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# **Exchange Rate Risk: Heads or Tails**

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## **Abstract**

More than forty years ago researchers started to reconsider the behavior of financial data. Since then, stylized facts about financial returns have become common knowledge in economics. Characteristics as fat-tailedness, leptokurtosis and serial dependence have been extensively analyzed. As the financial world became focused on risk management and prudential supervision, various risk models have been developed. However, the first generation of risk models is highly dependent on rough assumptions, empirically contradicted, but embraced by practitioners as they benefit from a fairly easy implementation. In the context of market risk, such a proxy was developed under the name of Value at Risk, which rapidly became a standard measure for both risk managers and supervisors. The current state of affairs brings us one step closer to the death of VaR. The need for a new approach is imperative.

This paper aims to bring new evidence to the limited performance of Value at Risk and test the fit of Extreme Value Theory as a complementary risk management tool for stressed market conditions, in the context of exchange rate risk. We use exchange rate returns of four currencies against the Euro and analyze the relative performance of several VaR models and Extreme Value Theory, respectively. We show that in extreme market conditions, extreme measures are required, and that no single measure can perform proper for both the centre and the tails of an exchange rate distribution.

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## 1 Introduction

Reality brings forward stressed market conditions and empirically proved *stylized facts*, as suggested in the early work of Mandelbrot (1963) or Fama (1963) or in the more recent work of Manganelli and Engle (2001): financial returns, especially exchange rate and interest rate returns, are not normally distributed, but fat-tailed, leptokurtic and skewed, suffer from volatility clustering and are not independent. Movements of 4-6 sigmas are rather common in financial markets, while normal distribution concentrates on movements of 2-3 sigmas<sup>1</sup>. Using Gaussian as a reference distribution in assessing market risk, one assumes that the probability of an extreme event is considerably lower than it is in fact, thus underestimating the true risk of an asset or portfolio.

Risk management is a key function within financial institutions and during the last decade financial markets have realized the importance of monitoring risks. Recent years brought significant instability in financial markets worldwide, mainly because of excessive risk appetite of market participants. In the context of current crisis, financial risk management has been very much challenged. The triggers of the crisis are various and still not fully known but the rapid spreading of the effects was the whistleblower of both risk managers and supervisory authorities. The development of high risk behaviour in financial markets, especially in the banking system and real estate (subprime lending, securitization, toxic assets, complex derivatives and deregulation) led to huge bankruptcies, bailouts and takeovers (e.g. Lehman Brothers U.S., Northern Rock U.K., Bear Stearns U.S., Merrill Lynch U.S.), and also to a generalized liquidity crisis, declining stock market prices and real estate values, numerous insolvencies and economic recession. The common lesson of financial disasters is that unbearable losses can occur because of poor supervision and risk management of financial risks.

One of the most affected sectors in the global economy is the banking system. The crisis has also put a very difficult charge on regulators. The advent of Basel Capital Accord back in 1996 brought into view the concern about quantifying risks, but its further amendments<sup>2</sup>, although incorporating new risks in risk management, also proved laxer, allowing internal risk management models for prudential capital requirements calculation. Under Basel II,

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<sup>1</sup> Blum, Dacorogna (2002) specifically consider daily fluctuations in FX returns up to 5% as highly significant.

<sup>2</sup> Followed by The Amendment to the Capital Accord in 1997 and the New Basel Capital Accord of 2006 (Basel II) issued by Basel Committee on Banking Supervision.

market risk became one of the priorities of risk management. The most widely used approach to measure market risk is Value at Risk (VaR). The directives ask for losses in trading portfolios to be covered over a 10-day time horizon, 99% of time. In practice, the calculation requires operating with minimum one year P&L historical data. The measure has become popular among practitioners, due to its simple implementation.

VaR is formally defined as the potential loss of a portfolio that would result if relatively large adverse price movements were to occur, over a certain time horizon, with a given probability. Historical VaR does not make any assumptions concerning the underlying distribution but it highly depends on historical data, ignoring out-of-sample events, thus implying that the future will be similar to the past. Various alternatives have been proposed. Some of these alternatives refer to the use of ARCH models introduced by Engle (1982) and Bollerslev (1986)<sup>3</sup>, with normal or Student-t distribution<sup>4</sup>, J.P. Morgan RiskMetrics (1996) Exponentially Weighted Moving Average volatility models, complementary Conditional VaR models, popularized by Rockafeller and Uryasev (1999, 2002), and others.

As recently suggested by researchers and practitioners<sup>5</sup>, VaR-based risk management framework must be reviewed. The presence of high volatility in financial markets and the occurrence of extreme events result into the need to develop new products and methods to deal with these issues. Risk managers and supervisors become more concerned with events occurring under extreme market conditions, events that produce huge, unexpected losses, that could affect their capital (i.e. solvency) and also lead to bankruptcies and hence, systemic risk.

In the last years financial risk modelling has easily incorporated a new framework for measuring risks, Extreme Value Theory, traditionally used in fields like hydrology and meteorology. A wide literature has been recently devoted to the study of extreme events in finance and insurance<sup>6</sup>. In practice, the framework has been adopted in operational risk and insurance loss modelling. EVT describes the behaviour of extreme returns rather than describing the behaviour of all returns. Unlike most VaR methods, there are no assumptions

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<sup>3</sup> Further developed by Nelson – EGARCH (1991), Glosten, Jagannathan and Runkle – GJR-GARCH (1993), Zakoian – TGARCH (1994), and others.

<sup>4</sup> Jorion (1996), and Yang and Brorsen (1995).

<sup>5</sup> See for example Colander et al. (2009), Einhorn (2008), Bernanke (2009); The High-Level Group on Financial Supervision in the EU (2009); Sir John Gieve (2008).

<sup>6</sup> See McNeil (1997a, 1997b, 1998, 1999), Huisman et al. (1998), Gencay et al. (2003), Embrechts et al. (1997) for instance.

about the nature of the underlying distribution. However, inference for very high quantiles is done at the expense of not modelling correctly moderate movements, precisely where VaR estimation intervenes, thus the two approaches are rather complementary. There are two general approaches under EVT: first, *Block Maxima*, stemming from the behaviour of the  $k$  largest order statistics within a block, which are assumed to follow a Generalized Extreme Value (GEV) distribution, and second, *Peaks over Threshold*, originating in observations exceeding a high threshold, which are considered to follow a Generalized Pareto Distribution (GPD). Most researchers are in favour of POT method since it uses data more efficiently.

The use of a certain proxy for market risk is somewhat reduced to a trade-off: regulators would prefer more conservative measures, which diminish systemic risk but results into inefficient supplementary capital allocation, and bank managers would prefer underestimated losses, with high risks but low capital requirements. This paper aims to test the performance of Extreme Value Theory (EVT) as a complementary risk measure for the analysis of extreme events, in the context of exchange rate risk<sup>7</sup>, using EUR/CHF, EUR/GBP, EUR/RON and EUR/USD exchange rate returns and underline the existing trade-off between coverage and efficiency. Our objectives are: i) analyze the presence of stylized facts in the data, ii) produce point estimates of potential losses from exchange rate positions using VaR and EVT, iii) modelling VaR to incorporate EVT and determine dynamic VaR measures, iv) backtest the results and conclude on the specific performance of employed measures.

The rest of the paper unfolds as follows: Section 2 makes a quick overview of the most referenced literature in the field of VaR and EVT; Section 3 presents the theoretical framework of different Value at Risk approaches and Extreme Value Theory; Section 4 describes the methodology; Section 5 deals with data analysis and empirical results and Section 6 states the concluding remarks and some directions for further research.

## **2 Literature Review**

A wide literature has been produced addressing market risk modelling, and most of the work refers to VaR modelling approaches. Hendricks (1996) analyses the performance of twelve different VaR models using historical data on exchange rate returns and finds that historical

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<sup>7</sup> Engel, Gizycki (1999) found that for four of the largest banks in Australia, FX risk accounts for over one third of market risk. This finding also applies to many Romanian banks, large and small.

simulation performs better at 95% than at 99% or higher confidence levels and Exponentially Weighted Moving Average is more reliable with 0.94 decay factor for daily returns. Duffie and Pan (1997) give a theoretical overview of VaR models applicable to market risk and their econometric and practical implications, without empirical evidence. Jorion (2001) offers a complete and detailed study of Value at Risk, of its application to different types of risks and portfolios, and also states the pitfalls of such models, although he very much supports the general approach. Engel and Gitzky (1999) develop the work of Hendricks and propose new tests for the performance, accuracy and efficiency. Rockafellar and Uryasev (2002) approach Conditional Value at Risk, as a measure who deals with some of the shortcomings of VaR, mainly the lack of subadditivity (coherence) and the limited information provided by VaR – it tells nothing about losses exceeding a certain threshold. Rockafellar and Uryasev (2000) also propose a method to optimize the CVaR measure. Alexander (2001) offers a very comprehensive overview of market risk models and also exemplifies their application using different software. Kaplanski and Levy (2009) approach VaR and underline the idea that existing regulation, allowing internal risk models, may induce excessive risk taking of banks and also distort capital allocation.

Standard VaR risk measures are generally derived by making distributional assumptions, the most common one being *normality*. The normality assumption imposes several restrictions on the underlying distribution of returns like symmetry and, most importantly, lack of excess kurtosis. However, many studies, like those of Mandelbrot (1963), Fama (1963), Mussa (1979), Andersen et al. (1999) and Manganelli and Engle (2001) have shown that this hypothesis is fairly unrealistic. Accordingly, papers like those of Hols and De Vries (1991), Huisman et al. (1997), Huisman et al. (1998), Wagner and Marsh (2003) and others have shown that financial data are fat-tailed and tested and proved the superior performance of EVT methodology in estimating tail risk. Moreover, financial data suffer from *volatility clustering*. In the sense of Mandelbrot (1963), this means that large changes in returns are followed by large changes of either sign and, correspondently, small changes are followed by small changes. This implies that, although raw returns may be uncorrelated, absolute or squared returns display a positive autocorrelation. In order to deal with some of these features, several alternative VaR approaches have been studied, like Student-t distribution, ARCH-GARCH models, pioneered by Engle (1982) and Bollerslev (1986) or mixtures of normal, also approached by Duffie and Pan (1997).

However, many authors have oriented their work towards more efficient tail-oriented models of risk, namely Extreme Value Theory (EVT) approach. The superiority of EVT has been extensively demonstrated by many researchers, in fields like insurance or financial risk management. Embrechts, Kluppelberg and Mikosch (1997) employ EVT tools for assessing fat tails of different time series, like hydrologic, insurance and financial data, supported by a very detailed and complex mathematical framework. Similar work is found in Resnick (2007), who studies extreme events in data networks, finance and insurance. McNeil (1997a, 1997b, 1998, 1999) also studies the performance of the methods in insurance and finance. His work focuses on the POT method<sup>8</sup>, i.e. fitting a Generalized Pareto Distribution to excesses over a high threshold. He also applies Block Maxima to financial time series (BMW returns)<sup>9</sup>. POT method is also preferred in the studies of Matthys and Beirlant (2000), Blum and Dacorogna (2002), Wagner and Marsh (2003), who compare the performance with Student-t and GARCH-t volatility models, Brooks et al. (2003), who employ GPD distribution to future contracts, and Gonzalo and Olmo (2004), who use simulated data to bootstrap for an optimal threshold. Block Maxima is preferred in papers of Caserta and De Vries (2003), who differentiate their analysis for minima and maxima of AEX index, Cotter and Dowd (2007), who apply the method to compare tail risks of limit and market orders considering the distribution of FX returns, Robert, Segers and Ferro (2008), who analyze the tail thickness of FTSE100 index return data. Many of these studies and others use Value at Risk to incorporate EVT framework, thus calculating higher quantiles than those computed for regular VaR methods. The general agreement is that EVT proves superior in analysing extreme movements in data.

Ample literature has also been dedicated to more specific issues of Extreme Value Theory, e.g. tail index and graphical tools of the framework, like mean excess function plot, Hill plot, QQ plots etc. Tail index estimation is yet a very widely debated problem of EVT. Starting with the work of Hill (1975) and Pickands (1975), many studies have tried to establish a measure of the tail thickness of fat-tailed distributions. Alternatives or improved approaches are offered in Dekkers et al. (1989), who extend the Hill estimator and prove consistency and asymptotic normality. Huisman et al. (2001), Segers (2005) and others offer different tail index estimators using Monte Carlo simulation, bootstrap methods or regressing Hill estimator on the number of order statistics. Despite the dedicated work, the literature is

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<sup>8</sup> McNeil (1997a), Mc Neil(1997b) and McNeil (1999).

<sup>9</sup> McNeil (1998).



divergent with respect to tail index estimation, especially since some of the underlying steps for the computation of such indexes (e.g. the choice of a threshold) are still subject to arbitrary methods and their relative efficiency depends on the characteristics of the data used for empirical analysis.

Researchers have also studied the behaviour of some usual graphical tools used in the preliminary analysis of data. Remarkable work has been done by Kratz and Resnick (1995), who analyze the information provided by QQ-plots, Embrechts et al. (1997), who offer a very detailed analysis of Hill plots, Mean Excess Plots and QQ plots, Drees et al. (1998), who approach Hill plot and several extensions. Similar work is found in Sousa and Michailidis (2004), and Embrechts and Resnick (2007). We will refer to some of the underlying methods and results in the following sections.

### 3 Theoretical Background

The purpose of this section is to offer an overall view on VaR measures and a more detailed presentation of basic EVT framework.

#### 3.1 Value at Risk

VaR is generally defined as the maximum potential loss on a portfolio or asset that would result over a time horizon, with a given probability, if relatively large adverse movements in market variables (price, interest rate, exchange rate) were to occur. More formally, the VaR of a portfolio at a confidence level  $\alpha \in (0;1)$  is the smallest number  $l$  such that the probability of a loss  $L$  exceeding  $l$  over a certain time horizon is smaller than or equal to  $(1-\alpha)$ :

$$VaR(\alpha) = \inf \{l \in R / P(L > l) \leq (1 - \alpha)\} = -(\mu + \sigma Z_{\alpha}) \quad (1)$$

where  $\mu$  is the mean value of the portfolio/asset,  $\sigma$  the respective standard deviation and  $Z_{\alpha}$  is the  $\alpha$ -percentile of normal distribution. Following J.P. Morgan RiskMetrics framework, most financial firms compute 5% VaR over a one-day holding period. For regulatory purposes, Basel proposed the calculation of 1% VaR for a ten-day period, based on a historical observation period of at least 1 year of data, which should then be multiplied by a *safety factor* of 3 in order to compute capital adequacy requirement. The safety factor was

introduced because the normal hypothesis for the P&L distribution is widely recognized as unrealistic but may lead to an overestimation of risk for medium movements in market variables, which results into significantly higher capital buffer, i.e. significant loss of efficiency or, on the contrary, to underestimated risk of very small or very large movements, affecting the solvency of the bank and even putting at risk an entire system.

### 3.1.1 Parametric Models

Models like RiskMetrics EWMA, launched in 1996 by J.P. Morgan and ARCH-GARCH, introduced by Engle (1982) and Bollerslev (1986), require a specific parameterization for the behaviour of prices.

Under **RiskMetrics approach**, the volatility is derived from the Exponentially Weighted Moving Average model, taking into account past information on returns and variance:

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2 \quad (2)$$

where:  $\sigma_t^2$  - variance at time t,  $r_t$  - return at time t,  $\lambda$  - decay factor, usually set at 0.94 or 0.97<sup>10</sup>. This approach uses the assumption of normally distributed standardized residuals.

The **EGARCH(1,1,1) model**, proposed by Nelson (1991), derives from the basic GARCH(1,1) of Engle (1982) and Bollerslev (1986) and incorporates the asymmetric response to shocks in the equation of conditional variance. The model has the following specification for the conditional mean and conditional variance, respectively:

$$\begin{aligned} r_t &= \beta_0 + \beta_1 r_{t-1} + \beta_2 \varepsilon_{t-1} + \varepsilon_t \\ \log \sigma_t^2 &= \alpha_0 + \alpha_1 \log \sigma_{t-1}^2 + \alpha_2 (|z_{t-1}| - E[|z_{t-1}|]) + \alpha_3 z_{t-1} \end{aligned} \quad (3)$$

where:  $z_t = \varepsilon_t / \sigma_t$  and  $\alpha_3$  measures the asymmetric impact of information, which is considered exponential and not quadratic. The model can be used in the general form, to include more AR or MA terms in the equation of the mean and more GARCH or ARCH terms in the equation of the variance. The i.i.d. assumption is needed in order to estimate the parameters of the model. The standard normal distribution of residuals can be replaced by alternatives like Student-t, a more appropriate approximation for financial data, or generalized error distribution (GED).

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<sup>10</sup> recommended values: 0.94 for daily data and 0.97 for monthly data.

Both normal EGARCH and RiskMetrics tend to underestimate the VaR measure for large movements in the data as the normality assumption is not consistent with the behaviour of financial returns. The main advantages of these methods are the fairly simple implementation and the fact that they allow a complete characterization of the distribution of returns. The use of Student-t approximation for innovations can also prove useful to improve the performance and we will use this specific method in this paper. On the other hand, disadvantages stem from three sources: specification of variance equation, assumptions on underlying distribution and the i.i.d. hypothesis on residuals. The impact of these flaws on VaR measure is highly dependent on empirical data and results.

### 3.1.2 Non-parametric Models

This class of methods requires no parameterization of price behaviour. The most common non-parametric approach is the *Historical Simulation* (HS). The procedure doesn't make any assumption on the underlying distribution of returns, this being one of its major advantages, apart from the very simple implementation. Basically, HS implies the choice of a window of observations for portfolio returns, which are then sorted in ascending order. The  $\alpha$ -quantile of interest is then chosen as the return that leaves  $\alpha\%$  of the observations under the respective value and  $(1-\alpha)\%$  above.

Historical VaR does not make any assumptions concerning the underlying distribution and eliminates the need of approximations that introduce inaccuracies into calculation. Although the measure can incorporate fat tails, it highly depends on historical data, ignoring out-of-sample events, thus implying that the future will be similar to the past. There are several other problems with this approach. First, the methodology is clearly inconsistent. Second, the quantile estimator is consistent only if the length of the chosen time window approaches infinity. Third, the choice of the time window implies the same distribution for all historical data, while there are several volatility clustering periods, which cannot be easily identified. So there is a trade-off between the choice of a large window which would make the estimator significant and a shorter window which would avoid the risk of taking observation outside the volatility cluster. Finally, HS is considered flawed since it puts the same weight on all observations, specifically not taking into account the clustering aspect.

In order to deal with this last disadvantage, a *Hybrid Approach* between HS and RiskMetrics EWMA was developed by Boudoukh, Richardson and Whitelaw (1998). The model applies

exponentially declining weights to past returns of the portfolio. Accordingly, to each return  $r_t$ ,  $r_{t-1}$ , ...,  $r_{t-k+1}$  in a  $k$ -length time window a weight is assigned in the range:  $\frac{1-\lambda}{1-\lambda^k}, \frac{(1-\lambda)\lambda}{1-\lambda^k}, \dots, \frac{(1-\lambda)\lambda^{k-1}}{1-\lambda^k}$ . Then the returns are ordered in ascending order. To compute the  $\alpha$ -quantile of the portfolio, the weights are summed until  $\alpha\%$  is reached and the corresponding return is considered.

The significant improvement of this approach is that it incorporates a more flexible specification of the data, thus resulting into more reliable figures for the VaR measure.

Some other models are also used in literature for the computation of tail risk. We remind **Conditional Value at Risk** (CVaR or Expected Shortfall), which is basically the mean size of losses exceeding VaR threshold and the semi-parametric model of VaR using **Extreme Value Theory**. We will consider the latter separately, as it is the approach that this paper focuses on and several technical issues have to be detailed.

### 3.2 Extreme Value Theory

The fundamental role that Extreme Value Theory plays in the modelling of maxima of a random variable is comparable to the role of Central Limit Theorem in modelling sums of random variables. More precisely, in both cases theory gives us the limiting distributions.

There are two main approaches when identifying extremes in real data. Let us consider the distribution of daily returns/losses. The first approach – under the generic name of **Block Maxima** – considers the maximum (minimum) values that returns take over successive periods of same length (blocks). The selected values (one maximum/minimum for each period in the time span of time series) are considered extreme events that constitute block maxima (minima). The second approach – **Peaks over Threshold** (POT) - focuses on returns that exceed a given high threshold. The two approaches are illustrated in Figure 1 below. Block Maxima is generally used in fields with seasonal data e.g. hydrology and has the disadvantage that it could overlook extreme events in the same block, as it only uses the largest observation in each block. Also, the choice of block length is subject to misspecification. POT method has the advantage to more efficiently use data but, on the other hand, relies on the choice of the high threshold, which is fairly subjective. Until the present day, no general agreement has been reached upon the best method for threshold selection nor have researchers developed a fully parametric algorithm.

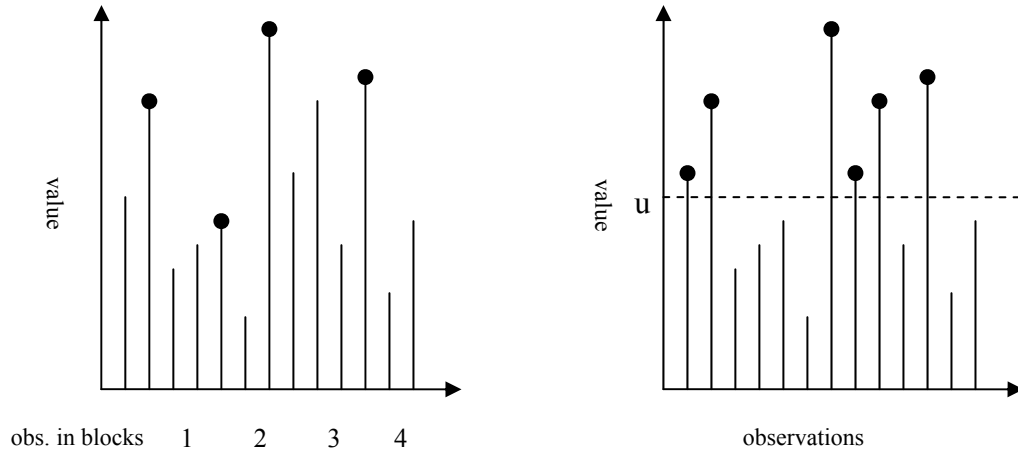


Figure 1. Block Maxima (left) and Peaks over Threshold (right)

### 3.2.1 Block Maxima. Distribution of Maxima (GEV)

Reconsider the sequence of daily returns with  $X_1, X_2, \dots, X_n$  i.i.d. and denote by  $M_n = \max(X_1, X_2, \dots, X_n)$  the block maxima, with  $n$  the size of the block. The limit law for the distribution of maxima is given by one of the two fundamental theorems of EVT – Fisher-Tippett Theorem, formally proved by Gnedenko.

#### Theorem 1. Fisher-Tippett (1928), Gnedenko (1943)

Let  $(X_n)$  be a sequence of i.i.d. random variables. If there exist constants  $c_n > 0$ ,  $d_n^{11} \in \mathfrak{R}$  and some non-degenerate function  $H$  such that

$$\frac{M_n - d_n}{c_n} \xrightarrow{d} H,$$

then  $H$  belongs to one of the three standard extreme value distributions:

$$\text{Fréchet:} \quad \Phi_\alpha(x) = \begin{cases} 0, & x \leq 0 \\ e^{-x^{-\alpha}}, & x > 0 \end{cases} \quad \alpha > 0 \quad (5)$$

$$\text{Weibull:} \quad \Psi_\alpha(x) = \begin{cases} e^{-(x)^{-\alpha}}, & x \leq 0 \\ 1, & x > 0 \end{cases} \quad \alpha > 0 \quad (6)$$

<sup>11</sup> Embrechts et. al (1997) state the common practice of taking  $d_n$  equal to 0.

$$\text{Gumbel:} \quad \Lambda(x) = e^{-e^{-x}} \quad x \in \mathfrak{R} \quad (7)$$

The Fisher-Tippett theorem suggests that the asymptotic distribution of maxima belongs to one of the three distributions above, regardless of the original distribution of observed data. By taking the reparameterisation  $\alpha=1/\xi$ , von Mises (1936) and Jenkinson (1955) represented the three distributions in one unified model with a single parameter, thus introducing Generalised Extreme Value Distribution (GEV):

$$H_{\xi}(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}}, & \xi \neq 0 \\ e^{-e^{-x}}, & \xi = 0 \end{cases} \quad (8)$$

where  $1+\xi x > 0$ ,  $\xi = 1/\alpha$  is the *shape parameter* and  $\alpha$  is the *tail index*. In practice, the name of tail index is mostly used for  $\xi$ , this being merely a convention, since if we have one of the values of  $\alpha$  or  $\xi$  we can immediately compute the other. Tail index  $\alpha$  represents the number of finite moments in the sample<sup>12</sup>, while shape parameter measures the degree of fatness in the tail. The smaller the tail index (less existing finite moments) the fatter the tails of the distribution.

The general form of GEV implies the three classes of functions mentioned above, considering the value of the shape parameter:

- $\xi < 0$  - Weibull, corresponding to short-tailed distributions, like the uniform, where the tail is bounded and has a finite right endpoint;
- $\xi = 0$  - Gumbel, corresponding to thin-tailed distributions, including the normal and exponential, with tails decaying exponentially;
- $\xi > 0$  - Fréchet or heavy-tailed distributions, like Cauchy, Student-t and Pareto, with tail values decaying like a power function (slower than the Gumbel class).

The latter, corresponding to heavy-tailed distributions, are more appropriate for financial data. Following the results of Gnedenko (1943), if the tail of the distribution decays like a

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<sup>12</sup> Huisman et al. (1998)

power function<sup>13</sup>, then the distribution is said to be in the maximum domain of attraction of the Fréchet.

### 3.2.2 Peaks over Threshold. Distribution of Exceedances (GPD)

Theorem 1 also underlies the approach of POT. The method allows us to extract the extremes of the sample by considering the exceedances over a high threshold  $u$ . Consider a sample of observations  $X_1, X_2, \dots, X_n$  with a distribution function  $F(x) = P(X_i \leq x)$  and a predetermined high threshold  $u$ , then an exceedance of the threshold  $u$  occurs when  $X_i > u$  for any  $i = 1, \dots, n$ .

An excess over  $u$  is defined by  $y = X_i - u$ . We are interested in estimating the *conditional excess distribution function (cedf)*  $F_u$  defined as:

$$F_u(y) = P(X - u \leq y \mid X > u), \quad 0 \leq y \leq x_F - u \quad (9)$$

which represents the probability of values of  $X$  exceeding the threshold  $u$ , by at most an amount  $y$ , given that  $X$  exceeds  $u$  and  $x_F \leq \infty$  represents the right endpoint of  $F$ . Writing  $F_u$  in terms of  $F$ , we can derive:

$$F_u(y) = \frac{F(u + y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \quad (10)$$

If the estimation of the portion between 0 and  $u$  is easy, as most observations lie in this area, we cannot say the same for the portion  $F_u$ , as we generally have very little observations left. This is another point where EVT proves useful, as it provides a very important result for the cdf, stated in the following theorem:

#### Theorem 2. Balkema and de Haan (1974), Pickands (1975)

For a large class of underlying distribution functions  $F$ , for a sufficiently high threshold  $u$ , the conditional excess distribution function  $F_u$  is well approximated by

$$F_u(y) \approx G_{\xi, \sigma}(y), \quad u \rightarrow \infty \quad (11)$$

where

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<sup>13</sup>  $1 - F(x) = x^{-1/\xi} L(x)$ , for some slow varying function  $L(x)$

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - (1 + \frac{\xi}{\sigma} y)^{-1/\xi}, \xi \neq 0 \\ 1 - e^{-y/\sigma}, \xi = 0 \end{cases} \quad \text{for } y \in \begin{cases} [0, (x_F - u)], \xi \geq 0 \\ [0, -\frac{\sigma}{\xi}], \xi < 0 \end{cases} \quad (12)$$

$G_{\xi,\sigma}$  is the **Generalized Pareto Distribution**<sup>14</sup> (GPD) with  $\xi$  the *shape parameter* and  $\sigma$  the *scale parameter*, which measures the statistical dispersion of the series. The higher the scale parameter, the more spread out the distribution. Embrechts et. al (1997) found that for financial data  $\alpha=1/\xi \in (3,4)$ , Beirlant and Matthys (2000) state that for exchange rate log-returns  $\alpha$  usually lies between (3,5) and Gençay and Selçuk (2003) found that for high frequency foreign exchange returns the estimates of  $\xi$  are usually less than 0.5 ( $\alpha > 2$ , implying finite variance). The parameters of GPD can be estimated through maximum likelihood (ML) or probability weighted method of moments (PWM). Hosking and Wallis (1987) found that for data with shape parameter greater than -0.5, the ML method holds. Rootzen and Tajvidi (1996), showed that for heavy-tailed data with shape parameter grater than 0.5, PWM method gives seriously biased estimates whereas ML estimates are consistent.

