# Some Non-Ricardian Agents Overturn the Taylor Principle: Pre-Volcker FED policy may have been better than you think.

FLORIN O. BILBIIE<sup>\*</sup> European University Institute, Florence

First version: October 2003; Comments solicited.

#### Abstract

This paper develops a simple framework to analyse monetary policy analytically in dynamic general equilibrium world with sticky prices when some ('non-Ricardian') agents have zero assets. We build on the approach of Gali, Lopez-Salido and Valles (2003) and look at how the presence of non-Ricardian agents alters some insights from the monetary policy literature. Some well-established results are overturned. Firstly, for simple interest rate rules, the usual Taylor principle is *generically* reversed: in order to ensure equilibrium uniqueness the central bank needs to pursue a passive rule. An interest rate peg can be fully consistent with a unique rational expectations equilibrium under some (more restrictive) conditions. Secondly, optimal and time consistent monetary policy in a non-Ricardian economy also implies passive policy. We then look at the effects of various shocks under the two scenarios, and provide new insights regarding pre-Volcker FED policy. If the economy was 'non-Ricardian' over that period, (i) policy implied a determinate equilibrium and ruled out sunspot fluctuations; (ii) policy might have been closer to optimal than conventional wisdom dictates; (iii) responses and variability of macro variables conditional upon fundamental shocks are close to their estimated counterparts. The conditions for these results to hold are found to be relatively mild in terms of the extent to which the economy is non-Ricardian, for otherwise standard parameterizations of the model.

<sup>\*</sup>Economics Department, European University Institute, Florence, Italy. florin.bilbiie@iue.it

<sup>&</sup>lt;sup>†</sup>I am highly indebted to Roberto Perotti for supervision and many useful comments. I am grateful to Giancarlo Corsetti in particular, and to Mike Artis, Fabrice Collard, Roger Farmer, Stephanie Schmitt-Grohe, Kris Nimark, Roland Straub and participants at the EUI Macro Workshop for comments and discussions at various stages. All errors are mine.

## 1 Introduction

A tremendous amount of research has grown recently studying monetary policy in optimizing, dynamic general equilibrium models. The importance of this research, form both a normative and positive standpoint, as well as its influence on real-life policymaking need not be stressed here further. An excellent and comprehensive overview of the state-of-the-art in the field is the recent book by Michael Woodford  $(2003)^1$ . Some normative conclusions of this literature which are robust across a variety of modelling strategies follow. First, the central bank needs to adopt an 'active' policy rule whereby it increases the nominal interest rate by more than inflation (and hence increases the real interest rate), for policy to be consistent with a unique rational expectations equilibrium; this is labeled 'the Taylor Principle' following Woodford 2001<sup>2</sup>. Secondly, optimal and time consistent (discretionary) policy, minimizing inflation and output variability, requires that the interest rate increase by more than inflation to contain aggregate demand. Thirdly, when there is no trade-off between output and inflation stabilization, full stabilization of both is possible by following a certain path for the interest rate, but a commitment to fulfill the Taylor principle is still required to ensure that this is the unique equilibrium. Relatedly, an interest rate peg (and any 'passive' policy rule) is inconsistent with unique equilibrium<sup>3</sup>, for any such policy would lead to multiple equilibria and stationary sunspot fluctuations (i.e. driven by beliefs and not fundamentals). Closely related to these (and other) theoretical results, there has been a subset of the literature interpreting various historical episodes through this literature's lenses, estimating policy rules dictated by these models' prescriptions, and assessing the effects of various fundamental shocks in the data and as predicted by models. From such positive a perspective, this literature tries to establish and explore the link between monetary policy and macroeconomic performance. Particularly, researchers in the field identified a change in monetary policymaking with the coming to office of Paul Volcker as a chairman of the FED in the US. Since macroeconomic performance (in terms of both response of macro variables to shocks, and variability) was also found to have changed, explaining the latter by the former (policy change) became the norm in the profession. Namely, many authors have argued that policy before Volcker was badly conducted along one or several dimensions, which led to worse macroeconomic performance in that period as compared to the Volcker-Greenspan era.

The scope of this paper is twofold. First, from a normative standpoint, we shall identify one assumption which, when relaxed 'enough', makes the theoretical insights presented above misfit for the conduct of 'good policy'. Indeed, we

 $<sup>^{1}</sup>$ Earlier overviews of these issues comprise., amongst others, Clarida, Gali and Gertler (1999) and Goodfriend and King (1997).

 $<sup>^{2}</sup>$ To be rigorous, this conclusion changes under some modelling choices. For example, in continuous time, Dupor (2000) shows that merely introducing capital invalidates the Taylor principle. A non-Ricardian fiscal policy in the sense of Woodford (1996) can also require a passive policy rule for equilibrium determinacy, as noted also by Leeper (1991).

 $<sup>^{3}</sup>$ As in the much celebrated paper by Sargent and Wallace (1975).

shall argue these policy prescriptions are overturned in such an economy. Secondly, from a positive standpoint, this paper raises the theoretical possibility that policy in the pre-Volcker era might have been better managed than conventional wisdom dictates. Instead, we shall argue that it might be a change in the structure of the economy (the same as the one making theoretical results above overturn) that caused the observed change in macroeconomic performance. Theoretical responses of the economy to various shocks, and calculated variability of macro variables under this scenario give *prima facie* support for such an (albeit partial) explanation.

The one dimension in which this paper departs from the standard analysis is the assumption that all agents have unlimited access to asset markets, and hence can smooth consumption perfectly. This last assumption is at the heart of the literature reviewed briefly above. We shall label such an economy 'Ricardian'<sup>4</sup>. In this paper, following an emerging literature reviewed above, it shall instead be assumed that some agents do not smooth consumption, being unable (constrained) or unwilling (myopic, uninformed) to participate to asset markets. We shall label these agents 'non-Ricardian'. This distinction has first been proposed by Mankiw (2000) for fiscal policy issues. Mankiw argues for such a modelling choice based on evidence in Campbell and Mankiw (1989) suggesting that about half of the US population does not act in a consumption-smoothing manner. Relatedly, comparing the wealth and income distributions for the US leads him to the same conclusion: since a significant fraction of the population has zero net worth, assuming that all agents are able to smooth consumption might not be fully justifiable. Many other papers, on micro or macro data, also tend to reject the permanent income hypothesis, and Mankiw reviews some theories put forward to explain such behavior. While having been already used to explain some puzzles in the fiscal policy literature<sup>5</sup>, this modelling choice has only recently been incorporated into the monetary policy literature. An insightful paper by Gali, Lopez-Salido and Valles (2003b, henceforth GLV) indeed argues that such a distinction makes the Taylor principle (the first standard conclusion enumerated above) not a good guide for policy<sup>6</sup>. Namely, GLV argue that if the central bank responds to current inflation via a simple Taylor rule, when the share of non-Ricardian agents is high enough the response coefficient has to be higher than that suggested by the Taylor principle. On the contrary, for a rule responding either to past or future expected inflation, GLV suggest, based on numerical

<sup>&</sup>lt;sup>4</sup>It is common knowledge that in such an economy without liquidity/borrowing constraints and infinitely-lived forward-looking agents Ricardian equivalence holds, as long as taxation is lump-sum and fiscal policy is also 'Ricardian' in the sense of Woodford 1996. That is, the timing of taxation does not matter and budget deficits have no aggregate demand effects.

<sup>&</sup>lt;sup>5</sup>Same assumption has recently been used by Gali, Lopez-Salido and Valles (2002), who argue that under some conditions, it can help explaining the effects of government spending shocks. See also Bilbiie and Straub (2003b) for different labor market and budgetary structures.

 $<sup>^{6}</sup>$ To be exact, Benassy (2002) is probably, to the best of our knowledge, the first paper to point out that the Taylor principle is not a good guide for determinacy in non-Ricardian economies. However, his model is different from ours, as is an overlapping generations one, *á* la Blanchard-Weil.

simulations, that for some parameter constellations the central banks needs to violate the Taylor principle to ensure equilibrium uniqueness/determinacy.

Our paper's first point is closest (although not identical) to this last point. We will argue that under some conditions, an 'Inverted Taylor principle' holds in general, no matter whether the policy rule responds to contemporaneous or future expected inflation. We shall derive this result analytically, and relate the necessary and sufficient conditions for this result to hold to a particular situation in the labor market. To obtain our results, we first exposit a standard dynamic sticky price model without capital incorporating the non-Ricardian feature just described. Then, we derive the reduced-form aggregate demand-aggregate supply system as is usually done in such models; since the resulting system is very simple (and includes as a special case the standard Ricardian sticky price model<sup>7</sup>), it might be of interest in itself to some researchers. Notably, we manage to describe the extent to which the economy is (generically) 'Ricardian' by a single parameter, which we label the 'Ricardianess' index. Whether the economy is Ricardian or not then has a close correspondence to what the situation in the labor market is: indeed, we shall argue that in a generically non-Ricardian economy the equilibrium wage-hours locus is upward sloping and cuts the labor supply curve from above (whereas it does so from below in a generically Ricardian economy). The rest of our results (overturning some benchmark theoretical results and helping explain some historical episodes) will follow directly from this intuition. The required share of non-Ricardian agents for our results to obtain will turn out to be small as compared to empirical estimates of Campbell and Mankiw (1989).

The rest of the paper proceeds as follows. Sections 2 and 3 introduce the non-Ricardian sticky-price model and its reduced log-linear aggregate demandsupply system. A discussion of the labor market useful for further intuition is also presented. Section 4 shows that an 'Inverted Taylor Principle' holds generically in a non-Ricardian economy and provides an intuitive discussion of this requirement in terms of constructing (or not) sunspot equilibria. Section 5 discusses optimal time-consistent (discretionary) monetary policy, and shows how this implies a passive instrument rule in a non-Ricardian economy. It is also suggested therein that an equilibrium with stable prices may be supported as the unique equilibrium in a non-Ricardian economy, without the need for the central bank to commit to respond to inflation. Section 6 calculates analytically the responses of the economy to fundamental (i.e. policy, natural interest rate and cost-push) and sunspot shocks under both a Ricardian and non-Ricardian economy, and for both determinate and indeterminate equilibria. Section 7 uses all the above results and looks at whether they can be used to shed new light on explaining the pre-Volcker period. Section 8 presents some tentative conclusions.

<sup>&</sup>lt;sup>7</sup>There is very high variance regarding a label for such a framework in the literature. This goes from 'New Keynesian' (Clarida Gali and Gertler 1999 - henceforth CGG) to 'New Neoclassical synthesis' (Goodfriend and King 1997) to 'Neomonetarist' (Kimball 1996) to 'optimizing IS-LM' (McCallum and Nelson 1999). Woodford (2003) refers to such a framework as 'Neo-Wicksellian'.

### 2 A Non-Ricardian Sticky-Price Model

The model we use draws on Gali, Lopez-Salido and Valles (2003), being a standard cashless dynamic general equilibrium sticky price model with Calvo-Yun pricing, augmented for the distinction between Ricardian and non-Ricardian households. There is a continuum of households, a single perfectly competitive final-good producer and a continuum of monopolistically competitive intermediategoods producers setting prices on a staggered basis. There is also a monetary authority setting its policy instrument, the nominal interest rate. The model is different from GLV in one main respect: we abstract from capital accumulation. This is explained further below, but allows us to obtain analytical solutions and does not affect the results qualitatively. Two other slight modifications are: (i) a slightly different utility function, useful for emphasizing the role of the labor supply; and (ii) a free parameter governing increasing returns to scale in the intermediate-goods sector (set to zero in GLV), which when set properly insures there are no long-run profits.

#### 2.1 Households

There is a continuum of households [0, 1]. A  $1 - \lambda$  share is represented by standard, neoclassical, 'Ricardian' households, who are forward looking and smooth consumption, being able to trade in all markets for state-contingent securities. The rest of the households on the  $[0, \lambda]$  interval is labeled 'non-Ricardian (or 'rule-of-thumb' as in GLV, or 'spenders' as in Mankiw 2000). For a variety of reasons, these households do not smooth consumption and act as if they solved a period-by period problem. Reasons could include constraints of participation to capital markets, myopia, extreme hyperbolic discounting, limited information (whereby current income is the most salient piece of information), etc.

**Ricardian Households** 

Each saver  $j \in [1 - \lambda, 1]$  chooses consumption, asset holdings and leisure solving the following standard intertemporal problem (we drop the j index as we look at the representative saver):

$$\max E_t \sum_{i=0}^{\infty} \beta^i U_S \left( C_{S,t+i}, 1 - N_{S,t+i} \right)$$
  
:  $U_S \left( C_{S,t}, 1 - N_{S,t} \right) = \ln C_{S,t} + \theta_S \frac{\left( 1 - N_{S,t} \right)^{1 - \gamma_S}}{1 - \gamma_S}$ 

subject to the sequence of constraints:

$$B_{S,t} \leq Z_{S,t} + W_t N_{S,t} + P_t D_{S,t} - P_t C_{S,t}$$

An S subscript stands for 'saver', i.e. a Ricardian household, and  $U_S(.,.)$  is saver's momentary felicity function, which takes the form considered here to be consistent with most DSGE studies<sup>8</sup>.  $\beta \in (0, 1)$  is the discount factor,  $\theta_S > 0$ 

 $<sup>^{8}</sup>$  This function is in the King-Plosser-Rebelo class and would lead to constant steady-state hours.

indicates how leisure is valued relative to consumption, and  $\gamma_S > 0$  is the coefficient of relative risk aversion to variations in leisure.  $C_{S,t}, N_{S,t}$  are consumption and hours worked by saver (time endowment is normalized to unity),  $B_{S,t}$  is the nominal value at end of period t of a portfolio of all state-contingent assets held by the Ricardian household, except for shares in firms.  $Z_{S,t}$  is begining of period wealth, not including dividend payoffs. Profits are rebated to these agents only as dividends  $D_{S,t}$  - that is to say that Ricardian households own the firms. We distinguish this from the rest of the assets since we do not model the equity market explicitly; we find the assumption of Ricardian households receiving the profits realistic since (i) the forward-looking behavior of firms modeled later would be hard to square with the static behavior of non-Ricardian households; (ii) we will use the stochastic discount factor of Ricardian households to value future income streams in the profit-maximizing pricing decision of firms.

