for inflation targeting regimes characterised by a high degree of transparency).

Chapter 4

Implementation of the cooperative outcome through delegation by contracts and targets and delegation incentives with no domestic inflation bias.

In this chapter we focus on a two-country model without a credibility problem in any of the countries. However, as we allow for policy spillovers, policy cooperation will always be Pareto optimal. We show how optimal contracts and targets can be designed to achieve the cooperative optimum when there is no inflation bias. Then we move to study the incentives that governments have to choose the contracts or targets at the delegation stage.

In a similar model, Dolado et al. (1994) argue that delegation to a Rogoff conservative central banker is the *optimal policy* even in the absence of credibility issues. However, we think delegation by a conservative central banker is unsatisfactory and far from optimal. This leads always to a tighter monetary policy (causes a deflationary bias), which causes a loss of welfare if spillovers are positive. We see this hard to reconcile with the idea of implementing a *cooperative optimum*. In fact, in the mentioned paper it is not shown how delegation can be used to actually *implement this optimum*

We use a version of their model for the purpose at hand. Although very schematic, the model can capture the type of question we are interested in. Moreover, it can be easily seen that the model is very similar to the reduced forms of elaborated models as the ones in Canzoneri and Henderson (1991) or Rogoff (1985) and the conclusions and insights of these models are essentially the same.

We consider a two-country model, where the two countries are engaged in a two-stage monetary policy game. The stages of the game are: (i) *delegation* –

each country chooses simultaneously and independently a central banker; (ii) the elected central bankers choose simultaneously the money growth rates of the two countries.

Let the preferences of the governments be represented by the welfare functions:

$$W_i^G = -\frac{1}{2}\left(y_i^2 + \mu\pi_i^2\right), \quad \text{i=1,2} \qquad (4.1)$$

Where we suppose without further loss of generalisation – since we are not interested in the inflation preferences parameter *per se*- that the weights on inflation are equal, in contrast to Dolado et al (1994).

Let the government delegate in the first stage of the game to an independent central banker, imposing a linear penalty for missing the target, i.e. an inflation contract, the welfare function of the central bank being:

$$W_i^{CB} = -\frac{1}{2}\left(y_i^2 + \mu\pi_i^2\right) - t_i\pi_{ii}, \quad \text{i=1,2} \qquad (4.2)$$

Alternatively, based on the result in Chapter 3, for the moment we also consider delegation to a central bank with an inflation target different than society's target:

$$W_i^{CBT} = -\frac{1}{2}\left(y_i^2 + \mu(\pi_i - \hat{\pi}_i)^2\right) \qquad (4.2')$$

Note that in this case, due to eq. (4.4), *inflation targeting* is actually equivalent to *money growth targeting*.

The two-country economy is described by (where the equations can be regarded as reduced forms of more complete models):

$$y_i = am_i + bm_j - z \qquad (4.3)$$

$$\pi_i = m_i \qquad (4.4)$$

where $i=1,2;\ i \neq j;\ i>a>/b/>0,\ z \geq 0$.

Deviations of output from the natural rate depend on the money growth rates in both countries and an adverse supply shock. Inflation depends only on money growth in the respective country, which simplifies the algebra (note that the welfare of one country is still affected by the other country's policy through the effect on y). Also a is assumed to be positive but no a priori sign can be imposed on b. In fact the sign of b gives the direction of the spillovers.

If b is positive, there are positive spillovers and the policies are *strategic substitutes*. It is easily shown (as first pointed out by Canzoneri and Gray, 1985) that in this case the Nash Equilibrium has a deflationary bias. If b is negative, there are negative spillovers and the policies are *strategic complements*, the Nash Equilibrium having an inflationary bias. Cooperation is thus in this symmetric two-country set-up always Pareto optimal, since there is no domestic credibility problem (thus the 'Rogoff' (1985b) result does not apply).

In this context we assume that delegation occurs to sustain this cooperative outcome (using the result of Fershtman et al., 1991 and Persson and Tabellini, 1995 presented earlier). Alternatively, one may think about sustaining the collusive outcome by reputation and trigger mechanisms if the game is repeated over time, as described in chapter 2. We do not deal with this case here.

First, we look at the contracts a benevolent social planner would like to design for implementation of the cooperative outcome. Then we look at the contracts that would result from the optimising behaviour of the players in the two-stage game by solving the game backwards and supposing Nash Playing at both stages, i.e. we look at the *Subgame Perfect Nash Equilibria* of the game. We then compare the two contracts to get an idea about the incentives the governments would have to sign the optimal contracts.

4.1 Optimal contracts and targets.

4.1.1 Equilibrium money growth rates

Given delegation by a pair of contracts t_I, the equilibrium money growth rates satisfy Lemma 1:

Lemma 4.1 *For any pair (t_1, t_2) there exists a unique symmetric Nash Equilibrium of the second-stage sub-game given by:*

$$m_1^{NE} = m_2^{NE} = m^{NE} = \frac{az - t}{ab + a^2 + \mu} \qquad i=1,2 \qquad (4.5)$$

If delegation is done by inflation targets (π_1,π_2), the corresponding Nash Equilibrium is:

$$m_1^{NE} = m_2^{NE} = m^{NE} = \frac{az + \mu \hat{\pi}}{ab + a^2 + \mu} \qquad (4.5')$$

Proof

Each central bank maximises its welfare function (4.2) after being assigned the contract or target, the set of first order conditions being:

For contracts:

$$\frac{\partial W_i^{CB}}{\partial m_i} = -ay_i - \mu\, \pi_i - t_i = 0 \qquad i=1,2 \qquad (4.6)$$

for targets:

$$\frac{\partial W_i^{CBT}}{\partial m_i} = -ay_i - \mu\, \pi_i + \mu \hat{\pi}_i = 0, \quad i=1,2, \qquad (4.6')$$

It is clear from (4.6) and (4.6') that in this case targets and contracts are equivalent. Hence, we won't provide all the derivations for targets but only the results, observing that in equilibrium

$$t_i = -\mu \hat{\pi}_i \qquad (4.7)$$

Substituting (4.3) and (4.4) in (4.6) we get:

$\left(a^2 + \mu\right)m_i + abm_j - az + t_i = 0$ *, i=1,2, i≠j*

and solving the system for m_i results in:

$$m_i^{NE} = \frac{az}{ab + a^2 + \mu} - \frac{a^2 + \mu}{\left(a^2 + \mu\right)^2 - (ab)^2} t_i + \frac{ab}{\left(a^2 + \mu\right)^2 - (ab)^2} t_j \qquad (4.5'')$$

Since the model is symmetric we study the symmetric equilibrium in which $m_1 = m_2$ thus from (4.5'') $t_1 = t_2$, resulting in (4.5)

The second order conditions ensure that (4.5) is indeed a maximum

$$\frac{\partial^2 W_i^{CB}}{\partial m^2{}_i} = -a^2 y_i - \mu^2 \pi_i < 0 .$$

4.1.2 The cooperative optimum

In order to find the optimal contracts and targets we have to compare the equilibrium money supplies from Lemma 1 to the cooperative outcome in which the two countries decide to maximise jointly an aggregate measure of their welfare $W^A = W_1^G + W_2^G$. The equilibrium is known as *the efficient equilibrium* (Canzoneri and Henderson 1991) and is symmetric due to the symmetric nature of the model.

Lemma 4. 2

The cooperative money supplies are given by

$$m_1^C = m_2^C = m^C = \frac{a+b}{\mu + (a+b)^2} z \qquad (4.8)$$

Moreover, they represent a Pareto optimum.

Proof

The countries (or an international social planner) maximise the aggregate welfare function, the first order conditions being:

$$\frac{\partial W^A}{\partial m_i} = -ay_i - \mu\pi_i - by_j = 0 \qquad i=1,2 \tag{4.9}$$

Substituting (4.3) and (4.4) in (4.9) and solving for m gives the two money supplies as given by (4.8).

The second order condition ensures this is a maximum:

$$\frac{\partial^2 W^A}{\partial m^2{}_i} = -a^2 y_i - \mu^2\pi_i - b^2 y_j < 0$$

The Pareto optimality of (4.8) results by observing that

$$\left.\frac{\partial W_i^G}{\partial m_i}\right|_{m_i=m^c} = -a[a+b+\mu]m^C + az = -a(a+b+\mu)\frac{a+b}{\mu+(a+b)^2} - az = \frac{(a+b)^2(\mu-1)-(a+b)\mu}{\mu(a+b)^2} < 0 \text{ ,}$$

since 0<μ<1. Thus, any deviation from this equilibrium reduces welfare.

By cooperating, the countries are able to internalise the externalities (either positive or negative) they impose on each other when playing Nash. When *b* is positive, the deflationary bias is eliminated and when *b* is negative the expansionary bias is reduced. The cooperative outcome can be achieved through a commitment technology. However, as we showed in the literature survey, there are incentives for the countries to deviate from this optimum. Supposing commitment (binding agreements) is not possible, the question of implementing the cooperative outcome in a decentralised fashion appears. As we mentioned earlier, we focus on institutional resolutions of the problem and not on reputational and trigger mechanisms.

4.1.3 Optimal contracts and targets

We show how delegation can achieve the cooperative optimum if delegation is done by means of either inflation contracts or targets.

Proposition 4.1

There exists a pair of contracts (t_1, t_2) or targets (π_1,π_2) that implement the cooperative optimum by delegation. The equilibria are symmetric and are given by:

$$t_1^* = t_2^* = t^* = -\frac{b\mu}{(a+b)^2+\mu} z \qquad (4.10)$$

$$\hat{\pi}_1^* = \hat{\pi}_2^* = \hat{\pi}^* = \frac{b}{(a+b)^2+\mu} z \qquad (4.10')$$

Proof

In the case of delegation with contracts, we compare the two first-order conditions for delegation and cooperation

$$-ay_i - \mu\,\pi_i - t_i = 0$$
$$-ay_i - \mu\pi_i - by_j = 0$$

from which it is clear that the contracts that implement the cooperative optimum are given by

$t_i = by_j^C = abm_j^C + b^2 m_i^C - bz$ *, i=1,2,*

where the m's are evaluated at the cooperative optimum (4.8). Substituting gives the optimal t's in (4.10).