### 3.2.3 Tail Estimates

For the heavy-tailed case ( $\xi > 0$ ), in terms of  $x = y + u$ , the GPD can be expressed as:

$$G_{\xi,\sigma}(x) = 1 - (1 + \xi \frac{x-u}{\sigma})^{-1/\xi} \quad (13)$$

From (10) we can derive the form of  $F(x)$  as following:

$$F(x) = (1 - F(u))F_u(y) + F(u) \quad (14)$$

After we have selected the high threshold  $u$ , the last term of the distribution can be estimated by  $(n - N_u) / n$ , where  $n$  is the number of observations and  $N_u$  is the number of excesses above  $u$ . Therefore, the tail estimate can be written as:

$$\hat{F}(x) = \frac{N_u}{n} (1 - G_{\xi,\sigma}(x)) + (1 - \frac{N_u}{n}) = 1 - \frac{N_u}{n} (1 + \frac{\xi}{\sigma} (x-u))^{-1/\xi} \quad (15)$$

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<sup>14</sup> Writing it in terms of  $x$ :  $G_{\xi,\sigma,\mu}(x) = \begin{cases} 1 - (1 + \frac{\xi}{\sigma} (x-\mu))^{-1/\xi}, \xi \neq 0 \\ 1 - e^{-(x-\mu)/\sigma}, \xi = 0 \end{cases}$ , where  $\mu$  is the location parameter.



As measures like VaR and Expected Shortfall relate to tail risks, more specifically to quantiles in the tails of the distribution, quantile estimates offered by EVT can be used to adapt such measures.

Considering a given desired probability  $p$ , inverting (15) we get the *extreme VaR estimate* using EVT, written as:

$$VaR_p = u + \frac{\hat{\sigma}}{\hat{\xi}} \left( \left( \frac{n}{N_u} (1-p)^{-\hat{\xi}} - 1 \right) \right) \quad (16)$$

where  $\hat{\sigma}$  and  $\hat{\xi}$  are the estimated values of  $\sigma$  and  $\xi$  respectively.

The definition of *Expected Shortfall* or *CVaR* is given by:

$$ES_p = VaR_p + E[X - VaR_p \mid X > VaR_p] \quad (17)$$

i.e. the Expected Shortfall is the mean value of the loss above VaR with a given probability, conditional on the loss exceeding VaR and the measure is also called Conditional Value at Risk.

Reinterpreting the above definition in terms of distribution F and a given high threshold  $u$ , we can imply that ES is the mean excess distribution  $F_{VaR}(y)$  over the threshold  $VaR_p$ . The *mean excess function* of GPD with  $\xi < 1$  is therefore:

$$e(z) = E[X - z \mid X > z] = \frac{\sigma + \xi z}{1 - \xi} \quad (18)$$

From (17) and (18), considering  $z = VaR_p - u$ , the Expected Shortfall with probability  $p$  using EVT is given by:

$$ES_p = VaR_p + \frac{\sigma + \xi(VaR_p - u)}{1 - \xi} = \frac{VaR_p}{1 - \xi} + \frac{\sigma - \xi u}{1 - \xi} \quad (19)$$

The CVaR measure offers supplementary information about the risk of losses given that the VaR threshold is exceeded. Moreover, the two measures only use extreme observations, thus they are more adapted to extreme market conditions.

## 4 Methodological Aspects

Practical implementation of EVT involves a number of challenging issues. After data collection, the early stage of data analysis is very important in determining whether the EVT framework for fat-tailed series can be applied or not. Several statistical and graphical methods can be used in order to conclude on this aspect. The sequent problem is the fact that the estimates of the limit distributions GEV and GPD highly depend on the number of extreme observations used and on the choice of a high threshold, respectively. Here, we will focus on POT method and GPD distribution. The threshold should be i) large enough to ensure that the data satisfies the conditions imposed by EVT, i.e. the threshold tends towards infinity, and, at the same time, ii) small enough to allow for sufficient observations to be taken into consideration. Finally, EVT relies on the i.i.d. assumption for observed returns, which is inconsistent with financial reality. In literature, stationarity is generally considered sufficient for weak consistency<sup>15</sup>. Also, in order to rely on i.i.d. observation, a common practice is to produce standardized residuals and use them in estimation.

### 4.1 Data Processing and Analysis

Before applying any measure or method to observed data we must process it. The market does not provide any information on returns of financial data on a time frame base. What we can observe from public data suppliers are realized values of indexes, exchange rates, interest rates etc., with a certain frequency, e.g. annual, monthly or daily. In order to assess market risk and, more specifically, exchange rate risk, on a daily basis, practitioners usually derive daily log-returns, computed as:

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}} \times 100 \approx \ln\left(\frac{S_t}{S_{t-1}}\right) \times 100 \quad (20)$$

where  $r_t$  denotes daily return for day  $t$  (logarithmic),  $S_t$  denotes daily exchange rate and  $\ln$  denotes the natural logarithm. Where no information is provided for the value of a daily return, given the discrete nature of observations, interpolation is applied in order to calculate the missing values.

As common practice we mentioned using standardized residuals as proxy for i.i.d. observations, instead of raw returns. We will fit specific models to the conditional mean and

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<sup>15</sup> Leadbetter et. al (1983)

variance of the returns and extract the correspondent residuals. Using the filtered residuals and the conditional standard deviations, we will produce i.i.d. standardized residuals, which we will further use in EVT estimation.

Another convention stems from the following: positive returns indicate data located in the right tail of the distribution, i.e. risk of increasing exchange rates, but in the special case of exchange rate risk, not only an extreme increase can generate losses in credit institutions portfolios, but also an extreme decrease, corresponding to the position taken. As EVT is designed to work with right fat tails, i.e. upper order statistics, in order to assess downside risk, a convention is used:

$$\text{Min}(r_1, r_2, \dots, r_n) = \text{Max}(-r_1, -r_2, \dots, -r_n)$$

Basically, the series of returns are multiplied by (-1) in order to adapt EVT to the study of minima.

Having derived the series of observations, one must analyse the data. This is a very important step and several tools are available in this area. Before applying VaR models and specific methods of EVT, the main characteristics of data have to be assessed, i.e. normality, heteroskedasticity, skewness, kurtosis, autoregressive terms etc. For heavy-tailed data, this will bring empirical evidence on stylized facts.

We start with the analysis of the main characteristics of the distribution: mean, variance, skewness, kurtosis etc. The skewness and, more importantly, the skewness and kurtosis offer very important information for the implementation of Extreme Value Theory, namely the degree of asymmetry and the peakness of data. Commonly used VaR models assume normally distributed returns, with 0 skewness and a kurtosis of roughly 3. In reality, financial data are known to be skewed and leptokurtic (with excess kurtosis, over the value of 3).

In order to sustain the stylized fact that financial data is not normally distributed, we use Jarque-Bera statistic. We assess the goodness-of-fit of normal distribution for our data, showing the departure from normality and the rejection of the null hypothesis of normally distributed data, which literature considers common for returns of financial data. In order to assess normality, we also use graphical tools: histograms and QQ-Plots.

Financial data are known to be heteroskedastic (with volatility changing over time) and affected by volatility clusters. We assess heteroskedasticity using a graphical output of the time series (log returns). We test for stationarity, autoregressive components and autocorrelation, using Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, correlograms and MLE estimation for AR terms. With respect to dependence, common knowledge indicates that financial data has persistent variance, thus we will extend the analysis of correlation to squared returns.

Our first goal is to demonstrate the stylized facts presented in Section 1 and the fat-tailedness of our data. Still, some further analysis is required in order to apply the results of EVT. Exploratory data analysis within EVT framework starts with two main graphical tools: QQ-plots against exponential distribution and the mean excess function plot.

#### 4.1.1 QQ-plots

Usually, one starts by exploring the histogram of the data. In practice, most of VaR methods use the approximation of a normal distribution. However, most financial data are fat-tailed. In order to assess which approximation is suited for the underlying distribution of returns, QQ-plots prove to be a very handful tool. The graph of quantiles makes it possible to assess the goodness of fit of the analyzed series to the parametric model.

First, we should define the graph of quantiles. Let  $X_1, X_2, \dots, X_n$  be a succession of i.i.d. random variables and  $X_{n,n} < X_{n-1,n} < \dots < X_{1,n}$  the decreasing order statistics, with  $F_n$  the empirical distribution, where  $F_n(X_{k,n}) = (n-k+1)/n$ , and  $F$  the parametric distribution. The **QQ-plot** (graph of quantiles) is defined as the set of points

$$\left\{ X_{k,n}, F^{-1}\left(\frac{n-k+1}{n}\right) \mid k = 1, \dots, n \right\} \quad (21)$$

If the parametric model fits the data, the graph should have a linear form. The more linear the QQ-plot, the more appropriate the model in terms of goodness of fit. We compare the empirical distribution with the normal. If the empirical distribution exhibits a curve to the top at the right end or to the bottom at the left end, then it has fatter tail than the empirical.

In the EVT framework, the quantiles of the empirical distribution are usually plotted against the exponential distribution. The graph should therefore show fatter tails for the underlying

distribution of data, i.e. a concave departure from the straight line of the empirical distribution.

Also, when the distribution of returns is more or less known, the QQ-plot is useful to detect possible outliers in the data<sup>16</sup>. The graph of quantiles can prove useful in assessing the fit of selected model (with estimated parameters) for the tail of the distribution, i.e. plotting the tail against the GPD.

#### 4.1.2 Mean Excess Plot

The mean excess plot is the graphical representation of the *mean excess function* (MEF). Let  $X$  be a random variable,  $u$  the sufficiently high threshold and  $x_F$  the right endpoint, then MEF is defined as:

$$e(u) = E[X - u \mid X > u], \quad 0 \leq u < x_F \quad (22)$$

The mean excess function is the average of excesses over the threshold  $u$  and describes the expected overshoot of  $u$  once an exceedance occurs.

If  $X$  follows an exponential distribution with parameter  $\lambda$ , the mean excess function is equal to  $\lambda^{-1}$ , for every  $u > 0$ . In the case of Generalized Pareto Distribution, the MEF is given by:

$$e(u) = \frac{\sigma + \xi u}{1 - \xi} \quad (23)$$

where  $\sigma + \xi u > 0$ . The mean excess function of a fat-tailed series is usually located between the constant MEF of the exponential and the linear GPD which tends towards infinity as the threshold  $u$  tends to infinity.

A graphical assessment of the behaviour of the tail can be performed using the plot of the sets of points:

$$\{(X_{k,n}, e_n(X_{k,n})) \mid k = 1, \dots, n\} \quad (24)$$

i.e. the *mean excess plot*, where  $e_n(u)$  is the *sample mean excess function*, defined as:

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<sup>16</sup> See Embrechts et. al (1997)

$$e_n(u) = \frac{\sum_{i=1}^n (X_i - u)}{\sum_{i=1}^n I_{\{X_i > u\}}} \quad (25)$$

where  $I_{\{X_i > u\}}$  is the indicator function which takes the value 1 if excesses occur and 0 otherwise.

If the mean excess function of the empirical dataset is a positively sloped line above a certain threshold  $u$ , data in the tail follow a Generalized Pareto Distribution with positive shape parameter  $\xi$ . Conversely, exponentially distributed data show a horizontal MEF (constant) and short-tailed data exhibit a negatively sloped line (corresponding to the negative shape parameter)<sup>17</sup>.

This is a very important graphical tool for the choice of the sufficiently high threshold  $u$ . Plotting MEF using the whole empirical distribution can help us choose the threshold  $u$  in the region where the curve is roughly linear, i.e. the data is well approximated by the GPD.

Another useful graphical tool is the Hill plot, i.e. plotting the order statistics of empirical data against different values of the Hill estimator of the tail index. As tail index estimation needs to be discussed prior to graphical results and this is a very complex and troublesome issue of the EVT framework, we will refer to Hill plot later.

## 4.2 Tail Index

The next natural step in EVT implementation is the selection of the threshold. Considering the theory stated above, the tail index refers to  $\alpha = 1/\xi$ , which could be interpreted as the number of existing finite moments of the empirical distribution, the number of degrees of freedom of an underlying Student-t distribution, the speed of decay in the tail or the inverse measure of tail fatness (fatter tail, smaller tail index and vice versa). In practice, this name often refers to the shape parameter  $\xi$ , which directly relates to tail fatness.

The literature is abundant in measures of tail index. New measures try to deal with some of the weaknesses of the existing ones but their behaviour has not been yet verified in datasets with different characteristics. Moreover, the quality of the estimates highly depends on

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<sup>17</sup> Gençay and Selçuk (2003)

empirical data: source, frequency of observations, volume, characteristics – e.g. independence or distribution. This is why no consensus has been reached in this matter.

#### 4.2.1 Hill Estimator

One of the most important and commonly used estimators of tail index/shape parameter is Hill estimator. The measure was introduced by Hill (1975) as a maximum likelihood estimator for the power coefficient of Pareto density:

$$h(x/x>u) = \alpha(x/u)^{-\alpha-1}u^{-1} \quad (26)$$

Let  $X_{1,n} \leq X_{2,n} \leq \dots \leq X_{n,n}$  be the order statistics of the empirical series and  $k$  the number of upper order statistics over the threshold  $u$ . Taking logarithms and differentiating with respect to  $\alpha$  in (26) yields

$$\partial \log h(x/x>u) / \partial \alpha = 1/\alpha - \log(x/u) \quad (27)$$

The Hill estimator is found by equating this first order condition to 0, replacing  $x$  with the order statistics  $X_{i,n}$  and applying the sum operator over this elements. Solving for  $\xi = 1/\alpha$  gives

$$\frac{1}{\alpha} = \frac{1}{k} \sum_{i=1}^k \log \frac{X_{i,n}}{u} \quad (28)$$

We can now derive the final form of the Hill estimator in

$$\xi_{k,n}^{Hill} = \frac{1}{k} \sum_{i=1}^k \log X_{n+1-i,n} - \log X_{n-k,n} \quad (29)$$

#### **Properties:**

- (1) Asymptotic normality (Mason, 1982):  $\sqrt{k}(\xi_{k,n}^{Hill} - \xi) \xrightarrow{d} N(0, \xi^2)$
- (2) (Weak) Consistency for  $\xi > 0$  in the following sense: If  $k_n, n \in \mathbb{N}$  is an intermediate sequence, that is  $k_n \rightarrow \infty, k_n/n \rightarrow 0$ , then  $\xi_{k,n}^{Hill} \xrightarrow{P} \xi$ .
- (3) Unbiased when  $\{X_n\}$  follows an exact Pareto distribution.

The second property relates to the choice of the sufficiently high threshold, leaving  $k$  upper order statistics above. It is not clear how to apply this result. Theoretically, one must try to minimize the asymptotic mean square error. In practice, this is usually done by choosing  $u$  based on the Hill plot, i.e. inferring on  $\xi$  from a stable region of the graph.

#### 4.2.2 Pickands Estimator

If Hill only holds for  $\xi > 0$ , Pickands estimator of tail index holds for  $\xi \in \mathfrak{R}$ . The estimator was introduced by Pickands (1975) and has the following form:

$$\xi_{k,n}^{Pick} = \frac{1}{\log 2} \log \left( \frac{X_{n-\lfloor k/4 \rfloor, n} - X_{n-\lfloor k/2 \rfloor, n}}{X_{n-\lfloor k/2 \rfloor, n} - X_{n-k, n}} \right) \quad (30)$$

where  $\lfloor x \rfloor$  denotes the largest integer not exceeding  $x$ . Pickands provides an estimator for all three types of limit laws within Generalized Extreme Value.

#### *Properties*<sup>18</sup>

1. Asymptotic normality.
2. Consistency for  $\xi \in \mathfrak{R}$ .

On the other hand, the estimator is quite volatile as a function of  $k$ . The same idea behind threshold selection for Hill estimator is applicable to Pickands estimator.

#### 4.2.3 Dekkers-Einmahl-de Haan Moment Estimator

The estimator was proposed by Dekkers, Einmahl and de Haan (1989) as a generalized form of the Hill estimator, in order to infer for values of  $\xi \in \mathfrak{R}$ .

$$\xi_{k,n}^{DEdH} = M_{k,n}^{(1)} + 1 - \frac{1}{2} \left( 1 - \frac{(M_{k,n}^{(1)})^2}{M_{k,n}^{(2)}} \right)^{-1} \quad (32)$$

DEdH is called the moment estimator because it is based on two measures (empirical moments):

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<sup>18</sup> Dekkers and de Haan (1989)



$$M_{k,n}^{(1)} = \frac{1}{k} \sum_{i=1}^k \log(X_{i,n} / X_{k+1,n}) \quad (33)$$

$$M_{k,n}^{(2)} = \frac{1}{k} \sum_{i=1}^k \log^2(X_{i,n} / X_{k+1,n}) \quad (34)$$

The measure in (33) is the estimator proposed by Hill (1975). DEdH estimator is consistent and asymptotically normal.

### **Comparative statements<sup>19</sup>:**

- Both Pickands and DEdH work for general  $\xi \in \mathfrak{R}$ , while Hill only for  $\xi > 0$ ;
- Pickands estimator is rather unstable;
- Hill estimator is very sensitive to dependence in the data<sup>20</sup>;
- Pickands and DEdH estimators converge faster;
- All estimators are biased, especially in small samples.

## **4.3 Graphing Techniques**

Using the measures presented in Sections 4.3.1 - 4.3.4 one can inspect the behaviour of tail index estimates for different order statistics. In practice, this is done by plotting Hill/Pickands/DeHaan estimates against values of  $k$ :

$$\{(k, \hat{\xi}_{k,n}), 1 \leq k \leq n\} \quad (35)$$

In order to establish the estimate of  $\xi$ , one should look for a stable region on the plot. This procedure is quite subjective, based on two considerations: the volatility of the plot, and the little time the plot spends in the true neighbourhood of the real value of  $\xi$ .

Theoretically speaking, tail index selection is based on the minimization of the asymptotic mean squared error, which is a measure of variance and bias, thus of the quality of the estimator in terms of variation and bias.

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<sup>19</sup> Embrechts et. al (1997).

<sup>20</sup> For ARMA or weakly dependent series the problem is usually treated by first fitting an ARMA model to data and then applying Hill estimator to the residuals.

#### 4.4 Threshold selection

This sub-section of the paper is dedicated to one of the most important steps in the implementation of Extreme Value Theory – the choice of the high threshold  $u$ . Although a wide literature has been devoted to the issue of tail index estimation, researchers haven't yet found a clear-cut answer to the question "What estimator should one use for the tail index?". However, threshold selection is crucial since the estimates of the shape parameter are sensitive to this choice.

Amongst the tools and methods for choosing the threshold in the heavy tail of some underlying fat-tailed distribution, graphical methods are apparently the most simple and widely used alternative<sup>21</sup>. In reality, the issue is quite problematic. A basic set of graphs should be always assessed in data analysis (see Section 4.1.). This basic set of graphs should, at least, comprise mean excess plot and Hill/Pickands/DEdH plot (or other plots of tail index estimates against  $k$  upper statistics).

The information *mean excess plot* offers should be resumed as it follows: choose the threshold in the area where the graph is roughly linear. This statement is backed by the simple fact that, according to the results of Pickands and Balkema-de Haan, excesses over a high threshold converge to GPD (Theorem 2 in Section 3.2.2.) and the GPD graph is perfectly linear. The slope of the curve also provides important information: if the slope is positive, then data follows the GPD above that certain threshold; if the slope is 0 then the underlying distribution follows the exponential and if the slope is negative, then the distribution is short-tailed.

Other graphical tools are the *Hill plot* or similar alternative plots. The idea behind the selection of the threshold using these graphs is the following: choose the threshold in the area where the graph is fairly stable. This is backed by the fact that all types of estimators above are GPD estimators as extreme distribution converges to GPD over a high threshold  $u$ .

#### 4.5 Bias-variance trade-off

As many authors suggest<sup>22</sup>, most alternative estimators for tail index are asymptotically unbiased but biased in small samples, generally leading to overestimates of tail risk. In order

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<sup>21</sup> This approach is applied by most of the referenced authors

<sup>22</sup> See for example Huisman et. al (1998), Blum and Dacorogna (2002).

to apply EVT tools, one has to choose a sufficiently high threshold. The threshold has to be low enough in order to reduce variance, as the reduction of variance relies on taking into account more observations i.e. variance reduces as the mean of a large number of excesses is considered. However, choosing the threshold too low, the bias of the estimators increases, because more observations situated far from the mean and, implicitly, from the linear part of the Pareto quantile plot are taken into account. This results in what literature calls *bias-variance trade-off* and the selection of the threshold has to best satisfy the two implications of this matter.

#### **4.6 Limitations and advantages of EVT**

Standard Value at Risk methods face a series of problems: normal distribution hypothesis, underestimation of risk; symmetry between tails assumption and focus on the centre of the distribution, i.e. events that happen in regular, manageable conditions. Moreover they provide no tools for extreme events or out of sample quantiles estimation. Their use is however common standard for risk managers, as the basic framework is easily implemented and supported by regulation.

In the light of new market conditions and proven stylized facts of financial data, especially in the case of exchange rates, interest rates and stock index returns, risk managers become more and more concerned with rare events, i.e. events occurring under extreme market conditions (as extreme events tend to become regular in current market behaviour). EVT is a powerful complementary tool because it provides more appropriate distributions to fit extreme events. Moreover, no assumptions are made about the nature of the original distribution of observations and the framework can be used to solve for very high quantiles (deriving extreme VaR measures), which is very useful in predicting extreme-losses.

However, EVT implementation faces many challenges. One of the most important ones is the fact that EVT is designed for independent data and financial data, exchange rate returns in our case, tend to be dependent. Many authors<sup>23</sup> suggest using standardized observations to deal with this problem. Other important issue of EVT is the choice of the high enough threshold, which we discussed previously. Although these issues are important and further research is required in order to solve them, the following section aims to prove the importance that EVT framework has in risk management.

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<sup>23</sup> See for example Embrechts et al.(1997), Bensalah (2000).

#### 4.7 Hybrid models

In order to produce estimates of potential high losses taking into account the whole distribution, not only the tails, and produce dynamic estimates of potential risk, EVT can be modelled to incorporate the standard VaR measures. The respective VaR values can be produced as it follows:

$$VaR_t(\alpha) = \sigma_t \times X_{k,n} \times \left( \frac{n}{k} \times \alpha \right)^{-\xi} \quad (36)$$

where the VaR at time  $t$  is computed using the variance at time  $t$  obtained by EWMA and EGARCH models, the threshold  $u$  which equals the  $k^{\text{th}}$  order statistic  $X_{k,n}$ , the ratio between the number of observations  $n$  and the number of order statistics in the tail  $k$  and not last, the shape parameter  $\xi$ . The method is proposed for example in Blum and Dacorogna (2002), Caserta and de Vries (2003). The multiplier of the variance is actually a higher order quantile which incorporates information about the tail fatness of the distribution. In literature, this quantile is often referred to as out-of-sample estimate.

#### 4.8 Backtesting

Researchers have developed many methods to test the performance of VaR models and a very comprising reviews of these methods can be found in Engel and Gitzky (1999). Such methods usually measure the performance of VaR models in terms of conservatism and accuracy. In this paper we use two methods: first, we analyze the conservatism through the percentage of failures and second, we compute the Mean Squared Error to assess the accuracy in estimation.

In order to determine the percentage of failures, we determine the number of failures for each model using the following loss function:

$$L_t = \begin{cases} 1, & r_t > VaR_t \\ 0, & r_t \leq VaR_t \end{cases} \quad (37)$$

Thus, a failure happens when the real return at time  $t$ , positive or negative, is not covered by the respective VaR value. Counting the number of failures for each model, applied to each series, we obtain a percentage of failures, which should not be larger than the significance

level for which the VaR is computed, e.g. at 99% we should not have more than 1% of failures.

The Mean Squared Error tests for the accuracy in predicting potential losses and we compute it as:

$$MSE = \frac{1}{n} \times \sum_{t=1}^n (VaR_t - r_t)^2 \quad (38)$$

The measure penalized overestimates and underestimates of observed returns and a model is chosen as to minimize the MSE.

## 5 Empirical Results

### 5.1 Data

We use daily returns of exchange rates of Swiss Franc, Great Britain Pound, Romanian New Leu and US Dollar, respectively, against EUR<sup>24</sup> for the time period between January 1999 and June 2009, each data set consisting in 2727 observations. Data is provided by The National Bank of Romania. Returns are computed as described in Section 4.1. For inference for lower tails, in order to transform minima into maxima, we use negative series. For our analysis on EVT performance we chose the POT method as it uses data more efficiently. The approach is in line with the studies of McNeil (1997a), McNeil(1997b) and McNeil (1999), Matthys and Beirlant (2000), Blum and Dacorogna (2002), Wagner and Marsh (2003), Brooks et al. (2003) and others.

In Section 1 we have underlined the fact that movements in financial market variables often occur within more that 2-3 sigmas and the normal distribution does not fit well such data. In Table 1 below we present the number of observations within intervals of sigmas up to 10 sigmas and above for our data. In Appendix 1. we present the evolution of exchange rates in the chosen time period.

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<sup>24</sup> As a convention, from this point on we will denote by CHF the price of a Euro in Swiss Francs, by GBP the price of a Euro in Great Britain Pounds and so on.

Series	Sigma (%)	Movements within ( $\pm$ )					
		2 sigmas	4 sigmas	6 sigmas	8 sigmas	10 sigmas	> 10 sigmas
<b>CHF</b>	0,2999	2613	93	13	6	1	1
<b>GBP</b>	0,5192	2591	125	10	1	0	0
<b>RON</b>	0,6322	2582	132	9	2	1	1
<b>USD</b>	0,6598	2585	137	5	0	0	0

Table 1. Sigma movements of exchange rate returns

Although data is concentrated in the middle of each distribution, it is obvious that important information lies beyond 2 sigmas and changes even above 10 sigmas occur (movements of CHF and RON vs. EUR). Such evidence of extreme movements gives incentive for the use of a stronger risk management framework, even more in the context of current crisis which has seriously weaken world's currencies.

To begin our analysis, we observe the stylized facts in our data, like fat-tailedness, skewness, leptokurtosis, heteroskedasticity, volatility clustering and dependence<sup>25</sup>.

The main statistics of the series show maximum daily changes up to almost 7% in one day for RON/EUR exchange rate (severe depreciation of RON back in 1999). Maximum currency appreciation of RON vs. EUR, i.e. decrease of exchange rate, is roughly 5%. The impact of current crisis on the other three currencies has pushed the exchange rates against the Euro to new historical maxima and minima: GBP depreciated with almost 3% against the EUR and suddenly appreciated, in just a few days, with 3.14%; CHF, known as a highly speculated exotic currency, dropped 3.4% against the Euro and regained 2.06% with several tendency changes in the last months; the American currency lost 3.72% against the Euro in 2009, after another maximum drop at the end of 2008 of roughly 3.5%. In Appendix 1. we also present the evolution of exchange rates in the period between January 1999 and June 2009.

In terms of skewness and kurtosis, all four series are positively skewed and exhibit *excess kurtosis*, up to over 14 for CHF, indicating fat-tails in the data. Considering this and the results of Jarque-Bera statistics (p-values equal 0), we can set grounds for rejecting the hypothesis of normally distributed returns. The Jarque-Bera statistic is distributed as  $\chi^2$  with 2 degrees of freedom under the null of normally distributed returns, with 1% critical value of 9.210. The main results are presented in Table 2. Histograms are presented in Appendix 2.

<sup>25</sup> Embrechts et. al (1997) stressed the importance of looking at the data before engaging in detailed statistical analysis.

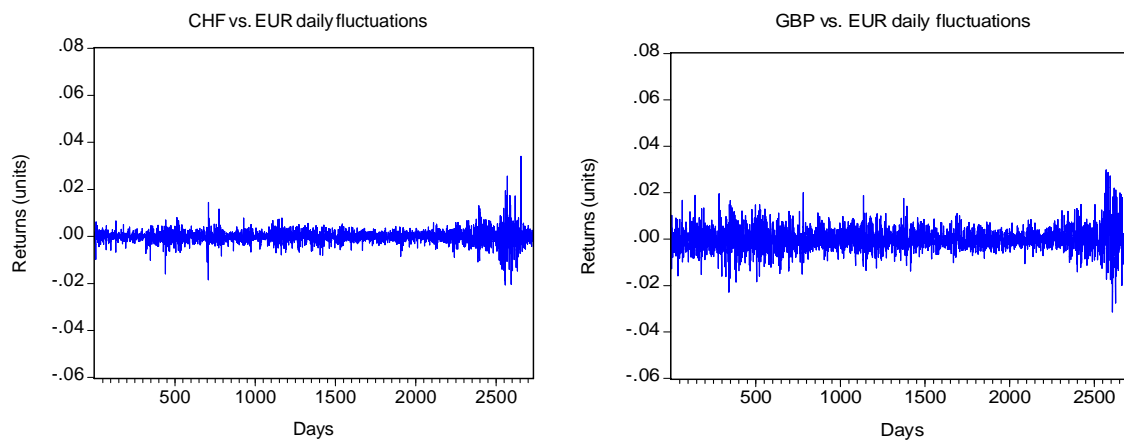
Series	CHF	GBP	RON	USD
Mean	-0.000026	0.000063	0.000431	0.000060
Maximum	0.034014	0.029818	0.068792	0.037188
Minimum	-0.020577	-0.031374	-0.051064	-0.027968
Std. Dev.	0.002999	0.005192	0.006322	0.006598
Skewness	0.297207	0.164338	0.652896	0.098952
Kurtosis	17.562900	6.388935	12.772760	4.458904
Jarque-Bera	24137.5*	1317.244*	11045.7*	246.2899*
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2727	2727	2727	2727

\*denotes significance at 1% level.

Table 2. Statistics of return series

We plot the quantiles of empirical series against the quantiles of normal distribution, in order to verify the fit. QQ plots show excess kurtosis of empirical series. The concave departure of the line proves the fact that exchange rate returns have fatter tails than the normal distribution. USD returns have the smallest departure from the normal as we can also observe in Table 1.: kurtosis a little over 3 and small skewness. QQ plots are presented in Appendix 3.