Absence of arbitrage implies that there exists a stochastic discount factor  $\Lambda_{t,t+1}$  such that the price at t of a portfolio with payoff  $Z_{S,t+1}$  at t+1 is:

$$B_{S,t} = E_t \left[ \Lambda_{t,t+1} Z_{S,t+1} \right] \tag{1}$$

The riskless gross short-term nominal interest rate  $R_t$  is a solution to:

$$\frac{1}{R_t} = E_t \Lambda_{t,t+1} \tag{2}$$

Substituting the no-arbitrage condition (1) into the wealth dynamics equation gives the flow budget constraint. Together with the usual 'natural' no-borrowing limit for *each* state, this will then imply the usual intertemporal budget constraint:

$$E_t \sum_{i=t}^{\infty} \Lambda_{t,i} P_i C_{S,i} \le Z_{S,t} + E_t \sum_{i=t}^{\infty} \Lambda_{t,i} W_i N_{S,i} + E_t \sum_{i=t}^{\infty} \Lambda_{t,i} P_i D_{S,i}$$
(3)

Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$\beta \frac{U_C \left( C_{S,t+1} \right)}{U_C \left( C_{S,t} \right)} = \Lambda_{t,t+1} \frac{P_{t+1}}{P_t}$$
$$\theta_S \left( 1 - N_{S,t} \right)^{-\gamma_S} = \frac{1}{C_{S,t}} \frac{W_t}{P_t}$$

along with (3) hold with equality (or alternatively flow budget constraint hold with equality and transversality condition ruling out overacummulation of assets and Ponzi games be satisfied:  $\lim_{i\to\infty} E_t [\Lambda_{t,t+i}Z_{S,t+i}] = 0$ ). Using (3) and the functional form of the utility function, the short-term nominal interest rate must obey:

$$\frac{1}{R_t} = \beta E_t \left[ \frac{C_{S,t}}{C_{S,t+1}} \frac{P_t}{P_{t+1}} \right]$$

Non-Ricardian agents

Non-Ricardian consumers also **optimize**. We prefer to think of these households as not participating to asset markets, either due to constraints or to their being shortsighted (case in which their optimal asset holdings are zero). One obvious generalization could treat these agents as saving a fixed (insensitive to interest rates) portion of their present income - it will become obvious that these would not change our results qualitatively. The problem these agents face then looks finally as a period-by-period one:

$$\max_{C_{H,t},N_{H,t}} \ln C_{H,t} + \theta_H \frac{(1 - N_{H,t})^{1 - \gamma_H}}{1 - \gamma_H}$$

$$C_{H;t} = \frac{W_t}{P_t} N_{H;t}$$
(4)

The first order condition is:

:

$$\theta_H \left( 1 - N_{H,t} \right)^{-\gamma_H} = \frac{1}{C_{H;t}} \frac{W_t}{P_t}$$
(5)

It is important to observe that the subsystem of these agents is then fully recursive, and given the optimal choice above, we can solve for reduced-form (functions only of  $\frac{W_t}{P_t}$ ) expressions for their consumption and notably labor supply, without the need to keep consumption (or marginal utility of income) constant, as this does not depend on saving decisions or any other intertemporal feature. Note that due to the very form of the utility function, hours are constant for this agents (the utility function is chosen to obtain constant hours in steady state, and this agent is 'as if' she were in the steady state always. In this case labour supply of non-Ricardian agents is fixed, no matter  $\gamma_H$ , as income and substitution effect cancel out. While this facilitates algebra, it is in no way necessary for our results (elastic labor supply will be discussed to some extent). Hours will be a solution to:

$$\left(1 - N_{H,t}\right)^{-\gamma_H} N_{H;t} = \frac{1}{\theta_H}$$

and then consumption will track the real wage to exhaust the budget constraint.

#### 2.2 Firms

The firms' problem is completely standard - see Gali (2002) or Woodford 2003 and can be skipped by some readers without loss of continuity (one slight difference/generalization is in the production function of intermediate goods).

#### Final Good Producers

The final good is produced by a representative competitive firm . The aggregation technology for producing final goods is of the CES form (constant elasticity of substitution  $\varepsilon$ ):

$$Y_t = \left(\int_0^1 Y_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

Final goods producers behave competitively, maximizing profit each period:

$$\max[P_t Y_t - \int_0^1 P_t(i) Y_t(i) di) \tag{7}$$

where  $P_t$  is the overall price index of the final good,  $P_t(i)$  are the prices index of the intermediate goods. The demand for each intermediate input and the price index can be shown to be:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t \tag{8}$$

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$
(9)

#### Intermediate Goods Producers

We assume that the intermediate firms face a technology which is linear in labor, for simplification:

$$Y_t(i) = \begin{cases} A_t N_t(i) - F(i), & \text{if } N_t(i) > F(i) \\ 0, & \text{otherwise} \end{cases}$$

F(i) is a firm-specific fixed cost: this will be a free parameter that can be chosen such that profits are zero in steady state and there are increasing returns to scale, consistent with evidence by Rotemberg and Woodford (1995). Alternatively, if the fixed cost is zero, there are steady-state profits (which is the case in GLV). We shall encompass both cases. Cost minimization taking the wage and the rental cost of capital as given implies the following conditions (written as relative factor demands and nominal marginal cost):

$$\frac{MC_t}{P_t} = \frac{W_t/P_t}{A_t} \tag{10}$$

When fixed cost is zero,  $Y_t(i)$  is a constant returns to scale function, and there will be positive steady state profits. When positive and properly chosen, there will be increasing returns and no profits in steady-state. The (nominal) profit function is given by:

$$P_t(i) O_t(i) = P_t(i)Y_t(i) - MC_t(Y_t(i) + F(i))$$

Price setting

Following Calvo (1983) and Yun (1996) intermediate good firms adjust their prices infrequently. The opportunity to adjust follows a Bernoulli distribution. We define  $\theta$  as the probability of keeping the price constant. This exogenous probability is independent of history. Thus each period there is a fraction of firms that keep their prices unchanged. The dynamic program of the firm is (maximizing discounted sum of future nominal profits, hence using the relevant stochastic discount factor  $\Lambda_{t,t+i}$  used as pricing kernel for nominal payoffs):

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\theta^s \Lambda_{t,t+s} \left[ P_t(i) Y_{t,t+s}(i) - M C_{t+i} Y_{t,t+s}(i) \right]$$

subject to the demand equation (at t+s, conditional upon price set s periods in advance):

$$Y_{t,t+s}(i) = \left(\frac{P_t(i)}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s}$$
(11)

The optimal price of the firm is then found as usually as a markup over a weighted average of expected future nominal marginal costs:

$$P_t^{opt}(z) = (1+\mu)E_t \sum_{s=0}^{\infty} \overline{\omega}_{t,t+s} MC_{t+s}$$
(12)  
$$\overline{\omega}_{t,t+s} = \frac{\theta^s \Lambda_{t,t+s} \left(\frac{1}{P_{t+s}}\right)^{(1-\varepsilon)} Y_{t+s}}{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \left(\frac{1}{P_{t+s}}\right)^{(1-\varepsilon)} Y_{t+s}}$$

In equilibrium each producer that chooses a new price  $P_t(i)$  in period t will choose the same price and the same level output. Then the dynamics of the price index given the aggregator above is:

$$P_{t} = \left( \left(1 - \theta\right) P_{t}^{opt} \left(i\right)^{1-\varepsilon} + \theta P_{t-1} \left(i\right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$
(13)

The combination of this two conditions leads in the log-linearized equilibrium to the well known New Keynesian Phillips curve given below. Profits will also be equal across producers, and equal to:

$$O_t = \left(1 - \frac{MC_t}{P_t}\right)Y_t - \frac{MC_t}{P_t}F$$

#### 2.3 Monetary policy

The monetary authority's problem will be discussed in some detail later, but we consider two policy frameworks prominent in the literature. First, we study *instrument rules* in the sense of a feedback rule for the instrument (short-term nominal interest rate) as a function of macro variables, mainly inflation. We focus on rules within the family (where the 'star' variables are natural levels of the corresponding variables defined below):

$$R_{t} = (R_{t}^{*})^{\phi^{*}} R^{1-\phi_{r}} R_{t-1}^{\phi_{r}} \left( E_{t} \frac{P_{t+k}}{P_{t-1+k}} \right)^{\phi_{\pi}} \left( E_{t} \frac{Y_{t+k}}{Y_{t+k}^{*}} \right)^{\phi_{y}} e^{\varepsilon_{t}}$$
(14)

We shall also consider targeting rules under discretionary policymaking, whereby the path of the nominal rates is found by optimization by the central bank this is described in detail in Section 6 below. Such a framework will also imply a behavioral relationship for the instrument rule, but this is only an *implicit instrument rule* in the language of Svensson (1999).

#### 2.4 Market Clearing

Market clearing and aggregation require:

$$N_t = \lambda N_{H,t} + (1 - \lambda) N_{S,t} \tag{15}$$

$$O_t = (1 - \lambda) D_{S,t} \tag{16}$$

$$C_t \equiv \lambda C_{H,t} + (1-\lambda) C_{S,t} = Y_t \tag{17}$$

Last equality (goods market clearing or economy resource constraint) holds by Walras' law, if we consider that state-contingent assets are in zero net supply, as is the case since markets are complete and agents trading in them (Ricardian agents) are identical.

# 3 A simplified linear aggregate demand-supply non-Ricardian model

We seek to express the above Non-Ricardian model in a form similar to models usually employed for monetary policy analysis (see CGG 1999, Woodford 2003); several substitutions of the log-linearized equations, exposited in detail in the Appendix, deliver a model of such a form:

$$AS : \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \text{ where } \kappa \equiv \psi \chi$$
(18)

$$AD : E_t x_{t+1} = x_t + \delta^{-1} \left[ r_t - E_t \pi_{t+1} - r_t^* \right]$$
(19)

Note that the reduced-form parameters can be explained in terms of deep parameters as:

$$\chi \equiv 1 + \varphi^{s} \frac{1}{1 + F_{Y}} \left( 1 + \frac{\lambda}{1 - \lambda} O_{Y} \right) \ge 1$$

$$\delta \equiv 1 - \varphi^{s} \frac{\lambda}{1 - \lambda} \frac{1}{1 + \mu}$$
(20)

Note that  $\chi = \delta + \varphi^s \frac{1}{1-\lambda} \frac{1}{1+F_Y} > \delta$ . The 18 equation is an usual Philips curve, where  $x_t$  denotes the deviations of output from its natural (flexible-price) level  $x_t \equiv y_t - y_t^*$ . Natural output is a function of technology level only in this model - permanent shock to technology have permanent effects on natural output:

$$y_t^* = \left[1 + F_Y\left(1 - \frac{1}{\chi}\right)\right]a_t$$

Following Clarida, Gali and Gertler (1999) or Gali (2002) we also introduce 'cost-push' shocks  $u_t$ , i.e. variations in marginal cost not due to variations in excess demand. These could come from the existence of sticky wages creating a time-varying wage markup, a time-varying elasticity of substitution among intermediate goods or other sources creating this inefficiency wedge between the efficient and natural levels of output. For details as to what these timevarying wedges could be, see Woodford (2003, Ch. 6). Hence, marginal cost variations will be given by:

$$mc_t = \chi x_t + u_t \tag{21}$$

As far as aggregate supply is concerned, the only difference from the 'Ricardian' framework is the dependence of  $\chi$  upon the share of non-Ricardian agents. Note that  $\varphi^S = \frac{\gamma^S N_S}{1-N_S}$  is the inverse of the Frisch elasticity of labor supply of Ricardian agents,  $F_Y$  the share of the fixed cost in total output in steady state (and the degree of increasing returns to scale) and  $O_Y \equiv \frac{\mu - F_Y}{1+\mu}$  is the share of profits in steady state output. This is further explained below.

Equation 19 instead is the non-Ricardian correspondent of a usual aggregate demand AD (or IS) function derived from the Euler equation of the forward-looking consumers, in the Ricardian case (please find Appendix for a detailed derivation). The main difference here is that to obtain a dynamic equation in terms of the aggregate output (gap), we need to express consumption of Ricardian agents as a function of output, where these are not related necessarily positively, as is the case in the benchmark model. Indeed, for some parameter configurations, we shall argue that  $\delta < 0$ . This is discussed below, and it will turn out to modify drastically determinacy properties of the model, its response to shocks, and the optimal design of interest rate rules. Finally, we can define the natural rate of interest (Wicksellian interest rate)  $r_t^*$  as the level of the interest rate consistent with output being at its natural level (and hence with zero inflation), as in Woodford 2003:

$$r_t^* = \left[1 + F_Y\left(1 - \frac{\delta}{\chi}\right)\right] \left[E_t a_{t+1} - a_t\right] \tag{22}$$

We can assume that  $\Delta a_t \equiv a_t - a_{t-1}$ , is given by an AR(1) process  $\Delta a_t = \rho^a \Delta a_{t-1} + \varepsilon_t^a$ , which implies shocks to technology have permanent effects (see Gali 1999, Gali, Lopez-Salido and Valles 2003a). We note that  $r_t^* = \left[1 + F_Y\left(1 - \frac{\delta}{\chi}\right)\right] \rho^a \Delta a_t$ , such that the natural interest rate increases with technology shocks.

# 3.1 A "*Ricardianess'* index' and a closer look at the labor market

We now seek to characterize the extent to which the reduced forms of our model are different from the standard Ricardian model, and how this relates to the deep parameters of the model. Take first aggregate supply AS. This differs only insofar as the presence of non-Ricardian agents modifies  $\chi$ , i.e. the elasticity of the marginal cost to movements in the output gap, and hence the response of inflation to aggregate demand variations. Moreover, wit increasing returns to scale such that the share of profits in output is zero in steady state, the AS curve does not modify at all<sup>9</sup>. Note, however, that even in case aggregate supply is not changed, it can still be the case that important differences occur with respect to the standard model, due to effects on aggregate demand, to which we now turn. We first define what may seem a somewhat artificial index, which we call the '*Ricardianess'* parameter.