Then, using (4.7) we immediately get the optimal targets as in (4.10').

The optimal contracts in (4.10) have an intuitive interpretation. If *b>0*, i.e. spillovers are positive and policies are strategic substitutes, the deflationary bias in the Nash equilibrium is reduced by imposing a **negative marginal**

penalty (a reward) for additional inflation. On the contrary, when spillovers are negative and policies are strategic complements the inflationary bias is eliminated by imposing a **penalty** for missing the inflation target. The larger the supply shock that generates the stabilisation game, the larger the penalty. A similar interpretation can be found for the targets in (4.10').

This is in sharp contrast to Dolado et al. (1994). In their case, delegation to a conservative central banker improves welfare *only* if spillovers are negative and even in that case it *does not achieve* the cooperative optimum. Furthermore, for positive spillovers delegation by a conservative central banker exacerbates the deflationary bias (p. 1062).

Thus, we have shown how a benevolent international planner can achieve the Pareto optimal cooperative outcome by delegating to an independent central banker by either inflation contracts or targets. This result is reminiscent of the folk theorem in delegation games mentioned before (Fershtman et al., 1994) used by Persson and Tabellini (1995). However, implementation is not entirely decentralised or non-cooperative since it implicitly assumes cooperation or at least some form of coordination at the delegation stage. For an entirely non-cooperative implementation, one needs to study the strategic incentives governments have to delegate at the first stage. We do this in the next section.

4.2 Strategic incentives for delegation

To study the strategic incentives we consider the case when governments act in a completely decentralised fashion, choosing contracts (or targets) for the central banks at the delegation stage. In other words, we try to find the *Subgame Perfect Equilibrium (SPE)* of the game and compare the results with the optimal contracts and targets. To do this we will solve the game by backward induction, where the governments will maximise at stage one the welfare functions in (4.2) with respect to *t*, where the contracts *t* appear in the solution due to their presence in the Nash equilibrium money supplies in the second-stage game. We note that formally these are dependent on both contracts as in (4.5''), although the symmetry of the model makes them equal and thus not apparent from the expressions (4.5).

Proposition 4.2

There exists an unique Subgame Perfect Equilibrium of the delegation game at stage one, whereby both governments delegate to central banks by imposing a linear contract given by:

$$t_1^{SPE} = t_2^{SPE} = t^{SPE} = \frac{ab^2\mu}{\left(a^2+\mu\right)\left(ab+a^2+\mu\right)-ab^2\left(a+b\right)}z \qquad (4.11)$$

If governments delegate by targets, the Subgame Perfect Equilibrium is:

$$\hat{\pi}_1^{SPE} = \hat{\pi}_2^{SPE} = \hat{\pi}^{SPE} = -\frac{ab^2}{\left(a^2+\mu\right)\left(ab+a^2+\mu\right)-ab^2\left(a+b\right)}z \qquad (4.11')$$

Proof

First step is finding the Nash equilibrium in the central banks' game, which we did in Lemma 4.1. Then by backward induction, we use the NE money supplies and substitute them back in the ***welfare functions of the government*** *(i.e. without the t's) and maximise these with respect to the t's. The first order conditions are given by:*

$$\frac{\partial W_i^G}{\partial t_i} = \frac{\partial W_i^G}{\partial m_i}\frac{\partial m_i^{NE}}{\partial t_i} + \frac{\partial W_i^G}{\partial m_j}\frac{\partial m_j^{NE}}{\partial t_i} = 0 \quad i, j=1,2;\ i\neq j \qquad (4.12)$$

We can solve this in two ways: first is by substituting everything using Lemma 4.1 and finding the t's, which is computationally more demanding. The second way is to use an envelope-like argument, which we pursue here.

We start from the first-order conditions:

Stage 2: $\frac{\partial (W_i^G - t_i \pi_i)}{\partial m_i} = 0 \Rightarrow \frac{\partial W_i^G}{\partial m_i} = t_i$ *(4.13)*

Stage 1: $\frac{\partial W_i^G}{\partial t_i} = \frac{\partial W_i^G}{\partial m_i}\frac{\partial m_i^{NE}}{\partial t_i} + \frac{\partial W_i^G}{\partial m_j}\frac{\partial m_{ij}^{NE}}{\partial t_i} = 0$ *i,j=1,2; i≠j* *(4.14)*

We also note that:

$$\frac{\partial W_i^G}{\partial m_i} = -\left(ay_i + \mu\pi_i\right)$$
$$\frac{\partial W_i^G}{\partial m_j} - by_i$$
(4.15)

By Lemma 4.1, equation (4.5''):

$$\frac{\partial m_i^{NE}}{\partial t_i} = \frac{a^2 + \mu}{a^2 b^2 - \left(a^2 + \mu\right)^2}$$
$$\frac{\partial m_j^{NE}}{\partial t_i} = -\frac{\partial m_i^{NE}}{\partial t_i}\frac{ab}{a^2 + \mu}$$
(4.16)

Now we can substitute (4.13), (4.15) and (4.16) into (4.14) to get:

$$t_i + \frac{ab^2}{a^2 + \mu} y_i^{NE} = 0 \qquad (4.17)$$

It is clear that y in (4.17) is evaluated at the Nash equilibrium. By substituting the NE money growth rates from Lemma 4.1 into (4.17) we get the result in (4.11), Proposition 4.2. A similar proof can be made for inflation targets.

4.4 Comparison of contracts

From now on we will focus the discussion only on inflation contracts, as the results regarding targets are equivalent.

At a first glance, it is obvious comparing the optimal contracts in (4.10) with the perfect equilibrium contracts in (4.11) that generally they are different. Thus, absent coordination from a supranational authority, the governments do not have the right incentives to delegate with those contracts that implement the cooperative optimum. We further try to study the relationship between them in a more rigorous way, stating the result in the following Proposition.

Proposition 4.3

The following inequalities hold:

(i) $t^{SPE}>0 \ \forall\, b\in\Re^*$. *(4.18)*

(ii) $t^{SPE}>t^{C}\,\forall\, b\in\Re$. *(4.19)*

Proof

(i) $t^{SPE}>0 \ \forall\, b\in\Re^*$.

The numerator is always greater than zero if b is different from zero. We focus on the denominator.

$$\left(a^2+\mu\right)\left(a(a+b)+\mu\right)>a^2a(a+b) \qquad \textit{since } \mu>0$$

But $aa^2(a+b)\geq ab^2(a+b)$, *since* $a\geq|b|$.

From these two inequalities, the result is demonstrated.

(ii) $t^{SPE}\geq t^{C}\,\forall\, b\in\Re$

The proof is by reductio ad absurdum.

Suppose that $\exists\, b\in\Re$ *s.t.* $t^{SPE}<t^{C}$.

This implies:

$$\therefore \frac{ab^2\mu}{(a^2+\mu)(ab+a^2+\mu)-ab^2(a+b)}z < -\frac{b\mu}{(a+b)^2+\mu}z$$

$$\therefore \frac{ab}{(a^2+\mu)(ab+a^2+\mu)-ab^2(a+b)} < -\frac{1}{(a+b)^2+\mu}$$

$$\therefore ab\left[(a+b)^2+\mu\right]+(a^2+\mu)(ab+a^2+\mu)-ab^2(a+b)<0$$

After decomposing and dividing by a^2 we get

$$\therefore b^2+2\left(a+\frac{\mu}{a}\right)b+a^2+2\mu+\left(\frac{\mu}{a}\right)^2<0$$

$$\therefore \left[b+\left(a+\frac{\mu}{a}\right)\right]^2<0$$

This is a contradiction, so the result is proved. Moreover, we observe that the contracts are equal only for $b=-\left(a+\frac{\mu}{a}\right)$. *But this solution is unfeasible since* $a>/b/$.

The results in Proposition 4.3 have an intuitive interpretation. First of all, the fact that the subgame perfect equilibrium contracts are always positive clearly shows their sub-optimality. The result is similar to that in Dolado et al. (1994) regarding delegation to a 'Rogoff' conservative central banker. When spillovers are positive and there is a contractionary bias delegation by the perfect equilibrium contract exacerbates the deflationary incentives of the governments as these fail to internalise the positive externality. It does this by imposing a penalty on the central bank when the optimal delegation parameter should in fact be a reward for additional inflation. Not surprisingly, the contract is equal to zero if there are no spillovers (*b=0)*, since there is no incentive to delegate if there is also no domestic credibility problem.

The second result shows the non-optimality of the subgame perfect equilibrium contracts no matter what the sign of the spillovers is. As we saw the intuition for positive spillovers, we focus now on the strategic complementarity case (*b<0*). In this case, a linear penalty is imposed on inflation to internalise some of the negative externalities the countries impose on each other. However, the size of the penalty is not optimal as is apparent from part (ii) of proposition 4.3. In fact, the penalty is in this case too large. There is no parameter value for which the two contracts are equal and non-cooperative playing combined with strategic complementarity of policies make the countries not achieve the cooperative solution in which they would internalise the externalities and achieve the Pareto optimum. Instead, they impose a too large marginal penalty on inflation.

4.5. Welfare comparison of equilibria

In this section we try to study the equilibria that emerge as a result of delegation through either of the two types of contracts studied, and also their welfare implications. We do this by deriving the reaction functions for the central banks when delegation occurred and they play Nash and showing the types of equilibria that will appear. We note that the reaction functions show how each country's money supply depends and the other country's policy. Moreover, they will also depend on the contracts.