Exchange rate returns are known to be *heteroskedastic*. To observe the daily evolution of exchange rates, we plot the returns on the time axis (Figure 2). It is obvious that the variances change in time and the series are heteroskedastic. We can also observe high and low changes. At the introduction of the Euro in 1999 the Romanian currency suffered a severe depreciation of more than 6% (RON/EUR return around the 50<sup>th</sup> tick). RON appears to be the most unstable currency in our data set and tends to peak once in 2-3 years. CHF follows very close the evolution of the EUR. Moreover, volatility clusters appear.



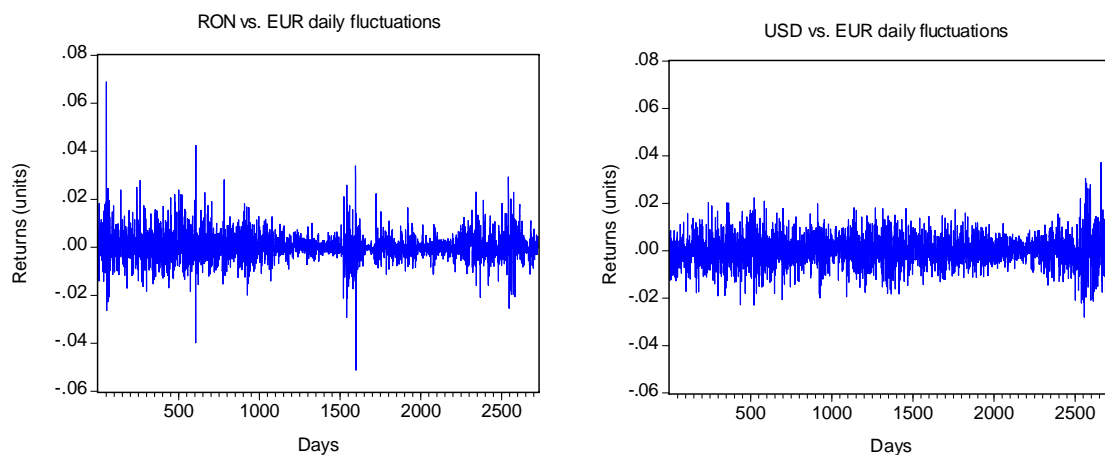


Figure 2. Daily fluctuation of exchange rates

In order to test for *stationarity*, we employ ADF and PP tests (estimation outputs available in Appendix 4.). Both tests reject the null of unit root existence at 1% (with p-values insignificantly different from 0), as critical values at 1%, 5% and 10% are constantly less negative than the ADF and PP statistics. The summary of the tests is presented in Table 3.

Series	CHF		GBP		RON		USD	
Test	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
ADF	-54.7158	0.0001	-52.1392	0.0001	-33.2409	0.0001	-53.3015	0.0001
PP	-55.0132	0.0001	-52.1392	0.0001	-48.5331	0.0001	-53.2919	0.0001
critical values			1%	-3.4326	5%	-2.8624	10%	-2.5673

Table 3. ADF and PP tests summary

Moving further, we test for autocorrelation, as Extreme Value Theory requires the observations to be approximately independent and identically distributed. Exchange rate returns are heteroskedastic and present some degree of autocorrelation. Below we plot the sample autocorrelation function for our data (Figure 3). Indeed, autocorrelation appears and it is obviously significant for RON series, first three lags. USD series seem to present the smallest degree of autocorrelation, with AC coefficients insignificantly different from 0.



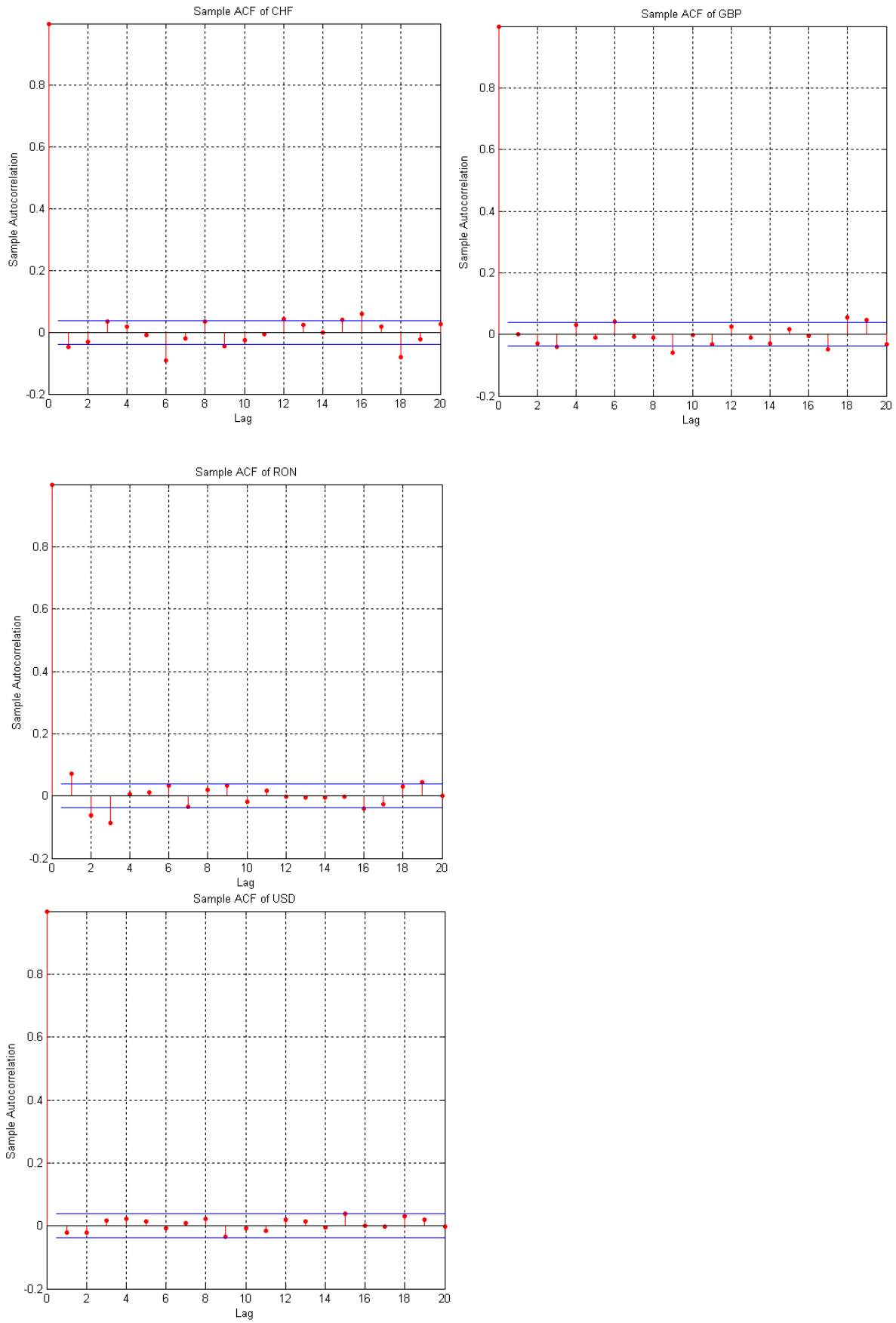


Figure 3. Sample ACF for exchange rate returns

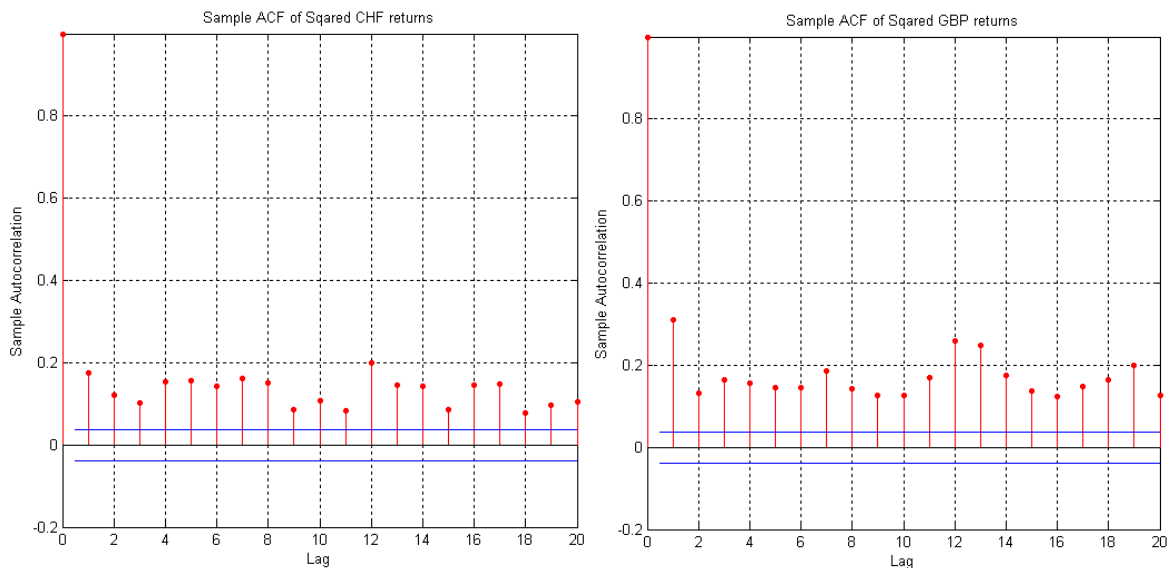
We also estimate AR(p) processes for all four series of returns with one lag for CHF, GBP and USD against EUR returns and with three lags for RON vs. EUR return. We cannot reject autoregressive terms for CHF and RON. In the special case of RON we fit up to three AR terms and since none of the models is rejected we make our choice based on AIC and SIC values (Table 4). We choose an AR(1) model for CHF and an AR(3) for RON.

Criterion	AR(1)	AR(2)	AR(3)
AIC	-7.293203	-7.296764	-7.302550
SIC	-7.288866	-7.290257	-7.293872

Table 4. AIC and SIC values for AR processes on RON

No further evidence of autocorrelation was found in the residuals. Inverted autoregressive roots are within the unit circle and thus we support the stationarity of our data. AR estimation outputs are shown in Appendix 5.

Even though GBP and USD series reject AR components, we plot the sample ACF of squared returns to illustrate the degree of persistence in variance, as the departure from i.i.d. structure impacts EVT framework. Indeed variances appear to be persistent, especially for GBP series. Plots of sample ACFs of squared returns are presented in Figure 4. below.



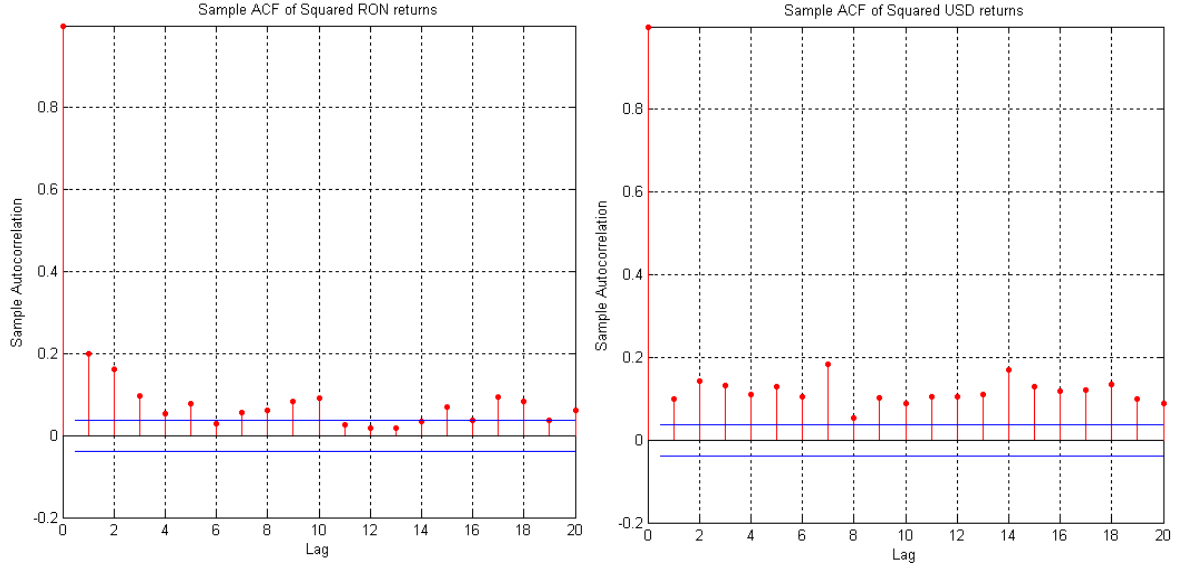
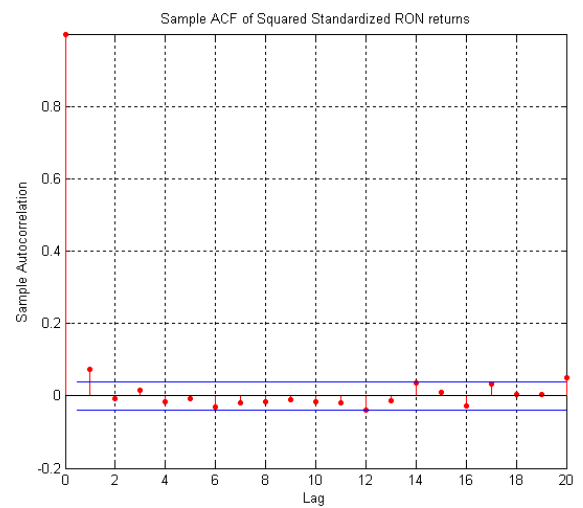
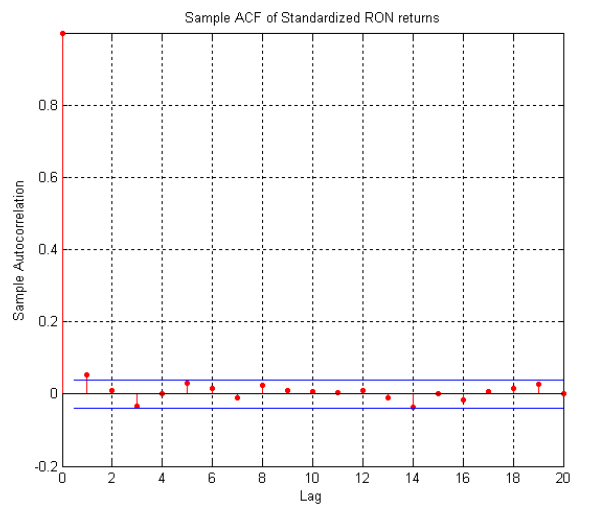
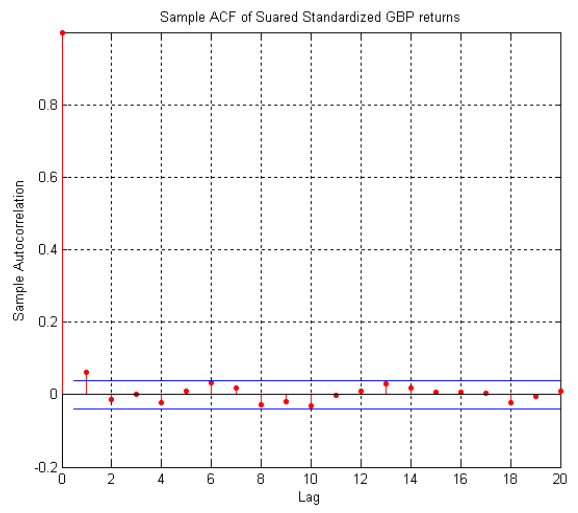
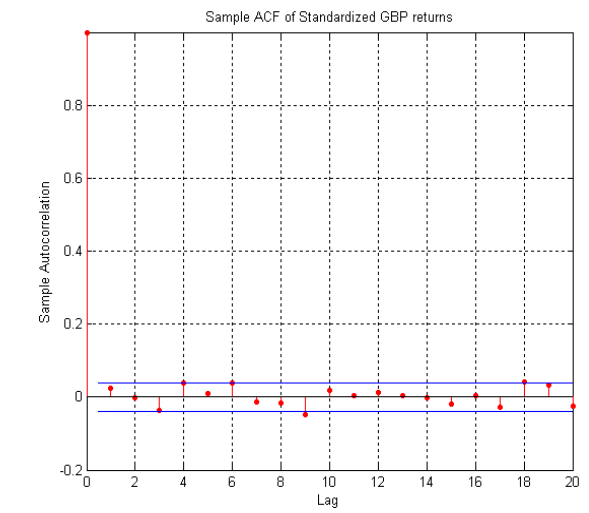
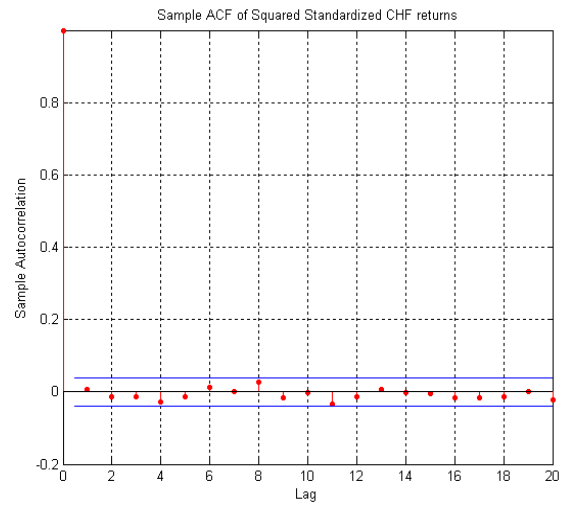
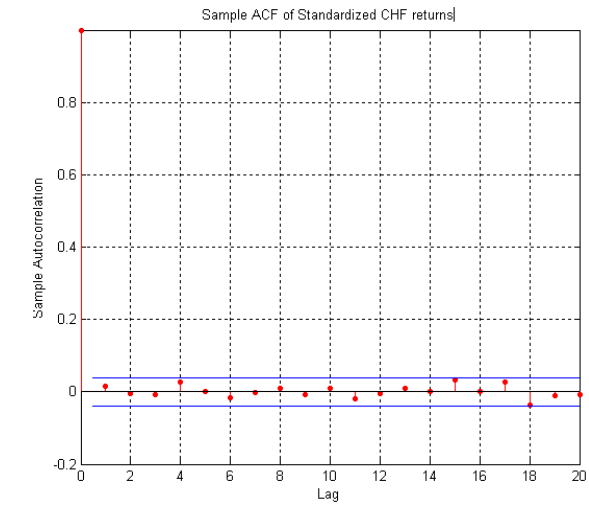


Figure 4. Sample ACF of squared returns

In order to apply EVT to our data we have to produce i.i.d. observations, i.e. we have to compensate for autocorrelation and heteroskedasticity in returns. To compensate for autocorrelation, we fit ARMA models to the conditional mean and to compensate for heteroskedasticity we fit EGARCH models to the conditional variance of all four series (outputs presented in Appendix 6). For EGARCH specification we employed Student-t distribution, as it is more appropriate for fat-tailed data. We extract the residuals and the conditional variances from the estimated models and we obtain approximately independent and identically distributed standardized distributions, computed as:

$$r(t) = \frac{\varepsilon(t)}{\sigma(t)}; i.i.d. \quad (39)$$

Resulting standardized series autocorrelation functions and ACFs of squared standardized series are plotted in Figure 5 below. Autocorrelation coefficients are roughly insignificant and persistence in variance has been removed. The standardized series are approximately i.i.d. and can be used in the EVT framework.



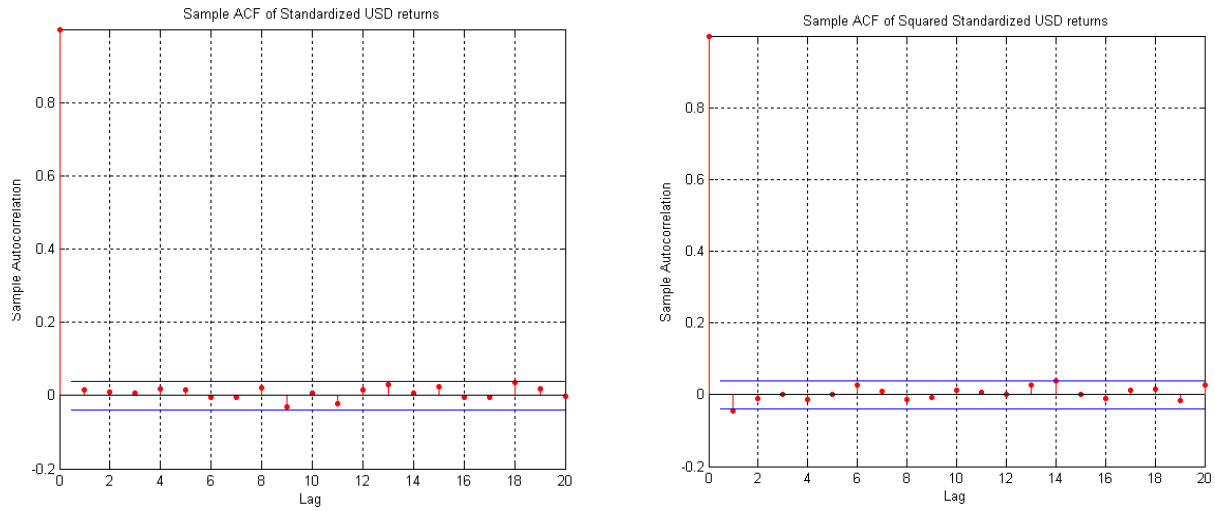


Figure 5. ACFs of Standardized Series and Squared Standardized Series

The conditional standard deviation and the correspondent residuals are plotted in Appendix 7. The main statistics of approximately i.i.d. standardized series are presented in Table 5 below.

Series	CHF	GBP	RON	USD
Mean	-0.000104	0.000166	0.000072	0.000207
Maximum	0.074168	0.044695	0.064260	0.041782
Minimum	-0.067014	-0.040348	-0.061674	-0.037682
Std. Dev.	0.010451	0.009976	0.009950	0.009951
Skewness	-0.331842	0.178408	0.335682	0.018966
Kurtosis	6.080639	3.860722	5.915937	3.531112
Jarque-Bera	1128.389*	98.64476*	1017.332*	32.21481*
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2727	2727	2727	2727

\*denotes significance at 1% level.

Table 5. Main statistics of standardized series

## 5.2 Results

### *Value at Risk*

For our non-normal, skewed, leptokurtotic and stationary non-standardized series, we apply Value at Risk framework for later comparison to VaR measures based on EVT models (extreme VaR).

We employ four VaR models: Historical Simulation (HS), Hybrid Historical Simulation (HHS), RiskMetrics EWMA with 0.94 decay factor for daily data and an EGARCH(1,1). The 99% and 99.9% Value at Risk and Expected Shortfall values for the 2728<sup>th</sup> day according to

each model employed are presented in Table 6. EGARCH model estimation is available in Appendix 6. We calculate the risk for right (upper) tail, denoted by U, and for the left (lower) tail, denoted by L.

Series	VaR (%)	HS	HHS	EWMA	EGARCH	ES (%)	HS	HHS	EWMA	EGARCH
CHF <sub>U</sub>	99%	0.71	0.47	0.37	0.54	99%	0.84	0.74	0.58	0.80
CHF <sub>L</sub>		0.82	0.48	0.37	0.54		1.26	0.77	0.58	0.80
GBP <sub>U</sub>		1.39	1.49	1.08	1.67		1.93	1.86	1.42	2.09
GBP <sub>L</sub>		1.37	1.13	1.08	1.67		1.76	1.59	1.42	2.09
RON <sub>U</sub>		1.84	0.86	0.48	0.69		2.55	1.31	1.01	1.48
RON <sub>L</sub>		1.50	0.76	0.48	0.69		2.10	1.27	1.01	1.48
USD <sub>U</sub>		1.72	1.67	1.39	0.91		2.15	1.94	1.71	2.45
USD <sub>L</sub>		1.73	1.81	1.39	0.91		1.97	2.31	1.71	2.45
CHF <sub>U</sub>	99.9%	1.79	0.72	0.49	0.72	99.9%	2.63	1.11	0.77	1.07
CHF <sub>L</sub>		1.86	0.79	0.49	0.72		1.99	1.38	0.77	1.07
GBP <sub>U</sub>		2.70	1.94	1.43	2.22		2.85	2.41	1.89	2.77
GBP <sub>L</sub>		2.27	1.92	1.43	2.22		3.48	2.31	1.89	2.77
RON <sub>U</sub>		3.07	1.11	0.64	0.91		3.70	1.59	1.33	1.96
RON <sub>L</sub>		2.71	1.10	0.64	0.91		3.02	1.68	1.33	1.96
USD <sub>U</sub>		2.80	1.78	1.83	2.14		3.20	2.01	2.26	3.25
USD <sub>L</sub>		2.22	2.35	1.83	2.14		3.18	2.87	2.26	3.25

Table 6. 99% and 99.9% VaR and ES (point estimates for day one out of the sample period).

Values in percents<sup>26</sup>. L stands for lower (left) tail and U stands for upper (right) tail.

The first observation to be made is the fact that VaR and ES values are constantly lower for CHF series. If we take a look at the daily movements in each exchange rate return data set (Figure 2. above), we have the immediate explanation: while CHF generally fluctuates in a narrow band of  $\pm 1\%$  against the Euro, the other currencies show higher changes, approaching  $\pm 2\%$  band, especially in the case of GBP and USD. VaR and ES values highly depend on this regular historical fluctuation.

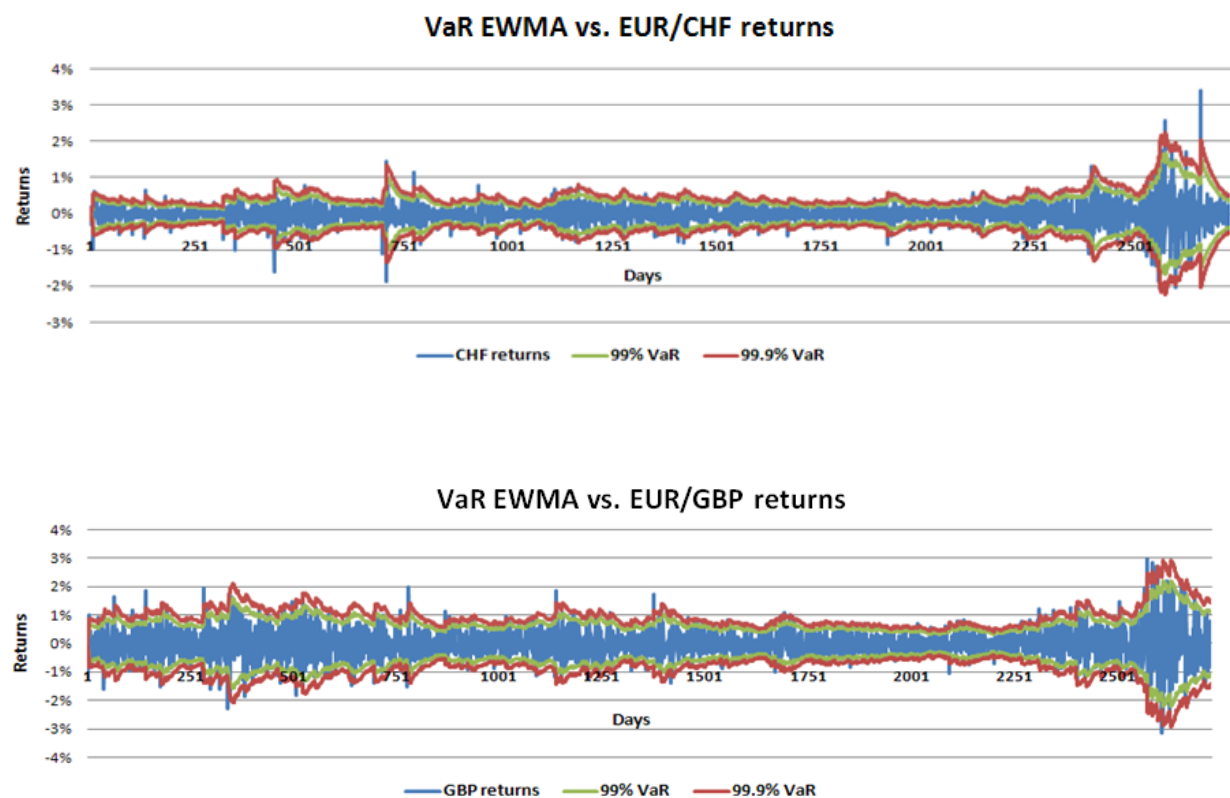
According to our result, the highest VaR and ES values are computed for Historical Simulation, which directly picks the 99% and 99.9% quantiles. Hybrid Approach shows lower values of potential loss but above those provided by RiskMetrics approach. HHS is expected to perform better for CHF, GBP and USD series, as the highest returns are rather recent. Conversely, HS and HHS could have never predicted returns of  $\pm 2\%$  in 2009 for CHF, GBP and USD since history offered significantly lower values for these exchange rate movements. EWMA takes into consideration past returns and volatility but with quadratic

<sup>26</sup> We refer to a positive percentage VaR as to a loss of x%.

impact and the slow adjustments may produce underestimates of risk. EGARCH model produces estimates in between those computed for HS and EWMA. The model takes into consideration the future impact of current information, considering it exponential and not quadratic.