**Definition 1** Let  $\delta \in (-\infty; 1)$  be the 'Ricardianess' parameter. We refer to  $\delta \in (0, 1]$  as a 'generically Ricardian economy' and to  $\delta \in (-\infty, 0)$  as 'generically non-Ricardian' economy. The economy will be 'more non-Ricardian' when  $\delta$  is smaller.<sup>10</sup>

This definition will be useful further. Note that  $\delta$  measures the extent to which variations in Ricardian consumption are ultimately related to variations in total consumption, and thereby output; hence, it does capture the extent to which the economy is Ricardian *ad litera*m. Also note how the '*Ricardianess*' parameter varies with the share of non-Ricardian consumers:

$$\frac{\partial \delta}{\partial \lambda} = -\varphi^s \frac{1}{\left(1-\lambda\right)^2} \frac{1}{1+\mu} \le 0$$

It is obvious that the only way for  $\delta$  to be independent of  $\lambda$  is for  $\varphi^s$  to be zero, i.e. labor supply of Ricardian agents be infinitely elastic<sup>11</sup>. When this is not the case, non-Ricardian agents have an impact upon aggregate demand. This is including the case whereby  $O_Y = 0$ , i.e. the degree of increasing returns is such

<sup>&</sup>lt;sup>9</sup>In this case consumption is equal across groups in steady state. The independence of  $\lambda$  follows directly by merely differentiating 20 to obtain  $\frac{\partial \chi}{\partial \lambda} = -\varphi^s \frac{1}{(1-\lambda)^2} \frac{\mathcal{O}_Y}{1+F_Y} \leq 0.$ 

 $<sup>^{10}{\</sup>rm We}$  shall sometimes loosely refer to a generically non-Ricardian economy as simply 'non-Ricardian'.

<sup>&</sup>lt;sup>11</sup>This is a direct corollary of a more general result in Bilbiie and Straub (2003) - with infinitely elastic labor supply, non-Ricardian consumers have no effect upon aggregate variables, and do not cause a failure of Ricardian equivalence per se. This instead happens because consumption of Ricardian agents becoes independent of wealth, and totally dependent upon wage income, as is consumption of non-Ricardian agents.

that profits are zero in steady state. As expected, the '*Ricardianess*' parameter is decreasing in  $\lambda$ , and more so, more inelastic is labor supply.

We shall now have a first glance at the magnitude of  $\lambda$  required for our results to hold, quantitatively. To that end, and for further use in simulations, we parameterize the model at quarterly frequency; the baseline case follows GLV (except for the mentioned differences) and most monetary policy studies. Namely, we set the discount factor  $\beta$  such that r = 0.01, the steady state markup  $\mu = 0.2$ corresponding to an elasticity of substitution of intermediate goods of 6. The fixed cost parameter (and degree of increasing returns to scale) is set to either 0 (steady-state profits) or  $F_Y = \mu = 0.2$ . The average price duration is one year, implying  $\theta = 0.75$ . As to parameterizing labour, this is somehow more delicate, for there is no data to the best of our knowledge disentagling various preferences for leisure, or equivalently hours worked, as a function of wealth. Here, as we have no priors for imposing otherwise, we assume both types work the same number of hours in steady state hence  $N = N_S = N_H = \frac{1}{3}$  as commonly assumed in the literature. Then, following the method in Bilbiie and Straub (2003), for each value of the elasticity of marginal utility of leisure to leisure  $\gamma_i$ we can find a level of  $\theta_j$ . This allows us to have a free parameter for the inverse Frisch elasticity of labor supply  $\varphi^S = \frac{\gamma^S N}{1-N}$ , a parameter for which we shall consider different values. Different values are also considered for the share of non-Ricardian agents  $\lambda$  since this is probably the most controversial parameter - empirical evidence by Campbell and Mankiw 1989 suggests this is around 0.5 for the US economy. In Figure 1, we plot the 'Ricardianess' parameter as a function of the share of non-Ricardian agents, and for different labor supply elasticities. The thick line illustrates our baseline parameterization  $\varphi^S = 10$ , where we see that the economy becomes non-Ricardian at around 0.1 share of non-Ricardian agents. The other two cases (presented for illustrative purposes) are the extremes of infinitely elastic labor supply (horizontal line), when the economy is Ricardian independently of the share of Ricardian agents; and of almost completely inelastic labor supply (almost vertical line), where the economy becomes non-Ricardian for a small measure of non-Ricardian agents<sup>12</sup>.

 $<sup>^{12}</sup>$ While a unit elasticity of intertemporal substitution has been considered here for simplifying exposition, this would not cancel our result. For that would show up in the IS curve as a multiplicative term on  $\delta$ , and hence in Figure 1 it would change both the intercept and the curvature of the index, but not th sign of the derivative.



Fig.1:The Ricardianess index as a function of the share of non-Ricardian agents, for different labor supply elasticities.

Key to understanding the results further obtained here is the equilibrium in the labor market. In system 23 we outline the labor supply and the equilibrium wage-hours locus. For labor supply, we only keep consumption of saver constant, for there is no intertemporal substitution for the non-Ricardian agent. The equilibrium wage-hours locus labeled WN is derived taking into account all equilibrium conditions, most notably how consumption is related to real wage in equilibrium. This schedule will be fixed in equilibrium (in fact, it will be shifted by technology shocks only) and hence not affected by policy.

$$WN : w_{t} = \left[ (1 + F_{Y}) \delta + \varphi^{s} \frac{1}{1 - \lambda} \right] n_{t} + (1 + F_{Y}) a_{t}$$
(23)  

$$LS : w_{t} = \varphi^{s} \frac{1}{1 - \lambda} n_{t} + c_{s,t}$$

Note that what we labeled as the non-Ricardian case ( $\delta < 0$ ) has an intuitive interpretation in labor market terms; for it implies that the equilibrium wage-hours locus is less upward sloping than (and hence cuts from above) the labor supply curve. Consider first the case when  $\lambda = 0$ ;  $\delta = 1$ , i.e. a fully Ricardian economy. Then, the wage-hours locus is more upward sloping than LS, the difference being given by the intertemporal elasticity of substitution in consumption, normalized to 1 in our case (multiplied by returns to scale  $1+F_Y$ ). Ceteris paribus, if the labor demand shifts out, labor supply also shifts leftward due to the usual income effect, since agents anticipate higher income and higher consumption. If instead labor supply shifts up due to a positive income effect, same effect makes labor demand shift out (due to sticky prices and countercyclical markups). This gives a WN locus more upward sloping than the labor supply curve LS. When  $\delta < 0$  this insight changes, and the threshold value for  $\lambda$  for this to happen is:

$$\lambda > \lambda^* = \frac{1}{1 + \varphi^s \frac{1}{1 + \mu}} \tag{24}$$

When the share of non-Ricardian agents is higher than this threshold (or equivalently for a given share, labor supply of savers is inelastic enough), the wagehours locus becomes less upward sloping than the labor supply. An intuition for that follows, where we assume that the real interest rate is constant along the equilibrium path for simplicity<sup>13</sup>.

Figure 2 illustrates the main mechanism. Take first an outward shift in labor demand, Ld moves to Ld1. Keeping supply fixed, there would be an increase in real wage and an increase in hours (their relative sizes depending on elasticity of labor supply as usual). Labor supply shifts left due to a positive income effect on Ricardian agents - indeed, if there were no non-Ricardian agents, this would be the end of the story and the equilibrium wage-hours locus would go through point R. But now, the increase in the real wage would boost consumption of non-Ricardian agents, henceforth amplifying the initial demand effect (Ld2). When labor supply is relatively inelastic, this increase in wage is large and the increase in hours is small compared to that necessary to generate the extra output demanded; note that the effect induced on demand is larger, higher the share of non-Ricardian agents. The only way for supply to meet demand is for labor supply to shift right to Ls2, and equilibrium obtains at point NR. This is insured in equilibrium by a fall in profits, resulting from: (i) increasing marginal cost (since wage increases) and (ii) the weak increase in hours and hence in output and sales. This is like and indirect negative income effect induced on Ricardian agents by the presence of non-Ricardian households. Next consider a shift in labor supply, for example leftward as would be the case if consumption of savers increased. Keeping demand fixed, wage increases and hours fall. The increase in wage (and the increase in consumption of savers itself) has a demand effect due to sticky prices. As labor demand shifts right, the real wage would increase by even more; hours would increase, but by little due to the relatively inelastic labor supply (the overall effect would again depend on the relative slopes of the two curves). The increase in the real wage means extra demand through the non-Ricardian consumption<sup>14</sup>. To meet this demand, only way for increasing output is an increase in labor supply, which instead obtains only if labor supply shifts right, which is insured as before by the fall in profits. This explains why in a non-Ricardian economy the wage-hours locus cuts the labor supply curve from above (in Figure 2 below we plot the wage-hours locus  $(w,n)^e$  with a thick line). This instead will help our intuition in explaining the further results<sup>15</sup>. Note that such a wage-hours locus implies that the model

 $<sup>^{13}</sup>$ How the nominal interest rate reacts to inflation, generated here by variations in demand, will be crucial in the further analysis.

<sup>&</sup>lt;sup>14</sup>The assumption on preferences of non-Ricardian agents delivering a fixed labor supply is less crucial than it might seem. For with an elastic labor supply, say  $n_{h,t} = \omega w_t$ ,  $\omega > 0$ , the elasticity of non-Ricardian consumption to wage will also increase  $c_{h,t} = (1 + \omega) w_t$ , amplifying the effect on labor demand itself. The parameter  $\omega$  would then matter for our results, but not change them qualitatively - this can be easily checked.

<sup>&</sup>lt;sup>15</sup>Note that the intuition for real indeterminacy to obtain in standard models (see e.g. Benhabib and Farmer 1994) requires the wage hours locus be upward sloping but cut the labor supply curve from *below*. This is also the case in standard sticky-price models, and gives rise to a certain requirement for the monetary policy rule to result into real determinacy

generates a higher equilibrium elasticity of hours to the real wage, and more so more negative  $\delta$  is. The model still generates a procyclical real wage, but less so than its Ricardian counterpart. Relatedly, it predicts a higher volatility of hours and lower volatility of the wage in response to shocks shifting the two curves.



Fig.2: The equilibrium wage-hours locus and labor supply curve in a non-Ricardian economy.

Having derived the equilibrium wage-hours locus gives us a simple way of thinking intuitively about the effects of shocks and of monetary policy in general; monetary policy, by changing nominal interest rates, modifies real interest rates and hence shifts the labor supply curve (by changing the intertemporal consumption profile of Ricardian agents). But such shocks have no effect on the wage-hours locus by construction, since this describes a relationship that holds in equilibrium always.

# 4 The Inverted Taylor Principle: Determinacy properties of interest rate rules

In this Section we study determinacy properties of simple interest rate rules. We shall consider for analytical simplicity only rules whereby the interest rate does not respond to the output gap, and there is no inertia (interest rate smoothing) - such extensions should be straightforward. We first consider rules involving a response to expected inflation, as done for example by CGG (2000). This specification provides simpler (sharper) determinacy conditions, and captures

<sup>-</sup> see below. Our intuition will be that having the wage-hours locus cut the labor suply from *above*, changes determinacy properties in a certain way.

the idea that the central bank responds to a larger set of information than merely the current inflation rate:

$$r_t = \phi_\pi E_t \pi_{t+1} + \varepsilon_t \tag{25}$$

where  $\varepsilon_t$  is the non-systematic part of policy-induced variations in the nominal rate. The dynamic system for the  $z_t \equiv (y_t, \pi_t)'$  vector of endogenous variables and the  $\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$  vector of disturbances is:

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \Psi \nu_t$$

The coefficient matrices are given by:

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 - \beta^{-1} \delta^{-1} \kappa (\phi_{\pi} - 1) & \delta^{-1} \beta^{-1} (\phi_{\pi} - 1) \\ -\beta^{-1} \kappa & \beta^{-1} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \delta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix}$$
(26)

Determinacy again requires that both eigenvalues be outside the unit circle. The determinacy properties of such a rule are emphasized in Proposition 2.

**Proposition 1** An interest rate rule such as 25 delivers a unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:

Case I: If 
$$\delta > 0, \phi_{\pi} \in \left(1, 1 + \delta \frac{2(1+\beta)}{\kappa}\right)$$
  
Case II: If  $\delta < 0, \phi_{\pi} \in \left(1 + \delta \frac{2(1+\beta)}{\kappa}, 1\right) \cap [0, \infty)$ 

A proof is in the Appendix. Case I can be viewed as the usual, 'Ricardian' case: the Taylor principle (Woodford 2001) is at work, and as noted in the previous literature the central bank should respond more than one-to-one to increases in inflation. It should also not respond 'too much', which is a well-established result first noted by Bernake and Woodford (1997).

Case II shall be labeled the 'Non-Ricardian' case<sup>16</sup>. In this case, the Central Bank should follow an 'inverted' Taylor principle: only passive policy is consistent with a unique rational expectations equilibrium. A result such as our Proposition 2 has first been noted (relying upon numerical simulations) by Gali et al 2003. The message of our paper, however, is different. For we provide analytical conditions for an inverted Taylor principle to hold in general, independent on the policy rule followed; while in Gali et al, it is only for a forward-looking rule that this result applies. In the Appendix we provide a Proposition A, which is the equivalent of Proposition 2 for a contemporaneous rule; there, an inverted Taylor principle holds too for high enough a share of

<sup>&</sup>lt;sup>16</sup>Note that for the determinacy region of the oplicy rule coefficient to be  $\phi_{\pi} \in [0, 1)$  we need exactly the conditions in Proposition 1; i.e., a high enough share of non-Ricardian agents, such that an exogenous interest rate as is the case when  $\phi_{\pi} = 0$  be consistent with a unique RE equilibrium.

non-Ricardian agents. The determinacy conditions are more complicated, and there is one amendment: in general, a very strong response to inflation will also ensure determinacy; but the required response is shown to be unrealistically high (e.g. around 35 under our baseline parameterization). It is in this sense that we say that the inverted Taylor principle holds 'generically'. Moreover, we shall now discuss the conditions under which one ends up in Case II of the above proposition, and relate them to our earlier discussion of the labor market.

In terms of deep parameters, the condition to end up in the non-Ricardian case is simple, and is the same as the condition making the wage-hours locus cut the labor supply curve from above, as noted previously:

$$\lambda > \frac{1}{1 + \varphi^s \frac{1}{1 + \mu}} \equiv \lambda^* \tag{27}$$

In the figure below, we plot the threshold  $\lambda$  and  $\varphi^s$ , such that values under the curve give the 'Ricardian' case, whereas above the curve we have the non-Ricardian case (the inequality above holds).



Fig.3: Threshold share of non-Ricardian agents as a function of inverse labor supply elasticity. Above the threshold economy is generically non-Ricardian.