We derive the reaction functions from the set of first order conditions (4.6) as follows:

$$\begin{cases} -ay_1-\mu\pi_1-t_1=0 \\ -ay_2-\mu\pi_2-t_2=0 \end{cases} \Rightarrow \begin{cases} -a(am_1+bm_2-z)-\mu m_1-t_1=0 \\ -a(am_2+bm_1-z)-\mu m_2-t_2=0 \end{cases} \Rightarrow$$

$$\begin{cases} R_1 \\ R_2 \end{cases} = \begin{cases} m_1=-\dfrac{ab}{a^2+\mu}m_2+\dfrac{a}{a^2+\mu}z-\dfrac{1}{a^2+\mu}t_1 \\ m_2=-\dfrac{ab}{a^2+\mu}m_1+\dfrac{a}{a^2+\mu}z-\dfrac{1}{a^2+\mu}t_2 \end{cases} \qquad (4.20)$$

The equilibria can be found at the intersection of R1 and R2, where different values of the t's give rise to different equilibria. We distinguish two cases:

b>0 and *b<0*, for b=0 no spillover being present and thus no delegation needed in the absence of domestic credibility problems in this model.

Case I: b>0

In this case spillovers are positive, i.e. policies are strategic substitutes. As we can see from (4.10), in this case the optimal contracts are *negative*, i.e. delegation takes place by imposing a marginal reward for additional inflation. We study the equilibria diagrammatically in Figure 1, which plots the reaction functions in the space (m_1, m_2).

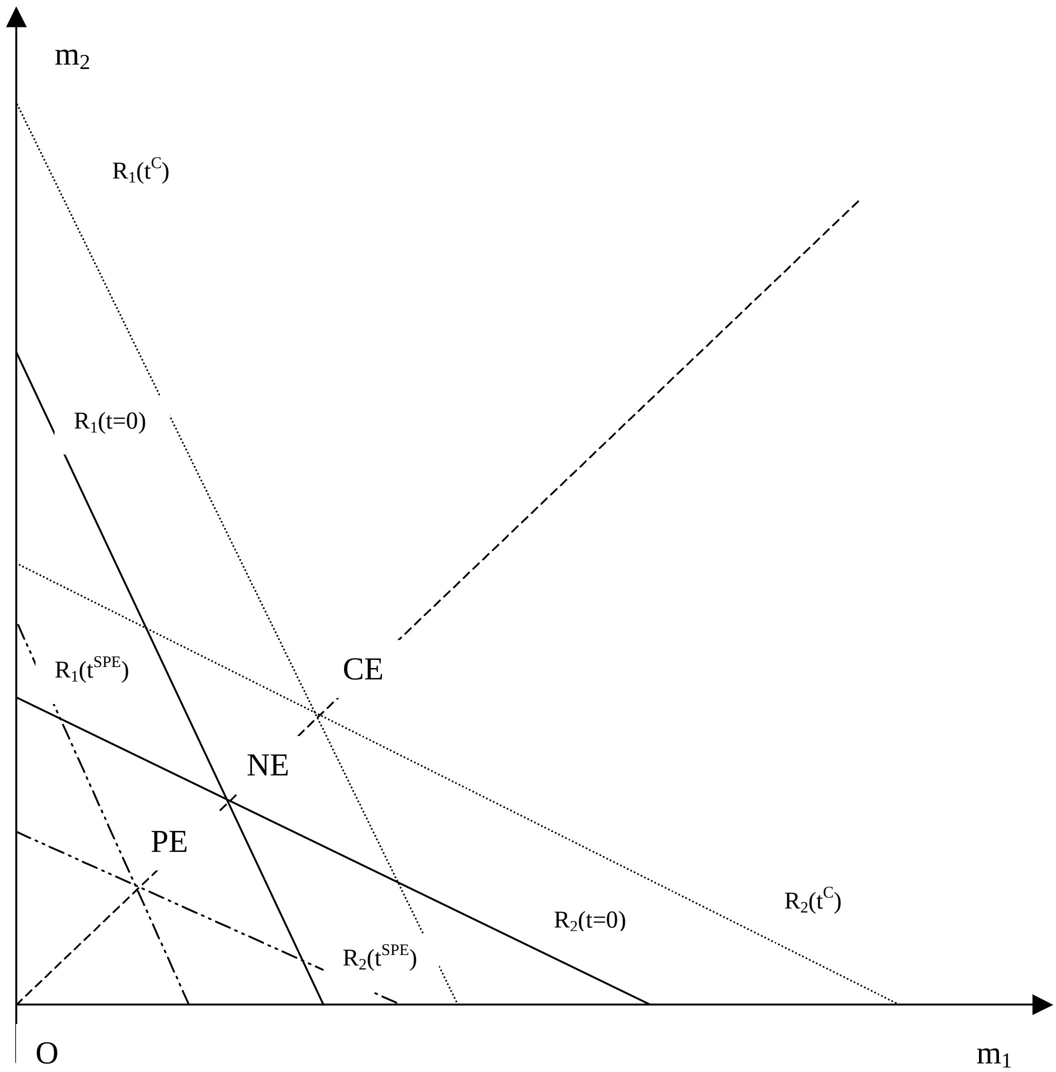


Fig. 1

The reaction functions have negative slopes in this case, due to strategic substitutability of policies. However, the slopes are greater than –1. We start from the case of non-coooperation, identifying the Nash Equilibrium (NE) at the intersection of the two reaction functions (4.20) when no delegation occurs (t=0). As we have shown, in this case the equilibrium has a contractionary bias. There is a region of improvement at the Northeast of NE, where both countries would expand more and CE denotes the symmetric cooperative optimum.

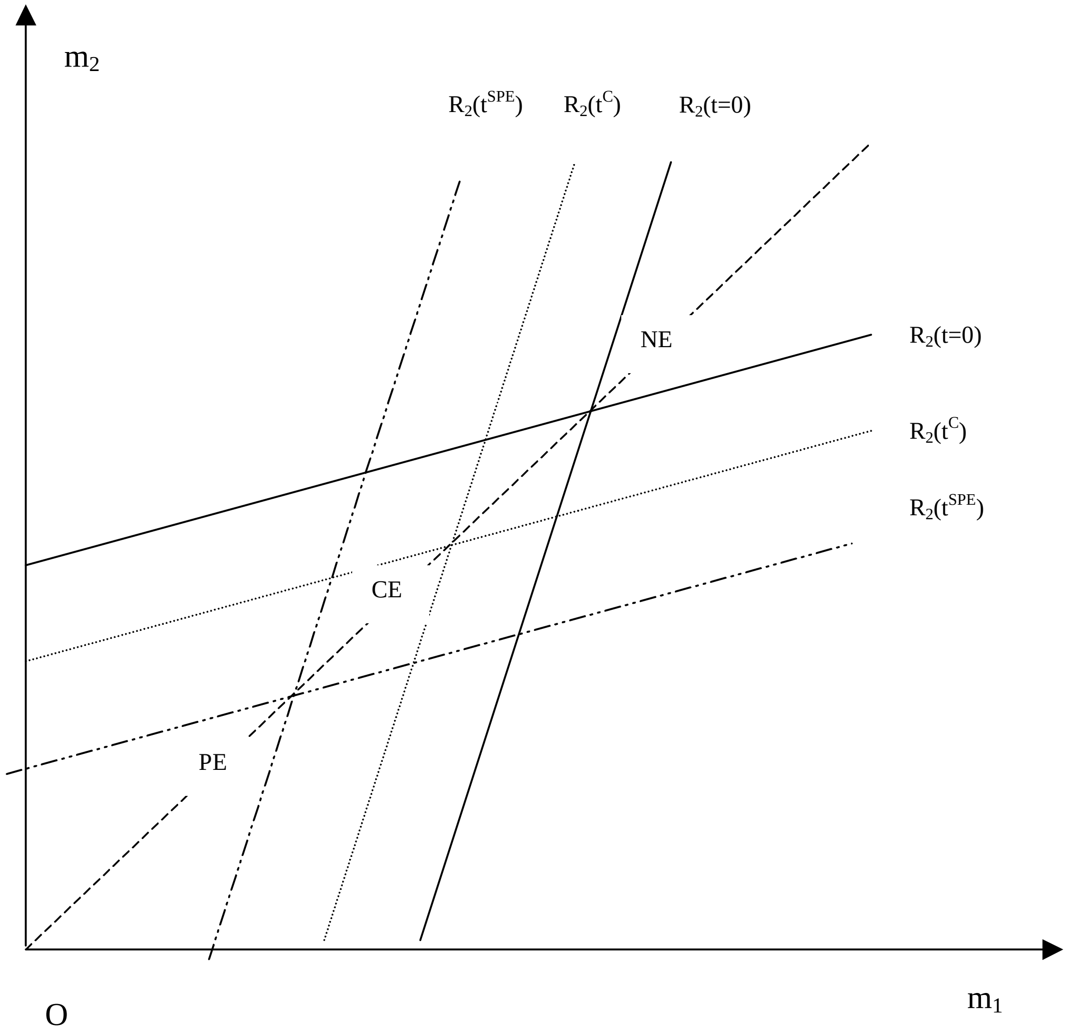
Starting from the reaction functions with no delegation, we observe that what delegation by a contract does, according to eqs. (4.20), is to cause parallel translations of the reaction functions without modifying the slope (if the t's are independent of the m's, as in (4.10) and (4.11)). Thus, delegation by the optimal contracts in (4.10) achieves the cooperative optimum CE in figure 1, as shown analytically in Proposition 1. It causes a translation of the reaction functions such that the equilibrium moves to the Northeast, since contracts are in this case negative.