In Figures 6 and 7 below, we show how daily estimates of VaR values computed with EWMA and EGARCH models fit actual returns. We show the symmetric coverage for both left and right tails. It is obvious that changes in exchange rates outside the  $\pm 2.5\%$  band are generally not captured by the two models. EGARCH shows little improvement compared to EWMA, especially at 99.9% confidence level. The high depreciation of CHF and USD vs. EUR in 2009 and that of RON vs. EUR back in 1999 are not captured by any of the two models.

Figure 6. VaR EWMA coverage for exchange rates appreciation (negative returns) and depreciation (positive returns) over the sample period: January 1999 – June 2009.



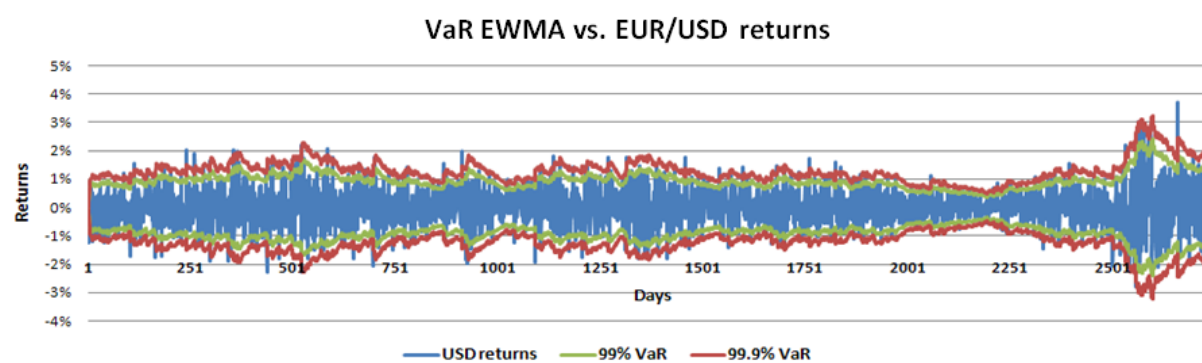
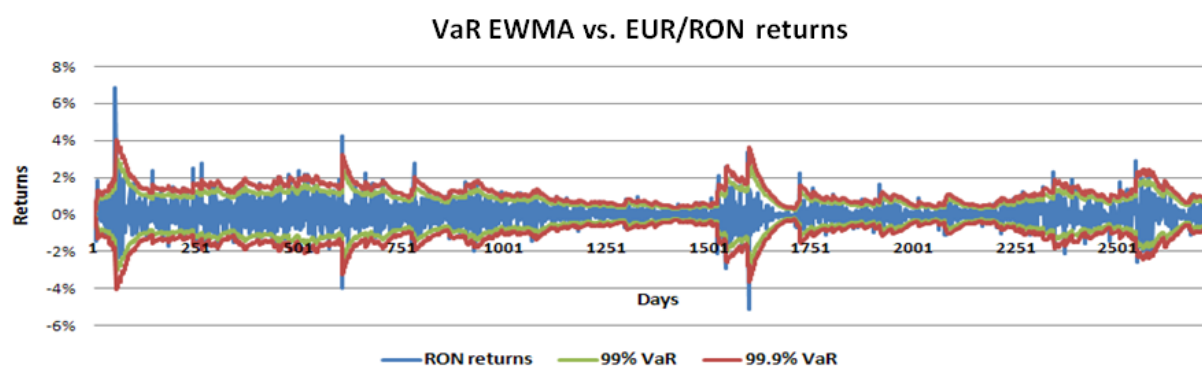
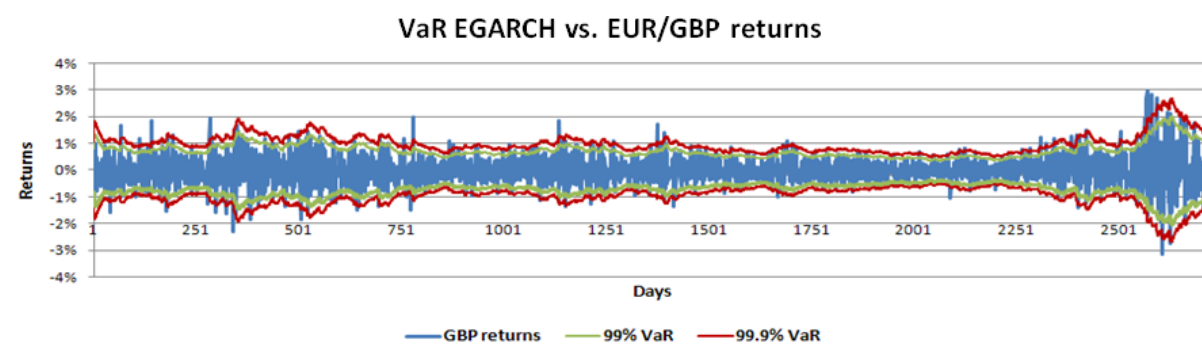
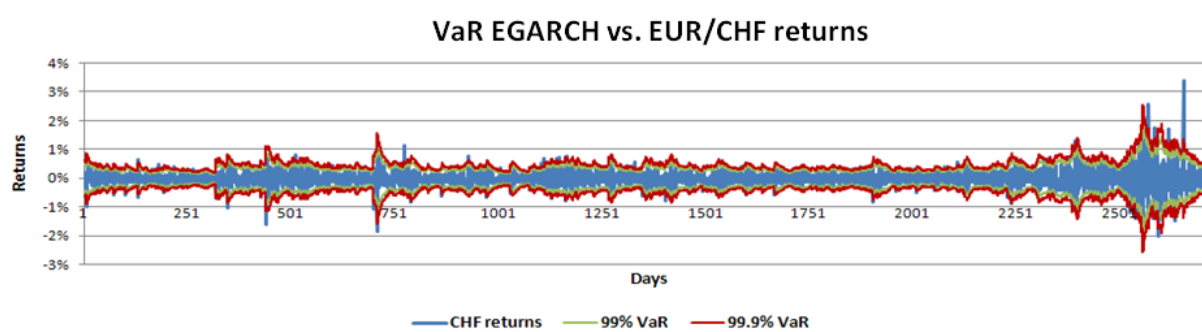
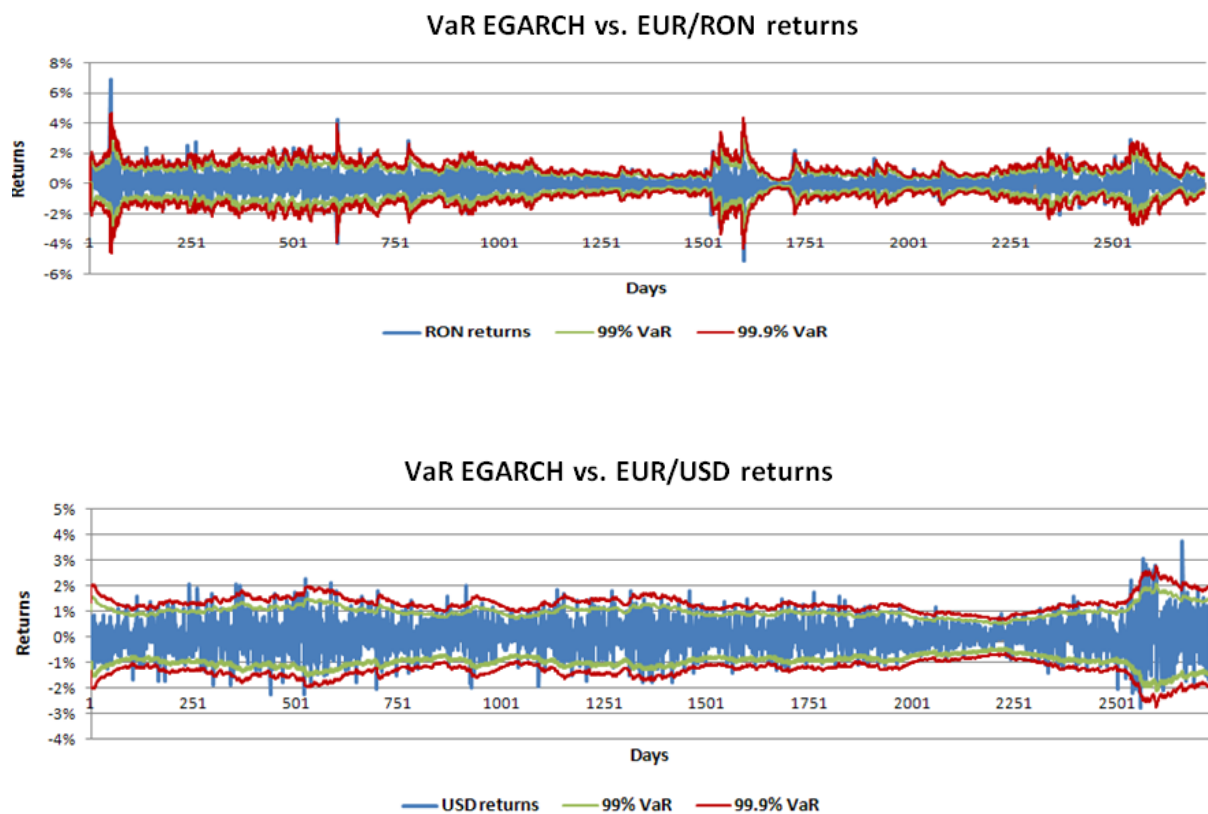


Figure 7. VaR EGARCH coverage for exchange rates appreciation and depreciation over the sample period: January 1999 – June 2009.







In Section 5.3.4. we will test the performance of the VaR methods employed and compare it to the performance of EVT models. We will prove that with respect to extreme events prediction, standard VaR models are clearly outperformed by extreme VaR measures, which incorporate the results of Extreme Value Theory.

### *Extreme Value Theory*

Objective: compute possible losses that have not yet been historically observed.

The first step in applying EVT is to explore the data<sup>27</sup>. This is usually done by plotting QQ graphs and the distribution of mean excess, as discussed in Sections 4.1.1. and 4.1.2. Because EVT is designed to analyze upper tails (positive values) of i.i.d. data, as stated before, we use positive standardized series to infer for upper tails and negative standardized series for lower tails.

First, we plot the quantiles of our series against the quantiles of exponential distribution, to verify the existence of heavy tails. Because exponential distribution decays faster than power function type distributions, the plot of the series against exponential quantiles should be curved at the bottom/upper end in order to prove heavier tails. As shown in Appendix 8., judging from the concave departures from the straight line, our datasets appear to have fatter tails than the exponential distribution. We expect GBP series to have a smaller degree of fat-tailedness than the others as they appear to have only a small departure from the exponential. In Appendix 8 some of the last order statistics have been removed in order to analyze how much distortion they induce to the graphs. Judging by the tendency of the data in the tail of USD to describe a convex curve against the straight line, we also expect these series to be considerably less fat-tailed.

Mean Excess (ME) plots in Appendix 9. generally show that for some threshold above 1%, the curves have an upward slope. Correspondently, data above that threshold will be approximated by a Generalized Pareto Distribution. Where the slopes are positive, GPD shape parameter will also be positive, i.e. the distributions are likely to be in the maximum domain of attraction (MDA) of the Fréchet (fat tailed). ME plots are generally unstable compared to the ideal case of an upward straight line, but in trend the graphs are positioned between the horizontal MEF of the exponential and the MEF of GPD, going towards infinity.

Hill plots in Appendix 10. show how estimates of the shape parameter  $\xi$  vary with the number of upper/lower order statistics and with the chosen threshold. For each tail, we plotted the last 300 order statistics (lower or upper). We consider this truncation as fair as it leaves more than 10% of data for analysis<sup>28</sup>. The graphs are roughly stable between 100 and 120 order statistics in the case of CHF right tail, between 100 and 150 for CHF left tail, around 100 order statistics for GBP right tail, between 80 and 100 order statistics for GBP left tail, between 120 and 150 order statistics for RON right tail, between 100 and 110 order

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<sup>27</sup> Embrechts et. al (1997)

<sup>28</sup> Peng et. al (2005) argued that the range of data left in the tail should be less than or equal to 10% of data.

statistics for RON left tail, between 130 and 150 order statistics for USD right tail and between 80 and 100 order statistics for USD left tail. In order to test the stability of the parameter in each of these areas, we computed ML shape parameter estimates for different numbers of observations left in the tail.

Based on the previous two graphing techniques we select the thresholds for each tail - we choose a number of order statistics  $k$  in order to obtain as small variance and bias as possible. Then we fit GPD distributions to each tail and obtain ML estimates<sup>29</sup> for shape parameter  $\xi$ . Goodness of fit is presented in Appendix 11. Results are presented in Table 7 below.

Tail	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
<b>k</b>	110	130	100	90	120	105	150	95
<b>Threshold (%)</b>	1.7068	1.7521	1.8526	1.7875	1.7834	1.6411	1.6453	1.8001
<b><math>\xi</math> estimates</b>	0.2031	0.1365	0.1176	0.0947	0.1402	0.1544	0.0962	0.1118

Table 7. ML estimates - threshold selected through graphing techniques

Using the above values for tail estimates as detailed in Section 3.2.3., we find that at 99% confidence level, extreme VaR estimates higher potential loss than the standard methods (Table 6 in Section 5.2.). Passing on to higher quantiles in the tail, like 99.9%, EVT VaR estimates depreciation of CHF as high as 4.03% (CHF series, right tail), not far from the value observed in March 2009 (3.40%), which came after a change of only 0.6% in CHF/EUR rate. The same observation can be made in the case of GBP at 99.9%, with an estimated extreme appreciation of 3.89% and a real, observed maximum appreciation of 3.14% in January 2009 or in the case of USD – estimated depreciation of 3.36% at 99.9% against observed 3.72% in March 2009. For RON series, extreme VaR still does not cover the maximum observed values of 6.89% depreciation in 1999 and 5.11% appreciation against EUR in 2005. VaR estimates are presented in Table 8.

VaR (%)	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
<b>99%</b>	<b>2.38</b>	<b>2.90</b>	<b>2.53</b>	<b>2.54</b>	<b>2.75</b>	<b>2.38</b>	<b>2.46</b>	<b>2.43</b>
<b>99.9%</b>	<b>4.03</b>	<b>5.11</b>	<b>4.00</b>	<b>3.98</b>	<b>4.75</b>	<b>4.07</b>	<b>3.36</b>	<b>3.37</b>
ES (%)	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
<b>99%</b>	<b>3.08</b>	<b>3.69</b>	<b>3.16</b>	<b>3.12</b>	<b>3.61</b>	<b>3.12</b>	<b>3.12</b>	<b>3.11</b>
<b>99.9%</b>	<b>5.15</b>	<b>6.25</b>	<b>4.83</b>	<b>4.71</b>	<b>5.93</b>	<b>5.12</b>	<b>4.12</b>	<b>4.17</b>

Table 8. Point estimates (values in percents) – extreme VaR and ES at 99% and 99.9% levels

As discussed in Section 4.3., we also computed tail index measures using Hill, Pickands and DEdH estimators. These estimates and corresponding VaR and ES measures are given in

<sup>29</sup> Hosking, Wallis (1987) show that for  $\xi > -0.5$  the ML method holds.

Table 9. Two observations have to be made: one, Pickands estimator produces lower VaR values than the other two estimators<sup>30</sup>, and two, for the same length of one particular tail, higher shape parameters result into higher VaR values<sup>31</sup>.

Estimator		CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
Hill		<b>0.2191</b>	<b>0.1765</b>	<b>0.1226</b>	<b>0.1134</b>	<b>0.1785</b>	<b>0.1806</b>	<b>0.1159</b>	<b>0.1268</b>
Pickands		<b>0.2078</b>	<b>0.1641</b>	<b>0.1197</b>	<b>0.1067</b>	<b>0.1680</b>	<b>0.1708</b>	<b>0.1126</b>	<b>0.1235</b>
DEdH		<b>0.2198</b>	<b>0.1769</b>	<b>0.1253</b>	<b>0.1175</b>	<b>0.1776</b>	<b>0.1797</b>	<b>0.1172</b>	<b>0.1283</b>
VaR 99%	Hill	2.77	3.67	3.12	2.86	3.35	3.10	2.66	2.64
	Pick	2.56	3.58	3.04	2.79	3.26	3.02	2.64	2.62
	DEdH	2.78	3.68	3.17	2.88	3.34	3.09	2.66	2.65
VaR 99.9%	Hill	5.41	5.81	4.26	4.06	5.34	5.11	3.68	3.63
	Pick	5.43	5.79	4.23	4.02	5.29	5.05	3.67	3.63
	DEdH	5.42	5.81	4.26	4.06	5.33	5.09	3.68	3.64
ES 99%	Hill	3.43	4.19	3.68	3.25	3.98	3.61	3.08	3.09
	Pick	3.35	4.17	3.58	3.17	3.92	3.47	3.07	3.08
	DEdH	3.43	4.19	3.73	3.29	3.98	3.63	3.09	3.09
ES 99.9%	Hill	6.36	6.75	4.94	4.40	6.10	5.75	4.12	4.11
	Pick	6.22	6.74	4.84	4.39	6.07	5.69	4.10	4.10
	DEdH	6.37	6.76	4.97	4.40	6.09	5.77	4.12	4.18

Table 9. Point estimates (%) - EVT VaR and ES using Hill, Pickands and DEdH estimators

We obtained higher estimators of the shape parameter and higher VaRs than by ML method. The 99.9% VaR for RON series is now considerably closer to the observed maxima and minima. On the other hand, USD extreme appreciation against EUR at 99.9% appears somewhat overestimated compared to the observed minima. For the chosen thresholds, CHF appears to be more fat-tailed than the other series. The second series in terms of tail-fatness appears to be RON, with a slightly fatter left tail, implying risk on Romanian Leu appreciation versus the Euro. GBP and USD appear to be less fat-tailed than the other two series. The largest potential extreme loss at 99% confidence level is expected for CHF series (around 3.6%) generated by an appreciation against the EUR. The recovery trend of CHF is actually consistent with real data, in and outside the sample period. At 99.9% confidence level, the CHF extreme appreciation is estimated at approximately 5.8%. For USD against the EUR, VaR values are roughly similar in the right and left tail. This is also consistent with the real trend as in the recent months the two currencies have been struggling to gain points against each other. As expected, for the RON higher losses are estimated for the upper tail, meaning extreme depreciation would be higher than correspondent appreciation. The values

<sup>30</sup> Brooks et. al (2003) also found that Pickands estimator usually results into 'slightly smaller' VaR values.

<sup>31</sup> We found similar results in the paper of Wagner and Marsh (2003).

for Expected Shortfall show how much can an investor lose on a short or long position if the extreme movement in exchange rates overshoots the VaR values.

### Hybrid models

According to this approach, we computed new VaR values and the point estimates for the first day out of sample are shown in Table 10 below:

<b>EVT EWMA</b>	<b>CHF<sub>U</sub></b>	<b>CHF<sub>L</sub></b>	<b>GBP<sub>U</sub></b>	<b>GBP<sub>L</sub></b>	<b>RON<sub>U</sub></b>	<b>RON<sub>L</sub></b>	<b>USD<sub>U</sub></b>	<b>USD<sub>L</sub></b>
99%	0.56	0.52	1.55	1.44	0.69	0.66	1.71	1.92
99.9%	0.90	0.71	2.04	1.79	0.96	0.94	2.14	2.49
<b>EVT EGARCH</b>	<b>CHF<sub>U</sub></b>	<b>CHF<sub>L</sub></b>	<b>GBP<sub>U</sub></b>	<b>GBP<sub>L</sub></b>	<b>RON<sub>U</sub></b>	<b>RON<sub>L</sub></b>	<b>USD<sub>U</sub></b>	<b>USD<sub>L</sub></b>
99%	0.55	0.51	1.61	1.49	0.66	0.62	1.77	2.20
99.9%	0.87	0.69	2.11	1.86	0.91	0.88	1.98	2.57

Table 10. Point estimates (%) - EVT VaR using EWMA and EGARCH

The point estimates for day 1 out of the sample period are slightly different from those computed with standard VaR models. Losses of less than 1% are expected for CHF and RON series. Highest losses are expected for USD against the EUR on long positions on EUR, around 2.49% at 99.9% confidence level. This method is likely to produce more biased estimates both in the tail and in the centre of the distribution, i.e. underestimate the tails (high losses) and overestimate the centre (small losses).

### 5.3 Backtesting

In order to assess the performance of VaR models we backtest VaR estimates against actual returns in order to determine the percentage of failure in VaR estimation. In this part of the analysis we only include dynamic results, for the whole distribution. Results for 99% and 99.9% confidence levels are presented in Table 11 and Table 12, respectively.

<b>Percentage of violations at 99% confidence level – fail if &gt; 1%</b>								
<b>Model</b>	<b>CHF<sub>U</sub></b>	<b>CHF<sub>L</sub></b>	<b>GBP<sub>U</sub></b>	<b>GBP<sub>L</sub></b>	<b>RON<sub>U</sub></b>	<b>RON<sub>L</sub></b>	<b>USD<sub>U</sub></b>	<b>USD<sub>L</sub></b>
EWMA	5.83	6.71	7.00	6.12	7.55	4.91	7.37	6.12
EGARCH	5.32	6.45	6.38	5.06	6.49	4.14	5.79	6.09
EVT EWMA	1.72	1.39	1.10**	2.34	0.92*	1.76	2.23	1.28
EVT EGARCH	1.32	0.73*	1.28	2.42	0.91*	2.53	1.90	1.28

\*Denotes accepted models at 99%

\*\*Denotes models close to acceptance at 99%

Table 11. Backtesting results: percentage of failures in VaR estimation at 99% level

Percentage of violations at 99.9% confidence level – fail if > 0.1%								
Model	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
EWMA	2.38	2.89	2.89	2.35	4.03	2.16	2.97	2.60
EGARCH	1.72	2.82	2.57	1.94	3.12	1.58	2.42	2.02
EVT EWMA	0.18	0.11**	0.07*	0.58	0.11**	0.69	0.25	0.11**
EVT EGARCH	0.07*	0.07*	0.07*	0.11**	0.11**	0.66	0.18	0.07*

\*Denotes accepted models at 99.9%

\*\*Denotes models close to acceptance at 99.9%

Table 12. Backtesting results: percentage of failures in VaR estimation at 99.9% level

Taking into consideration the average failures in predicting future losses at 99%, EVT EWMA and EVT EGARCH seem to perform better than regular VaR models. Performance at 99.9% is even better for these two approaches, especially for EVT EGARCH. Still, this may be a result of overestimation of risk, rather than of reliability. EWMA and EGARCH seem to be rejected both at 99% and 99.9% confidence levels, with a better performance at 99.9%, especially for EGARCH.

Next, we compute Mean Squared Error for all the models employed (excepting HS and HHS). For pure EVT models we analyze how much bias they produce in estimating tail risk, and for regular VaR models, EVT EWMA and EVT EGARCH we want to determine the bias of the estimates for the whole distribution.

Mean Squared Error for 99% confidence level models								
Model	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
Prediction at tails								
EVT ML	<b>0.3454</b>	<b>0.1982</b>	<b>0.1045</b>	<b>0.0560</b>	<b>0.0667</b>	<b>0.0566</b>	<b>0.0650</b>	<b>0.0762</b>
EVT Hill	0.3945	0.3199	0.1308	0.0784	0.1129	0.0796	0.0914	0.0966
EVT Pick	0.3736	0.2464	0.1119	0.0762	0.1069	0.0797	0.0916	0.0964
EVT DEdH	0.3907	0.3181	0.1176	0.0837	0.1119	0.0778	0.0933	0.0987
Prediction for complete distribution								
EWMA	0.3037	0.3069	0.9278	<b>0.8934</b>	<b>1.4598</b>	<b>1.2703</b>	<b>1.4726</b>	<b>1.4686</b>
EGARCH	<b>0.2814</b>	<b>0.2823</b>	<b>0.9273</b>	0.9022	1.5093	1.3058	1.4966	1.4943
EVT EWMA	0.5887	0.5186	1.6299	1.4019	2.5382	2.0457	2.0255	2.4314
EVT EGARCH	0.5359	0.4706	1.6352	1.4155	2.6363	2.1144	2.0627	2.4799

\*Minimum MSE in bold

Table 13. Mean Squared Error at 99% level

Mean Squared Error for 99.9% confidence level models								
Model	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
Prediction at tails								
EVT ML	<b>2.6021</b>	<b>1.2894</b>	<b>0.7185</b>	<b>0.3614</b>	<b>1.0851</b>	<b>1.1454</b>	<b>0.4972</b>	<b>0.5903</b>
EVT Hill	3.1963	2.4670	0.9583	0.5486	2.0117	1.7118	0.7741	0.7962
EVT Pick	3.1281	2.4585	0.8693	0.5126	1.9120	1.5734	0.7867	0.7813
EVT DEdH	3.1478	2.4478	0.8353	0.5961	1.9892	1.6641	0.7942	0.8179
Prediction for complete distribution								
EWMA	0.4655	0.4697	<b>1.4164</b>	<b>1.3708</b>	<b>2.2165</b>	<b>1.9656</b>	<b>2.2533</b>	<b>2.2481</b>
EGARCH	<b>0.4259</b>	<b>0.4271</b>	1.4206	1.3847	2.3003	2.0309	2.2958	2.2927
EVT EWMA	1.3633	0.8925	2.5976	2.0280	4.4050	3.8277	2.9112	3.7766
EVT EGARCH	1.2269	0.8033	2.6087	2.0476	4.5868	3.9733	2.9694	3.8576

\*Minimum MSE in bold

Table 14. Mean Squared Error at 99.9% level

Both at 99% and 99.9% confidence levels, the best prediction in the tails is obtained with EVT ML, thus with lower tail indexes. Appears that bias in the Hill, Pickands and DEdH estimators induces bias in the quantile estimation. Prediction for the whole distribution is split between EWMA and EGARCH, but slightly better for EWMA, as differences in MSE are not significantly different. EVT EWMA and EVT EGARCH are clearly outperformed in terms of MSE by regular VaR models, thus applying some form of EVT for the whole distribution is not desirable as it clearly overestimates small size changes in exchange rates. ES models were not included in this analysis as we only use them for the purpose of orientation.

### Scenario Setting

Last, but not least, we want to produce some sort of scenario. We know for sure how exchange rates have evolved in the last ten years. So we extract the maximum and minimum historical returns for each series, as well as the returns observed in the day prior to these changes. Then we compare them with the 99.9% VaR vales obtained by different models for day one out of the sample period and with returns in the last day of the sample period, respectively, as shown in Table 15 below.

Values in %	CHF <sub>U</sub>	CHF <sub>L</sub>	GBP <sub>U</sub>	GBP <sub>L</sub>	RON <sub>U</sub>	RON <sub>L</sub>	USD <sub>U</sub>	USD <sub>L</sub>
Worst daily movement	<b>+3.40</b>	<b>-2.06</b>	<b>+2.98</b>	<b>-3.14</b>	<b>+6.88</b>	<b>-5.11</b>	<b>+3.72</b>	<b>-2.80</b>
Return in previous day	<b>+0.57</b>	<b>-1.87<sup>32</sup></b>	<b>+2.70<sup>33</sup></b>	<b>+0.07</b>	<b>+1.90</b>	<b>+0.49</b>	<b>+0.36</b>	<b>+0.99</b>
Return in last day of sample	<b>-0.08</b>	<b>-0.08</b>	<b>+0.48</b>	<b>+0.48</b>	<b>-0.02</b>	<b>-0.02</b>	<b>+0.26</b>	<b>+0.26</b>
VaR HS	1.79	1.86	2.70	2.27	3.07	2.71	2.80	2.22
VaR HHS	0.72	0.79	1.94	1.92	1.11	1.10	1.78	2.35
VaR EWMA	0.49	0.49	1.43	1.43	0.64	0.64	1.83	1.83
VaR EGARCH	0.72	0.72	2.22	2.22	0.91	0.91	2.14	2.14
VaR EVT EWMA	0.90	0.71	2.04	1.79	0.96	0.94	2.14	2.49
VaR EVT EGARCH	0.87	0.69	2.11	1.86	0.91	0.88	1.98	2.57
Average VaR EVT	<b>5.07</b>	<b>5.63</b>	<b>4.19</b>	<b>4.03</b>	<b>5.18</b>	<b>4.83</b>	<b>3.60</b>	<b>3.57</b>

Table 15. Realized vs. estimated extremes

Searching on what generated these extremes movements, we found the following information:

- EUR/CHF +3.40% on March 12<sup>th</sup> 2009 – Swiss National Bank lowers interest rates by 25bps and adopts quantitative ease.
- EUR/CHF -2.06% on October 27<sup>th</sup> 2008 – effects of the crisis – extra burden to the slowing export-dependent economy.
- EUR/GBP +2.98% on November 13<sup>th</sup> 2008 – a report from Bank of England leads to expectations of further cuts into interest rates (after a 150bps cut in the previous week);
- EUR/GBP -3.14% on January 5<sup>th</sup> 2009 – European Central Bank cuts 50bps of interest rate overnight.

<sup>32</sup> With +0.55% change in previous day

<sup>33</sup> With -0.25% change in previous day



- EUR/RON +6.89% on March 17 1999 – the largest commercial bank in Romania almost bankrupt; overwhelming external debt (almost 30% of the total mid and long-term debt); effects of the Russian crisis.
- EUR/RON -5.11% on February 22<sup>nd</sup> 2005 – expectations that National Bank of Romania would enter the market and buy excess foreign currency.
- EUR/USD +3.72% on March 19<sup>th</sup> 2009 – Federal Open Market Committee announces quantitative ease.
- EUR/USD -2.80% on October 27<sup>th</sup> – The Fed announces another economic stimulus – optimism in the market.