This means that for the Taylor principle to work (to end up in the Ricardian case), the Frisch elasticity of labor supply (and of intertemporal substitution in labor supply), should be high, and higher, the higher the share of non-Ricardian people  $\lambda$  is. For a range of  $\varphi^s$  between 1 (unit elasticity) and 10 (0.1 elasticity) the threshold share of non-ricardian people should be lower than 0.5 to as low as around 0.1 respectively. This shows once more that the required share of non-Ricardian agents for the standard results to be overturned is not that large, after all.

#### 4.1 A simple Taylor rule

For completion we also study determinacy properties of a simple Taylor rule. This is done to further illustrate the differences of our results to Gali et al (2003), where there is a dramatic distinction between forward-looking and contemporaneous rules. For a contemporaneous rule to be compatible with a unique equilibrium, they note that the central bank should respond to increases in inflation more strongly (and indeed very strongly under some parameter constellations). Our results have the same flavor as for a forward-looking rue: an inverted Taylor principle holds generically, i.e. if we exclude some extreme values for some of the parameters. We consider rules of the form:

$$r_t = \phi_\pi \pi_t + \varepsilon_t \tag{28}$$

Replacing this in the AD equation (60) and using the same method as previously (described at length in the Appendix) we obtain the following Proposition.

**Proposition 2** An interest rate rule such as 28 delivers a unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:

Case I: If  $\delta > 0, \phi_{\pi} > 1$  (the 'Taylor Principle') Case II: If  $\delta < 0$ ,

$$\phi_{\pi} \in \left[0, \min\left\{1, \delta \frac{\beta - 1}{\kappa}, \delta \frac{-2\left(1 + \beta\right)}{\kappa} - 1\right\}\right) \cup \left(\max\left\{1, \delta \frac{-2\left(1 + \beta\right)}{\kappa} - 1\right\}, \infty\right)$$

In the Appendix we prove this and distinguish a few cases for the implied condition on the policy rule coefficient as a function of deep parameters of the model. It turns out that the 'inverted Taylor principle' holds in Case II for a somewhat larger share of non-Ricardian agents than was the case under a forward-looking rule. It is also the case that a policy rule responding to current inflation very strongly would insure equilibrium uniqueness<sup>17</sup>. But we also argue that the implied response ( $\phi_{\pi} = 35$  under the baseline parameterization): (i) is much larger than any plausible empirical estimate; (ii) would imply that zero bound on nominal interest rates be violated for even small deflations; (iii) would have little credibility. This is in clear contrast with Gali et al (2003), who do not look at a possible inversion of the Taylor principle in their numerical analysis of such rules, but instead argue that for a large share of non-Ricardian agents making the required policy response too strong under a Taylor rule, the central bank should switch to a passive forward-looking rule.

What is missing is an intuitive explanation as to why is it that a passive rule is compatible with a unique equilibrium in a non-Ricardian economy, whereas an active rule is generally not. This is what we try to study next<sup>18</sup>.

#### 4.2 Intuition: sunspot equilibria in non-Ricardian economies

It is useful to conduct a mental experiment trying to construct a sunspot equilibrium - in the last section, we will compute sunspot equilibria formally. In a

 $<sup>^{17}</sup>$ This is not the case under a forward looking rule, since there, even in a fully Ricardian economy too strong a response leads to indeterminacy - see Bernake and Wodford 1997.

<sup>&</sup>lt;sup>18</sup>Note that the type of policy studied in the previous section is merely a particular case of a passive rule, whereby there is no feedback whatsoever from inflation to the policy instrument.

Ricardian economy, this is possible if monetary policy is passive, and not if it is active, which is a well-established result in the literature in a model like our Case I. There, a non-fundamental increase in expectations about inflation and/or output matched by too weak a policy response (a fall in the real rate) causes an increase in consumption of Ricardian agents today; since total demand is equal to consumption of Ricardian agents, this boosts aggregate demand and increases inflation, which in turn makes the initial inflationary expectation self-fulfilling.

In a Non-Ricardian economy (Case II), this is reversed. First, we cannot construct sunspot equilibria with a *passive* policy rule  $\phi_{\pi} < 1$ . The crucial difference in our non-Ricardian case is that aggregate demand is no longer completely forward-looking, i.e. linked to its Ricardian component. Suppose for simplicity and without losing generality that the sunspot is located in inflationary expectations. A non-fundamental increase in expected inflation causes a fall in the real interest rate. This leads to an increase in consumption of Ricardian agents, and an increase in the demand for goods; but note these are now partial effects. Indeed, to work out the overall effects one needs to look at the non-Ricardian component of the aggregate demand, and hence at the labor market. The partial effects identified above would cause an increase in the real wage (and a further boost to consumption of non-Ricardian agents) and a fall in hours. Increased demand, however, means that (i) some firms adjust prices upwards, bringing about a further fall in the real rate (as policy is passive); (ii) the rest of firms increase labor demand, due to sticky prices. Note that the real rate will be falling along the entire adjustment path, amplifying these effects. But since this would translate into a high increase in the real wage (and marginal cost) and a low increase in hours, it would lead to a fall in profits, and hence a negative income effect on labor supply. The latter will then not move, and no inflation will result, ruling out the effects of sunspots. This happens when the economy is non-Ricardian 'enough' in a way made explicit by 27.

Next, consider non-Ricardian policy with an *active* interest rate rule  $\phi_{\pi} > 1$ . We suggest that a sunspot equilibrium is always possible to construct in this case. Consider the same thought experiment as above, which now leads to a fall in the consumption of Ricardian agents (real rate increases). This implies a rightward shift of labor supply, and hence a fall in wage and increase in hours. Consumption of non-Ricardian agents also falls one-to-one with the wage, and hence aggregate demand falls by more than it would in a Ricardian economy. Firms who can adjust prices will adjust them downwards, causing deflation, and a further fall in the real rate. Firms who cannot adjust prices will cut demand, causing a further fall in the real wage and a small fall in hours (since labor supply is inelastic). But this will mean higher profits (since marginal cost is falling), and eventually a positive income effect on labor supply of Ricardian agents. As labor supply starts moving leftward, demand starts increasing, its increase being amplified by the sensitivity of non-Ricardian agents to wage increases. The economy will establish at a point on the wage-hours locus consistent with the overall negative income effect on labor supply of Ricardian agents, i.e. with higher inflation and real activity. Hence, the initial inflationary expectations become self-fulfilling.

The above analysis suggests that in an economy with binding borrowing constraints, underdeveloped financial markets, low shareholding and/or a high share of myopic agents, the central bank would, by adopting an active rule, leave room for sunspot-driven real fluctuations. The size of these fluctuations would depend upon the exact size of the sunspot shocks (something impossible to quantify in practice), but this would unambiguously increase the variances of real variables. If such variance is welfare-damaging, as is almost uniformly believed to be the case and assumed in the literature, it is clear that such policies would be suboptimal since sunspot fluctuations themselves would be welfare-reducing. In contrast, in the same 'generically non-Ricardian' economy, a passive rule would rule out such fluctuations and would be closer to optimal policy. Our next task is to characterize some form of 'optimal' policy rules, when variability of inflation and output gap are costly.

### 5 Optimal time consistent monetary policy

We seek to establish whether and how does the presence of non-Ricardian agents alter the *optimal* design of monetary policy rules in the simple Non-Ricardian IS-AS model introduced above. To keep things simple, we shall only focus on the discretionary, and not fully optimal (commitment) solution to the central banker's problem. This case can be argued to be more realistic in practice, as do CGG  $(1999)^{19}$ . There is another sense in which we cannot treat our solution as an 'optimal' rule. The objective function we shall use will be a quadratic loss function in inflation and the output gap. While in the Ricardian case this can be derived as a second-order approximation to the representative agent's utility (as is done in Woodford 2003 Ch. 6), this welfare metric would modify in our case, for there is no representative agent in the first place<sup>20</sup>. But our approach could be justified if one sees relative price distortions as dominating any other distortions from a welfare standpoint<sup>21</sup>. We shall henceforth assume that the central bank has the following intertemporal objective function, standard in the literature:

$$-\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\alpha x_{t+i}^2 + \pi_{t+i}^2\right]\right\}$$
(29)

<sup>&</sup>lt;sup>19</sup>Moreover, gains from commitment are likely to be comparable to the standard Ricardian case, since they usually come from an improvement of the responses of variables to shocks to aggregate supply; but the aggregate supply of our model is not that different from that of the prototypical model analyzed in CGG (1999) or Woodford (2003), hence we have nothing to add to this debate.

 $<sup>^{20}</sup>$ Amato and Laubach (2003) do calculate the proper welfare function in a somehow related model with 'rule-of-thumb' households; however, their non-standard consumers' rule merely equates present to last period's consumption, which is not the case in our model.

 $<sup>^{21}</sup>$ This is indeed found to be the case by Woodford (2003, Ch 6), who introduces a series of other distortions in a welfare-maximizing framework. Schmitt-Grohe and Uribe (2003) do not use second-order approximations but the Ramsey approach to optimal policy, and reach a similar conclusion.

The optimal discretionary rule  $\{r_t^o\}_0^\infty$  is found by maximizing this objective function taking as a constraint the IS-AS system, and re-optimizing every period. Note that by usual arguments this equilibrium will be time-consistent. This is, up to interpretation of the solution, isomorphic to the standard problem in CGG (1999). Hence, for brevity, we skip solution details available elsewhere and go to the result:

$$x_t = -\frac{\kappa}{\alpha} \pi_t \tag{30}$$

When inflation increases the central bank has to act in order to contract demand, and expand it in case of deflation. Assuming an AR(1) process for the cost-push shock  $E_t u_{t+1} = \rho_u u_t$  for simplicity, we obtain the following reduced forms for inflation and output from the aggregate supply curve:

$$\pi_t = \alpha \frac{1}{\kappa^2 + \alpha (1 - \beta \rho_u)} u_t$$

$$x_t = -\kappa \frac{1}{\kappa^2 + \alpha (1 - \beta \rho_u)} u_t$$
(31)

Substituting these expressions into the aggregate demand curve, we obtain the *implicit instrument rule* consistent with the optimal time consistent equilibrium, written in terms of expected inflation for comparison with our previous instrument  $rule^{22}$ :

$$r_t^o = r_t^* + \phi_\pi^o E_t \pi_{t+1}$$

$$\phi_\pi^o = \left[ 1 + \frac{\delta \kappa}{\alpha} \frac{1 - \rho_u}{\rho_u} \right]$$
(32)

We can see that some of the (by now) classical results of CGG (1999) obtained in a Ricardian economy carry over: from the existence of a trade-off between inflation and output stabilization, to convergence of inflation to its target under the optimal policy. Shocks affecting only the natural rate of output (to the extent they would exist, which is not the case in our model) should not cause a policy reaction. Also, real disturbances affect nominal rates only insofar as they affect the Wicksellian interest rate, as discussed in detail for example by Woodford 2003 p.250. There is one important exception however, emphasized in the following Proposition.

**Proposition 3** (i) In a generically non-Ricardian economy ( $\delta < 0$ ) the implied instrument rule for optimal policy is passive  $\phi_{\pi}^{o} < 1$ . The optimal response to inflation is decreasing in the share of non-Ricardian agents  $\frac{\partial \phi_{\pi}^{o}}{\partial \lambda} < 0$ .

The above Proposition shows the exact way in which the central bank has to change its instrument in order to meet the targeting rule 30: contract demand when inflation increases, but move nominal rates such that the real rate

<sup>&</sup>lt;sup>22</sup>A positive policy response to inflation requires  $\alpha \geq -\delta \kappa \frac{1-\rho_u}{\rho_u}$ . This is obviously satisfied in the Ricardian case, but not necessarily in the non-Ricardian one, unless the central bank aims for output stabilization enough.

decreases in case the economy is non-Ricardian. This happens because part of aggregate demand (given by the non-Ricardian agents) is insensitive to interest rate changes, and the intuition is the same as provided before when ruling out sunspot equilibria with a passive rule. As consumption of non-Ricardian agents moves one-to one with (and hence overreacts to increases in) the wage, the other part of aggregate demand becomes oversensitive to interest rate changes though the channel emphasized repeatedly above. A decrease in the real rate is optimal, since otherwise (if the real rate increased) there would be too strong a fall in consumption of Ricardian agents, violating the optimality condition 30.

Note that responses to shocks are independent of the share of non-Ricardian agents under optimal policy 31. This is merely an implication of our ad-hoc objective function - it is likely that an utility-based welfare objective would at least have  $\alpha$  depend on the share of non-Ricardian agents. This is a natural next tackle but is beyond this paper's scope.

#### 5.1 Optimal policy without trade-off

It is clear from the above analysis that another insight of the monetary policy literature (see again i.a. Woodford's Ch 4 and CGG 1999) carries over in our setup - when cost-push shocks are absent (and so is the inflation-output stabilization trade-off), the flexible-price allocation can be achieved. This is done by having  $r_t^o = r_t^*$ , i.e. the nominal rate equal the Wicksellian rate at all times. However, there is one major difference with the usual literature: aside from being optimal, tracking the Wicksellian rate may also imply a unique rational expectations equilibrium under some conditions.

**Proposition 4** A policy rule whereby the nominal interest rate tracks the Wicksellian natural rate:

(i) implements the flexible-price allocation in the non-Ricardian model (when cost-push shocks are absent);

(ii) supports the optimum with stable prices as a unique rational expectations equilibrium if the parameters satisfy the condition:

$$\lambda \ge \frac{1 + \frac{1}{1 + F_Y} \varphi^S \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}}{1 + \frac{1}{1+\mu} \varphi^S} \tag{33}$$

Part (i) follows directly by inspecting the IS-AS system. Part (ii) is just a corollary of Proposition 1, Case II: just consider  $\phi_{\pi} = 0$ . The condition for this to be consistent with an unique equilibrium is then:  $1 + \delta \frac{2(1+\beta)}{\kappa} \leq 0$  which gives (33) above. Note also that  $r_t^o = r_t^*$  can also be optimal under the existence of the cost-push shocks, but this implies restrictions on the preferences of the central bank. Namely, such happens if and only if  $\alpha = -\delta \kappa \frac{1-\rho_u}{\rho_u}$  (which is around 0.65 under the baseline parameterization).