However, delegation by the Subgame Perfect Equilibrium contracts achieves equilibrium PE, which makes both countries worse of as the contracts are positive as shown in proposition 3(i). A penalty is suboptimally imposed while the optimal policy would be to reward additional inflation to reduce the contractionary bias since there are positive spillovers. The positive externality not being internalised at either of the stages, this results in an aggravation of the contractionary bias. In this case, the Pareto ranking of the equilibria is:

$$CE \succ NE \succ PE \qquad (4.21)$$

Case 2: b<0

In this case spillovers are negative, policies are strategic complements and optimal contracts in (4.10) are positive. Figure 2 deals with the diagrammatic representation of the equilibria in this case. The slopes of the reaction functions are positive and less than one. Again, we start from the Nash equilibrium at the intersection of the reaction functions in the no-delegation case.



The region of improvement is in this case at the Southwest of NE since the Nash Equilibrium has an inflationary bias: both countries would be better of had they expanded less. To internalise this negative externality (*beggar-thy-*

neighbour policy) and achieve the cooperative outcome countries delegate monetary policy to an independent central bank by imposing the optimal linear contract given by (4.10). Diagrammatically, this causes a parallel translation of the reaction functions such that the equilibrium moves to the Southwest since contracts are in this case positive.

Delegation by the subgame perfect equilibrium contracts, however, imposes a too large marginal penalty as shown in Proposition 3(ii). The equilibrium that is achieved through this delegation is SPE, in which both countries contract too much compared to the optimal cooperative equilibrium. Due to the negative externalities and to the non-cooperative behaviour at both stages, there is a *kick-on effect,* making the two countries impose too large penalties on their central banks.

Chapter 5.

Delegation incentives in a two-country model with policy spillovers and credibility problems.

In this chapter we use a model similar to that of Persson and Tabellini (1995, 1996) to study the strategic incentives that the governments would have to delegate monetary policy by imposing a linear contract (or an inflation target) on their central banks. Like in the previous chapter we do this by comparing the optimal and the subgame perfect equilibrium contracts. Given the equivalence result of Chapter 3, we will focus the analysis only on contracts for the ease of comparison with Persson and Tabellini (1996). We note, however, that the results are similar if governments are to delegate policy to inflation targeting central banks.

5.1. The model

The model we use is a simplified version of the one in chapter 2. More specifically, we suppose that there are neither velocity nor speculative shocks ($v=0$ in eq. 3 and $\phi=0$ in eq. 5). Moreover, we assume that the countries share the same stochastic output (or employment) target, i.e. $\theta^*=\theta$ in the loss function (6)) and there is no exchange rate target ($\xi=\xi^*=0$ in the loss function). These assumptions are made to simplify the algebra. Optimal contracts and targets being derived in chapter 3, the presence of shocks does not seem to bring many insights when studying the contracting incentives. We note that we still have the supply shocks that give rise to the stabilisation game. Also, by making the targets equal, we are making the model symmetric, which helps the ease of calculation without affecting the nature of the results. Keeping an output target different from zero preserves the inflation bias. Supposing that the exchange rate target is zero does not seem unrealistic. Svensson (1999, section 5) provides arguments for this assumption. As the EMU is a large and closed economy, it is known that the ESCB does not have exchange rate stability among its goals. Moreover, the specification in the Maastricht treaty is unambiguous: 'the Council, acting unanimously, may conclude formal agreements on an exchange rate system for the Euro' (Article 109(1)). Also, the Council may 'adopt, adjust or abandon the central rates of the Euro within the exchange rate system' by qualified majority vote. Thus, Svensson argues that 'the time-inconsistency problem may not be dead in Europe' (p. 36) since short-term manipulation of monetary policy may occur via exchange-rate management decisions or political pressure on the Council.

Considering that the exchange rate-regime seems to be at the discretion of the Council in the case of the European Union, together with the reduced importance of the exchange rate channel in a closed economy, we decided not to include an exchange rate target in the loss functions.

Our exercise is relevant when studying the cooperation and delegation incentives for countries for which no organisation can *a priori* act as a principal imposing a certain regime at the delegation stage, i.e. for large closed economies like the EMU, USA and Japan. By contrast, Persson and Tabellini's (1996) study was focused on the arrangements between the ins and outs of the EMU within EU and thus the presence of an exchange rate target can be argued.

In our model, the elimination of the exchange rate target eliminates one of the systematic biases (the competitive depreciation bias – second term in eq. 8), leaving the inflation bias unchanged. However, nothing else will change in the model and this should not affect the insights regarding the optimality of delegation.

Specifically, we will work with the following model:

$$z = s + q - q^* \qquad (5.1)$$

$$p = q + \beta z \qquad (5.2)$$

$$q = m \qquad (5.3)$$

$$x = \gamma(q - q^e) - \varepsilon \qquad (5.4)$$

$$z = \delta(x - x^*) \qquad (5.5)$$

$$L = \frac{1}{2}\left[p^2 + \lambda(x - \theta)^2\right] \qquad (5.6)$$

The explanations of the reduced forms are the same as in Chapter 3, taking into account the simplifications we made. The foreign country would be modelled in the same way, with the variables having an asterisk (except for the output target). The timing of the game is the same as in chapter three and we restate it here:

(i) policy targets $\tau=\theta$ are revealed, (ii) private expectations (q^e, q^{*e}) are formed, (iii) structural shocks $\omega=(\varepsilon,\varepsilon^*)$ are revealed, (iv) policies (m, m^*) are simultaneously set, (v) macroeconomic outcomes are realised.

5.2 Optimal policy

As we did in chapter 3, we try to find the equilibrium in which we suppose that the two authorities decide *to cooperate* with each other and *commit ex ante* to follow the optimal state-contingent policy rules. We then will use this as a benchmark to find the *contracts* that would implement this optimum. In this case the authorities minimise the expected value of the joint loss function $E(L+L^*)$ taking as a constraint the fact that private sector has rational expectations $q^e=E(q/\theta)$, $q^{e*}=E(q^*/\theta)$ formed conditionally on the observed value of the output target.

Lemma 5.1

In the cooperative and ex-ante commitment optimum monetary authorities choose their money supplies to fulfil the following rules:

$$\begin{aligned} &p(\tau,\omega)+\lambda\gamma x(\tau,\omega)+\beta\delta\gamma(p(\tau,\omega)-p^*(\tau,\omega))=0 \\ &p^*(\tau,\omega)+\lambda\gamma x(\tau,\omega)+\beta\delta\gamma(p^*(\tau,\omega)-p(\tau,\omega))=0 \end{aligned} \tag{5.8}$$

The policy rules are given by:

$$\begin{aligned} m^{cc} &= \frac{\lambda\gamma}{1+\lambda\gamma^2}\varepsilon+\beta\delta\frac{(1+2\beta\delta\gamma-\gamma)}{(1+\lambda\gamma^2)\left((1+2\beta\delta\gamma)^2+\lambda\gamma^2\right)}(\varepsilon-\varepsilon^*) \\ m^{cc*} &= \frac{\lambda\gamma}{1+\lambda\gamma^2}\varepsilon^*-\beta\delta\frac{(1+2\beta\delta\gamma-\gamma)}{(1+\lambda\gamma^2)\left((1+2\beta\delta\gamma)^2+\lambda\gamma^2\right)}(\varepsilon-\varepsilon^*) \end{aligned} \tag{5.9}$$

Proof *– please find Appendix for Chapter 5.*

Due to the absence from the loss function of the exchange rate, there is no direct effect on the losses induced through the exchange rate directly. However, the indirect effects through inflation are still present. The rest of the interpretation is as in chapter 3, eq. 3.7.

In the optimal equilibrium (5.9) the authorities stabilise both domestic supply shocks and relative foreign supply shocks. That is because domestic shocks have a direct effect on output and inflation and the difference between home and foreign shocks affect the variables through the real exchange rate appreciation or depreciation.

5.3. Non-cooperative and discretionary equilibrium

The fourth-best equilibrium is again obtained when countries cannot precommit to either each other or the private sector to deliver m^{cc}. Minimisation of the loss function is done taking both foreign policy and expectations as given (as in section 3.3 and Appendix ch. 3), the first order conditions being:

$$
\begin{aligned}
&p(\tau,\omega)+\lambda\gamma x(\tau,\omega)+\beta\delta\gamma\left(p(\tau,\omega)-p^*(\tau,\omega)\right)=\lambda\gamma\theta-\beta\delta\gamma p^*(\tau,\omega)\\
&p^*(\tau,\omega)+\lambda\gamma x^*(\tau,\omega)-\beta\delta\gamma\left(p(\tau,\omega)-p^*(\tau,\omega)\right)=\lambda\gamma\theta-\beta\delta\gamma p(\tau,\omega)
\end{aligned}
\qquad (5.10)
$$

It is clear that compared to Lemma 3.2 in the more general model in chapter 3 some of the terms have disappeared. However, the two incentives are represented by the two terms on the right hand side. The credibility constraint leads to the inflation bias (first term), whereas the 'individual rationality' constraint distorts stabilisation of shocks when the countries do not take into account the spillovers they generate when responding to shocks by real appreciation / depreciation.

5.4. Optimal inflation contracts

As in chapter 3.4, we assume an international benevolent principal imposes linear performance contracts on both central banks to which monetary policy is delegated, where the penalties are linear of the form $T(p)=tp$ and $T^*(p^*)=t^*p^*$. The central banks will minimise the sum E($L+T$) in a discretionary and non-cooperative manner. Again, all this does is to introduce in the first order conditions (5.10) an additional term on the left hand side equal to $(1+\beta\delta\gamma)t$, respectively $(1+\beta\delta\gamma)t^*$.

$$
\begin{aligned}
&p(\tau,\omega)+\lambda\gamma x(\tau,\omega)+\beta\delta\gamma\left(p(\tau,\omega)-p^*(\tau,\omega)\right)+t(1+\beta\delta\gamma)=\lambda\gamma\theta-\beta\delta\gamma p^*(\tau,\omega)\\
&p^*(\tau,\omega)+\lambda\gamma x^*(\tau,\omega)-\beta\delta\gamma\left(p(\tau,\omega)-p^*(\tau,\omega)\right)+t^*(1+\beta\delta\gamma)=\lambda\gamma\theta-\beta\delta\gamma p(\tau,\omega)
\end{aligned}
\qquad (5.10')
$$

Comparing (5.10') with (5.8), Proposition 5.1 becomes straightforward.