Now looking back at Table 15 and considering what were the main drivers of these extreme moves and the current situation in the market, one question arises: Are such extreme scenarios that improbable? The answer is up to the reader but what is certainly clear is that underestimating risk one could suffer severe losses, moreover considering the fact that the current market conditions are highly unpredictable.

## 6 Concluding remarks

Under the current regulation of Basel II, banks are allowed to use internal risk models to calculate capital requirements for market risk, in order to cover their trading positions. The most common approach in computing expected losses is Value at Risk. Under Basel approach, VaR should be computed for a 10-day holding period at 99% confidence level, using minimum one year of historical data. Then the capital requirements are computed using a multiplication factor of 3. The implementation is simple but significantly flawed. The basic assumption in VaR computation is that returns in financial data are normally distributed. According to this assumption, the size of a one in 1000 days extreme event is considerably underestimated. In reality, a return of 5% or more is observed once a few years, and returns of roughly 3% even more frequently. In theory, the normality assumption is contradicted by stylized facts like fat-tailedness and leptokurtosis. In order to adapt VaR to characteristics of the data and improve the vanilla Historical Simulation, some new approaches have been proposed, like EWMA, Hybrid Historical Simulation, GARCH models, with normal or

Student-t distributions etc. Indeed, these new models seem to better fit the behaviour of financial data, but still fail to predict some extreme, unexpected changes in market variables, that produce huge losses. In the last years, a new framework has been adopted in financial risk modelling, Extreme Value Theory. First used in fields like hydrology or meteorology, EVT has been recently adopted in insurance and operational risk modelling. The theory is used to describe the behaviour of extreme historical returns and can be used to compute very high quantiles, i.e. estimate extreme losses with very low probability, once in 1000 days or more. Many studies have analyzed the performance of EVT in describing the behaviour of exchange rates. Generally, the studies concluded that indeed EVT is more fit to estimate extreme movements in exchange rates. However, inference for very high quantiles is done at the expense of not modelling correctly moderate movements, precisely where VaR estimation intervenes, thus the two approaches are rather complementary.

In this paper we used four exchange rate returns series and analyzed the performance of four Value at Risk models, namely Historical Simulation, Hybrid Historical Simulation, Exponentially Weighted Moving Average and EGARCH. We also employed EVT Peaks over Threshold method to estimate potential extreme losses on exchange rate positions and two hybrid models between EWMA and EGARCH, respectively, and Extreme Value Theory. We treated each tail separately, as exchange rate risk refers not only to negative movements in the data, but also to positive ones, corresponding to the long or short position taken on a currency. Among VaR models, Historical Simulation seems to produce high values for expected losses but it highly depends on historically observed returns. If we were to eliminate the last 300 observations in each data set, HS would have predicted considerably lower losses. Moreover, HS and also Hybrid HS can only be used to produce point estimates as we refer to one single data set, with a predetermined time length. EWMA and EGARCH can be used for dynamic computation of VaR and build the variance at each step taking into account past information on returns and variance. Their performance differs between right and left tails, as they produce symmetric VaR values, whereas the tails contain asymmetric information. On the other hand, EVT is very sensitive to the choice of threshold and this choice can prove very difficult as the tails can show significant departures from the Pareto distribution. Also, VaR estimates using EVT are apparently more reliable when using ML estimates for the shape parameter, than using other estimators, like Hill, Pickands or Dekkers-Einmahl-DeHaan, as these estimators are known to be biased in small samples. The hybrids of

EVT and standard VaR models perform well in terms of failure percentage but very poor in terms of Mean Squared Error.

Based on backtesting results, we consider that the tails and rest of the distribution should be modelled separately and the information from both should be used in risk management. We back this statement by the simple scenario proposed in the last part of our analysis, which does not appear so improbable in the current market conditions. However there is a trade-off when considering how this information should be used: regulators would prefer more conservative measures, which diminish systemic risk but results into inefficient supplementary capital allocation; on the other hand, bank managers would assume the risks but prefer those models which result into low capital requirements. It is obvious that no regulation could ask banks to put aside as much capital as based on EVT VaR values, although, roughly speaking, if we consider a historical VaR of 1% and multiply it by 3, we may need to put aside as much capital as EVT indicates. The use of EVT should orientate on stress testing or limit setting for long or short positions, as the limits set for transactions highly depend on the probability of an extreme loss, that may not be easy to cover and EVT, by contrast to VaR, tells that such a probability is considerably high. We underline this two issues as possible orientation for future research.

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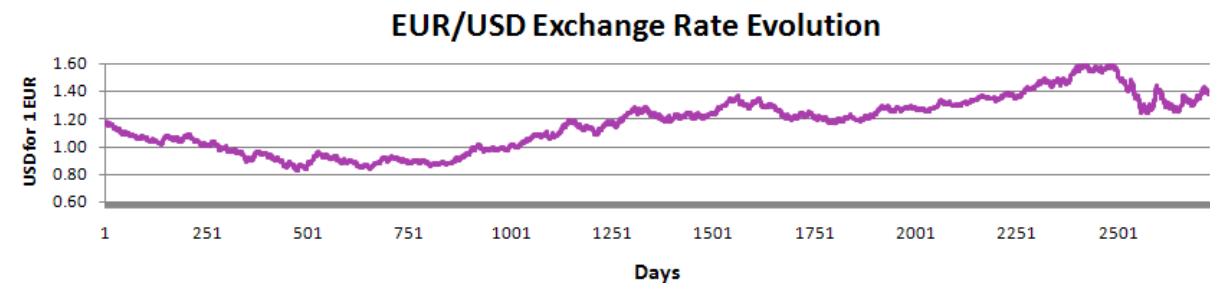
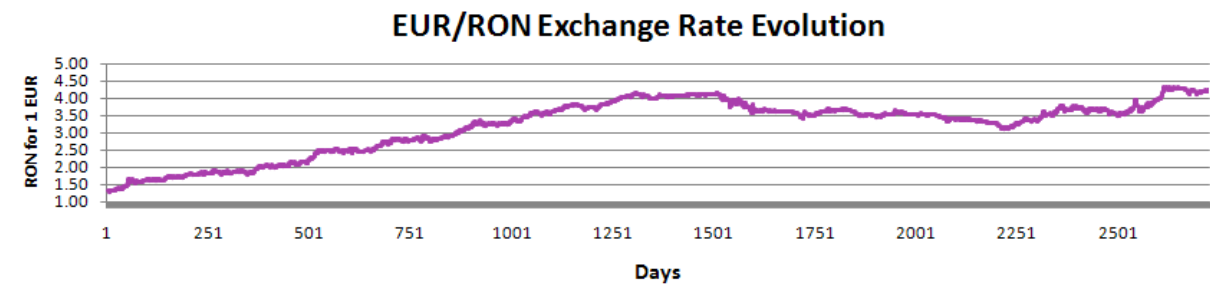
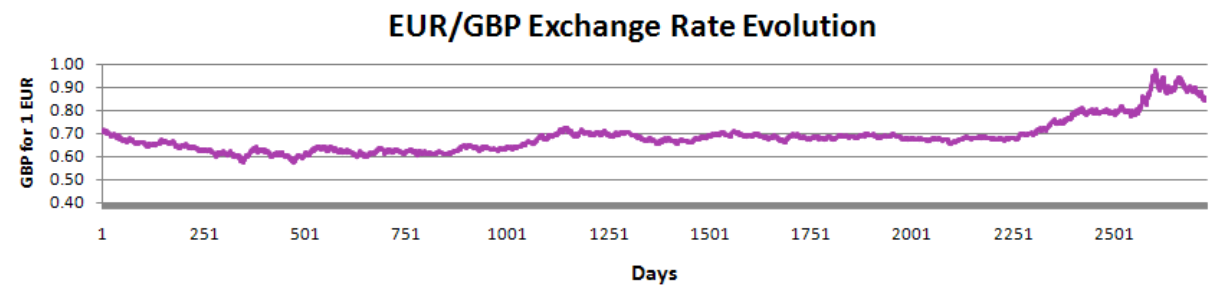
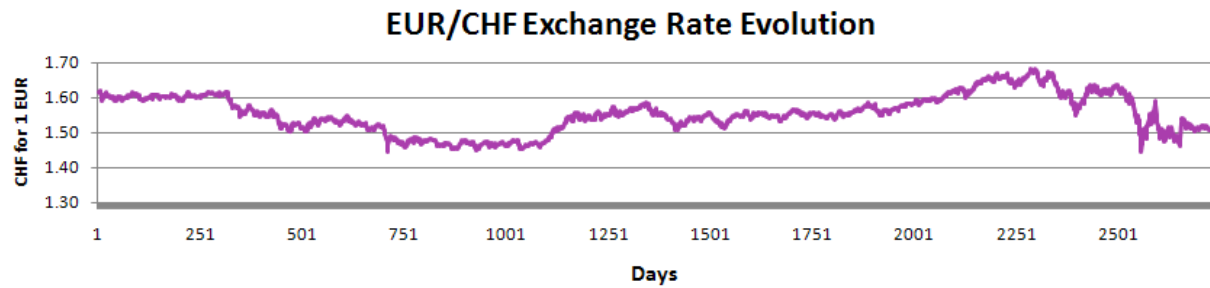
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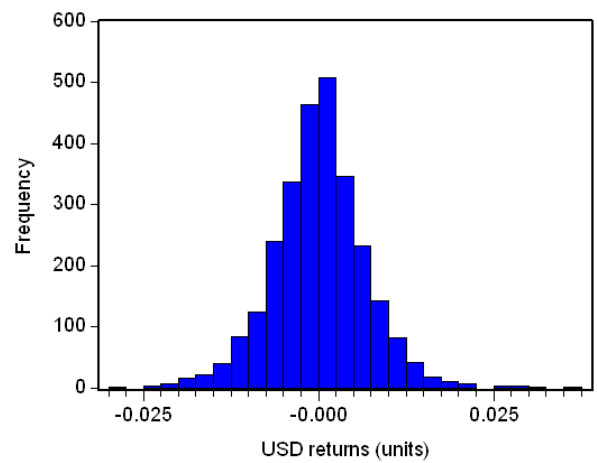
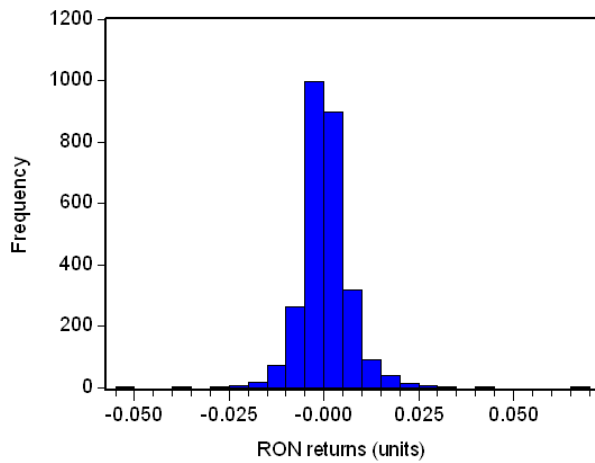
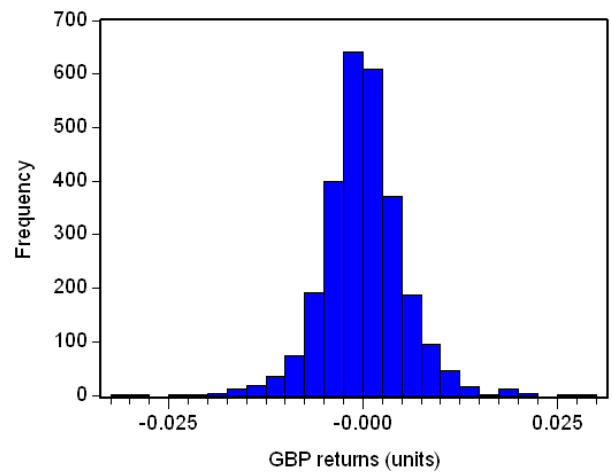
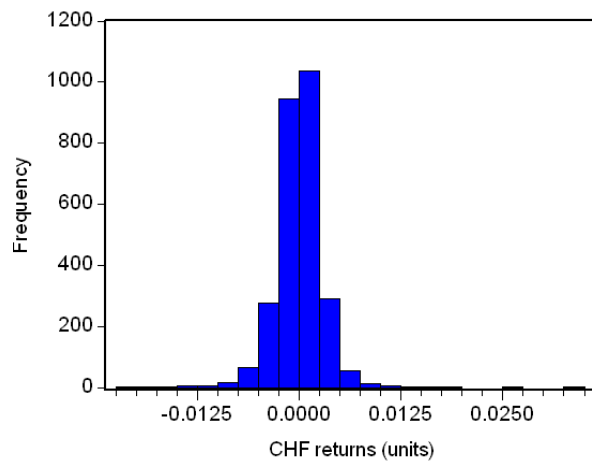
## Appendixes

**1. Exchange rates evolution between January 1999 and June 2009.** Effects of the current crisis can be observed in the last 200 observations.

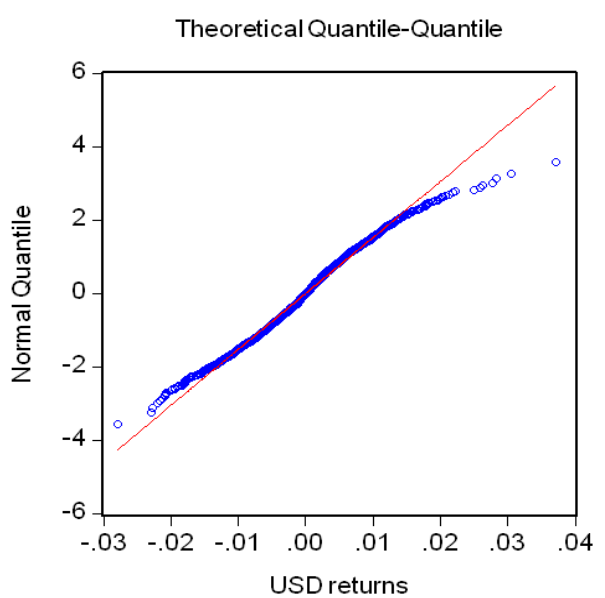
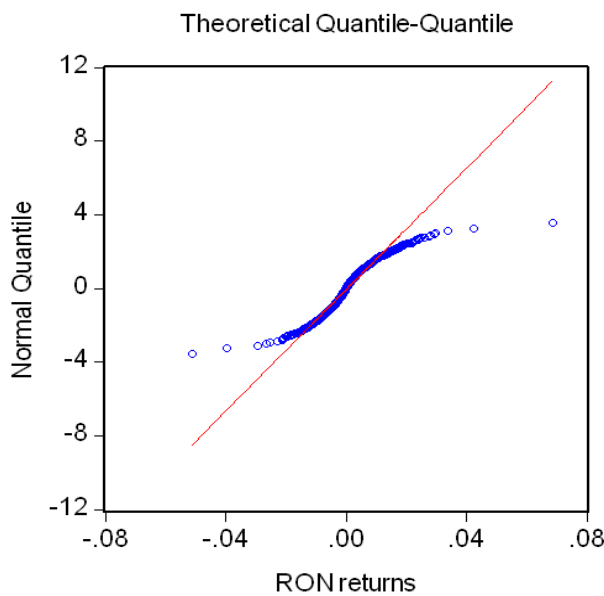
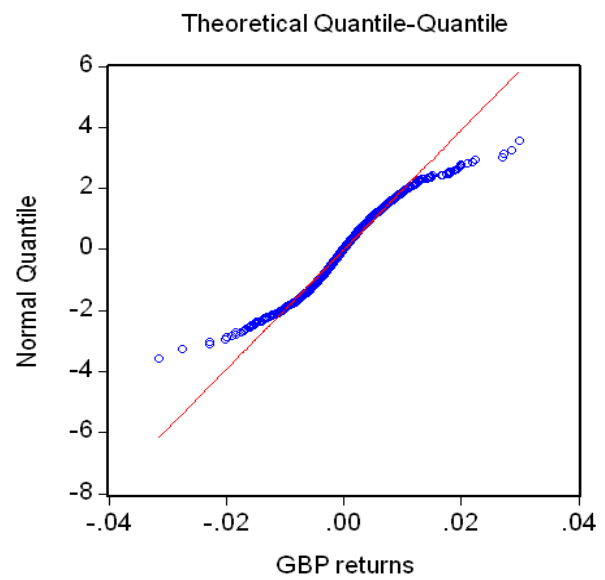
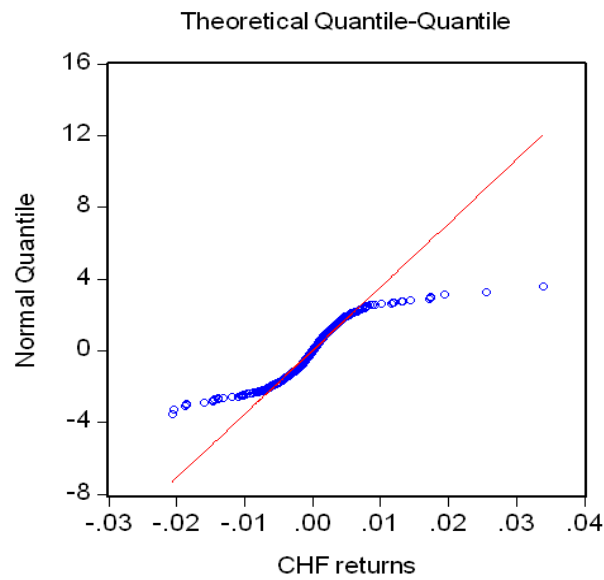




**2. Histograms of exchange rate returns series.** The distributions are skewed and leptokurtic. Generally, right tails are longer, i.e. more severe depreciation than appreciation against the Euro.



**3. QQ-plots: Exchange rate returns vs. Normal distribution.** Series are not normally distributed. USD exhibits the smallest departure from the Normal.



**4. Unit root tests.** Unit root hypothesis is rejected at 1% confidence level by both tests in all four cases. Series are stationary.

**Augmented Dickey-Fuller unit root test.  $H_0$ : data has a unit root**

Null Hypothesis: CHF has a unit root Exogenous: Constant Lag Length: 0 (Automatic based on SIC, MAXLAG=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-54.71576	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

Null Hypothesis: GBP has a unit root Exogenous: Constant Lag Length: 0 (Automatic based on SIC, MAXLAG=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-52.13922	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

Null Hypothesis: RON has a unit root Exogenous: Constant Lag Length: 2 (Automatic based on SIC, MAXLAG=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-33.24085	0.0000
Test critical values: 1% level	-3.432557	
5% level	-2.862402	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

Null Hypothesis: USD has a unit root Exogenous: Constant Lag Length: 0 (Automatic based on SIC, MAXLAG=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-53.30146	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

**Phillips-Perron unit root test.  $H_0$ : data has a unit root**

Null Hypothesis: CHF has a unit root Exogenous: Constant Bandwidth: 15 (Newey-West using Bartlett kernel)		
	Adj. t-Stat	Prob.*
Phillips-Perron test statistic	-55.01316	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

Null Hypothesis: GBP has a unit root Exogenous: Constant Bandwidth: 1 (Newey-West using Bartlett kernel)		
	Adj. t-Stat	Prob.*
Phillips-Perron test statistic	-52.13920	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

Null Hypothesis: RON has a unit root Exogenous: Constant Bandwidth: 9 (Newey-West using Bartlett kernel)		
	Adj. t-Stat	Prob.*
Phillips-Perron test statistic	-48.53310	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

Null Hypothesis: USD has a unit root Exogenous: Constant Bandwidth: 11 (Newey-West using Bartlett kernel)		
	Adj. t-Stat	Prob.*
Phillips-Perron test statistic	-53.29189	0.0001
Test critical values: 1% level	-3.432557	
5% level	-2.862401	
10% level	-2.567273	
*MacKinnon (1996) one-sided p-values.		

**5. AR(p) estimates for exchange rate returns.** AR(1) estimation for CHF, GBP and USD series and AR(3) estimation for RON series. Rejected: GBP and USD. Accepted: CHF, at 5%, and RON at 1%.

Dependent Variable: CHF  
Method: Least Squares  
Date: 06/24/09 Time: 10:09  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.57E-05	5.48E-05	-0.469169	0.6390
AR(1)	-0.047202	0.019139	-2.466293	0.0137
R-squared	0.002228	Mean dependent var	-2.57E-05	
Adjusted R-squared	0.001862	S.D. dependent var	0.003000	
S.E. of regression	0.002997	Akaike info criterion	-8.781843	
Sum squared resid	0.024463	Schwarz criterion	-8.777507	
Log likelihood	11971.65	F-statistic	6.082601	
Durbin-Watson stat	2.003044	Prob(F-statistic)	0.013713	
Inverted AR Roots	-.05			

Dependent Variable: RON  
Method: Least Squares  
Date: 06/24/09 Time: 21:55  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000430	0.000130	3.311166	0.0009
AR(1)	0.070283	0.019112	3.677371	0.0002
R-squared	0.004940	Mean dependent var	0.000430	
Adjusted R-squared	0.004575	S.D. dependent var	0.006323	
S.E. of regression	0.006308	Akaike info criterion	-7.293203	
Sum squared resid	0.108397	Schwarz criterion	-7.288866	
Log likelihood	9942.635	F-statistic	13.52305	
Durbin-Watson stat	1.990558	Prob(F-statistic)	0.000240	
Inverted AR Roots	.07			

Dependent Variable: RON  
Method: Least Squares  
Date: 06/24/09 Time: 10:37  
Sample (adjusted): 4 2727  
Included observations: 2724 after adjustments  
Convergence achieved after 2 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000432	0.000112	3.840845	0.0001
AR(1)	0.069998	0.019108	3.663268	0.0003
AR(2)	-0.061008	0.019117	-3.191238	0.0014
AR(3)	-0.078483	0.019108	-4.107380	0.0000
R-squared	0.015544	Mean dependent var	0.000432	
Adjusted R-squared	0.014459	S.D. dependent var	0.006322	
S.E. of regression	0.006276	Akaike info criterion	-7.302550	
Sum squared resid	0.107152	Schwarz criterion	-7.293872	
Log likelihood	9950.073	F-statistic	14.31616	
Durbin-Watson stat	1.996154	Prob(F-statistic)	0.000000	
Inverted AR Roots	.22+.41i	.22-.41i	-.36	

Dependent Variable: GBP  
Method: Least Squares  
Date: 06/23/09 Time: 15:33  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 2 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.46E-05	9.96E-05	0.649292	0.5162
AR(1)	0.000935	0.019161	0.048815	0.9611
R-squared	0.000001	Mean dependent var	6.46E-05	
Adjusted R-squared	-0.000366	S.D. dependent var	0.005192	
S.E. of regression	0.005193	Akaike info criterion	-7.682199	
Sum squared resid	0.073465	Schwarz criterion	-7.677862	
Log likelihood	10472.84	F-statistic	0.002383	
Durbin-Watson stat	1.998599	Prob(F-statistic)	0.961071	
Inverted AR Roots	.00			

Dependent Variable: RON  
Method: Least Squares  
Date: 06/25/09 Time: 08:42  
Sample (adjusted): 3 2727  
Included observations: 2725 after adjustments  
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000429	0.000122	3.527489	0.0004
AR(1)	0.074923	0.019122	3.918038	0.0001
AR(2)	-0.067019	0.019122	-3.504794	0.0005
R-squared	0.009401	Mean dependent var	0.000429	
Adjusted R-squared	0.008673	S.D. dependent var	0.006323	
S.E. of regression	0.006296	Akaike info criterion	-7.296764	
Sum squared resid	0.107893	Schwarz criterion	-7.290257	
Log likelihood	9944.841	F-statistic	12.91596	
Durbin-Watson stat	2.009145	Prob(F-statistic)	0.000003	
Inverted AR Roots	.04-.26i	.04+.26i		

Dependent Variable: USD  
Method: Least Squares  
Date: 06/23/09 Time: 15:34  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.23E-05	0.000124	0.503235	0.6148
AR(1)	-0.020934	0.019154	-1.092925	0.2745
R-squared	0.000438	Mean dependent var	6.24E-05	
Adjusted R-squared	0.000071	S.D. dependent var	0.006598	
S.E. of regression	0.006598	Akaike info criterion	-7.203326	
Sum squared resid	0.118590	Schwarz criterion	-7.198989	
Log likelihood	9820.133	F-statistic	1.194485	
Durbin-Watson stat	2.000623	Prob(F-statistic)	0.274523	
Inverted AR Roots	-.02			

**6. Fitting EGARCH models to exchange rate return series.** All conditional variance coefficients are positive.

Series CHF: accept ARMA(1,1) for conditional mean and EGARCH(1,1,1) for conditional variance.

Dependent Variable: CHF  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 10:30  
Sample: 1 2727  
Included observations: 2727  
Convergence achieved after 11 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3)*\text{ABS}(\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1))) + \text{C}(4)*\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1)) + \text{C}(5)*\text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	2.07E-05	3.86E-05	0.536030	0.5919
Variance Equation				
C(2)	-0.257319	0.048930	-5.258933	0.0000
C(3)	0.143329	0.019498	7.350947	0.0000
C(4)	-0.056637	0.013363	-4.230888	0.0000
C(5)	0.987730	0.003437	287.3979	0.0000
T-DIST. DOF	6.145385	0.636835	9.649892	0.0000
R-squared	-0.000239	Mean dependent var	-2.57E-05	
Adjusted R-squared	-0.002077	S.D. dependent var	0.002999	
S.E. of regression	0.003002	Akaike info criterion	-9.257799	
Sum squared resid	0.024523	Schwarz criterion	-9.244793	
Log likelihood	12629.01	Durbin-Watson stat	2.093871	

Dependent Variable: CHF  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 10:51  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 6 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4)*\text{ABS}(\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1))) + \text{C}(5)*\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1)) + \text{C}(6)*\text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.839199	0.091496	9.171929	0.0000
MA(1)	-0.865764	0.083567	-10.36015	0.0000
Variance Equation				
C(3)	-0.413658	0.015238	-27.14674	0.0000
C(4)	0.199075	0.018499	10.76147	0.0000
C(5)	0.077335	0.010438	7.409207	0.0000
C(6)	0.978822	0.001329	736.2464	0.0000
T-DIST. DOF	6.868935	0.670691	10.24159	0.0000
R-squared	0.002867	Mean dependent var	2.57E-05	
Adjusted R-squared	0.000667	S.D. dependent var	0.003000	
S.E. of regression	0.002999	Akaike info criterion	-9.253032	
Sum squared resid	0.024447	Schwarz criterion	-9.237854	
Log likelihood	12618.88	Durbin-Watson stat	2.044984	

Dependent Variable: CHFL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 20:41  
Sample: 1 2727  
Included observations: 2727  
Convergence achieved after 12 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3)*\text{ABS}(\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1))) + \text{C}(4)*\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1)) + \text{C}(5)*\text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	-2.07E-05	3.86E-05	-0.536132	0.5919
Variance Equation				
C(2)	-0.257359	0.048921	-5.260662	0.0000
C(3)	0.143352	0.019495	7.353312	0.0000
C(4)	0.056518	0.013362	4.229880	0.0000
C(5)	0.987728	0.003436	287.4443	0.0000
T-DIST. DOF	6.145131	0.636864	9.649044	0.0000
R-squared	-0.000239	Mean dependent var	2.57E-05	
Adjusted R-squared	-0.002077	S.D. dependent var	0.002999	
S.E. of regression	0.003002	Akaike info criterion	-9.257799	
Sum squared resid	0.024523	Schwarz criterion	-9.244793	
Log likelihood	12629.01	Durbin-Watson stat	2.093871	

Dependent Variable: CHFL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 10:53  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 6 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4)*\text{ABS}(\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1))) + \text{C}(5)*\text{RESID}(-1)/\text{SQRT}(\text{GARCH}(-1)) + \text{C}(6)*\text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.839199	0.091496	9.171929	0.0000
MA(1)	-0.865764	0.083567	-10.36015	0.0000
Variance Equation				
C(3)	-0.413658	0.015238	-27.14674	0.0000
C(4)	0.199075	0.018499	10.76147	0.0000
C(5)	0.077335	0.010438	7.409207	0.0000
C(6)	0.978822	0.001329	736.2464	0.0000
T-DIST. DOF	6.868935	0.670691	10.24159	0.0000
R-squared	0.002867	Mean dependent var	2.57E-05	
Adjusted R-squared	0.000667	S.D. dependent var	0.003000	
S.E. of regression	0.002999	Akaike info criterion	-9.253032	
Sum squared resid	0.024447	Schwarz criterion	-9.237854	
Log likelihood	12618.88	Durbin-Watson stat	2.044984	

Series GBP: accept ARMA(1,1) for conditional mean and EGARCH(1,1,0) for conditional variance.