Point (ii) above means that price stability can be achieved *in principle* by having the Central Bank follow variations in the Wicksellian rate, and that would result in an unique rational expectations equilibrium, with no need for committing to react to inflation. This is in clear contrast with the Ricardian case studied in detail by Woodford (2003 Ch.4). There, the bank needs to commit to respond to inflation by fulfilling the Taylor principle  $r_t^o = r_t^* + \phi_\pi \pi_t, \phi_\pi > 1$  in order to pin down a unique equilibrium. But since such inflation would never occur in equilibrium, one then wonders whether one can estimate a Taylor rule of this type.

One further implication is that an interest rate peg, or any exogenous path for the interest rates, will result in a determinate equilibrium if the same condition is satisfied ((33) above). But is this parameter condition unrealistically restrictive<sup>23</sup>? Not quite: assuming usual numbers for the price stickiness and inverse Frisch elasticity of labor supply, namely  $\theta = 0.75, \varphi^S = 10$ , and zero steady-state profits  $F_Y = \mu$ , the threshold value of the share of non-Ricardian people is 0.126; compared to the empirical estimates of Campbell and Mankiw of around 0.5, this is a quite small number. Note that this threshold level is decreasing with price stickiness as illustrated in Figure 3 for two labor supply elasticities:  $\varphi^S = 10$  and 5. In the first case (thick line), while for flexible prices this condition cannot be fulfilled, for a high degree of price stickiness a very small share of non-Ricardian agents is enough to render the equilibrium determinate under an exogenous interest rate. Under a 1/2 elasticity (which is can be seen as an upper bound empirically - see discussion in King and Rebelo 2000), the required non-Ricardian share is higher as can be seen from the dashed thin line.



Fig.4: Threshold share of non-Ricardian agents for Proposition 4 to hold, as a function of price stickiness, for different labor supply elasticities.

However, the ability of the central bank to achieve full price stability as the *unique* equilibrium by tracking the Wicksellian interest rate applies to the simple model assumed here. Furthermore, it relies upon the ability/willingness of the bank to monitor the natural rate of interest and match its movement one-to-one

<sup>23</sup>In fact, the parameter subspace where this condition is fulfilled is non-empty if and only if  $\frac{1+\frac{1}{1+F_Y}\varphi^S \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}}{1+\frac{1}{1+\mu}\varphi^S} < 1$ , implying  $\frac{2\theta(1+\beta)}{(1+\theta)(1+\beta\theta)} > \Omega_Y = \frac{\mu-F_Y}{1+\mu}$ , which is always satisfied as long as prices are sticky to some (no matter how small) extent and the steady-state share

as long as prices are sticky to some (no matter how small) extent and the steady-state share of profits  $\Omega_Y$  is zero.

by movements in the nominal rate. So usual caveats of such a policy proposal emphasized by Woodford apply, where one can add that the natural interest rate can sometimes be negative. In practice moreover, central banks do respond to movements in macroeconomic aggregates, as a huge and important literature emphasized, by following interest rate rules, whether resulting from optimization or not, as the ones we studied previously.

### 6 The dynamic effects of shocks

In this section we go back to the simple instrument rule and try to compute analytically the effects of fundamental and sunspot shocks under determinacy and indeterminacy, distinguishing between the Ricardian and non-Ricardian cases. Our interest in this exercise is twofold. First, it might be of interest in itself to understand the effects of shocks in a determinate non-Ricardian model. One obvious historical candidate for such a case could be found in the pre-Volcker era; it is fairly well established (see e.g. CGG 2000, Taylor 1999, Lubik and Schorfheide 2003b) that the response of monetary policy in that period implied a (long-run) response to inflation of less than one. But if we allow for the possibility that the economy were non-Ricardian, this would not imply that policy was inconsistent with a unique equilibrium; hence, we will be able to assess the effects of fundamental shocks, an impossible task under indeterminacy (more below). Indeed, we shall argue that in this case, such shocks can explain stylized facts of the pre-Volcker period (impulse responses to shocks and moments) quite well. Secondly, there is the mirror image of the above argument. Estimates of policy rule coefficients in the Volcker-Greenspan era for the US (and similarly, for most other industrialized countries) such as e.g. CGG 2000 indicate a response of nominal rates to inflation larger than one. Coupled with the possibility of a non-Ricardian economy, this would instead imply indeterminacy. Hence, it might be of interest to assess the effects of various (fundamental and sunspot) shocks in an indeterminate equilibrium.

We follow the new method proposed by Lubik and Schorfheide (2003a) to compute sunspot equilibria by decomposing expectational errors, building upon the approach of Sims (2000). The IS-AS system can be written, in terms of the defined variables  $\xi_t^z \equiv E_t z_{t+1}$ , so  $\xi_t \equiv (\xi_t^y, \xi_t^\pi)$  and define the expectational errors  $\eta_t^z \equiv z_t - E_{t-1} z_t$ .

$$\xi_t = \mathbf{\Gamma}\xi_{t-1} + \mathbf{\Psi}\nu_t + \mathbf{\Gamma}\eta_t$$

The coefficient matrices  $\Gamma, \Psi$  are given in (26). We replace  $\Gamma$  by its Jordan decomposition  $\Gamma = JQJ^{-1}$  and define the auxiliary variables  $z_t = J^{-1}\xi_t$  and rewrite the above model as

$$z_t = Q z_{t-1} + J^{-1} \Psi \varepsilon_t + J^{-1} \Gamma \eta_t \tag{34}$$

The eigenvalues of  $\Gamma$  are found to be:

$$q_{\pm} = \frac{1}{2} \left[ tr\Gamma \pm \sqrt{\left(tr\Gamma\right)^2 - 4\det\Gamma} \right]$$
(35)

(where the determinant and trace are det  $\Gamma = \beta^{-1} > 1$ ,  $tr\Gamma = 1 + \beta^{-1} - \beta^{-1} \delta^{-1} \kappa (\phi_{\pi} - 1)$ ). The corresponding eigenvectors are stacked in the J matrix:

$$J = \begin{bmatrix} \frac{1}{\kappa} \left(1 - \beta q_{-}\right) & \frac{1}{\kappa} \left(1 - \beta q_{+}\right) \\ 1 & 1 \end{bmatrix}$$

#### 6.1 Determinacy

The equilibrium under determinacy is easily calculated, since the only stable solution is  $\xi_t = 0$ , obtained for:

$$\Psi \nu_t + \Gamma \eta_t = 0$$

Hence, the expectation errors are determined exclusively by fundamental shocks (and sunspot shocks would have no effect on dynamics) by  $\eta_t = -\Gamma^{-1}\Psi\nu_t$ , namely:

$$\eta_t = -\delta^{-1} \begin{bmatrix} 1\\ \kappa \end{bmatrix} (\varepsilon_t - r_t^*) + \begin{bmatrix} \kappa \delta^{-1} \beta^{-1} (1 - \phi_\pi) \\ 1 - \kappa \delta^{-1} \beta^{-1} (\phi_\pi - 1) \end{bmatrix} u_t$$
(36)

The initial impact on output and inflation is also given by the same expression. Since both roots are eliminated under determinacy, all after-shock dynamics will come from the persistence of exogenous shock processes. Note the sharp differences for the two subcases identified above, showing asymmetric effects of some shocks depending on whether the economy is Ricardian or not.

Case I: (Generically) Ricardian economy,  $\delta > 0, \phi_{\pi} \in \left(1, 1 + \delta \frac{2(1+\beta)}{\kappa}\right)$ : The effects at work are as usual and we should not insist upon their interpretation. A policy-induced interest rate cut or an increase in the natural rate of interest (coming here only from shocks to technology growth) increase both the output gap and inflation. Cost-push shocks have a negative effect on the output gap, and generally inflationary effects depending upon the policy response<sup>24</sup>.

**Case II: (Generically) Non-Ricardian economy**,  $\delta < 0, \phi_{\pi} \in \left(1 + \delta \frac{2(1+\beta)}{\kappa}, 1\right)$ . In contrast to the standard Ricardian case, a monetary contraction (positive  $\varepsilon_t$ ) has expansionary effects, and causes inflation. This follows directly from our intuition above regarding the labor market equilibrium. A monetary contraction shifts the labor supply curve right, making agents want to work more at the same wage, which instead leads to an increase in hours and the real wage, and in output and inflation thereby (we do not insist upon the whole mechanism making this happen, as this was emphasized above). An increase in the natural rate of interest driven by technology results in a recession and deflation - this shall be addressed in some detail below. It is clear then that a policy response increasing the nominal rate by more than the natural rate  $\varepsilon_t > r_t^*$  increases both output and inflation, whereas when it falls short of doing so, it has deflationary

<sup>&</sup>lt;sup>24</sup>There is an inflationary effect if  $\phi_{\pi} < 1 + \frac{\beta}{\kappa}\delta$ , and a deflationary effect if  $\phi_{\pi} \in \left(1 + \frac{\beta}{\kappa}\delta, 1 + \delta\frac{2(1+\beta)}{\kappa}\right)$ . As the latter case implies unrealistically high policy coefficients, we can conclude that in general cost-push shocks have inflationary effects.

effects, and causes a fall in output. Cost-push shocks have still a negative effect on the output gap, and an inflationary effect in general<sup>25</sup>. While the magnitude of the response is changed in the non-Ricardian economy, depending both upon deep parameters and policy coefficient, the same intuition as in the Ricardian case applies.

#### 6.2 Indeterminacy

In this case one of the roots  $q_{\pm}$  will be inside the unit circle. In this case sunspot shocks can have real effects, and the responses to fundamental shocks change too, in a way made explicit below. We confine ourselves to the case whereby the smaller root is inside the unit circle and the larger one is greater than one, i.e.  $q_{-} \in (-1,1)$  and  $q_{+} > 1$ . This can be shown to be the case if either (i)  $\delta > 0, \phi_{\pi} < 1$  or (ii)  $\delta < 0, \phi_{\pi} > 1^{26}$  Since in this case there is one-dimensional indeterminacy, the stability condition for 34 modifies: expectation errors are not spanned by fundamental shocks, but by both fundamental and sunspot shocks.

We can apply the results in Proposition 1 in Lubik and Schorfheide to solve for the full solution set for the expectational errors. This is described in some detail in the Appendix, and the solution is:

$$\eta_{t} = -\frac{\kappa\delta^{-1}}{d^{2}} \begin{bmatrix} \kappa q_{+} \\ 1 - q_{+} \end{bmatrix} (\varepsilon_{t} - r_{t}^{*}) + \frac{1}{d^{2}} \begin{bmatrix} \kappa\beta^{-1}(1 - q_{+}) \\ (1 - q_{+})(q_{-} - \beta^{-1}) \end{bmatrix} u_{t} + \frac{1}{d} \begin{bmatrix} q_{+} - 1 \\ \kappa q_{+} \end{bmatrix} (M_{1}\nu_{t} + \varsigma_{t}^{*})$$
(37)

where  $M_1$  is an arbitrary 2×2 matrix and  $\varsigma_t^*$  is a reduced-form sunspot shock, which can be interpreted as a belief-induced increase in output and/or inflation of undetermined size. First thing to note is that a positive realization of this shock will increase output and inflation no matter whether the economy is Ricardian or non-Ricardian since  $q_+ > 1$  as established above. This conforms our intuitive construction of sunspot equilibria when discussing determinacy properties of interest-rate rules.

On the other hand, the effects of fundamental shocks become ambiguous, and depend crucially upon the choice of  $M_1$ . Unfortunately, there is nothing to pin down a choice for this matrix, which captures the well-known problem of indeterminate equilibria - the effects of fundamental shocks cannot be studied without further restrictions. Two leading possibilities to restrict the  $M_1$  matrix are suggested by Lubik and Schorfheide:

 $<sup>^{25}</sup>$ In fact, the effect on inflation is positive for  $\phi_{\pi} > 1 + \frac{\beta}{\kappa} \delta$  and negative otherwise. However, note that under our baseline parameterization  $1 + \frac{\beta}{\kappa} \delta$  is negative, and as the policy coefficient is positive; so there will *in general* be an inflationary effect of cost-push shocks, as in the Ricardian case.

<sup>&</sup>lt;sup>26</sup>For the rest of the parameter regions where there is indeterminacy we would have  $q_+ \in (-1, 1)$  and  $q_- < -1$ , but this can be shown to imply very restrictive conditions on the deep parameters and the policy rule coefficient.

#### 6.2.1 Orthogonality

The two sets of shocks are orthogonal in their contribution to the forecast error, and hence  $M_1 = 0$  in 37. The effect of a cost-push shock is of the same sign whether the economy is Ricardian or not, as is independent of  $\delta$ . A positive realization of this shock would increase inflation (since  $(1 - q_+) (q_- - \beta^{-1}) > 0$ ) and decrease output  $(q_+ > 1)$ , as it did under determinacy for reasonable policy responses. The effects of policy shock, and of shocks to the natural rate of interest, are again different depending on which case we consider:

Ricardian case,  $\delta > 0$ : An interest rate increase keeping constant the natural rate decreases output under its natural level but causes inflation as  $1 - q_+ < 0$  (this is also found by Lubik and Schorfheide for a contemporaneous rule). An increase in the natural rate without a discretionary policy response increases the output gap and causes deflation.

Non-Ricardian case  $\delta < 0$ : A policy-induced interest rate increase increases output and causes deflation. An increase in the natural rate not matched by policy depresses output and causes inflation. In either case, the overall effect on inflation and the output gap depends on whether the policy response is stronger or weaker than the variation in the natural rate.

#### 6.2.2 Continuity

In order to preserve continuity of the impulse responses to the fundamental shock when passing from determinacy to indeterminacy,  $M_1$  can be chosen such that it implies that the response to the fundamental shock is the same, i.e.:

$$\begin{split} \eta_t &= -\delta^{-1} \begin{bmatrix} 1\\ \kappa \end{bmatrix} (\varepsilon_t - r_t^*) + \begin{bmatrix} \kappa \delta^{-1} \beta^{-1} (1 - \phi_\pi) \\ 1 - \kappa \delta^{-1} \beta^{-1} (\phi_\pi - 1) \end{bmatrix} u_t \\ &+ \frac{1}{d} \begin{bmatrix} q_+ - 1\\ \kappa q_+ \end{bmatrix} \varsigma_t^* \end{split}$$

This happens for a very particular  $M_1$  matrix and implies that the effects of fundamental shocks are as under determinacy, namely in the non-ricardian case a contractionary policy shock increases both output and inflation. While continuity is an attractive feature, there is nothing to insure that the  $M_1$  takes exactly the form necessary to get this result.