Proposition 5.1

There exists a unique pair of state-contingent inflation contracts implementing the cooperative optimum, the marginal penalties being given by:

$$t^{cc}(\tau,\omega)=\frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma\theta-\beta\delta\gamma p^*(\tau,\omega)\right)$$
$$t^{cc*}(\tau,\omega)=\frac{1}{1+\beta\delta\gamma}\left(\lambda\gamma\theta-\beta\delta\gamma p(\tau,\omega)\right) \tag{5.11}$$

where p and p are evaluated at the cooperative and commitment equilibrium. In terms of shocks, these are expressed as:*

$$t^{cc}(\tau,\omega)=\frac{b}{1+a}\theta-\frac{ab}{(1+a)(1+b\gamma)}\varepsilon^*-\frac{a^2(1+b+2a)}{\left((1+2a)^2+b\gamma\right)(1+b\gamma)(1+a)}(\varepsilon-\varepsilon^*)$$
$$t^{cc*}(\tau,\omega)=\frac{b}{1+a}\theta-\frac{ab}{(1+a)(1+b\gamma)}\varepsilon+\frac{a^2(1+b+2a)}{\left((1+2a)^2+b\gamma\right)(1+b\gamma)(1+a)}(\varepsilon-\varepsilon^*) \tag{5.12}$$

where $a=\beta\delta\gamma$, $d=\beta\delta$, and $b=\lambda\gamma$ to simplify the notation.

***Proof** – (5.11) results by comparing (5.10') with (5.8) and (5.12) by direct substitution of m^{CC}'s in (5.11).*

Again, the complicated expressions in (5.12) have an intuitive interpretation. First of all, the state-contingency of contracts is preserved. The first term makes the penalty eliminate the systematic inflation bias. The other two terms correct the failure of the central banks to internalise the policy spillovers by sub-optimally responding to shocks. The penalty is weaker, e.g. for the home country if the foreign country suffers an adverse supply shock or a less severe supply shock as compared to the home country's. In the two cases ($\varepsilon^*>0$ and $\varepsilon-\varepsilon^*>0$) $p^*>0$ at the equilibrium and there is a contractionary bias of home country's policy and a smaller penalty is needed to compensate for this bias.

We note that even if the model were perfectly symmetric, the state-contingency of contracts would still be needed. If we suppose $\varepsilon=\varepsilon^*$, as in most of the policy-cooperation literature, the contracts in (5.12) would still comprise the first two terms. Failure to internalise the externalities (suboptimal response to the common supply shock due to neglecting the effect on foreign inflation of changing the real exchange rate) gives rise to the need for eliminating the stabilisation bias by the second term, which would be the same for both countries.

These results are similar to those presented in Chapter 3. However, we now make the same argument as in Chapter 4. For the cooperative and commitment optimum to be implemented in a fully decentralised manner, we see no reason for the two countries to cooperate at the delegation stage such that they would both delegate with the optimal contracts given by (5.11) or (5.12). This is clearly inconsistent with the notion of *'non-cooperative implementation of the cooperative optimum'* argued by Person and Tabellini (1995, 1996). We do the same type of exercise as in Chapter 4, i.e. we study the strategic incentives of the governments at the delegation stage.

5.5 Strategic incentives for delegation

In this section we distinguish between the two stages of the monetary policy game, as in Chapter 4. The stages of the game are: (i) *delegation* – each government chooses independently and simultaneously a central bank with a certain form of delegation; (ii) the central banks elected at stage (i) choose the policies. At stage (ii), the time sequence of the subgame is that presented in section 5.1.

To study the delegation incentives, we have to find the *Subgame Perfect Equilibrium* in terms of contracts of the two-stage game by solving the game by backward induction as described in 4.2. We will compare these with the optimal contracts derived earlier.

5.5.1. Nash Equilibrium money supplies

To find the subgame perfect equilibrium we first have to solve for the equilibrium policies at stage (ii) supposing delegation by contracts took place

(t's are given) and central banks choose policies non-cooperatively. The solution method is as follows: one has to take conditional (on observed targets) expectations of the first order conditions (5.10') to find the expected values of the money supplies. Then these are substituted back in (5.10') and the resulting two-equation system is solved for m^{NE} *and* m^{NE*} giving the result in Lemma 5.2

Lemma 5.2

Given delegation by a pair of contracts (t,t) at stage one, there exists an unique Nash Equilibrium of the game of the central banks at stage two, given by (5.13):*

$$m^{NE} = -t + \frac{\lambda\gamma}{1+\beta\delta\gamma}\theta + \frac{\lambda\gamma}{1+\beta\delta\gamma+\lambda\gamma^2}\varepsilon + \beta\delta\frac{(1+\beta\delta\gamma)^2}{(1+\beta\delta\gamma+\lambda\gamma^2)((1+\beta\delta\gamma)(1+2\beta\delta\gamma)+\lambda\gamma^2)}(\varepsilon-\varepsilon^*)$$

$$m^{NE}{}^* = -t^* + \frac{\lambda\gamma}{1+\beta\delta\gamma}\theta + \frac{\lambda\gamma}{1+\beta\delta\gamma+\lambda\gamma^2}\varepsilon^* - \beta\delta\frac{(1+\beta\delta\gamma)^2}{(1+\beta\delta\gamma+\lambda\gamma^2)((1+\beta\delta\gamma)(1+2\beta\delta\gamma)+\lambda\gamma^2)}(\varepsilon-\varepsilon^*)$$

Proof *– please find Appendix for ch. 5.*

Ignoring the t's for the moment, we observe that the absence of pre-commitment makes the inflation bias appear in the policy rule. Also, the lack of cooperation and thus of internalisation of the externalities makes the countries respond sub-optimally to shocks (one may see that the coefficients on ε, ε^* *and* ε-ε^* are smaller in the Nash Equilibrium than in the cooperative and commitment equilibrium.

As to the way the t's appear in the policy rules it may seem somehow counterintuitive that each country's money supply depends only on its own marginal penalty despite the policy spillovers. More realistically, given strategic interaction each country's policy rule should depend on both contracts. Using the original specification in Persson and Tabellini (1996) does not modify this feature of the model, i.e. introducing velocity and speculative shocks and non-homogeneity of targets leaves the result unchanged.

This property of the model comes from another assumption, i.e. from equation (5.3) in which it is supposed that there is a one-to-one relationship between money supply and inflation. If the coefficient on the money supply would be constrained to be an arbitrary constant k where $k \neq 1$ the policy rule in each country would depend on both marginal penalties. However, this complicates the algebra without modifying the basic idea behind this exercise.

5.5.2. Subgame Perfect Equilibrium Contracts

Given the Nash equilibrium money supplies, we move one stage backward to study the problem of the governments choosing the marginal penalties in a decentralised, non-cooperative manner. At this stage each government minimises its loss function with respect to t. To do this, we substitute the Nash equilibrium money supplies (5.13) in the loss functions (5.6) and minimise (5.6) with respect to t.

Proposition 5.2.

There exists an unique subgame perfect Nash equilibrium at stage one in which both governments delegate monetary policy to a central bank imposing a linear inflation contract with the marginal penalties given by (5.15), where the first order conditions (5.14) are fulfilled.

$$\begin{aligned} -p\big|_{\substack{m=m^{NE} \\ m^*=m^{NE*}}} &= 0 \\ -p^*\big|_{\substack{m=m^{NE} \\ m^*=m^{NE*}}} &= 0 \end{aligned} \tag{5.14}$$

$$t^{SPE}=\frac{b}{1+a}\theta+\frac{b}{1+a+b\gamma}\varepsilon*+\frac{b\left[(1+a)^2+b\gamma\right]}{(1+a+b\gamma)\left[(1+a)(1+2a)+b\gamma\right]}(\varepsilon-\varepsilon*)$$
$$t^{SPE}*=\frac{b}{1+a}\theta+\frac{b}{1+a+b\gamma}\varepsilon-\frac{b\left[(1+a)^2+b\gamma\right]}{(1+a+b\gamma)\left[(1+a)(1+2a)+b\gamma\right]}(\varepsilon-\varepsilon*) \tag{15}$$

where $a=\beta\delta\gamma$, $d=\beta\delta$, *and* $b=\lambda\gamma$ to simplify the notation.

Proof *– please find Appendix for chapter. 5.*

Note that in this equilibrium:

$$\frac{\partial p^{NE}}{\partial t}=\frac{\partial p*^{NE}}{\partial t*}=-1$$
$$\frac{\partial x^{NE}}{\partial t}=\frac{\partial x*^{NE}}{\partial t*}=0$$

Increasing the penalty in one country causes a one-to-one decrease in the country's inflation but does not affect the other country's inflation or either of the outputs.

Comparing the subgame perfect equilibrium t's (5.15) and the cooperative and commitment t's (5.10) two conclusions arise.

Firstly, we observe that the term eliminating the systematic domestic inflation bias is the same and equal to $\frac{\lambda\gamma}{1+\beta\delta\gamma}\theta$. Thus, we conclude that the presence of policy spillovers and strategic interaction does not influence the delegation of monetary policy in what concerns elimination of domestic incentives. Incentives to delegate for this reason are the same whether policy is conducted cooperatively or purely decentralised (non-cooperatively).

On the contrary, however, stabilisation of shocks is different. It is apparent from (5.15) that the subgame perfect equilibrium contracts have sub-optimal responses to shocks.