Dependent Variable: GBP  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 11:44  
Sample: 1 2727  
Included observations: 2727  
Convergence achieved after 15 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) * \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) * \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) * \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	-2.15E-06	7.82E-05	-0.027469	0.9781
Variance Equation				
C(2)	-0.122609	0.029373	-4.174282	0.0000
C(3)	0.095476	0.014149	6.747792	0.0000
C(4)	0.007368	0.009597	0.767714	0.4427
C(5)	0.995482	0.002270	438.4503	0.0000
T-DIST. DOF	10.32808	1.886154	5.475733	0.0000
R-squared	-0.000159	Mean dependent var	6.34E-05	
Adjusted R-squared	-0.001997	S.D. dependent var	0.005192	
S.E. of regression	0.005197	Akaike info criterion	-7.933746	
Sum squared resid	0.073488	Schwarz criterion	-7.920741	
Log likelihood	10823.66	Durbin-Watson stat	1.997340	

Dependent Variable: GBP  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 10:54  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 21 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) * \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(5) * \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(6) * \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.907780	0.056315	16.11963	0.0000
MA(1)	-0.927660	0.049804	-18.62612	0.0000
Variance Equation				
C(3)	-0.127417	0.029635	-4.299540	0.0000
C(4)	0.097950	0.014150	6.922347	0.0000
C(5)	0.004982	0.008442	0.590057	0.5552
C(6)	0.995211	0.002318	429.2809	0.0000
T-DIST. DOF	10.22935	1.854872	5.514853	0.0000
R-squared	0.000790	Mean dependent var	6.46E-05	
Adjusted R-squared	-0.001415	S.D. dependent var	0.005192	
S.E. of regression	0.005196	Akaike info criterion	-7.935315	
Sum squared resid	0.073407	Schwarz criterion	-7.920137	
Log likelihood	10822.83	Durbin-Watson stat	1.958966	

Dependent Variable: GBPL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 10:56  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 15 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) * \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(5) * \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.909177	0.055283	16.44585	0.0000
MA(1)	-0.928901	0.048789	-19.03913	0.0000
Variance Equation				
C(3)	-0.129788	0.030247	-4.290922	0.0000
C(4)	0.100320	0.014002	7.164778	0.0000
C(5)	0.995153	0.002368	420.3012	0.0000
T-DIST. DOF	10.24143	1.860298	5.505264	0.0000
R-squared	0.000792	Mean dependent var	6.46E-05	
Adjusted R-squared	-0.001045	S.D. dependent var	0.005192	
S.E. of regression	0.005195	Akaike info criterion	-7.935922	
Sum squared resid	0.073406	Schwarz criterion	-7.922913	
Log likelihood	10822.66	Durbin-Watson stat	1.959274	

Dependent Variable: GBPL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 20:49  
Sample: 1 2727  
Included observations: 2727  
Convergence achieved after 14 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) * \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) * \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) * \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	2.15E-06	7.82E-05	0.027464	0.9781
Variance Equation				
C(2)	-0.122595	0.029372	-4.173891	0.0000
C(3)	0.095467	0.014149	6.747337	0.0000
C(4)	-0.007370	0.009597	-0.767897	0.4425
C(5)	0.995482	0.002270	438.4611	0.0000
T-DIST. DOF	10.32831	1.886190	5.475755	0.0000
R-squared	-0.000159	Mean dependent var	-6.34E-05	
Adjusted R-squared	-0.001997	S.D. dependent var	0.005192	
S.E. of regression	0.005197	Akaike info criterion	-7.933746	
Sum squared resid	0.073488	Schwarz criterion	-7.920741	
Log likelihood	10823.66	Durbin-Watson stat	1.997340	

Dependent Variable: GBPL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 10:57  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 21 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) \cdot \text{ABS}(\text{RESID}(-1)/\sqrt{\text{GARCH}(-1)}) + \text{C}(5) \cdot \text{RESID}(-1)/\sqrt{\text{GARCH}(-1)} + \text{C}(6) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.907851	0.056405	16.09509	0.0000
MA(1)	-0.927727	0.049891	-18.59504	0.0000
Variance Equation				
C(3)	-0.127416	0.029636	-4.299319	0.0000
C(4)	0.097945	0.014151	6.921427	0.0000
C(5)	-0.004985	0.008444	-0.590433	0.5549
C(6)	0.995211	0.002318	429.2668	0.0000
T-DIST. DOF	10.22914	1.854953	5.514501	0.0000
R-squared	0.000790	Mean dependent var	-6.46E-05	
Adjusted R-squared	-0.001415	S.D. dependent var	0.005192	
S.E. of regression	0.005196	Akaike info criterion	-7.935315	
Sum squared resid	0.073407	Schwarz criterion	-7.920137	
Log likelihood	10822.83	Durbin-Watson stat	1.958973	

Dependent Variable: GBPL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 10:58  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 15 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) \cdot \text{ABS}(\text{RESID}(-1)/\sqrt{\text{GARCH}(-1)}) + \text{C}(5) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.909177	0.055283	16.44585	0.0000
MA(1)	-0.928901	0.048789	-19.03913	0.0000
Variance Equation				
C(3)	-0.129788	0.030247	-4.290922	0.0000
C(4)	0.100320	0.014002	7.164778	0.0000
C(5)	0.995153	0.002368	420.3012	0.0000
T-DIST. DOF	10.24143	1.860298	5.505264	0.0000
R-squared	0.000792	Mean dependent var	-6.46E-05	
Adjusted R-squared	-0.001045	S.D. dependent var	0.005192	
S.E. of regression	0.005195	Akaike info criterion	-7.935922	
Sum squared resid	0.073406	Schwarz criterion	-7.922913	
Log likelihood	10822.66	Durbin-Watson stat	1.959274	

Series RON: accept ARMA(3,3) for conditional mean and EGARCH(1,1,0) for conditional variance.

Dependent Variable: RON  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 10:46  
Sample: 1 2727  
Included observations: 2727  
Convergence achieved after 13 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1)/\sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{RESID}(-1)/\sqrt{\text{GARCH}(-1)} + \text{C}(5) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	-4.57E-05	7.27E-05	-0.629158	0.5292
Variance Equation				
C(2)	-0.361676	0.058939	-6.136431	0.0000
C(3)	0.278797	0.025894	10.76678	0.0000
C(4)	-0.029686	0.014469	-2.051738	0.0402
C(5)	0.984784	0.004854	202.8602	0.0000
T-DIST. DOF	4.950726	0.489416	10.11557	0.0000
R-squared	-0.005690	Mean dependent var	0.000431	
Adjusted R-squared	-0.007538	S.D. dependent var	0.006322	
S.E. of regression	0.006345	Akaike info criterion	-7.716287	
Sum squared resid	0.109558	Schwarz criterion	-7.703262	
Log likelihood	10527.16	Durbin-Watson stat	1.848878	

Dependent Variable: RON  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:00  
Sample (adjusted): 4 2727  
Included observations: 2724 after adjustments  
Convergence achieved after 16 iterations  
MA backcast: 1 3, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(7) + \text{C}(8) \cdot \text{ABS}(\text{RESID}(-1)/\sqrt{\text{GARCH}(-1)}) + \text{C}(9) \cdot \text{RESID}(-1)/\sqrt{\text{GARCH}(-1)} + \text{C}(10) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.477354	0.091670	-5.207309	0.0000
AR(2)	0.685295	0.049817	13.75638	0.0000
AR(3)	0.782486	0.093929	8.330587	0.0000
MA(1)	0.513902	0.083396	6.162220	0.0000
MA(2)	-0.679124	0.045229	-15.01510	0.0000
MA(3)	-0.816649	0.085646	-9.535188	0.0000
Variance Equation				
C(7)	-0.371859	0.059311	-6.269636	0.0000
C(8)	0.279972	0.025789	10.85626	0.0000
C(9)	-0.028999	0.014536	-1.994894	0.0461
C(10)	0.984261	0.004867	202.2381	0.0000
T-DIST. DOF	5.229225	0.530055	9.865439	0.0000
R-squared	0.013206	Mean dependent var	0.000432	
Adjusted R-squared	0.009568	S.D. dependent var	0.006322	
S.E. of regression	0.006292	Akaike info criterion	-7.722739	
Sum squared resid	0.107407	Schwarz criterion	-7.698874	
Log likelihood	10529.37	Durbin-Watson stat	1.923623	

Dependent Variable: RON  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:09  
Sample (adjusted): 4 2727  
Included observations: 2724 after adjustments  
Convergence achieved after 9 iterations  
MA backcast: 1 3, Variance backcast: ON  
LOG(GARCH) = C(7) + C(8)\*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(9)\*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.538314	0.034433	-15.63346	0.0000
AR(2)	0.675962	0.034263	19.72880	0.0000
AR(3)	0.851814	0.015515	54.90107	0.0000
MA(1)	0.571394	0.030660	18.63654	0.0000
MA(2)	-0.670068	0.029619	-22.62289	0.0000
MA(3)	-0.882555	0.009695	-91.03334	0.0000

Variance Equation				
C(7)	-0.381469	0.058568	-6.513283	0.0000
C(8)	0.275307	0.025202	10.92415	0.0000
C(9)	0.982993	0.004829	203.5809	0.0000

T-DIST. DOF	5.101459	0.505438	10.09315	0.0000
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R-squared	0.012389	Mean dependent var	0.000432
Adjusted R-squared	0.009114	S.D. dependent var	0.006322
S.E. of regression	0.006293	Akaike info criterion	-7.721621
Sum squared resid	0.107495	Schwarz criterion	-7.699926
Log likelihood	10526.85	Durbin-Watson stat	1.917723

Dependent Variable: RONL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:13  
Sample (adjusted): 4 2727  
Included observations: 2724 after adjustments  
Convergence achieved after 13 iterations  
MA backcast: 1 3, Variance backcast: ON  
LOG(GARCH) = C(7) + C(8)\*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(9)\*RESID(-1)/@SQRT(GARCH(-1)) + C(10)\*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.415224	0.111848	-3.712385	0.0002
AR(2)	0.660859	0.059804	11.05043	0.0000
AR(3)	0.744944	0.108530	6.863935	0.0000
MA(1)	0.453658	0.103543	4.381334	0.0000
MA(2)	-0.656114	0.055053	-11.91795	0.0000
MA(3)	-0.779663	0.100950	-7.723282	0.0000

Variance Equation				
C(7)	-0.366666	0.058921	-6.223040	0.0000
C(8)	0.276887	0.025709	10.76995	0.0000
C(9)	0.029438	0.014204	2.072576	0.0382
C(10)	0.984441	0.004827	203.9461	0.0000

T-DIST. DOF	5.136528	0.512437	10.02372	0.0000
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R-squared	0.013264	Mean dependent var	-0.000432
Adjusted R-squared	0.009627	S.D. dependent var	0.006322
S.E. of regression	0.006292	Akaike info criterion	-7.722629
Sum squared resid	0.107400	Schwarz criterion	-7.698764
Log likelihood	10529.22	Durbin-Watson stat	1.927712

Dependent Variable: RONL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 20:58  
Sample: 1 2727  
Included observations: 2727  
Convergence achieved after 15 iterations  
Variance backcast: ON  
LOG(GARCH) = C(2) + C(3)\*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)\*RESID(-1)/@SQRT(GARCH(-1)) + C(5)\*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.56E-05	7.27E-05	0.627381	0.5304

Variance Equation				
C(2)	-0.361614	0.058945	-6.134825	0.0000
C(3)	0.278796	0.025894	10.76663	0.0000
C(4)	0.029695	0.014469	2.052320	0.0401
C(5)	0.984790	0.004855	202.8442	0.0000

T-DIST. DOF	4.950529	0.489451	10.11446	0.0000
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R-squared	-0.005687	Mean dependent var	-0.000431
Adjusted R-squared	-0.007535	S.D. dependent var	0.006322
S.E. of regression	0.006345	Akaike info criterion	-7.716287
Sum squared resid	0.109558	Schwarz criterion	-7.703282
Log likelihood	10527.16	Durbin-Watson stat	1.848884

Dependent Variable: RONL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:14  
Sample (adjusted): 4 2727  
Included observations: 2724 after adjustments  
Convergence achieved after 9 iterations  
MA backcast: 1 3, Variance backcast: ON  
LOG(GARCH) = C(7) + C(8)\*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(9)\*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.538314	0.034433	-15.63346	0.0000
AR(2)	0.675962	0.034263	19.72880	0.0000
AR(3)	0.851814	0.015515	54.90107	0.0000
MA(1)	0.571394	0.030660	18.63654	0.0000
MA(2)	-0.670068	0.029619	-22.62289	0.0000
MA(3)	-0.882555	0.009695	-91.03334	0.0000

Variance Equation				
C(7)	-0.381469	0.058568	-6.513283	0.0000
C(8)	0.275307	0.025202	10.92415	0.0000
C(9)	0.982993	0.004829	203.5809	0.0000

T-DIST. DOF	5.101459	0.505438	10.09315	0.0000
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R-squared	0.012389	Mean dependent var	-0.000432
Adjusted R-squared	0.009114	S.D. dependent var	0.006322
S.E. of regression	0.006293	Akaike info criterion	-7.721621
Sum squared resid	0.107495	Schwarz criterion	-7.699926
Log likelihood	10526.85	Durbin-Watson stat	1.917723



Series USD: accept AR (1) for conditional mean and EGARCH(1,1,0) for conditional variance.

Dependent Variable: USD  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 11:47  
Sample: 1 2727  
Included observations: 2727  
Convergence not achieved after 500 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000163	0.000110	1.473451	0.1406
Variance Equation				
C(2)	-0.104340	0.029337	-3.556536	0.0004
C(3)	0.072798	0.012220	5.957088	0.0000
C(4)	-0.002988	0.007213	-0.414221	0.6787
C(5)	0.995239	0.002451	406.0809	0.0000
T-DIST. DOF	12.16923	3.110471	3.912341	0.0001
R-squared	-0.000240	Mean dependent var	6.03E-05	
Adjusted R-squared	-0.002078	S.D. dependent var	0.006598	
S.E. of regression	0.006605	Akaike info criterion	-7.327145	
Sum squared resid	0.118701	Schwarz criterion	-7.314140	
Log likelihood	9996.562	Durbin-Watson stat	2.041065	

Dependent Variable: USD  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:27  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 41 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(5) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(6) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.446191	0.333098	1.339518	0.1804
MA(1)	-0.475186	0.327265	-1.451991	0.1465
Variance Equation				
C(3)	-0.104311	0.029720	-3.509830	0.0004
C(4)	0.071191	0.012143	5.862817	0.0000
C(5)	-0.004191	0.006839	-0.612885	0.5400
C(6)	0.995094	0.002481	401.0212	0.0000
T-DIST. DOF	11.63379	2.899213	4.012742	0.0001
R-squared	0.000879	Mean dependent var	6.24E-05	
Adjusted R-squared	-0.001326	S.D. dependent var	0.006598	
S.E. of regression	0.006603	Akaike info criterion	-7.327263	
Sum squared resid	0.118538	Schwarz criterion	-7.312085	
Log likelihood	9994.059	Durbin-Watson stat	1.984349	

Dependent Variable: USD  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:26  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence not achieved after 500 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.037808	0.020181	-1.873487	0.0610
Variance Equation				
C(2)	-0.110833	0.031036	-3.571155	0.0004
C(3)	0.073948	0.012550	5.892298	0.0000
C(4)	-0.003886	0.007144	-0.544021	0.5864
C(5)	0.994663	0.002598	382.8221	0.0000
T-DIST. DOF	11.34089	2.762001	4.106040	0.0000
R-squared	0.000058	Mean dependent var	6.24E-05	
Adjusted R-squared	-0.001780	S.D. dependent var	0.006598	
S.E. of regression	0.006604	Akaike info criterion	-7.326972	
Sum squared resid	0.118636	Schwarz criterion	-7.313962	
Log likelihood	9992.662	Durbin-Watson stat	1.967382	

Dependent Variable: USD  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:28  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence not achieved after 500 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.037941	0.020166	-1.881414	0.0599
Variance Equation				
C(2)	-0.107823	0.029846	-3.612661	0.0003
C(3)	0.074249	0.012493	5.943228	0.0000
C(4)	0.994988	0.002470	402.8092	0.0000
T-DIST. DOF	11.22892	2.717225	4.132496	0.0000
R-squared	0.000053	Mean dependent var	6.24E-05	
Adjusted R-squared	-0.001417	S.D. dependent var	0.006598	
S.E. of regression	0.006603	Akaike info criterion	-7.327596	
Sum squared resid	0.118636	Schwarz criterion	-7.316755	
Log likelihood	9992.513	Durbin-Watson stat	1.967122	

Dependent Variable: USDL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/25/09 Time: 21:02  
Sample: 1 2727  
Included observations: 2727  
Convergence not achieved after 500 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000163	0.000110	-1.473249	0.1407
Variance Equation				
C(2)	-0.104306	0.029350	-3.553908	0.0004
C(3)	0.072784	0.012226	5.953432	0.0000
C(4)	0.002985	0.007215	0.413683	0.6791
C(5)	0.995242	0.002452	405.9247	0.0000
T-DIST. DOF	12.17344	3.108430	3.916266	0.0001
R-squared	-0.000240	Mean dependent var	-6.03E-05	
Adjusted R-squared	-0.002078	S.D. dependent var	0.006598	
S.E. of regression	0.006605	Akaike info criterion	-7.327145	
Sum squared resid	0.118701	Schwarz criterion	-7.314140	
Log likelihood	9996.562	Durbin-Watson stat	2.041065	

Dependent Variable: USDL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:20  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence achieved after 36 iterations  
MA backcast: 1, Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(5) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(6) \cdot \text{LOG}(\text{GARCH}(-1))$

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.446239	0.333102	1.339646	0.1804
MA(1)	-0.475231	0.327269	-1.452112	0.1465
Variance Equation				
C(3)	-0.104312	0.029720	-3.509840	0.0004
C(4)	0.071192	0.012143	5.862809	0.0000
C(5)	0.004192	0.006839	0.612918	0.5399
C(6)	0.995094	0.002481	401.0190	0.0000
T-DIST. DOF	11.63385	2.899325	4.012605	0.0001
R-squared	0.000879	Mean dependent var	-6.24E-05	
Adjusted R-squared	-0.001326	S.D. dependent var	0.006598	
S.E. of regression	0.006603	Akaike info criterion	-7.327263	
Sum squared resid	0.118538	Schwarz criterion	-7.312085	
Log likelihood	9994.059	Durbin-Watson stat	1.984355	

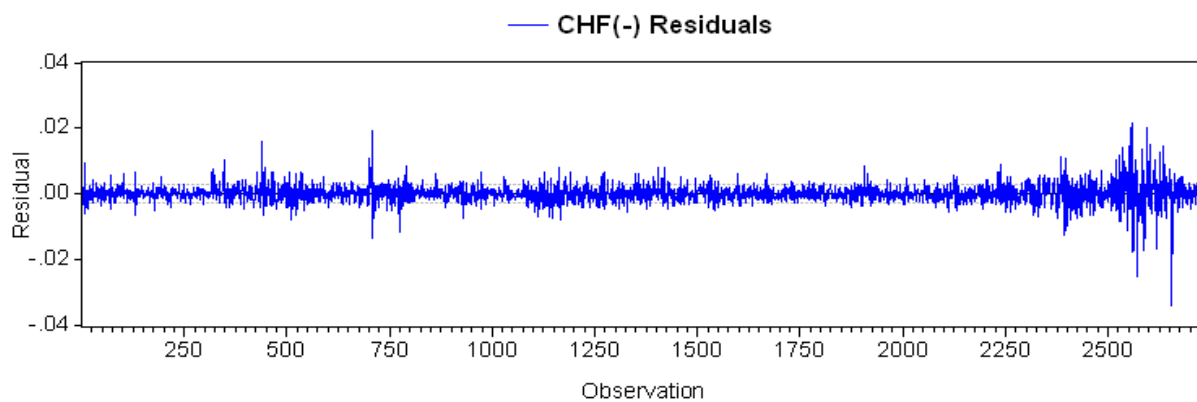
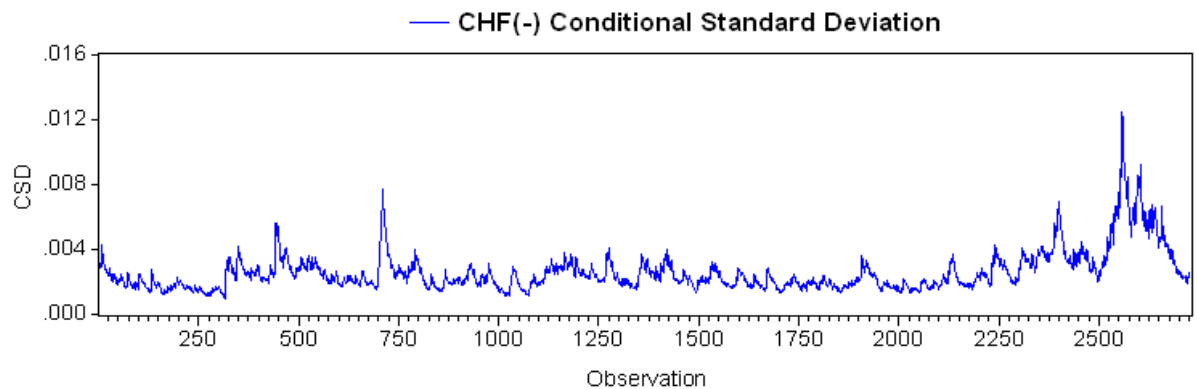
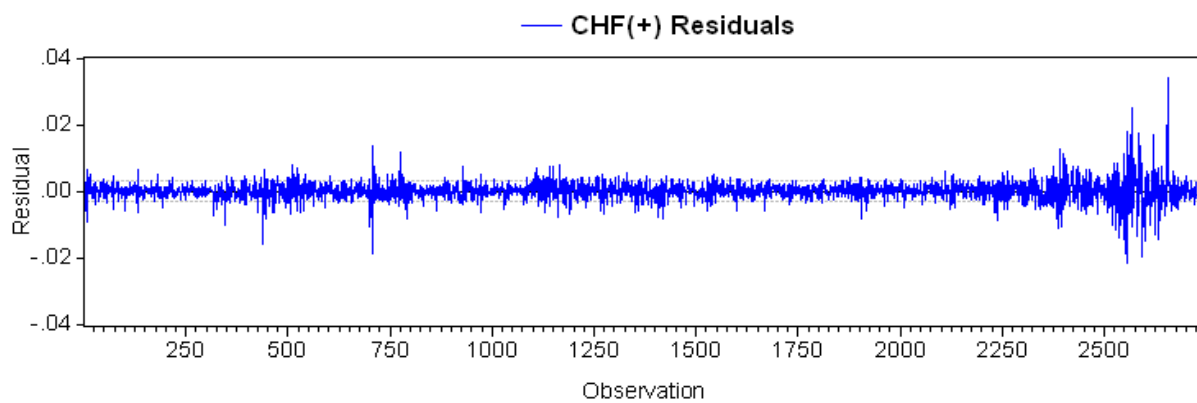
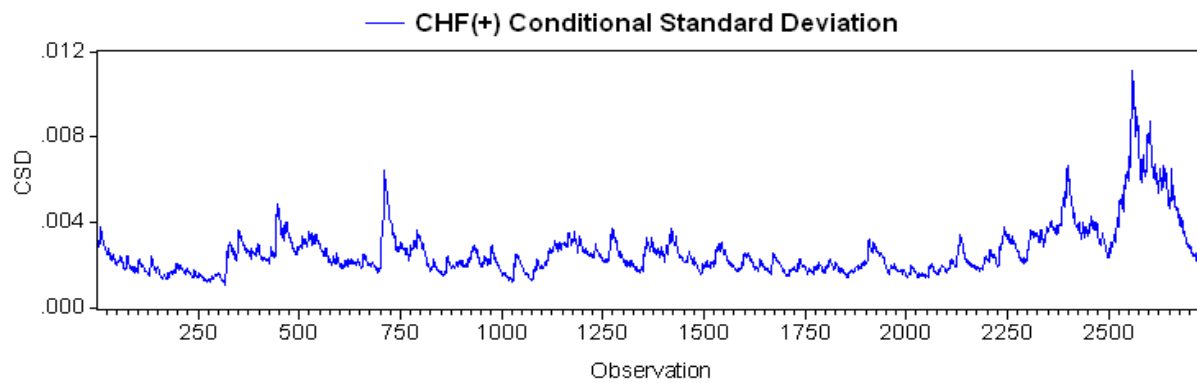
Dependent Variable: USDL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:19  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence not achieved after 500 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) \cdot \text{LOG}(\text{GARCH}(-1))$

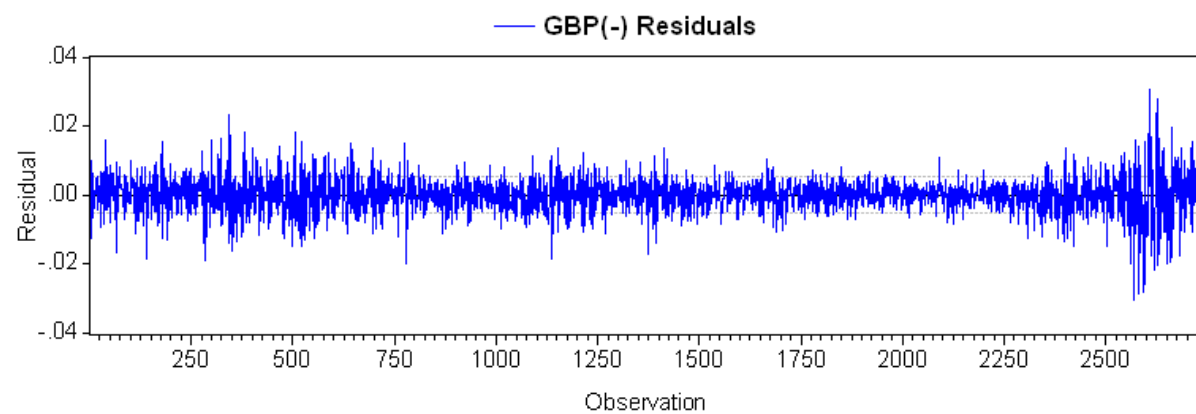
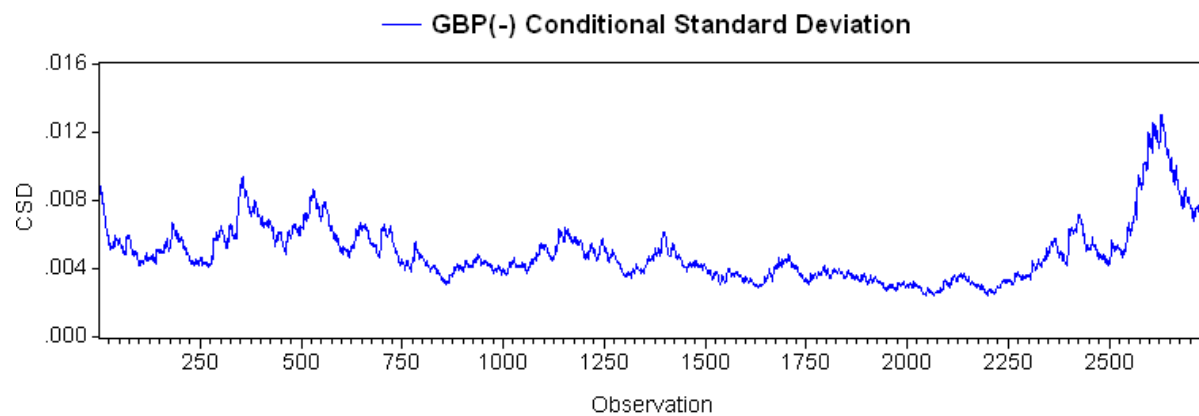
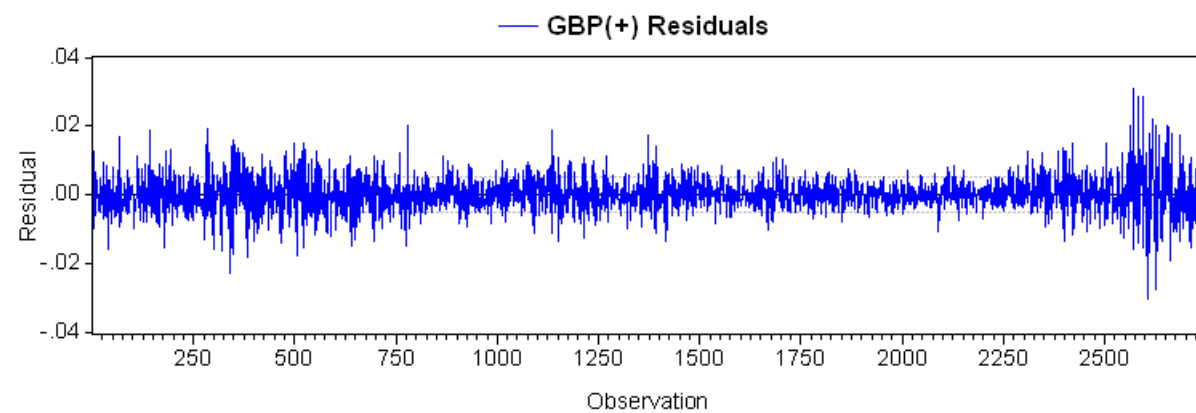
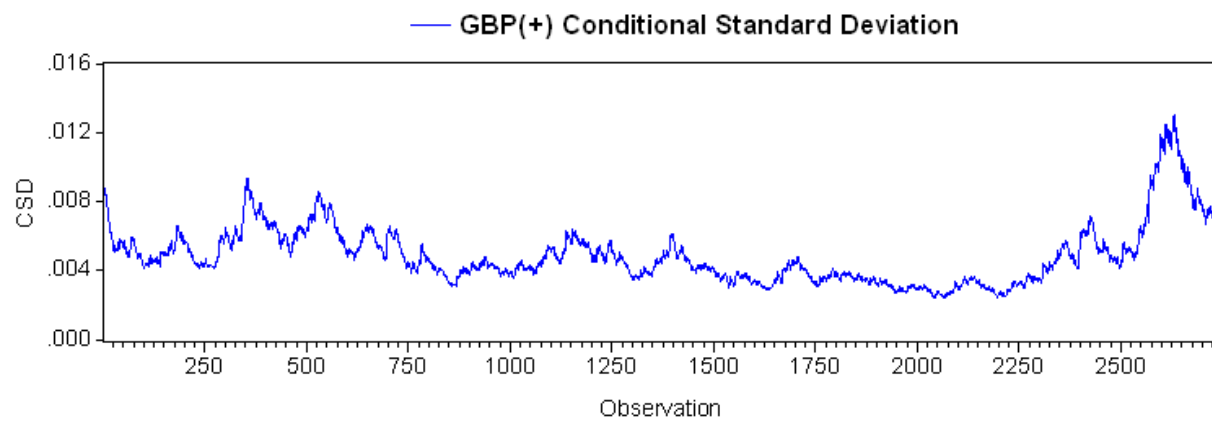
	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.037809	0.020181	-1.873498	0.0610
Variance Equation				
C(2)	-0.110801	0.031046	-3.568881	0.0004
C(3)	0.073935	0.012554	5.889122	0.0000
C(4)	0.003884	0.007146	0.543527	0.5868
C(5)	0.994665	0.002599	382.6987	0.0000
T-DIST. DOF	11.34402	2.760661	4.109169	0.0000
R-squared	0.000058	Mean dependent var	-6.24E-05	
Adjusted R-squared	-0.001781	S.D. dependent var	0.006598	
S.E. of regression	0.006604	Akaike info criterion	-7.326972	
Sum squared resid	0.118636	Schwarz criterion	-7.313962	
Log likelihood	9992.662	Durbin-Watson stat	1.967380	