# 7 Revisiting the pre-Volcker period: new insights.

This section looks at the pre-Volcker period using the theoretical insights developed above. We will argue that the possibility of the economy being non-Ricardian in that period may help explain both why policy was conducted as it was, and the effects of policy on macroeconomic variables and their variability. Furthermore, we shall argue that such a possibility makes uncertainty spanned by fundamental shocks be enough in generating more aggregate variability.

It is an almost consensual view that monetary policymaking changed with the coming to office of Paul Volcker. One instance of this is a change in estimated coefficients of interest rate rules. CGG (2000), Taylor (1999), Lubik and Schorfheide (2003) and Cogley and Sargent (2002) all reach such a conclusion. One is then tempted to attribute (at least part of) the change in dynamics of macro variables (mainly inflation and output) and their variability to such a change in policy<sup>27</sup>. Most importantly, since a passive rule leads to an indeterminate equilibrium in the models of CGG and Lubik and Schorfheide, these authors argue that part of inflation variability can be accounted by sunspot shocks. However, the same authors show that sunspot shocks drive up both inflation and output (and this was he case even in our non-Ricardian economy above). If one wants to find an explanation for high inflation and recessions (features of the 1960-1980 period) sunspot shocks may not be a good candidate. Fundamental shocks, on the other hand, cannot be studied in an indeterminate equilibrium as the one with a passive rule in the standard models: they can have virtually any effects<sup>28</sup>. On the other hand, this is possible in the model just developed, if one assumes that some high enough fraction of agents were non-Ricardian. Hence, we should look at the responses and moments of macro variables under two different scenarios, using the parametrization described earlier. We shall consider the pre-Volcker period as a period with a passive rule, and a high enough share of non-Ricardian agents to make the economy generically non-Ricardian. We then consider the Volcker-Greenspan period with an active rule and very low share of non-Ricardian agents, corresponding to a view whereby financial markets became more deregulated, constraints relaxed, consumers were more informed, shareholding became more common, etc. The policy rules are parameterized using estimates by CGG and Taylor, while the share of non-Ricardian agents in the pre-Volcker period is taken to be the lower bound of the estimates of Campbell and Mankiw 1989,  $\lambda = 0.4$ . Notably, we keep the variances of shocks unchanged across the two periods.

The first experiment is a unit-cost push shock. This exercise is all the more relevant, in our view, given the recent findings of Peter Ireland (2003) suggesting that these shocks were the main cause of fluctuations in the pre-Volcker era. The responses of various variables under the two scenarios are plotted in Figure 5 (circles for non-Ricardian and triangles otherwise). Indeed, the responses conform both conventional wisdom and what we view as a good test for a theory purported to explain dynamics in that period: higher inflation, low

<sup>&</sup>lt;sup>27</sup>Many authors have emphasized that increased variability may come from a different distribution from which shocks were drawn in that period - see Sargent 2002 and the studies by Sims and Bernake and Mihov quoted therein. This is likely to be an important explanation. But a change in variances of shocks, however, would not generate a change in responses to shocks in itself.

 $<sup>^{28}</sup>$ CGG(2000) argue that even variability as explained by cost-push shocks is increased in a 'near-determinate' equilibrium, whereby the coefficient on inflation is slightly above one. Hence, this would explain increased variability and higher inflation from fundamentals. But this merely explains why in a determinate equilibrium with an active rule responding less to inflation results in higher variability of the latter. Dynamics in the indeterminate equilibrium are not pinned down.

Table 1: Conditional standard deviations, cost-push shock

	Pre- Volcker	Volcker- Greenspan
$\sigma_x$	0.83898	1
$\sigma\pi$	4.7762	1
$\sigma_r$	2.5473	1

real rates, and negative comovement of inflation and the output gap. Moreover, responses of output and inflation have the same sign in both cases, as shown analytically above. But note that the response of inflation is much larger in the non-Ricardian scenario. The response of output is not much different, and the real rate is negative as expected, since the policy rule is passive. The Wicksellian rate is of course unchanged. Table 2 looks at conditional standard deviations of output gap, inflation and interest rates, normalizing standard deviations in the Ricardian scenario to 1.



 $\phi_{\pi} = 0.8$ ; line with triangles has  $\lambda = 0.05$  and  $\phi_{\pi} = 1.5$ . Otherwise baseline parameterization.

The implied standard deviation of inflation and interest rates are much

 Table 2: Implicit preferences making the estimated rules optimal

 PreVolcker

 Volcker 

		Greenspan
$\alpha^*$	0.437	0.0769

higher for the parameterized pre-Volcker period, confirming conventional wisdom and empirical findings, while the standard deviation of the output gap (and implicitly output) is slightly lower. Note that these results are obtained in a determinate equilibrium, keeping constant the variance of shocks across the two periods, and changing only the share of non-Ricardian agents, and the policy responses.

We next want to ask whether such a difference in responses could have resulted even if the FED was following optimal policy, and what did this imply in terms of preference parameters. First thing to note is that for the estimated Taylor-CGG policy rules, and for the other deep parameters of the model given, we can track down one weight on output stabilization  $\alpha$  which would make the estimated rule exactly the same as the optimal rule; this is done by merely solving for  $\alpha$  in the equation  $\phi_{\pi} = \left[1 + \frac{\delta \kappa}{\alpha} \frac{1-\rho_u}{\rho_u}\right]$ , for the two scenarios considered above, and the results are in Table 2. These implicit weights are:

Even if estimated policy rules were *optimal*, difference in macroeconomic performance could be explained by a change in preferences of the FED. This squares with the general view that the FED was less 'conservative' in Rogoff's 1985 sense, i.e. it cared more about output stabilization<sup>29</sup> in the pre-Volcker period (see e.g. Sargent 2002). While this is specific to the parameterization considered, the general message it captures carries over to alternatives as long as we preserve the Ricardian-non-Ricardian distinction. The estimated monetary policy response in the pre-Volcker period may not have been totally inconsistent with optimal policy, if the preference for output stabilization was relatively high.

#### 7.1 Technology shocks

Can the model exposed here help us explain the responses to technology shocks also? A recent paper by Gali, Lopez-Salido and Valles (2003a) estimates the empirical responses of various macroeconomic variables in the two sub-periods. They find that in the pre-Volcker era, a positive shock to technology growth (identified as having permanent effects using the method of Gali 1999) was associated with a fall in output below potential and a fall in inflation. We find it worth re-emphasizing that such empirical responses cannot be compared with their theoretical counterpart in the standard Ricardian models, whereby the effects of fundamental shocks cannot be assessed when the policy rule is passive; the equilibrium is in that case indeterminate. But this is possible in our framework. Figure 6 plots the responses of the economy under the non-Ricardian

<sup>&</sup>lt;sup>29</sup>Whether this can be justified on welfare-maximization grounds, given the 'Non-Ricardian' nature of the economy, is in our view an interesting question.

parameterization, compared to the benchmark case of optimal policy whereby the central bank tracks the Wicksellian rate.



Fig.6: Impulse responses to unit shock to technology growth. Estimated rule is with circles, optimal policy with triangles.

The model fits remarkably well the stylized facts mentioned above: both inflation and the output gap decrease. The central bank responds to inflation (and deflation) without internalizing the effect on the natural interest rate. The nominal rate declines since there is deflation (and recession), but this response is suboptimal. Note that the response is not suboptimal because it is too weak in the sense that the nominal rate does not decrease enough to make real rates decline! Indeed, that would lead to indeterminacy of equilibrium, which is at the heart of our Inverted Taylor Principle. Instead, the response is suboptimal because it has the *wrong sign*! The optimal response (plotted in the circle lines) requires nominal rate increases to accommodate the increase in Wicksellian rate brought about by the positive technology shock. While the responses conform empirical findings, it is hard to argue that technology shocks were in fact driving fluctuations in inflation and output gap in that period, since what one wants to explain is high inflation coupled with recessions. Indeed, based on variance decompositions from a DSGE model estimated by maximum likelihood, Ireland (2003) finds that technology shocks did not play an important role in the pre-Volcker era in driving such fluctuations.

Without deviating from optimal policy, but at the same time letting the technology shocks have effects, can we explain high inflationary episodes in the present model? We argue that this is possible, if one resorts to an argument already put forward by DeLong (1997) and Orphanides (2002). These authors argue that the FED was overestimating the natural rate of output and was keeping real rates too low because it was implicitly underestimating the relative level of actual output, which it seeked to stabilize. Such policy response can be accommodated in our model. To see a simple instance of this, consider that the FED was following what it thought to be optimal policy, but it was overestimating the natural interest rate (and the natural output) systematically over that period<sup>30</sup>. Hence, it was systematically moving the interest rate by changing the intercept in the policy rule  $\varepsilon_t$  more than required, e.g.  $\varepsilon_t = \hat{r}_t^* = 1.1r_t^*$ , where a hat means the estimate of the central bank. Such a case (compared with optimal policy without estimation errors  $\varepsilon_t = \hat{r}_t^* = r_t^*$ ) is plotted in Figure 7 in the graphs with triangles.



Fig.7: Estimated responses to unit technology growth shock. In the 'circles' economy, natural rate of interest is systematically overestimated.

Overestimating the natural interest rate creates inflation, and higher volatility of inflation if compared to optimal rule, but the mechanism is quite different from the usual one; indeed, real rates here increase too much when compared to optimal policy, which leads to inflation by the mechanism stressed throughout this paper when the economy is non-Ricardian. However, by the same mechanism, this would also generate a positive output gap. Moreover, for significant

 $<sup>^{30}</sup>$  For simplicity, and just to make the point, we assume here that the FED does not actually learn the true process for the natural rate, neither that it is extracting a signal from a noisy variable. This could be easily accommodated.

departures from optimal policy to obtain, estimation errors should be quite large. One could conclude that cost-push shocks' role in driving fluctuations and output might have indeed been important in the pre-Volcker era.

The results presented above rely upon a very simple model; we find it worth stressing, however, that they are robust to further complications, and are only depending crucially on whether the economy can ever be 'non-Ricardian' or not. But insofar as the economy were 'non-Ricardian' in the sense of this paper, business cycle fluctuations might have well not changed during the 80's because of 'better' policy. While monetary policy did change with the coming to office of Paul Volcker, this might have not been the *cause* of the business cycle change (this is argued forcefully by Stock and Watson 2002, 2003). What might have changed are structural features of the economy such as the ones making the economy 'more Ricardian' (more access to asset markets, less credit constraints, etc.). Policy, instead, might have been quite well managed even before Volcker - for if such financial frictions were predominant (and in the way considered by this paper), responding more actively to inflation would have led to great aggregate instability. Greater variability in macroeconomic aggregates might result exactly from this structural change, let alone the most likely change in the distribution of shocks (see Sargent 2002).

### 8 Tentative conclusions

Interest rate changes modify the intertemporal consumption and labor supply profile of ('Ricardian') agents who have access to complete asset markets and can smooth consumption. This affects the real wage, and the demand thereby of ('non-Ricardian') agents who have no asset holdings, are oversensitive to real wage changes, and insensitive directly to interest rate changes. If the share of non-Ricardian agents is high enough and/or and the elasticity of labor supply is low enough, this last effect works to offset the interest rate effects on demand of Ricardian agents. For in such a case, variations in the real wage mean variations in marginal costs, which instead lead to variations in profits (and hence dividend income) that can offset (and indeed overturn) the initial impact of interest rates on aggregate demand. The equilibrium wage-hours locus is upward sloping but cuts the labor supply curve from above. This is the main mechanism identified by this paper to change drastically the effects of monetary policy as compared to a standard, Ricardian case, whereby aggregate demand is completely composed of forward-looking agents. The required share of non-Ricardian agents for the economy to become 'generically non-Ricardian' is found to be relatively mild for parameterizations usually employed in the literature, and far below empirical estimates of Campbell and Mankiw (1989).

This paper analyzes such changes, and thereby challenges some results dictated by conventional wisdom in the field. Its scope is very limited: to make a small contribution to the literature emphasizing the role of non-Ricardian consumers in shaping macroeconomic policy and helping towards a better understanding of the economy. In that respect, we just seek to add to a new developing literature analyzing the role of non-Ricardian agents in dynamic general equilibrium models, initiated by Mankiw (200) and Gali, Lopez-Salido and Valles (2002) for fiscal policy and Gali, Lopez-Salido and Valles (2003) for monetary policy. The main contributions of the paper are two-fold:

- 1. Normative: Our results imply that central banks should arguably pay attention to the demand side of the economy when designing interest rate rules. Notably, the extent to which agents participate to asset markets and hence smooth consumption would become an important part of the policy input. While the degree of development of financial markets might make this not a concern in present times in the developed economies, central banks in developing countries with low participation in financial markets might find this of some practical interest. The theoretical results hinting to such policy prescriptions are that: (i) In a 'non-Ricardian' economy an 'Inverted Taylor Principle' holds generically<sup>31</sup>; the central bank needs to adopt a passive policy rule to ensure equilibrium uniqueness and rule out the possibility of self-fulfilling sunspot-driven fluctuations; (ii) Optimal and time-consistent (discretionary) monetary policy also implies that the central bank should move nominal rates such that real rates decline in a generically non-Ricardian economy. For it is by decreasing the real rate that aggregate demand is contracted when inflation increases, as required by the targeting rule, which is different from the standard Ricardian case as explained in text.
- 2. **Positive**: Our theoretical findings may offer new insights on interpreting episodes whereby empirical estimates of interest rate rules would imply equilibrium indeterminacy, as is the case for the pre-Volcker era. If one is ready to adopt a view whereby the economy in that period was characterized by underdeveloped financial markets, one might be able to suggest that pre-Volcker policy was consistent with a determinate equilibrium, and hence did not leave room for non-fundamental fluctuations. One is then able to study the effects of fundamental shocks, which is a notoriously impossible task when equilibrium is indeterminate. We explore such a possibility, and find that theoretical responses to fundamental shocks conform empirically estimated responses. Notably, we find that cost-push shocks (found by others to have been the primary source of fluctuations in that period) generate higher inflation and inflation variability in the non-Ricardian economy than they do in the Ricardian counterpart (where we use the policy responses estimated by CGG 2000). We also suggest

<sup>&</sup>lt;sup>31</sup>This result depends only to a small extent on whether the rule is specified in terms of current or expected future inflation. As discussed in text in more detail, this is in contrast to Gali, Lopez-Salido and Valles (2003) who, while having noted the possibility to violate the Taylor principle for a forward-looking rule, also argue that a strengthening of the Taylor principle is required for a contemporaneous rule to result in equilibrium uniqueness. A very strong response to current inflation would also insure determinacy in our model, but we find the implied coefficient is higher than any plausible estimates, makes policy non-credible and qould lead to violation of the zero lower bound in case of small deflations.

that pre-Volcker policy could have well approximated optimal policy, if we accept that output stabilization was more important in that period (which is dictated by conventional wisdom). Whether that is legitimate from a welfare perspective is a question with yet no answer. As to technology shocks, the theoretical responses in our non-Ricardian economy match empirical responses estimated by Gali et al 2003a, leading to deflation and output below potential. Too inflationary a policy in response to technology shocks results in our non-Ricardian world if the central bank overestimates the natural rate of interest, despite following an otherwise optimal policy. This conforms the view of some authors (e.g. Orphanides 2002) about pre-Volcker policy. All in all, our results may contribute towards a partial explanation of the change in business cycles based on a change on the structure of the economy (in this case, developing financial markets and hence better consumption smoothing), rather than 'better policy'; Stock and Watson (2002, 2003) provide empirical evidence favoring such a view.