Consider again the case where $\varepsilon*>0$ (an adverse supply shock in the foreign country) and $\varepsilon-\varepsilon*>0$ (a less severe a larger favourable supply shock in the foreign country) so that in equilibrium $p^*>0$ and there is a contractionary bias of home country's monetary policy. Both shock stabilisation coefficients in the optimal contract of the home country (5.10) are negative, i.e. the optimal penalty decreases to eliminate this contractionary bias. By contrast, the non-cooperative contract for the home country (5.15) has both shock stabilisation coefficients positive, i.e. the penalty is increasing in ε^* and ε-ε^*. This aggravates the contractionary bias of home policy by giving incentives to reduce inflation. Playing Nash at both stages, countries fail to internalise the externalities they impose on each other. They ignore the spillover effects (i.e. the impact on the other country's loss function) they generate when responding to shocks by real exchange rate appreciation or depreciation.

Another way to see this is to look at the first order conditions for the delegation stage (5.14). In the perfect equilibrium, the inflation in both countries is zero. However, in the presence of shocks as in the example considered above the optimal response could be a positive inflation. In the perfect equilibrium, to achieve this zero-inflation a positive increasing penalty is needed.

Of course that in the converse case, where there is a favourable supply shock in the foreign country ($\varepsilon^*<0$) and a relatively larger adverse shock (or relatively smaller favourable shock) in the foreign country (ε-$\varepsilon^*<0$) the reverse is true. There is an expansionary bias of the home monetary policy generated by negative spillovers, which is eliminated by the optimal contracts by imposing larger penalties. In the perfect equilibrium however, the spillovers are ignored and the penalties are sub-optimally low compared to the optimal ones. In this case a smaller penalty is imposed to achieve a zero inflation as in the FOC's (5.14) when in fact at the equilibrium $p^*<0$.

Even when the shocks are perfectly correlated (i.e. in the symmetric case *ε=ε*)* the responses to the common shock are suboptimal.

$$t^{cc} = t^{cc*} = \frac{b}{1+a}\theta + \frac{b}{1+a+b\gamma}\varepsilon \qquad (5.16)$$

An adverse common supply shock generates an increase in the inflation penalty, although the optimal response (given by (5.12)) would be to reduce the penalty to eliminate the deflationary bias that arises as a result of such a shock.

However, we recall the arguments of Chapter 2 and 3 regarding the difficulties to enact state-contingent contracts (or targets) and we analyse the case in which only state-independent contracts are feasible. In this case the contracts are given by:

$$t^{cc} = t^{cc*} = t^{SPE} = t^{SPE*} = \frac{\lambda\gamma}{1+\beta\delta\gamma}E(\theta) \qquad (5.17)$$

Contracts in this case are equal and they eliminate the inflation bias but do not affect shock stabilisation. However, this is not surprising since the only source of strategic interaction in the model consists of countries' responses to supply shocks through real exchange rate appreciation/depreciation. Ignoring that, it is natural to obtain equality of contracts as if the two countries were not linked at all.

One may perform the same type of exercise as in Chapter 4 comparing the equilibria that arise in the two cases with delegation and in the no-delegation case. This is done by substituting the cooperative and commitment *t's* and the subgame perfect equilibrium *t's* in the reaction functions given by Lemma 5.2. While optimal contracts make these equal to m^{CC}*'s*, it is clear that delegation by t^{SPE} induces suboptimal responses to shocks compared to the optimal *m's* due to the lack of internalisation of the externalities. The externalities are a result of countries not taking into account the impact, through real exchange rate appreciation/depreciation, their policy actions have on the other country's loss function.

To implement the cooperative optimum, *decentralised delegation is thus not efficient*. In a way resembling McCallum's (1985) critique, in a two-country framework the cooperation problem is merely relocated by the delegation solution from the policy rules choosing to the delegation stage.

6. Conclusions

Can the cooperative equilibrium in international monetary policy games be implemented in a non-cooperative set-up by delegation? What is the difference between delegating with inflation contracts and inflation targets? If there exist optimal contracts and targets implementing the collusive outcome, is implementation entirely non-cooperative, i.e. do governments have the appropriate incentives to delegate with exactly those contracts and targets that achieve the desired outcome? If not, what may make them do so and how *non-cooperative* will implementation be?

The answer to the first question appears to be affirmative. It is a general result in game theory known as *the Folk Theorem in Delegation Games*, derived by Fershtman e al. (1991) and extended by Polo and Tedeschi (1999). Its application to international monetary policy cooperation suggests that there are indeed *state-contingent linear contracts* implementing the cooperation (and ex-ante commitment) optimum as shown by Persson and Tabellini (1995). We tried to summarise this type of results in the first part of chapter 3.

However, provided the difficulties with implementing even simple linear contracts listed in section 2.4 we use Svensson's (1995, 1997) idea and show that the same outcome can be achieved by both governments *delegating to inflation-targeting central banks*. Moreover, the *targets will be state-contingent*. This result reminds of Lockwood's (1997) and Svensson's (1997) results in a closed-economy but dynamic context, where dynamics is introduce through persistence in a real variable. However, in our model inflation targets and contracts are *perfectly equivalent*, whereas in Svensson's model a state-contingent inflation target has to be augmented with a "Rogoff' conservative central banker to achieve the same *second-best* *equilibrium* as a state-contingent inflation contract. In our model no need for such conservativeness (in the Rogoff 1985a sense) of the central banks arises.

The result is also different from Persson and Tabellini (1996), who show in the same model that zero inflation targets achieve a third-best equilibrium when studying optimal monetary policy arrangements between the 'ins' and 'outs' of the EMU inside the European Union.

Finally, we think this result may be interpreted as another application of the folk theorem for delegation games in Fershtman et al. (1991). Delegating to an inflation-targeting central bank can be viewed as delegating to an agent with a *distorted utility function*, where the agent's utility function is public information, which is true for inflation-targeting regimes characterised by a high degree of transparency.

The answer to the third question, however, requires another type of exercise. All we showed up to now is that there *exist* inflation targets and contracts implementing the cooperative outcome. However, implementation by exactly these optimal contracts requires *cooperation at the delegation stage* or *coordination from a benevolent international principal* who chooses between the multiplicity of possible delegation schemes. We see this hard to reconcile with the idea of implementing the cooperative equilibrium by *decentralised, non-cooperative mechanisms* argued by Persson and Tabellini (1995, 1996). Thus, in chapters 4 and 5 we study the incentive-compatible contracts by which governments would delegate when countries are playing non-cooperatively at both stages (delegation and policy instrument-choosing stages).

In chapter 4 we do this type of exercise in a simple model with policy spillovers but no domestic inflation bias (thus cooperation is ex-post Pareto optimal since the Rogoff 1985b result does not apply). We first show that state-contingent contracts or targets can be designed to achieve the Pareto optimal cooperative outcome and delegation arises even in the absence of domestic dynamic inconsistency problems. We then study the S*ubgame Perfect Nash Equilibrium* of the two-stage game by solving the game by backward induction and finding the contracts consistent with the governments' incentives when playing Nash at both stages. We then compare the two contracts and conclude that governments would not have the right strategic incentives to delegate by the contracts or targets that implement the cooperative optimum. The perfect equilibrium contracts are always positive, while in the case of positive spillovers the optimal contract is in fact a negative penalty, i.e. a reward for additional inflation. Imposing a penalty would only exacerbate the deflationary bias. Moreover, the perfect equilibrium marginal penalty is always higher than the optimal one, independent of the sign of the spillovers. When spillovers are negative, the penalty is too large since there is a *kick-on effect* induced by non-cooperation and non-internalisation of the negative externalities at either of the stages. We show this type of results in a more intuitive set-up, by studying the welfare implications of different equilibria diagrammatically.

Our results are in contrast with those of Dolado et al (1994) who study delegation to a 'Rogoff' conservative central banker in the same type of model. However, while they show that the need for delegation arises, they do not show how the cooperative optimum can be implemented by delegation. Our results are similar in one respect: the perfect equilibrium in our case (delegation by contracts and targets) aggravates the deflationary bias when there are positive spillovers, which is also the case with delegation to inflation-averse central bankers studied by Dolado et al. However, this is not the case with optimal contracts and targets studied in our case.

The introduction of a domestic inflation bias does not change the essence of the results. We perform the same type of comparison in a model with both policy spillovers and domestic credibility problems (an adaptation of Persson and Tabellini, 1996). The non-equivalence of contracts is preserved. There is again a failure to internalise the externalities that leads to suboptimal responses to shocks as described in the text. However, there is one qualification to this result. Following a point made by, *i.a.*, Beetsma and Jensen (1998) that state-contingent contracts or targets may be impossible to enact we observe that state-independent inflation contracts would be in our case equivalent. The term leading to the elimination of the inflation bias is the same for both perfect equilibrium and optimal contracts. This is not surprising since the only source of strategic interaction is given by responses to shocks through exchange rate appreciation/depreciation. However, state-independent contracts and targets induce suboptimal responses to shocks, thus there is no internalisation of inter-country externalities but only of the externality each policymaker imposes on private expectations.

Our results suggest that the presence of an international principal would be needed to insure the cooperative outcome is implemented by delegation. This way however, the game is *not completely non-cooperative* but implies, if not cooperation, at least *coordination* by the international principal at the delegation stage. Thus, it seems delegation by targets and contracts merely relocates the problem from the policy choosing to the *delegation stage*. This resembles McCallum's (1995) critique concerning contracts and targets in the domestic policy context.

Why then wouldn't this same principal intervene to impose a commitment technology so that countries cooperate in the first place? We think it is far more plausible to assume that countries would commit with respect to their agents (i.e. Central Banks) than to assume they would enter binding agreements that may sometimes lower their welfare and make them give up sovereignty. Moreover, the need of delegation also arises independently of the presence nature of the spillovers when an inflation bias is present.