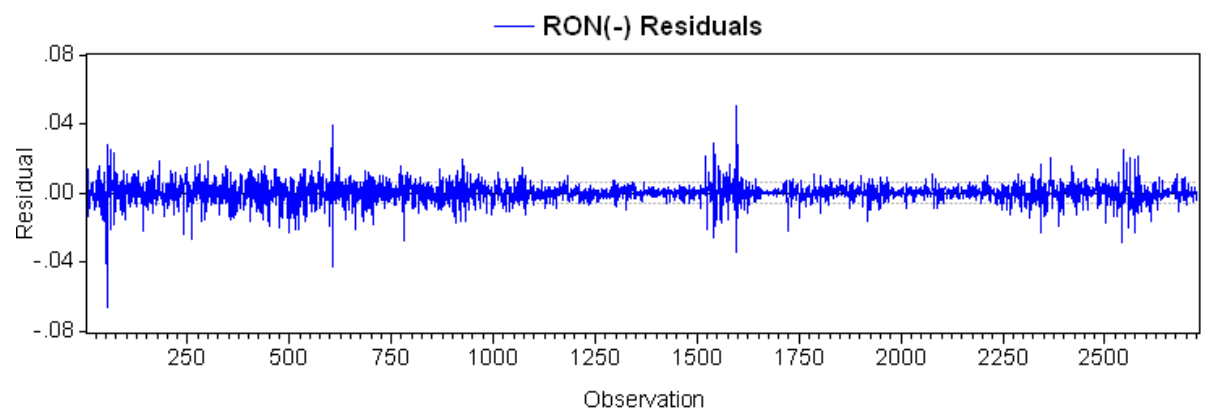
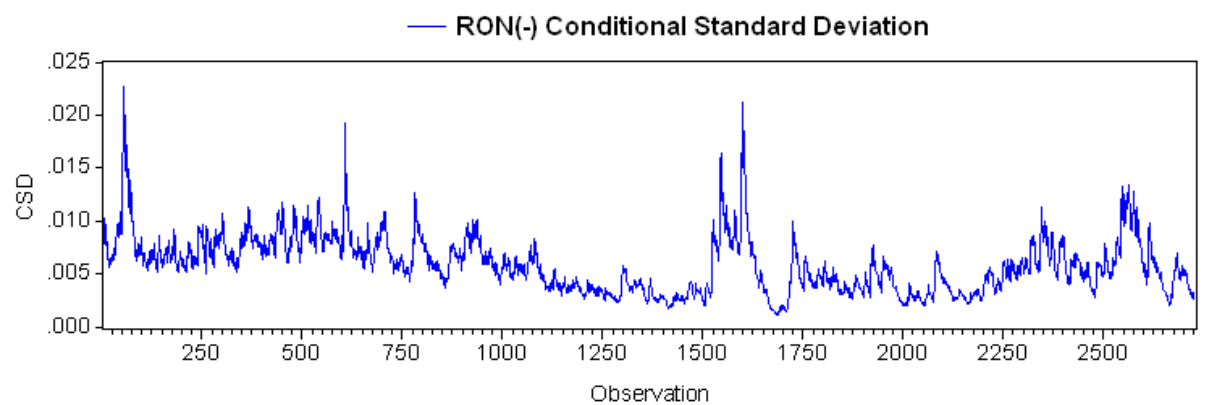
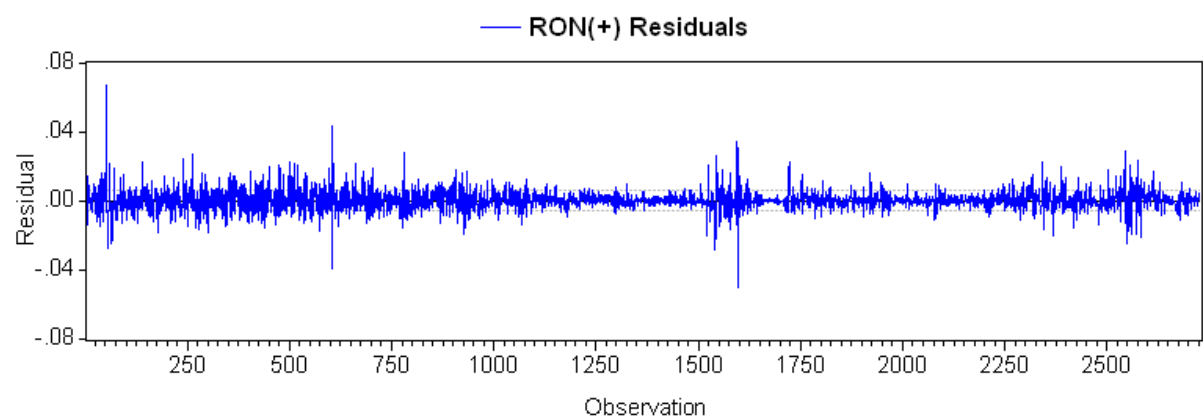
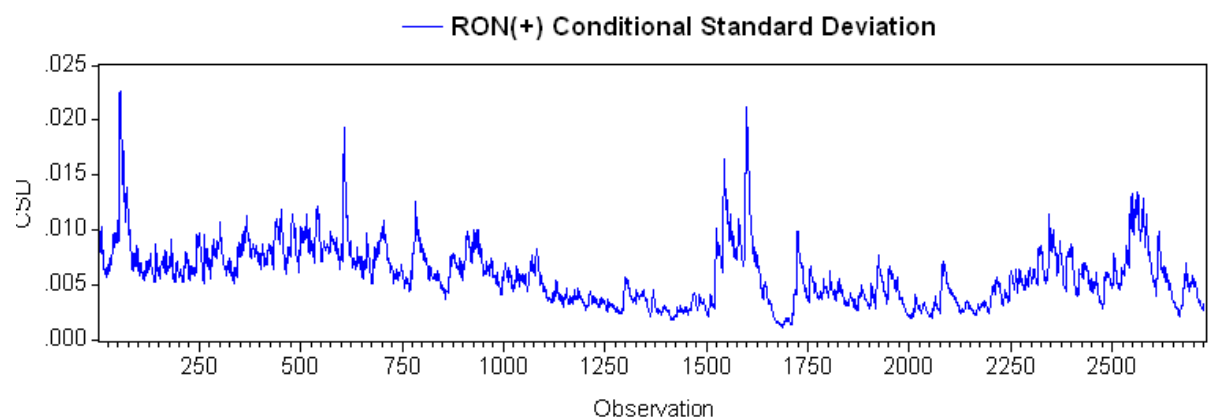
Dependent Variable: USDL  
Method: ML - ARCH (Marquardt) - Student's t distribution  
Date: 06/27/09 Time: 11:22  
Sample (adjusted): 2 2727  
Included observations: 2726 after adjustments  
Convergence not achieved after 500 iterations  
Variance backcast: ON  
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) \cdot \text{LOG}(\text{GARCH}(-1))$

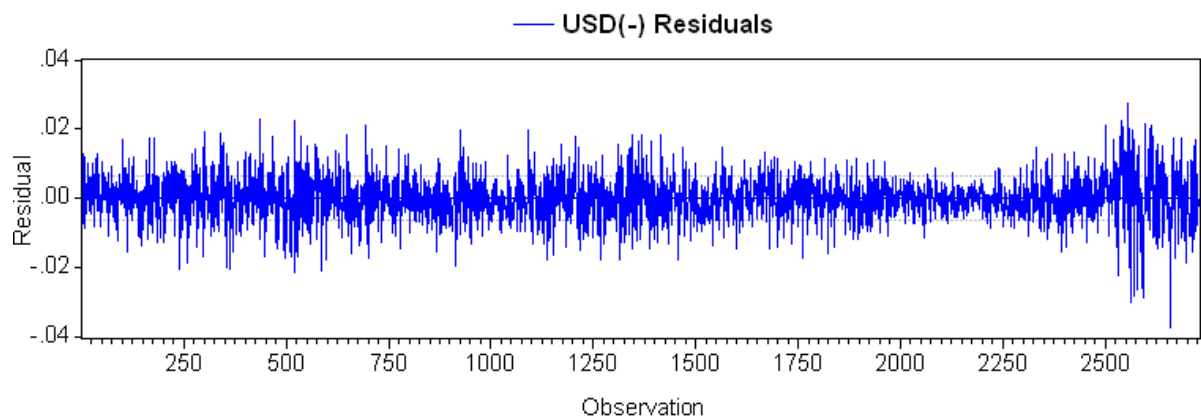
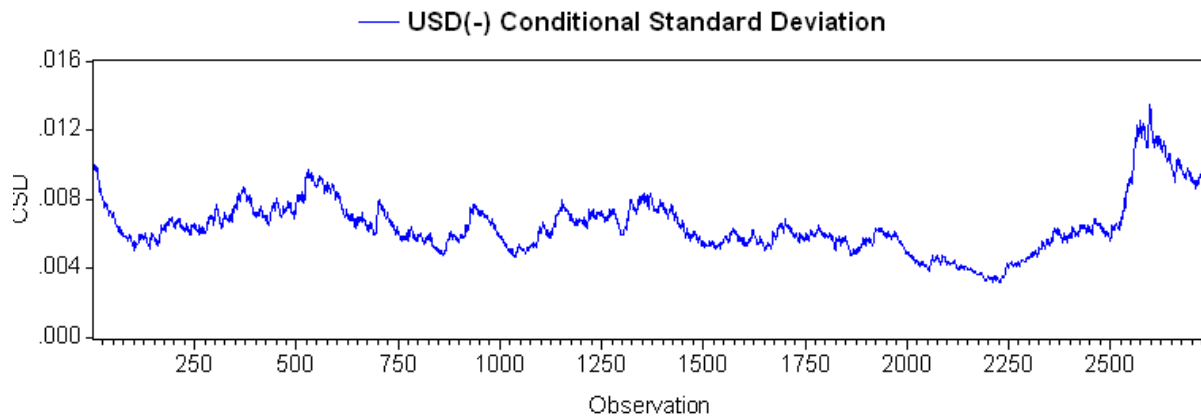
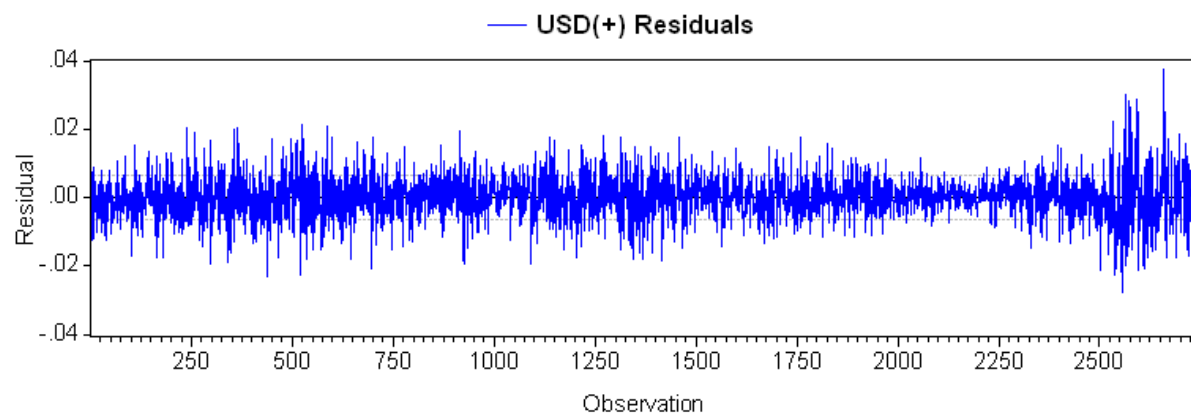
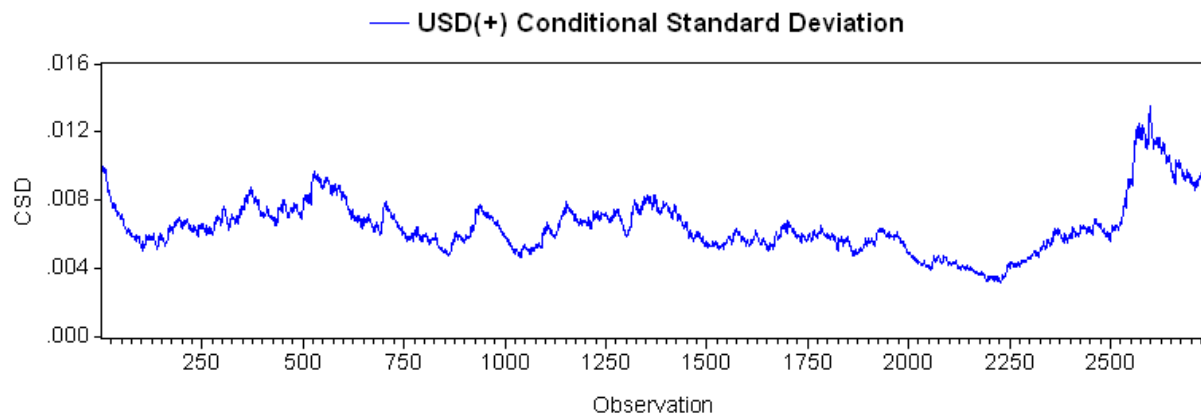
	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.037941	0.020166	-1.881414	0.0599
Variance Equation				
C(2)	-0.107823	0.029846	-3.612661	0.0003
C(3)	0.074249	0.012493	5.943228	0.0000
C(4)	0.994988	0.002470	402.8092	0.0000
T-DIST. DOF	11.22892	2.717225	4.132496	0.0000
R-squared	0.000053	Mean dependent var	-6.24E-05	
Adjusted R-squared	-0.001417	S.D. dependent var	0.006598	
S.E. of regression	0.006603	Akaike info criterion	-7.327596	
Sum squared resid	0.118636	Schwarz criterion	-7.316755	
Log likelihood	9992.513	Durbin-Watson stat	1.967122	

## 7. Conditional Standard Deviations and Filtered Residuals from EGARCH estimation.

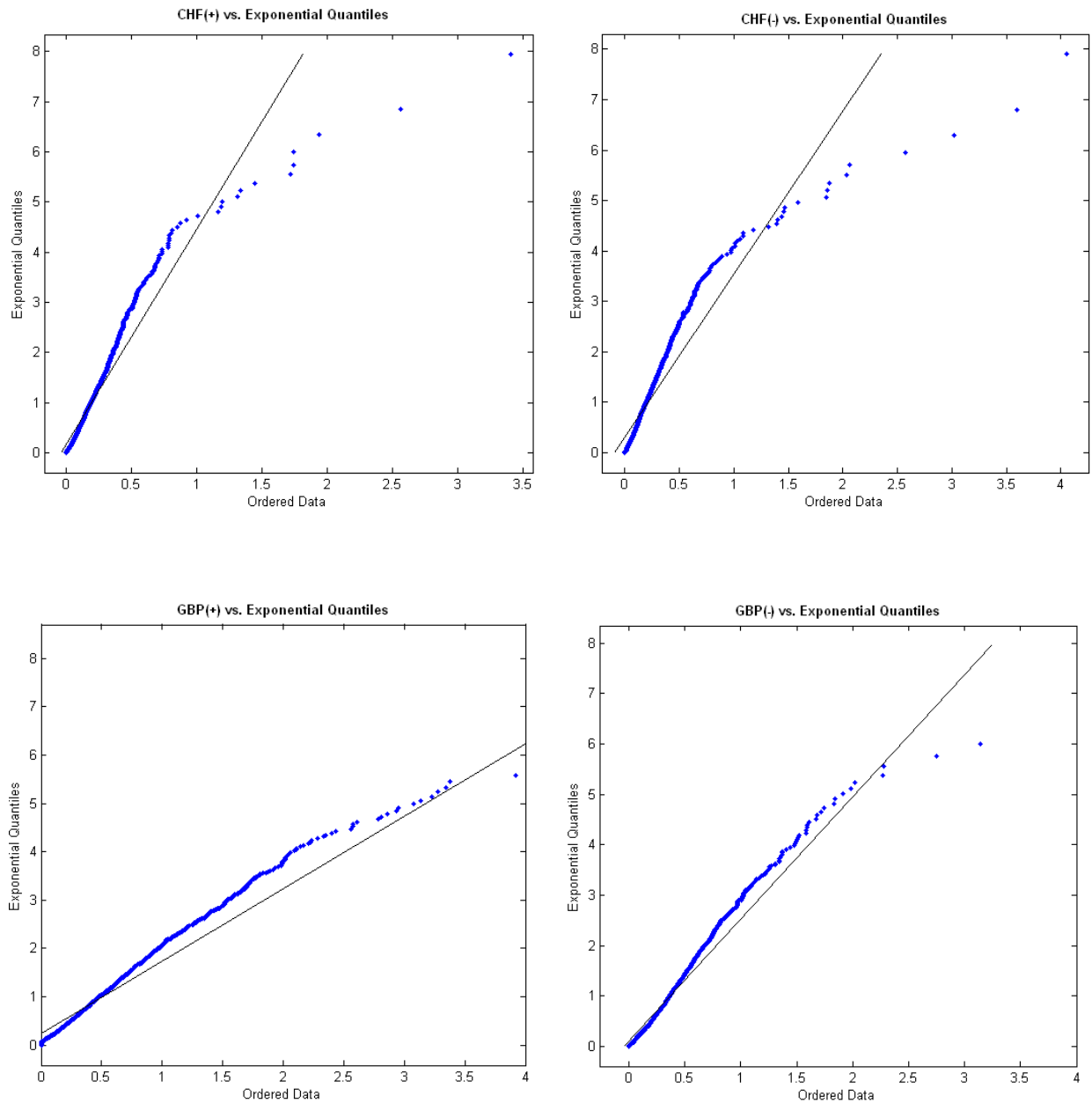


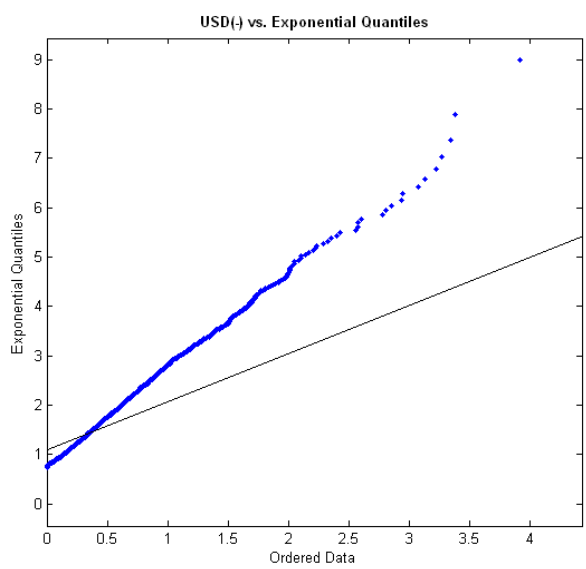
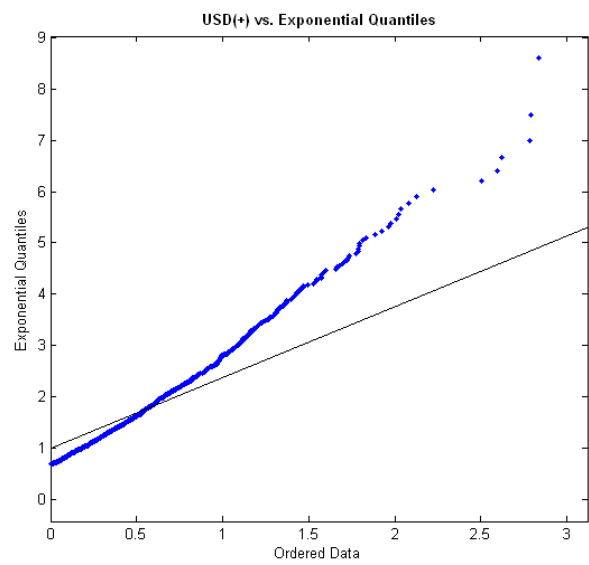
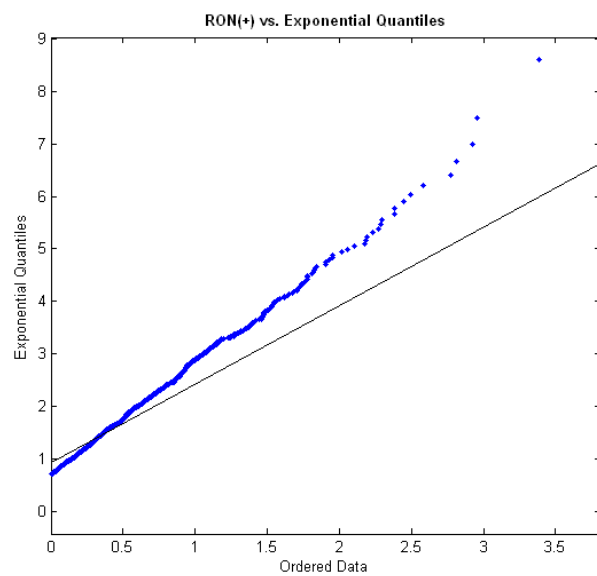
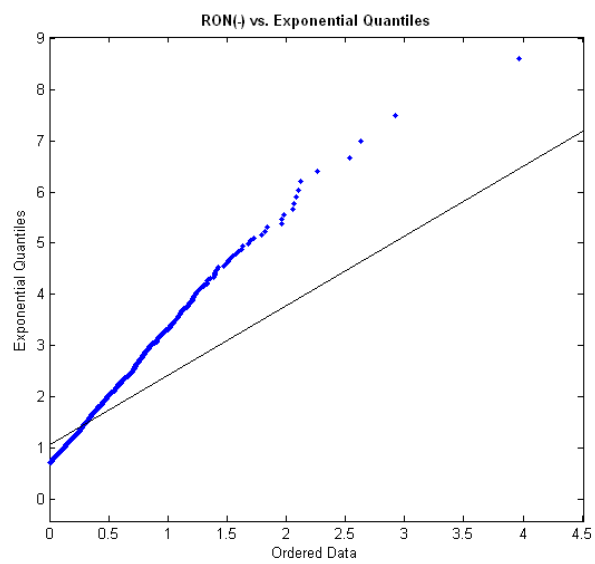






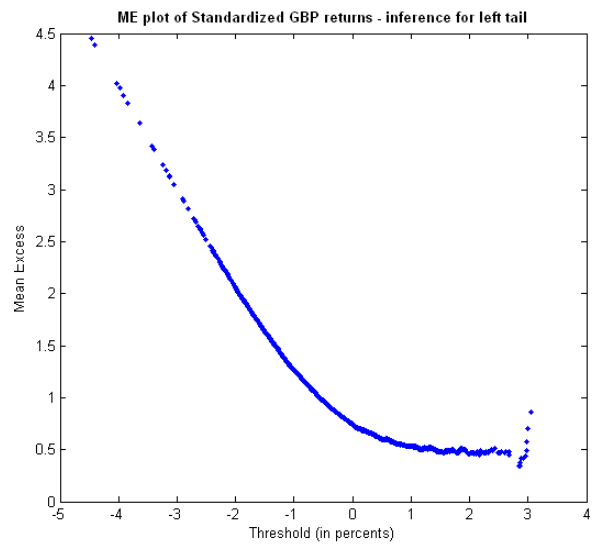
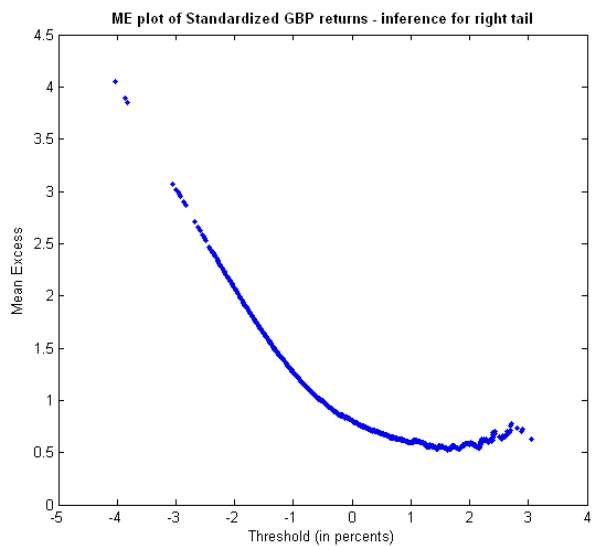
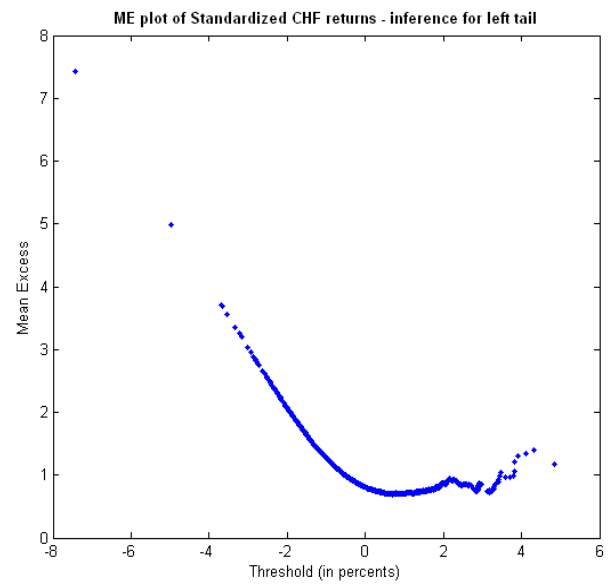
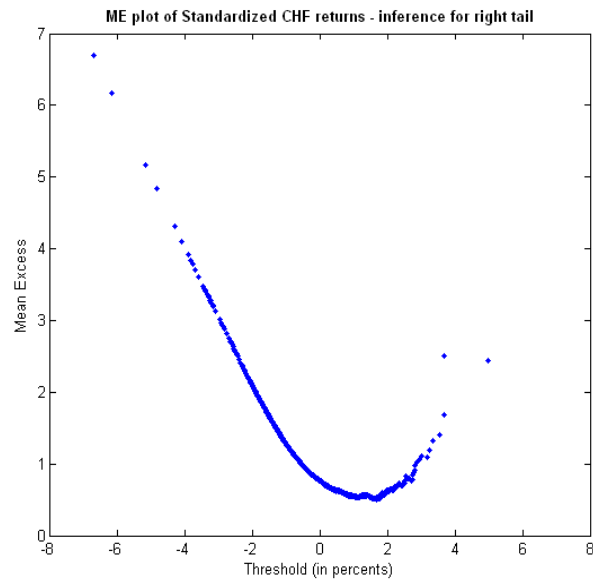
**8. QQ plots against exponential distribution.** Right tails in left panel and left tails in right panel. Concave departures from the straight line indicate heavy tails. Data is truncated in the case of GBP(+), RON and USD series in order to eliminate information that distorts the plots.