The model presented has the advantage of simplicity; indeed, we show how to analyze non-Ricardian economies analytically in the same type of framework used in standard, Ricardian analyses. However, this simplicity can potentially also be a shortcoming, for it implies many realistic features have been left out. How these insights would modify if one incorporates other realistic features in the model economy is an interesting question for future research. More important still, in our view, would be to relax the way we modelled for non-Ricardian households, which is a shortcoming we share with the rest of the literature, as this very literature emphasizes (see Gali et al 2002, 2003). The present approach is only justified for tractability, but potentially important insights can be gained by an explicit modelling of microfoundations for non-'*Ricardianess*' (or demand-side frictions). Crucially, where this could help is in deriving a proper welfare metric in order to analyze optimal policy choices meaningfully. This is potentially a cumbersome exercise, but to our mind worth of all further investigation<sup>32</sup>. Lastly, assessing empirically the extent to which some agents do not smooth consumption, the evolution of this over time, and its implications at aggregate level, is in our view a necessary step for understanding business cycle dynamics, which we intend to pursue further.

# A Loglinearized equilibrium

#### Ricardian:

Euler equation, intratemporal and budget constraint ( $o_t$  are profits as a share of steady-state GDP,  $o_t = \frac{O_t - O}{V}$ ):

 $<sup>^{32}\</sup>mathrm{A}$  related paper by Amato and Laubach (2002) calculates a welfare metric for an economy with 'rule-of-thumb' consumers, although their approach in modelling rules of thumb is different from the one taken here.

$$E_t [c_{s,t+1}] - c_{s,t} = r_t - E_t [\pi_{t+1}]$$
(38)

$$\varphi_s n_{s,t} = w_t - c_{s,t} \text{ where } \varphi_S = \left[\frac{\gamma_S N_s}{1 - N_s}\right]$$
 (39)

$$\frac{C_S}{Y}c_{s,t} = \frac{W}{P}\frac{N_s}{Y}\left(w_t + n_{s,t}\right) + \frac{1}{1-\lambda}o_t \tag{40}$$

Non-Ricardian

Intratemporal and budget constraint:

$$n_{h,t} = 0 \tag{41}$$

$$c_{h,t} = w_t \tag{42}$$

Firms:

$$y_t = (1+F_Y) n_t + (1+F_Y) a_t$$
(43)

$$mc_t = w_t - a_t \tag{44}$$

$$o_t = -\frac{1+F_Y}{1+\mu}mc_t + \frac{\mu}{1+\mu}y_t$$
(45)

$$\pi_t = \beta E_t \pi_{t+1} + \psi m c_t, \ \psi = \frac{(1-\theta)(1-\theta\beta)}{\theta}$$
(46)

Market clearing

Labour market (n = labour demand by firms)

$$n_t = \frac{(1-\lambda)N_s}{N}n_{s,t} \tag{47}$$

Aggregate consumption is:

$$c_t = \frac{\lambda C_H}{C} c_{h,t} + \frac{(1-\lambda) C_s}{C} c_{s,t}$$
(48)

Aggregate resource constraint - equilibrium in goods market, holds by Walras' law, and is redundant.

$$y_t = c_t \tag{49}$$

Monetary policy rule:

$$r_t = \phi_r r_{t-1} + (1 - \phi_r) \phi_\pi E_t \pi_{t+k} + (1 - \phi_r) \phi_y E_t y_{t+q}$$
(50)

#### A.1 Steady state

$$R = \frac{1}{\beta} \text{where } R \equiv 1 + r$$
$$\frac{W}{P} = \frac{Y + F}{N} \frac{MC}{P} = \frac{Y}{N} \frac{1 + \frac{F}{Y}}{1 + \mu}$$
$$profits : \frac{O}{Y} = \frac{\mu - F_Y}{1 + \mu}$$

We assume hours are the same for the two groups in steady state only,  $N_H = N_S = N$ . Then, for the loglinear budget constraints of both agents the coefficients are fully determined:

$$\frac{W}{P}\frac{N_s}{Y} = \frac{1+F_Y}{1+\mu}; \ \frac{C_S}{Y} = \frac{1+F_Y}{1+\mu} + \frac{\mu-F_Y}{1+\mu}\frac{1}{1-\lambda} = \frac{1}{1-\lambda}\left(1-\lambda\frac{1+F_Y}{1+\mu}\right)$$
$$\frac{WN_H}{PY} = \frac{C_H}{Y} = \frac{1+F_Y}{1+\mu}$$

# **B** Deriving the IS-AS system

Since Walras' law holds I will use the economy resource constraint instead of the Ricardian budget constraint in the derivation. We seek to express everything in terms of aggregate variables, and then use the two dynamic equations to get dynamics only in terms of output, inflation and interest rate. First, try to express consumption of Ricardian household as function of aggregate variables, from 42, 47, 48 using the steady state coefficients just calculated:

$$c_{s,t} = \frac{1}{1-\lambda} \frac{C}{C_s} y_t - \frac{\lambda}{1-\lambda} \frac{C_h}{C_s} w_t \tag{51}$$

Substituting this, together with 47 into 39 and using the production function we get:

$$w_t = \chi y_t - (1 + F_Y) (\chi - 1) a_t = \chi (1 + F_Y) n_t + (1 + F_Y) a_t$$
(52)

where 
$$\chi \equiv \left[1 + \varphi^s \frac{C_s}{C} \frac{1}{1 + F_Y}\right] = \left[1 + \varphi^s \frac{1}{1 - \lambda} \frac{1}{1 + F_Y} \left(1 - \lambda \frac{1 + F_Y}{1 + \mu}\right)\right]$$
 (53)

Note that we have always  $\chi \ge 1$ . Substituting back into 51 and using the steady state consumption shares we get consumption of saver as a function of output:

$$c_{s,t} = \delta y_t + (1+F_Y)(1-\delta)a_t = \frac{\delta}{\chi}w_t + (1+F_Y)\left(1-\delta + \frac{\chi-1}{\chi}\right)a_t$$
$$\delta \equiv 1 - \varphi^s \frac{\lambda}{1-\lambda}\frac{1}{1+\mu}$$

Note  $\chi = \delta + \varphi^s \frac{1}{1-\lambda} \frac{1}{1+F_Y}$ 

The elasticity (share)  $\delta$  will turn out to play an important role for determinacy properties and dynamics. Having done this, we just need to replace these last two equations in the Euler and New Keynesian Philips curve to obtain a system in output and inflation. As we have technology shocks, it is easier to write the whole system in terms of the output gap (difference of actual output from output under flexible prices) as is usually done in the literature. Real marginal cost is given by

$$mc_t = \chi y_t - \left[ (1 + F_Y) \left( \chi - 1 \right) + 1 \right] a_t \tag{54}$$

Since in the flexible-price equilibrium the markup is constant (and so is the real marginal cost) we see directly from 54 that natural output is:

$$y_t^* = \left[1 + F_Y\left(1 - \frac{1}{\chi}\right)\right]a_t$$

So marginal cost is related to the output gap  $x_t \equiv y_t - y_t^*$  by:

$$mc_t = \chi \left( y_t - y_t^* \right) = \chi x_t \tag{55}$$

Following Clarida, Gali and Gertler (1999) or Gali (2002) we also introduce cost-push shocks  $u_t$ , i.e. variations in marginal cost not due to variations in excess demand. These could come from the existence of sticky wages creating a time-varying wage markup, or other sources creating this inefficiency wedge although we do not model this explicitly here. Hence, marginal cost variations are given by

$$mc_t = \chi x_t + u_t \tag{56}$$

Substituting consumption of Ricardian agents in the Euler equation, we can write

$$\delta E_t x_{t+1} = \delta x_t + [r_t - E_t \pi_{t+1}] + (1 + F_Y) (1 - \delta) [a_t - E_t a_{t+1}] - \delta [E_t y_{t+1}^* - y_t^*]$$
(57)

We can define the natural rate of interest (Wicksellian interest rate)  $r_t^*$  as the level of the interest rate consistent with output being at its natural level (and hence with zero inflation), as in Woodford 2003. Solving from 57 we obtain:

$$r_t^* = \left[1 + F_Y\left(1 - \frac{\delta}{\chi}\right)\right] \left[E_t a_{t+1} - a_t\right]$$
(58)

Assuming  $a_t$  is given by an AR(1) process such that  $E_t a_{t+1} = \rho^a a_t$ , we note that  $r_t^* = -\left[1 + F_Y\left(1 - \frac{\delta}{\chi}\right)\right](1 - \rho^a)a_t$ , such that the natural interest rate varies negatively with technology.

Using 55 and 58 into the New Keynesian Philips curve and the 57 we get the reduced system:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \text{ where } \kappa \equiv \psi \chi \tag{59}$$

$$E_t x_{t+1} = x_t + \delta^{-1} \left[ r_t - E_t \pi_{t+1} - r_t^* \right]$$
(60)

For simplicity, we consider only monetary policy rules involving inflation stabilization and no inertia, of the form:

$$r_t = \phi_\pi E_t \pi_{t+k} + \varepsilon_t$$

where  $\varepsilon_t$  are policy shocks, i.e. movements in nominal rates coming form anything else than systematic response to inflation.

## C Proof of Proposition 2

Necessary and sufficient conditions for determinacy in such systems are (given in Woodford Appendix to Chapter 4):

Either: A: (A1) det 
$$\Gamma$$
 > 1; (A2) det  $\Gamma - tr\Gamma > -1$  and (A3) det  $\Gamma + tr\Gamma > (61)$   
Or: B: (B1) det  $\Gamma - tr\Gamma < -1$  and (B2) det  $\Gamma + tr\Gamma < -1$  (62)

For our forward -looking rule case, the determinant and trace are:

$$\det \mathbf{\Gamma} = \beta^{-1} > 1$$

$$tr \mathbf{\Gamma} = 1 + \beta^{-1} - \beta^{-1} \delta^{-1} \kappa \left(\phi_{\pi} - 1\right)$$
(63)

Imposing the determinacy conditions in Case A above (where Case B can be ruled out due to sign restrictions), we obtain the requirement for equilibrium uniqueness:

$$\delta^{-1} \left( \phi_{\pi} - 1 \right) \in \left( 0, \frac{2\left( 1 + \beta \right)}{\kappa} \right)$$

This implies the two cases in the Proposition:

Case I:  $\delta > 0, \phi_{\pi} \in \left(1, 1 + \delta \frac{2(1+\beta)}{\kappa}\right)$ , which is a non-empty interval. Case II:  $\delta < 0, \phi_{\pi} \in \left(1 + \delta \frac{2(1+\beta)}{\kappa}, 1\right)$ . Notice that (i)  $1 + \delta \frac{2(1+\beta)}{\kappa} < 1$  so the interval is non-empty; (ii)  $1 + \delta \frac{2(1+\beta)}{\kappa} > 0$  implies instead that we can rule out an interest rate peg, whereas a peg is consistent with a unique REE for  $1 + \delta \frac{2(1+\beta)}{\kappa} < 0$ . The last condition instead holds if and only if  $\lambda \geq \frac{1 + \frac{1}{1+F_V}\varphi^S \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}}{1 + \frac{1}{1+\mu}\varphi^S} \geq \frac{1}{1 + \frac{1}{1+\mu}\varphi^S}$  which is the condition in proposition 1. Indeed, one can impose  $\phi_{\pi} = 0$  here, and obtain the proof of Proposition 1.

When this condition is not fulfilled, we have  $0 < 1 + \delta \frac{2(1+\beta)}{k} < 1$ , so there still exist policy rules  $\phi_{\pi} \in \left(1 + \delta \frac{2(1+\beta)}{k}, 1\right)$  bringing about a unique rational expectations equilibrium. But in this case an interest rate peg, and any policy rule with too weak a response  $\phi_{\pi} \in \left[0, 1 + \delta \frac{2(1+\beta)}{k}\right)$  is not compatible with a unique equilibrium.