Appendix for Chapter 3

Proof of Lemma 3.1

Rewrite (1) and (5) as a function of m's and m^e's:

$$\begin{aligned}
p &= m + v + \beta\delta\gamma\left(m + v - m^e - m^* - v^* + m^{e*}\right) - \beta\delta(\varepsilon - \varepsilon^*) + \beta\phi \\
p^* &= m^* + v^* - \beta\delta\gamma\left(m + v - m^e - m^* - v^* + m^{e*}\right) + \beta\delta(\varepsilon - \varepsilon^*) - \beta\phi \\
z &= \delta\gamma\left(m + v - m^e - m^* - v^* + m^{e*}\right) - \delta(\varepsilon - \varepsilon^*) + \phi
\end{aligned} \qquad (A1.1)$$

The authorities minimise E(L+L) subject to the constraint that expectations are formed rationally conditional on observed targets:*

$$\begin{aligned}
&\min_{m, m^e, m^*, m^{e*}} E(L + L^*) \\
&s.t. \\
&m^e = E[m|\tau] \\
&m^{e*} = E[m^*|\tau]
\end{aligned} \qquad (A1.2)$$

The Lagrangean of (A1.2) is:

$$\Lambda = E(L + L^*) - \rho\left(m^e - E[m|\tau]\right) - \rho^*\left(m^{e*} - E[m^*|\tau]\right) \qquad (A1.3)$$

Take for example the home country, the first order conditions with respect to m and m^e are:

$$\begin{aligned}
&\frac{\partial\Lambda}{\partial m} = (1 + \beta\delta\gamma)p + \lambda\gamma(x - \theta) + \mu\delta\gamma(z - \xi) + \rho - \beta\delta\gamma p^* + \mu\delta\gamma(z + \varepsilon^*) = 0 \\
&\frac{\partial\Lambda}{\partial m^e} = E\left[-\beta\delta\gamma p - \lambda\gamma(x - \theta) - \mu\delta\gamma(z - \xi) - \rho + \beta\delta\gamma p^* - \mu\delta\gamma(z + \varepsilon^*)|\tau\right] = \\
&= -\beta\delta\gamma m^e + \lambda\gamma\theta + \mu\delta\gamma\xi - \rho + \beta\delta\gamma m^{e*} - \mu\delta\gamma\xi^*
\end{aligned} \qquad (A1.4)$$

Eliminating the Lagrangean multiplier ρ from (A1.4) gives:

$$\begin{aligned}
&(1 + \beta\delta\gamma)p + \lambda\gamma(x - \theta) + \mu\delta\gamma(z - \xi) - \beta\delta\gamma p^* + \mu\delta\gamma(z + \varepsilon^*) + \beta\delta\gamma\left(m^{e*} - m^e\right) + \\
&\lambda\gamma\theta + \mu\delta\gamma(\xi - \xi^*) = p + \beta\delta\gamma(p - p^*) + \lambda\gamma x + 2\mu\delta\gamma z - \beta\delta\gamma\left(m^e - m^{e*}\right) = 0
\end{aligned} \qquad (A1.5)$$

Taking conditional expectations of (A1.5) we get

$$m^e - \beta\delta\gamma\left(m^e - m^{e*}\right) + \beta\delta\gamma\left(m^e - m^{e*}\right) = 0 \Rightarrow m^e = 0 \qquad (A1.6)$$

A similar derivation for the foreign country gives $m^{e*}=0$, so (A1.5) becomes equation (3.7) and Lemma 3.1 is demonstrated.

Proof of Lemma 3.2

Monetary authorities play Nash, minimising their own loss function taking as given both the other country's policy and private expectations.

The first order conditions are:

$$\left.\frac{\partial E(L)}{\partial m}\right|_{m^e, m^*, m^{e}*} = (1+\beta\delta\gamma)p + \lambda\gamma(x-\theta) + \mu\delta\gamma(z-\xi) = 0$$
$$\left.\frac{\partial E(L^*)}{\partial m^*}\right|_{m^e, m, m^{e}*} = (1+\beta\delta\gamma)p^* + \lambda\gamma(x^* - \theta^*) - \mu\delta\gamma(z+\xi^*) = 0 \qquad (A1.7)$$

Taking expectations of these results in:

$$(1+\beta\delta\gamma)m^e = \lambda\gamma\theta + \mu\delta\gamma\xi$$
$$(1+\beta\delta\gamma)m^{e}* = \lambda\gamma\theta^* + \mu\delta\gamma\xi^*$$

Substituting these back in (A1.7) and rearranging gives (3.8) in Lemma 3.2.

Appendix for Chapter 5

Proof of Lemma 5.1

We can substitute everything so that we get p and z as a function of m, m, m^e and m^{e*} we have the following equations:*

$$\begin{aligned} &p = (1+\beta\delta\gamma)m - \beta\delta\gamma m* - \beta\delta\gamma\left(m^e - m^e*\right) - \beta\delta(\varepsilon - \varepsilon*) \\ &z = \delta\gamma\left(m - m^e - m* + m^e*\right) - \delta(\varepsilon - \varepsilon*) \\ &p* = (1+\beta\delta\gamma)m* - \beta\delta\gamma m + \beta\delta\gamma\left(m^e - m^e*\right) + \beta\delta(\varepsilon - \varepsilon*) \end{aligned} \qquad (A3.1)$$

To get the first order conditions we simply observe that this is a special case of Lemma 3.1 and we refer to Appendix Ch. 3 for the proof, the expressions being those given in (5.8). By the same argument as there, m^e=m^{e}=0.*

To get the policy rules (5.9) we substitute (A3.1) in the first order conditions using m^e=m^{e}=0 to obtain:*

$$\begin{aligned} &\left[(1+\beta\delta\gamma)^2 + (\beta\delta\gamma)^2 + \lambda\gamma^2\right]m - 2\beta\delta\gamma(1+\beta\delta\gamma)m* - \lambda\gamma\varepsilon - \beta\delta(1+2\beta\delta\gamma)(\varepsilon - \varepsilon*) = 0 \\ &\left[(1+\beta\delta\gamma)^2 + (\beta\delta\gamma)^2 + \lambda\gamma^2\right]m* - 2\beta\delta\gamma(1+\beta\delta\gamma)m - \lambda\gamma\varepsilon* + \beta\delta(1+2\beta\delta\gamma)(\varepsilon - \varepsilon*) = 0 \end{aligned} \qquad (A3.2)$$

We solve this by eliminating m, using the same change of notation as in Chapter 3, i.e. a=βδγ, b=λγ, d=βδ, and observing that dγ=a to get:*

$$\begin{aligned} &\left\{\left[(1+a)^2 + a^2 + b\gamma\right]^2 - 4a^2(1+a)^2\right\}m = b\left[(1+a)^2 + a^2 + b\gamma\right]\varepsilon + 2ab(1+a)\varepsilon* - \\ &- d(1+2a)\left[(1+a)^2 + a^2 + b\gamma - 2a(1+a)\right](\varepsilon - \varepsilon*) \\ &\therefore m^{CC} = \frac{b}{1+b\gamma}\varepsilon + \frac{d(1+2a) - a}{\left[(1+2a)^2 + b\gamma\right](1+b\gamma)}(\varepsilon - \varepsilon*) \end{aligned}$$

After changing back the notation, (5.9) is obtained.

Proof of Lemma 5.2

The first order conditions with delegation by a contract are given by:

$$\begin{aligned} &(1+\beta\delta\gamma)p + \lambda\gamma x - \lambda\gamma\theta + (1+\beta\delta\gamma)t = 0 \\ &(1+\beta\delta\gamma)p* + \lambda\gamma x* - \lambda\gamma\theta* + (1+\beta\delta\gamma)t* = 0 \end{aligned} \qquad (A3.3)$$

Take expectations of these conditional on observed targets and use the change of notation described above to get:

$$m^e = \frac{b}{1+a}\theta - t$$
$$m^e* = \frac{b}{1+a}\theta - t* \qquad (A3.4)$$

Substitute (A3.4.) in (A3.3) and change notation to get:

$$\left[(1+a)^2 + b\gamma\right]m - a(1+a)m* + \left[(1+a)^2 + b\gamma\right]t - a(1+a)t* - b\frac{1+a+b\gamma}{1+a}\theta - b\varepsilon -$$
$$- d(1+a)(\varepsilon - \varepsilon*) = 0$$
$$\left[(1+a)^2 + b\gamma\right]m* - a(1+a)m + \left[(1+a)^2 + b\gamma\right]t* - a(1+a)t - b\frac{1+a+b\gamma}{1+a}\theta* - b\varepsilon* +$$
$$+ d(1+a)(\varepsilon - \varepsilon*) = 0 \qquad (A3.5)$$

Eliminating m to solve for m and rearranging gives:*

$$m^{NE} = -t + \frac{b}{1+a}\theta + \frac{d(1+a)}{(1+a)(1+2a)+b\gamma}(\varepsilon - \varepsilon*) - \frac{ab(1+a)}{[(1+a)(1+2a)+b\gamma](1+a+b\gamma)}(\varepsilon - \varepsilon*) +$$
$$\frac{b}{1+a+b\gamma}\varepsilon$$

Solving and changing notation gives the result in Lemma 5.2.

Note that the independence of m of t and of m* of t comes from stage (A3.5), where the coefficients are perfectly symmetric and thus will be reduced. This is however, as specified in the text, a special case due to the unit coefficient of m in the equation of p. Changing this makes the money supplies dependent on both contracts.*

Proof of Proposition 5.2

After finding the Nash Equilibrium at stage two we substitute the NE money supplies in the loss functions without delegation and minimise with respect to t and t. To simplify notation, we use:*

$$m^{NE} = -t + R$$
$$m^{NE}* = -t* + R* \qquad (A3.6),$$

where R and R are the rest of the terms in the NE money supplies, R and R* being independent of t and t*.*

Substitute using (A3.6) and (A3.4) in the loss function to get:

$$L = \frac{1}{2}\left\{ \begin{array}{l} \left[(1+\beta\delta\gamma)(-t+R) - \beta\delta\gamma(-t^*+R^*) - \beta\delta\gamma(t^*-t) - \beta\delta(\varepsilon-\varepsilon^*)\right]^2 + \\ \lambda\left[\gamma(-t+R+t) - \varepsilon - \theta\right]^2 \end{array} \right\} = \qquad (A3.7)$$

$$= \frac{1}{2}\left\{\left[-t+R-\beta\delta\gamma(R^*-R) - \beta\delta(\varepsilon-\varepsilon^*)\right]^2 + \lambda(R-\varepsilon-\theta)^2\right\}$$

The first order condition with respect to t and t are:*

$$\begin{aligned} \frac{\partial E(L)}{\partial t} &= -\left.p\right|_{NE} = 0 \\ \frac{\partial E(L^*)}{\partial t^*} &= -\left.p^*\right|_{NE} = 0 \end{aligned} \qquad (A3.8)$$

Note that $\frac{\partial x^{NE}}{\partial t} = \frac{\partial x^{*NE}}{\partial t^*} = 0$ *(A3.9),*

so the Nash Equilibrium output is not affected by the delegation parameter.

To obtain the subgame perfect equilibrium contracts substitute the NE m's from Lemma 5.2 in (A3.9):

$$\begin{aligned} & m^{NE} + a\left(m^{NE} - m^{NE*}\right) - a(t^*-t) - d(\varepsilon-\varepsilon^*) = 0 \\ & \therefore -t + \frac{b}{1+a}\theta + \frac{b}{1+a+b\gamma}\varepsilon + \frac{d\left(1+a^2\right)}{\left[(1+a)(1+2a)+b\gamma\right](1+a+b\gamma)}(\varepsilon-\varepsilon^*) + \\ & + a\left[t^*-t+\frac{b}{1+a+b\gamma}(\varepsilon-\varepsilon^*) + \frac{2d\left(1+a^2\right)}{\left[(1+a)(1+2a)+b\gamma\right](1+a+b\gamma)}(\varepsilon-\varepsilon^*)\right] - \\ & - a(t^*-t) - d(\varepsilon-\varepsilon^*) = 0 \end{aligned}$$

$$\begin{aligned} \therefore t^{SPE} = & \frac{b}{1+a}\theta + \frac{b}{1+a+b\gamma}\varepsilon^* + \frac{b(1+a)}{1+a+b\gamma}(\varepsilon-\varepsilon^*) + \\ & \frac{d\left(1+a^2\right)(1+2a)}{\left[(1+a)(1+2a)+b\gamma\right](1+a+b\gamma)}(\varepsilon-\varepsilon^*) - d(\varepsilon-\varepsilon^*) \end{aligned} \qquad (A3.10)$$

solving this and the corresponding expression for t results in the expressions in Proposition 5.2.*

References

[1] Alesina, A. and Grilli, V., 1992. ‘On the Feasibility of a One- or Multi-Speed European Monetary Union’; in Eichengreen, B. et al. eds. 1994

[2] Alesina, A. and Grilli, V., 1992. ‘The European Central Bank: Reshaping Monetary Politics in Europe’; in Canzoneri, M. et al eds. 1992

[3] Barro, R. and Gordon, D., 1983. ‘Rules, Discretion and Reputation in a Model of Monetary Policy’, *Journal of Monetary Economics* 12, 101-22.

[4] Beetsma, R. and Jensen, H., 1998. ‘Optimal inflation targets, ‘conservative’ central bankers and linear inflation contracts: Comment’, *American Economic Review*, forthcoming.

[5] Blinder, A., 1997. ‘Central banking in theory and practice’, Cambridge University Press.

[6] Buiter, W and Marston, W. (eds), 1985. International economic policy coordination, Cambridge University Press, Cambridge, UK

[7] Canzoneri, M. and Gray, J.A., 1985. ‘Monetary Policy Games and the Consequences of Non-co-operative behaviour’, *International Economic Review*, vol. 26, pp. 547-64

[8] Canzoneri, M. and Henderson, D., 1991. ‘Monetary Policy in Interdependent Economies: A Game-Theoretic Approach’, MIT Press, Cambridge MA

[9] Canzoneri, M.; Grilli, V. and Mason, P. eds., 1992. ‘Establishing a Central Bank: Issues in Europe and lessons from the US’, Cambridge University Press

[10] Canzoneri, M.; Nolan, C. and Yates, A., 1995, 'Mechanisms for achieving monetary stability: Inflation targeting versus the ERM', *CEPR Discussion Paper.*

[11] Cooper, Richard, 1969. 'Macroeconomic policy adjustment in interdependent economies', *Quarterly Journal of Economics*, 83, pp. 1-24.

[12] Debelle, G. and Fisher, S., 1994. 'How independent should a Central Bank be?' in Fuhrer, J., ed., 'Goals, guidelines and constraints facing monetary policymakers', *Federal Reserve Bank of Boston*

[13] Dolado, J.; Griffiths, M. and Padilla, J., 1994. 'Delegation in international monetary policy games', *European Economic Review*, 38, 1057-69

[14] Fershtman, C.; Judd, K. and Kalai, 1991. 'Observable contracts: strategic delegation and cooperation' *International Economic Review*, 32, 551-9.

[15] Friedman, J., 1986. 'Game Theory with Applications to Economics'' New York: Oxford University Press

[16] Ghosh, A.; Masson, P., 1994. 'Economic cooperation in an uncertain world', Blackwells, Oxford

[17] Goodhart, C. and Vinals, J. 'Strategy and tactics of monetary policy: examples from Europe and the Antipodes' in Fuhrer, J., ed., 'Goals, guidelines and constraints facing monetary policymakers', *Federal Reserve Bank of Boston*

[18] Hamada, K., 1976. 'A strategic analysis of monetary interdependence'', *Journal of Political Economy* 84, 677-700

[19] Herrendorf, B. and Lockwood, B., 1997. 'Rogoff's 'conservative' central banker restored', *Journal of Money, Credit and Banking*, 29, 476-95.

[20] Jensen, H., 1997. 'Monetary policy cooperation may not be counterproductive', *Scandinavian Journal of Economics*, 99(1), 73-80

[21] Jensen, H., 1999. 'Monetary policy cooperation and multiple equilibria', *Journal of Economic Dynamics and Control*, 23, 1133-53

[22] Kohler, M., 1998. 'Coalition formation in international monetary policy Games', *Bank of England Working Paper*

[23] Kydland, F. and Prescott, E., 1977. 'Rules rather than discretion: the inconsistency of optimal plans', *Journal of Political Economy* 85, 473-490

[24] Laskar, D., 1989. 'Conservative central bankers in a two-country world', *European Economic Review*, 33, pp. 1575-95

[25] Lockwood, B., 1997. 'State-contingent inflation contracts and unemployment persistence', *Journal of Money, Credit and Banking*, 29(3), 286-99.

[26] Lockwood, B. and Philippopoulos, A., 1994. 'Insider power, unemployment dynamics and multiple inflation equilibria', *Economica 61*, 59-77

[27] Lockwood, B.; Miller, M. and Zhang, L., 1995. 'Designing monetary policy when unemployment persists', *Economica* 65, 327-45

[28] Lohmann, S., 1992, 'The optimal degree of commitment: credibility and flexibility' *American Economic Review*, 82, 273-286

[29] Mas-Colell, A.; Whinston, M. and Green, J., 1995. 'Microeconomic Theory', ch.7-9 and 18, Oxford University Press.

[30] McCallum, B., 1995, 'Two fallacies concerning central-bank independence', *American Economic Review*, 85, 207-211

[31] Miller, M and Salmon, M., 1985. 'Policy coordination and dynamic games', in Buiter, W. and Marston, R. (eds), 1985

[32] Nolan, C. and Schaling, E., 1996. 'International monetary policy co-ordination: some lessons from the literature', *Bank of England Quarterly Bulletin*, November

[33] Obstfeld, M. and Rogoff, K., 1996. 'Foundations of international macroeconomics', MIT Press, Cambridge, MA

[34] Oudiz, G and Sachs, J., 1985. 'International policy co-ordination in Dynamic macroeconomic models', in Buiter, W. and Marston, R. (eds.)

[35] Persson, T. and Tabellini, G., 1993. 'Designing Institutions for Monetary Stability' *Carnegie-Rochester Conference Series on Public Policy* 39, 53-84

[36] Persson, T. and Tabellini, G., 1996. 'Monetary cohabitation in Europe', *American Economic Review Papers and Proceedings*, 111-17

[37] Persson, T. and Tabellini, G., 1995. 'Double-edged incentives: Institutions and policy coordination', in Grossman, G. and Rogoff, K. (eds) *Handbook of International Economics*, Vol. III, North-Holland

[38] Polo, M. and Tedeschi, P., 1999. Equilibrium and renegotiation in delegation games', Working Paper, IGIER - Bocconi University

[39] Rogoff, K., 1985a, 'The optimal degree of commitment to a monetary target', *Quarterly Journal of Economics*, 100, 1169-90

[40] Rogoff, K., 1985b. 'International monetary policy co-ordination may be counterproductive', *Journal of International Economics* 18, 199-217

[41] Svensson, L.E.O., 1995. 'Optimal inflation targets, conservative central bankers and linear inflation contracts', NBER Working Paper No. 5251

[42] Svensson, L.E.O., 1997. 'Optimal inflation targets, conservative central bankers and linear inflation contracts', *American Economic Review*, 87-1, pp. 99-115

[43] Svensson, L.E.O., 1999. 'Monetary policy issues for the Eurosystem', NBER Working Paper No. 7177

[44] von Hagen, J. and Suppel, R., 1994. 'Central bank constitutions for federal monetary unions', *European Economic Review* 38, 774-82

[45] Walsh, C., 1995. 'Optimal Contracts for Central Bankers' *American Economic Review* 85(1), pp.150-67

[46] Walsh, C., 1998. 'Monetary theory and policy', MIT Press, Cambridge, MA