#### Determinacy properties of a simple Taylor D rule

# Proof of Proposition 3

Substituting the Taylor rule in the IS equation and writing the dynamic system in the usual way for the  $z_t \equiv (y_t, \pi_t)'$  vector of endogenous variables and the  $\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$  vector of disturbances :

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \Psi \nu_t$$

The coefficient matrices are given by:

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 + \beta^{-1} \delta^{-1} \kappa & \delta^{-1} \left( \phi_{\pi} - \beta^{-1} \right) \\ -\beta^{-1} \kappa & \beta^{-1} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \delta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix}$$

Determinacy requires that both eigenvalues of  $\Gamma$  be outside the unit circle. Note that:

$$\det \mathbf{\Gamma} = \beta^{-1} \left( 1 + \delta^{-1} \kappa \phi_{\pi} \right)$$
$$tr \mathbf{\Gamma} = 1 + \beta^{-1} \left( 1 + \delta^{-1} \kappa \right)$$

For Case A we have: (A1) implies:

$$\delta^{-1}\phi_{\pi} > \frac{\beta - 1}{\kappa}$$

(A2) implies

$$\delta^{-1} \left( \phi_{\pi} - 1 \right) > 0$$

(A3) implies

$$\delta^{-1}(1+\phi_{\pi}) > \frac{-2(1+\beta)}{\kappa}$$

A comparison with the standard model could help. Notice that we implicitly assumed unit elasticity of substitution in consumption. The standard determinacy condition (labeled by Woodford 2001 'Taylor principle') is  $\phi_{\pi} > 1$ , which also holds here, of course, for  $\lambda = 0$  (this can be seen by direct substitution in the expressions for  $\delta$  and  $\gamma$ , obtaining 1, respectively  $1 + \frac{\varphi^s}{1 + F_Y}$ ). The 'Taylor principle' comes from the second requirement above, since for the standard case requirements 1 and 3 are automatically satisfied (quantities on the right-hand side are negative, and those on the left-hand side positive in this 'Ricardian' case).In the non-Ricardian case, however, this is no longer true. Instead, the determinacy requirements are as follows. First, note that (2) merely requires that  $\delta^{-1}$  and  $(\phi_{\pi} - 1)$  have the same sign. Hence, we can distinguish two cases: Case I:  $\delta^{-1} > 0, \phi_{\pi} > 1$ . As we shall see, the Ricardian case is encompassed

herein, and the Taylor principle is at work as one would expect. The other

conditions are automatically satisfied, since both  $\delta^{-1}\phi_{\pi}$  and  $\delta^{-1}(1+\phi_{\pi})$  are positive, and  $\frac{\beta-1}{\kappa}, \frac{-2(1+\beta)}{\kappa} < 0$ . In terms of deep parameters, the requirement for the sufficiency of the Taylor principle is:

$$\varphi^s < \frac{1-\lambda}{\lambda} \left(1+\mu\right)$$

**Proof.** Case II:  $\delta^{-1} < 0, \phi_{\pi} < 1$ . Hence, we are looking at the parameter sub-space whereby:

$$\varphi^s > \frac{1-\lambda}{\lambda} \left(1+\mu\right)$$

Condition 1 implies (note that since  $\delta < 0$  the right-hand quantity will be positive):

$$\phi_{\pi} < \delta \frac{\beta - 1}{\kappa}$$

The third requirement for uniqueness implies:

$$\phi_{\pi} < \delta \frac{-2\left(1+\beta\right)}{\kappa} - 1$$

Since  $\phi_{\pi} \geq 0$ , this last requirement implies a further condition on the parameter space, namely  $\delta \frac{-2(1+\beta)}{\kappa} - 1 \geq 0$ . Overall, the requirement for determinacy when  $\delta^{-1} < 0$  is hence:

$$0 \le \phi_{\pi} < \min\left\{1, \delta \frac{\beta - 1}{\kappa}, \delta \frac{-2\left(1 + \beta\right)}{\kappa} - 1\right\}$$
(64)

Case B, instead, involves fulfilment of the following conditions: B1 implies

$$\delta^{-1} \left( \phi_{\pi} - 1 \right) < 0$$

(B2) implies

$$\delta^{-1} \left( 1 + \phi_{\pi} \right) < \frac{-2 \left( 1 + \beta \right)}{\kappa}$$

Note that in Case I, i.e. the Ricardian case whereby  $\delta^{-1} > 0$ , these conditions cannot be fulfilled due to sign restrictions (this is the case in a standard economy as in Woodford 2003, e.g.). In Case II however, the two conditions imply:

$$\phi_{\pi} > \max\left\{1, \delta \frac{-2\left(1+\beta\right)}{\kappa} - 1\right\} \tag{65}$$

64 and 65 together imply the following overall determinacy condition for the policy parameter:

$$\phi_{\pi} \in \left[0, \min\left\{1, \delta \frac{\beta - 1}{\kappa}, \delta \frac{-2\left(1 + \beta\right)}{\kappa} - 1\right\}\right) \cup \left(\max\left\{1, \delta \frac{-2\left(1 + \beta\right)}{\kappa} - 1\right\}, \infty\right)$$

Note that  $\delta$  and  $\kappa$  are functions of the deep parameters.

To assess the magnitude of policy responses needed for determinacy as a function of deep parameters, we can distinguish a few cases for different parameter regions (note that we are always looking at the subspace whereby  $\delta^{-1} < 0$ ):

$$\begin{split} & \text{Non-Ricardian share} \quad \text{Determinacy condition} \\ & \lambda < \bar{\lambda}_1 & \phi_\pi > 1 \\ & \lambda \in [\bar{\lambda}_1, \bar{\lambda}_2) & \phi_\pi \in \left[0, \delta \frac{-2(1+\beta)}{\kappa} - 1\right) \cup (1, \infty) \\ & \lambda \in [\bar{\lambda}_2, \bar{\lambda}_3) & \phi_\pi \in \left[0, \delta \frac{\beta-1}{\kappa}\right) \cup (1, \infty) \\ & \lambda \in [\bar{\lambda}_3, \bar{\lambda}_4) & \phi_\pi \in \left[0, \delta \frac{\beta-1}{\kappa}\right) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty\right) \\ & \lambda \in [\bar{\lambda}_4, 1) & \phi_\pi \in [0, 1) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty\right) \\ & \text{where} \\ & \bar{\lambda}_i = \frac{1 + \frac{1}{1+F_Y} \varphi^S \frac{(1-\theta)(1-\beta\theta)}{h_i(\theta)}}{1 + \frac{1}{1+\mu} \varphi^S} \end{split}$$

 $h_{1}\left(\theta\right)=\left(1+\theta\right)\left(1+\beta\theta\right);h_{2}\left(\theta\right)=1+\beta\theta^{2}+2\beta\theta;h_{3}\left(\theta\right)=1+\beta\theta^{2};h_{4}\left(\theta\right)=1-\beta\theta^{2}$ 



Fig.8: Threshold value for non-Ricardian share making determinacy conditions for Taylor rule closest to the inverted Taylor Principle.

We plot the last case  $\lambda \in [\bar{\lambda}_4, 1)$ ;  $\phi_{\pi} \in [0, 1) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$  in Figure 8 above, where the region above the curve and below the horizontal line gives parameter combinations compatible with the above condition. The different curves correspond to different labor supply elasticities ( $\varphi^S = 1$  dotted line and  $\varphi^S = 10$  thick solid line). Note that for parameters most often assumed in the literature (e.g.  $\theta = 0.75$ ;  $\varphi^S = 10$ ), a non-Ricardian share as low as 0.25 would bring us in this region. Hence, in view of usual estimates of lambda in the literature (see e.g. Campbell and Mankiw) we shall consider this case as the most plausible. Whenever these parameter restrictions are met, determinacy is insured by either a violation of the Taylor principle, or for a strong response to inflation. However, note that the lower bound on the inflation coefficient then becomes very large (35. 433 under the baseline calibration), which is far from any empirical estimates. Indeed, the threshold inflation coefficient is sharply increasing in the share of non-Ricardian consumers, elasticity of labor supply,

as can be seen by merely differentiating  $\delta \frac{-2(1+\beta)}{\kappa}-1$  with respect to all these parameters.

# E Computing sunspot equilibria

The stability condition in the case of indeterminacy is - see Lubik and Schorfheide 2003, p. 278 (where  $[A]_2$  denotes the second row of the A matrix, attached to the explosive component):

$$\left[J^{-1}\Psi\right]_{2\cdot}\nu_t + \left[J^{-1}\Gamma\right]_{2\cdot}\eta_t = 0$$

Straightforward algebra to calculate

$$\begin{bmatrix} J^{-1}\Psi \end{bmatrix}_{2.} = -\frac{1}{q_{+}-q_{-}} \begin{bmatrix} \kappa \delta^{-1} & \beta^{-1}-q_{-} \end{bmatrix}$$
$$\begin{bmatrix} J^{-1}\Gamma \end{bmatrix}_{2.} = \frac{1}{q_{+}-q_{-}} \begin{bmatrix} -\kappa q_{+} & q_{+}-1 \end{bmatrix}$$

delivers the stability condition as:

$$-\kappa\delta^{-1}\left(\varepsilon_{t}-r_{t}^{*}\right)-\left(\beta^{-1}-q_{-}\right)u_{t}-\kappa q_{+}\eta_{t}^{y}+\left(q_{+}-1\right)\eta_{t}^{\pi}=0$$

Since only one root is suppressed, there is endogenous persistency of the effects of shock (which was not the case under determinacy).

Following Lubik and Schorfheide (2003) we compute a singular value decomposition of  $(q_+ - q_-) [J^{-1}\Gamma]_{2}$ :

$$\begin{bmatrix} J^{-1}\Gamma \end{bmatrix}_{2.} = 1 \cdot \begin{bmatrix} d & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{-\kappa q_{+}}{d} & \frac{q_{+}-1}{d} \\ \frac{q_{+}-1}{d} & \frac{\kappa q_{+}}{d} \end{bmatrix}$$
$$d = \sqrt{(\kappa q_{+})^{2} + (q_{+}-1)^{2}}$$

Using also  $(q_+ - q) [J^{-1}\Psi]_{2.} = [-\kappa \delta^{-1} \quad q_- - \beta^{-1}]$  we get the full set of stable solutions as described in text.

### References

- Amato, J. and Laubach, T. 2003 'Rule-of-thumb behavior and monetary policy', European Economic Review, forthcoming
- [2] Benassy, Jean-Pascal 2002 'Interest rate rules and price level determinacy in a non-Ricardian world', mimeo CEPREMAP Paris
- [3] Benhabib, J. and Farmer, R. 1999 'Indeterminacy and Sunspots in Macroeconomics', in Taylor, J. and Woodford, M. eds, 'Handbook of Macroeconomics, Volume 1b, North-Holland.

- [4] Bernake, B. and Woodford, M. 1997 'Inflation forecasts and monetary policy', Journal of Money, Credit and Banking, 24: 653-684, 1997
- [5] Bilbiie, F.O. and Straub, R. 'Ricardian Economy with non-Ricardian Agents? Fiscal policy, heterogeneity and labor supply.', Mimeo, EUI
- [6] Bilbiie, F.O. and Straub, R. 'Fiscal Policy Multipliers and Transmission Mechanism with Heterogenous Agents and Distortionary Taxation' Mimeo, EUI
- [7] Blanchard, Olivier J. and Charles M. Kahn, "The solution of linear difference models under rational expectations," Econometrica, 48, 5, 1980, 1305-12
- [8] Bullard, J. and Mitra, K., 2002. "Learning About Monetary Policy Rules" Journal of MOnetary Economics, 49:1105-29
- [9] Calvo, G., "Staggered prices in a utility-maximizing framework," Journal of Monetary Economics, 12, 1983, 383-98.
- [10] Campbell, John Y. and Mankiw, N. G. 1989. "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," NBER Macro Annual 185-216.
- [11] Clarida, R., J. Gali, and M. Gertler, 1999, 'The science of monetary policy: a New Keynesian perspective', Journal of Economic Literature, 37, 1661-1707.
- [12] Clarida, R., J. Gali, and M. Gertler 2000: "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," Quarterly Journal of Economics 115: 147-180
- [13] DeLong, B. 1997 'America's only peacetime inflation: the '70s'
- [14] Dupor, W. 2000 'Investment and Interest Rate Policy." Journal of Economic Theory
- [15] Farmer, R. 1999 'The macroeconomics of self-fulfilling prophecies' MIT Press
- [16] Gali, J. 1999 'Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?' American Economic Review, 249-271
- [17] Gali, J., 2002, "New perspectives on Monetary Policy, Inflation and the Business Cycle", NBER WP 8767.
- [18] Gali, J., J. D. López-Salido and J. Vallés, 2002 "Understanding the Effects of Government Spending on Consumption", mimeo CREI.

- [19] Galí, J., Lopez-Salido D. and J.Valles 2003a, Technology Shocks and Monetary Policy: Assessing the Fed's Performance', Journal of Monetary Economics, vol 50, 723-743,
- [20] Galí, J., Lopez-Salido D. and J.Valles 2003b, Rule-of-Thumb Consumers and the Design of Interest Rate Rules, mimeo, UPF and Bank of Spain
- [21] Goodfriend, M. and R. King, 1997, "The New Neoclassical Synthesis and the Role of Monetary Policy" in Bernanke and Rotemberg, eds., Macroeconomics Annual, Cambridge: MIT Press, pp. 231-282
- [22] King, R. and Rebelo, S. 2000 'Resuscitating Real Business Cycles', in Taylor and Woodford, eds, Handbook of Macroeconomics, North-Holland
- [23] Leeper, Eric M. 1991 "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies.", Journal of Monetary Economics 27, February, 129-147
- [24] Lubik, T. and Schorfheide F. 2003a "Computing Sunspot Equilibria in Linear Rational Expectations Models" Journal of Economic Dynamics and Control, 28(2), pp. 273-285.
- [25] Lubik, T. and Schorfheide, F. 2003b "Testing for Indeterminacy: An Application to U.S. Monetary Policy" forthcoming in American Economic Review.
- [26] Mankiw, N. G. 2000. "The Savers-Spenders Theory of Fiscal Policy," American Economic Review, Vol. 90 (2) pp. 120-125.
- [27] Rotemberg, J. and Woodford, M. 1995. "Dynamic General equilibrium Models and Imperfect Product Markets', in Cooley, Th., ed 'Frontiers of Business Cycle Research', Princeton University Press.
- [28] Sargent, T. 2002 'Reactions to the 'Berkeley Story', Mimeo Hoover Institution, Stanford
- [29] Stock, J. and Watson, M. 2002 'Has the Business Cycle Changed and Why?' NBER Macro Annual
- [30] Stock, J. and Watson, M. 2002 'Has the Business Cycle Changed? Evidence and explanations.' Mimeo Princeton University
- [31] Svensson, L. E.O. 1999, "Inflation Targeting as a Monetary Policy Rule," Journal of Monetary Economics 43, 607-654.
- [32] Taylor, J. B. 1993, "Discretion versus Policy Rules in Practice," Carnegie-Rochester Conference Series on Public Policy 39, 195-214.
- [33] Taylor, J. B. 1999, "A Historical Analysis of Monetary Policy Rules," in Taylor, John B., ed., Monetary Policy Rules, Chicago: Univ. of Chicago Press.

- [34] Woodford, M. 1996, "Control of the Public Debt: A Requirement for Price Stability?" NBER working paper no. 5684
- [35] Woodford, M. 2001, "The Taylor Rule and Optimal Monetary Policy" American Economic Review 91(2): 232-237
- [36] Woodford, M., 2003, Interest and prices: foundations of a theory of monetary policy, Princeton University Press
- [37] Yun, T. 1996, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles", Journal of Monetary Economics, 37, 2, 345-70