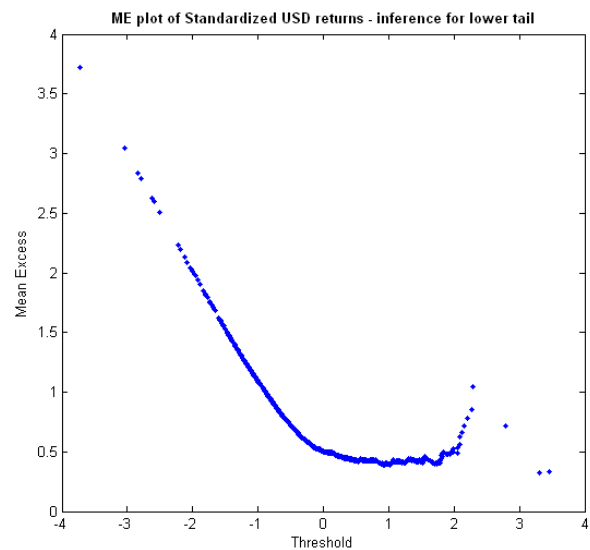
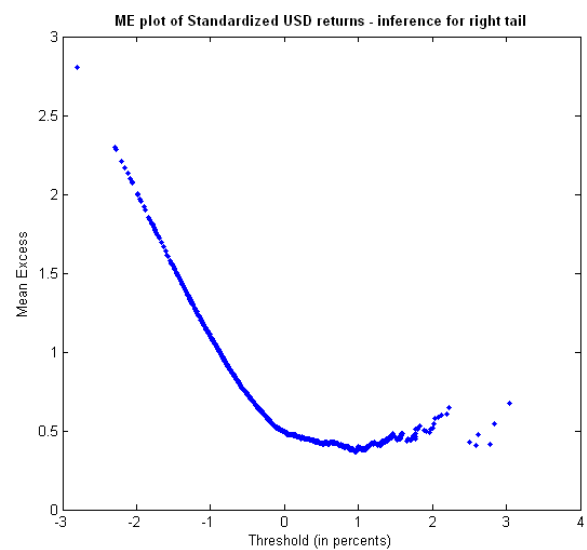
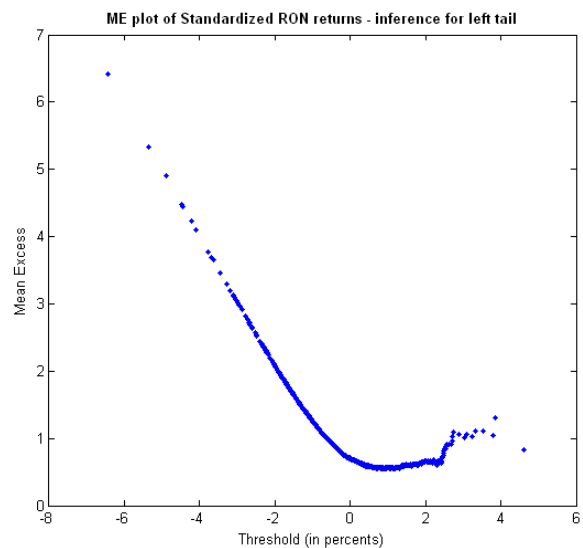
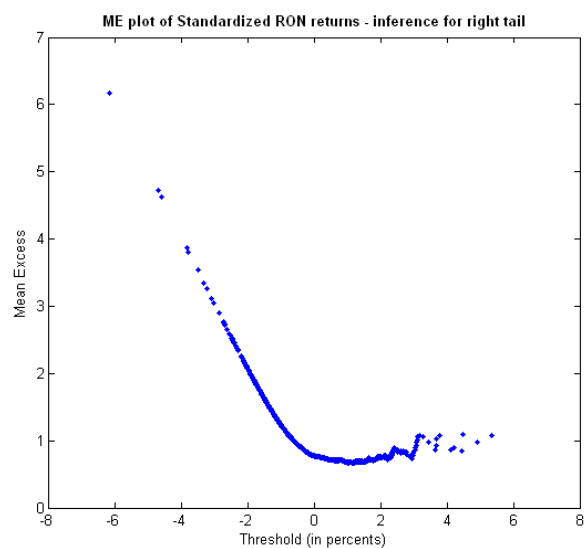




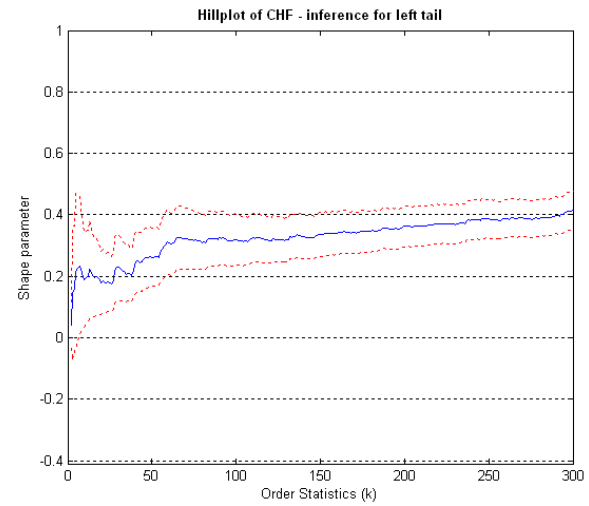
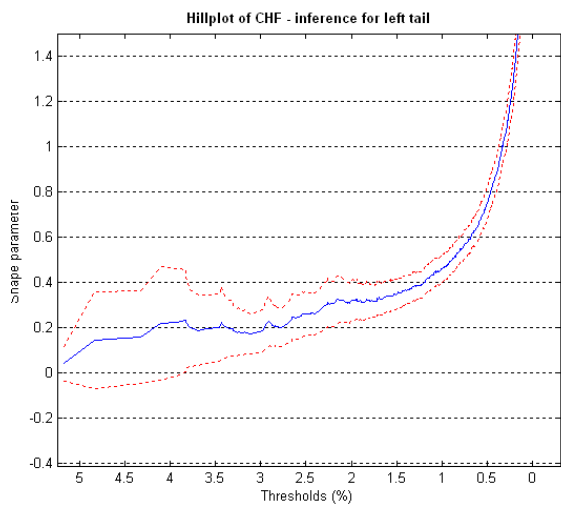
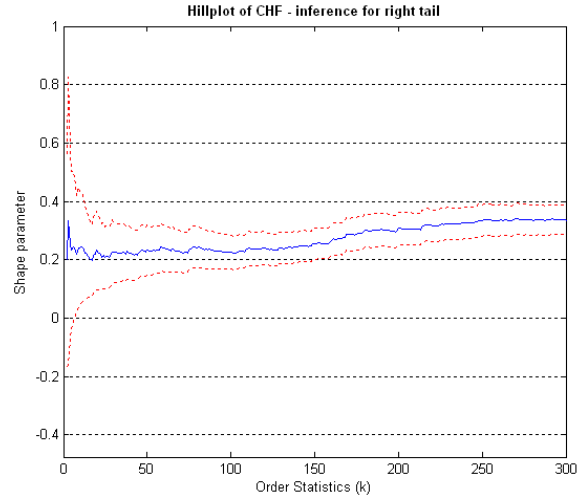
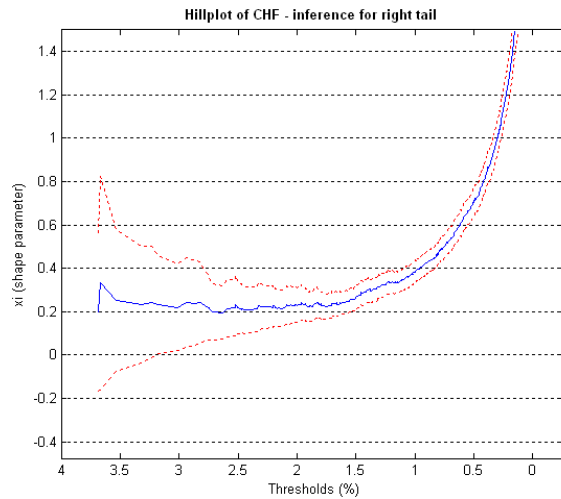


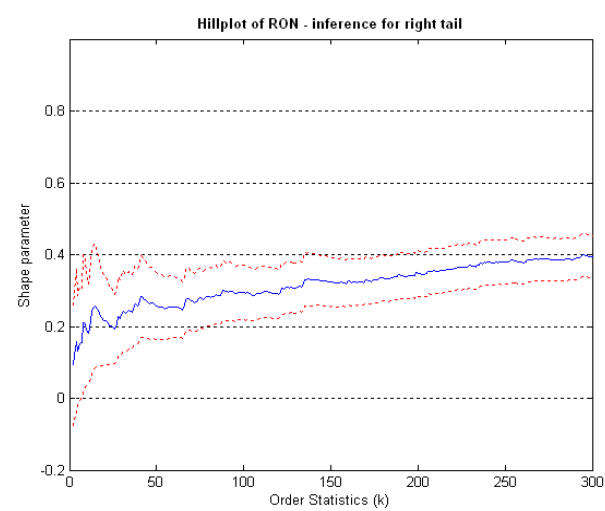
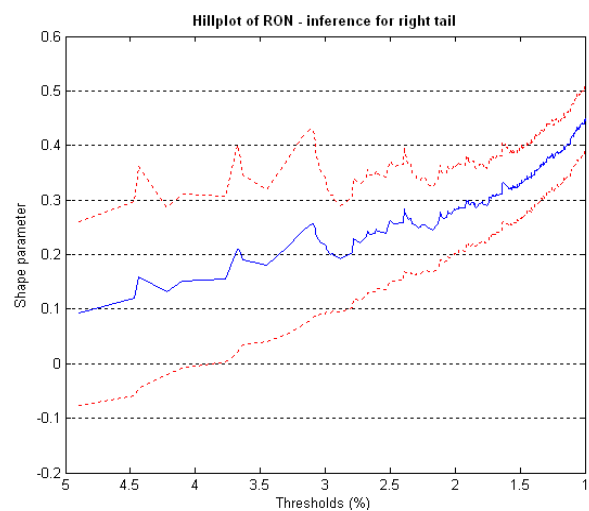
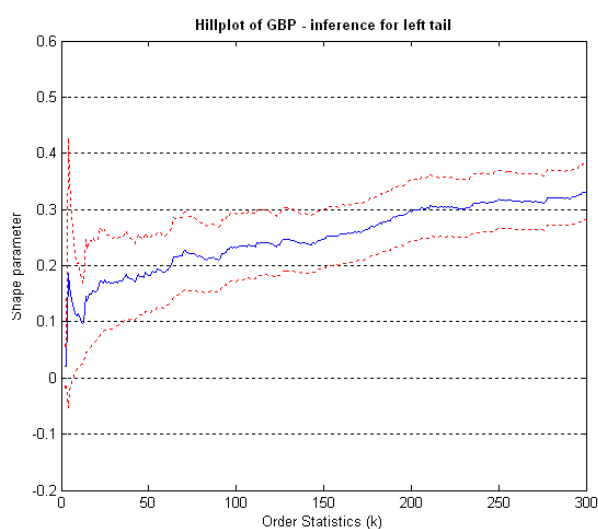
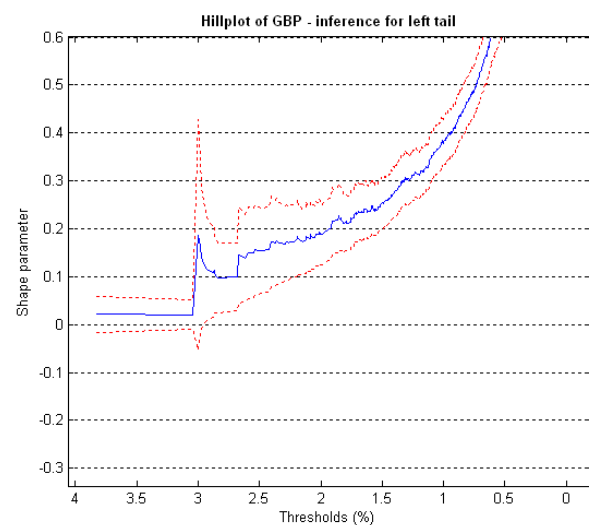
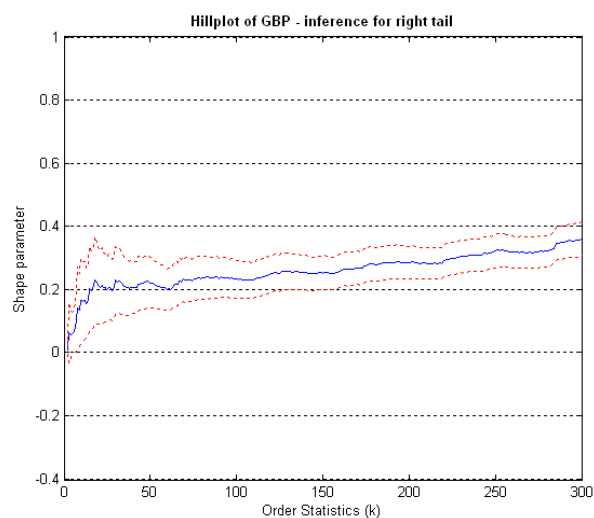
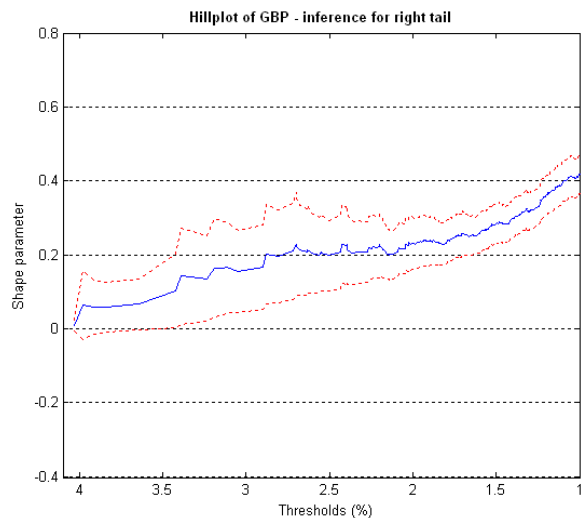
**9. Mean Excess plots of return series against threshold values.** X Axis: threshold values (returns in percents). Y Axis: mean excess function value. Objective: find a threshold above which the graph has a positive slope; ideal case: the graph above the threshold is linear.

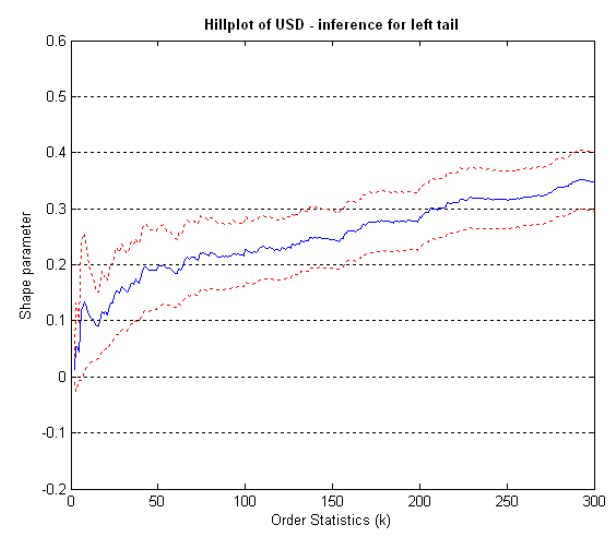
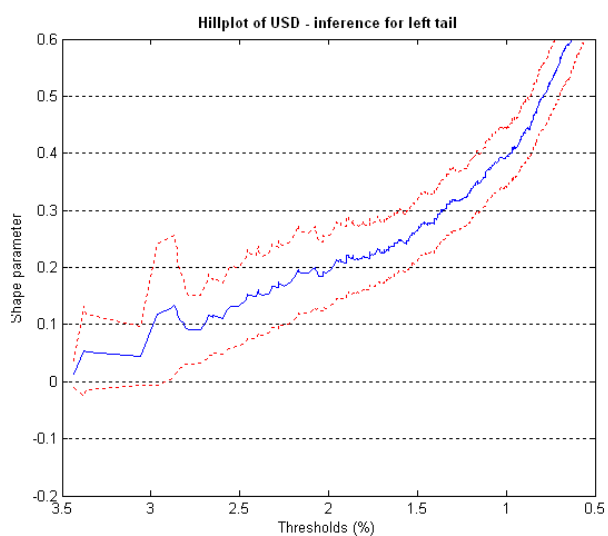
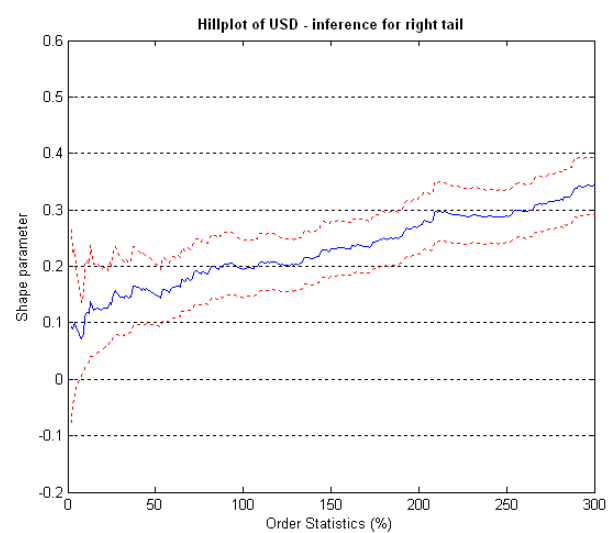
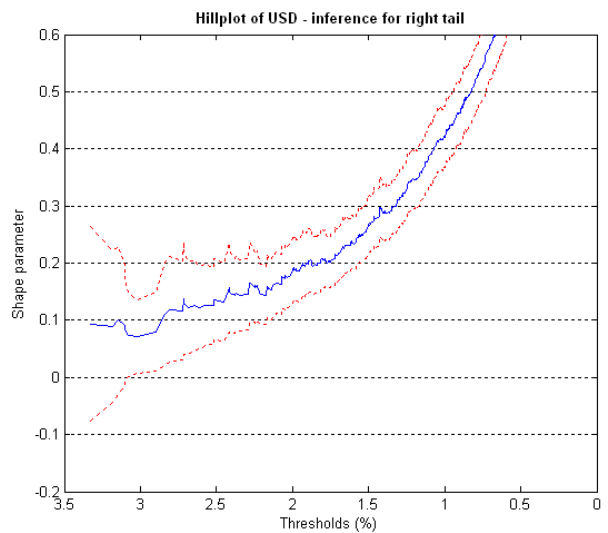
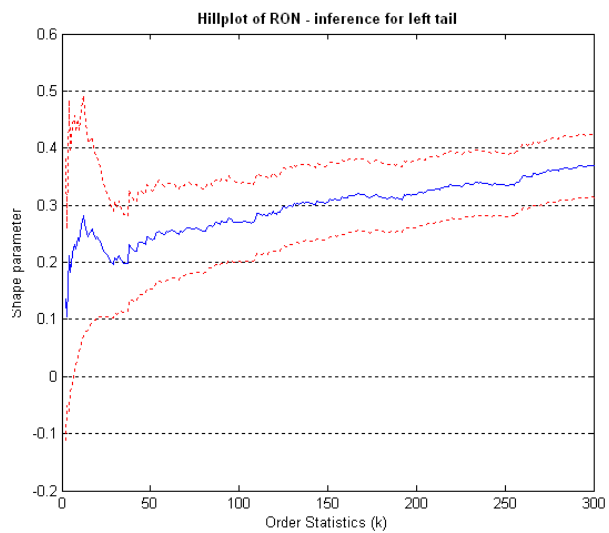
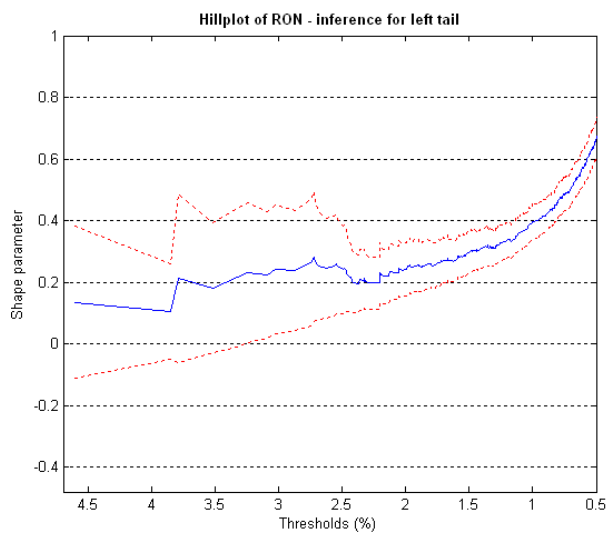




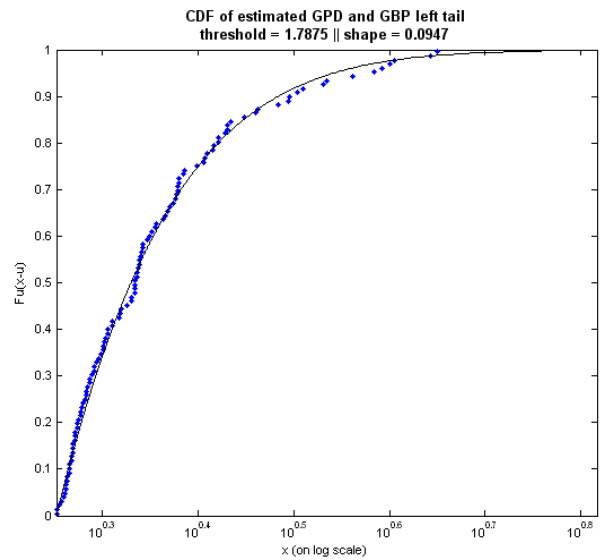
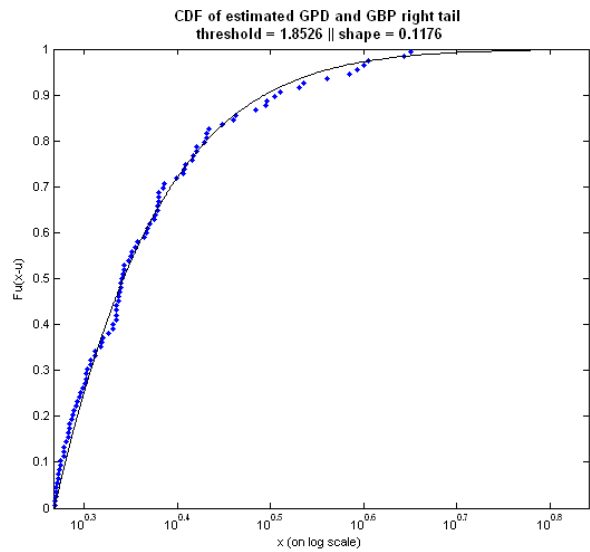
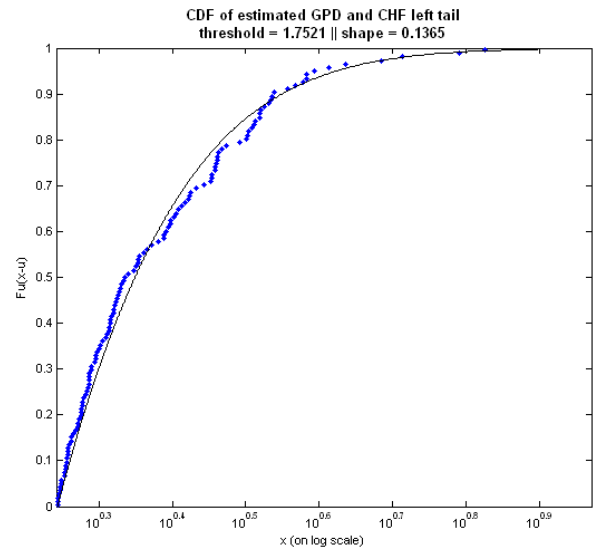
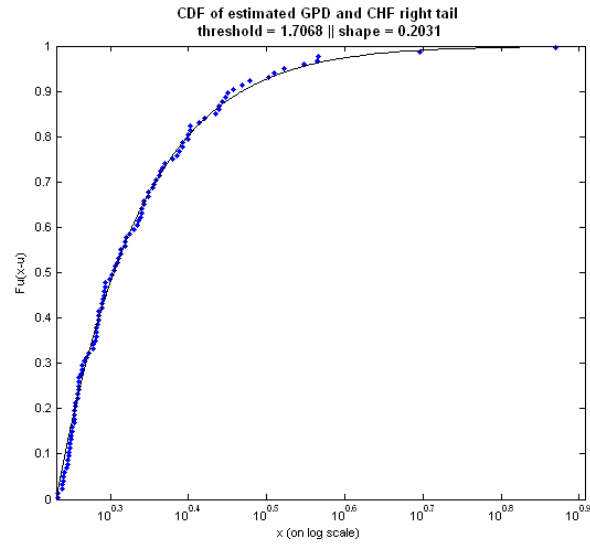
**10. Hill plots:** left panel – shape parameter against threshold values; right panel – shape parameter against number of observations in the tail; 99% confidence intervals in red. Objective: find a relatively stable area on the graph. The threshold is likely to lie in that area.

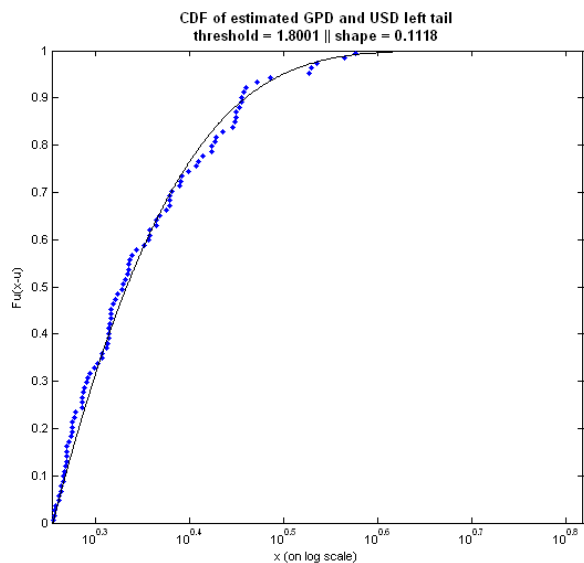
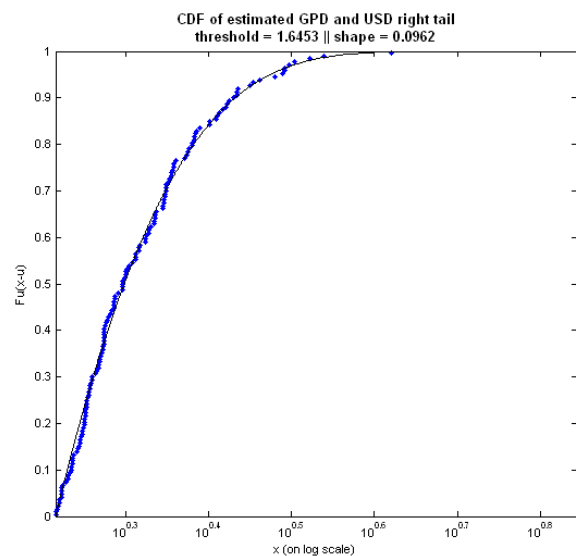
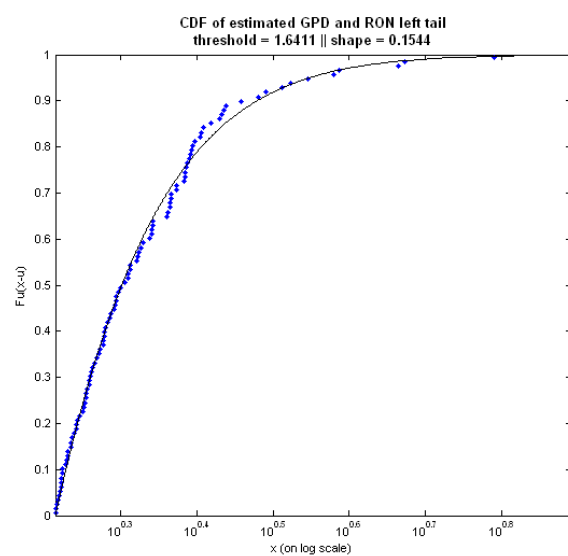
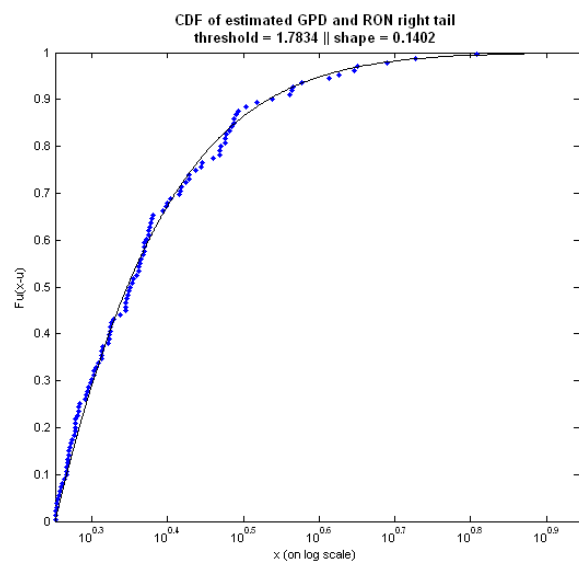




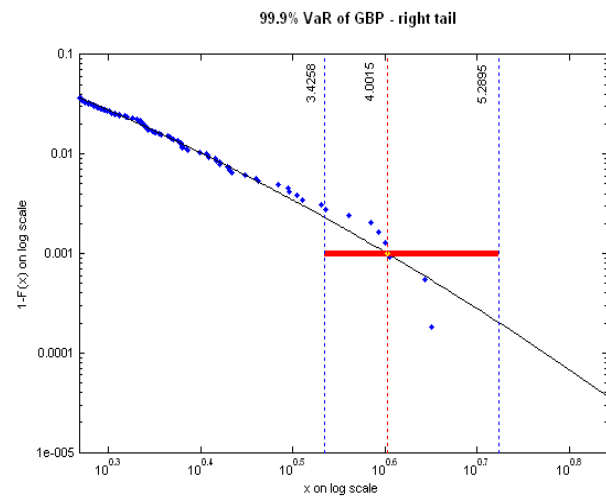
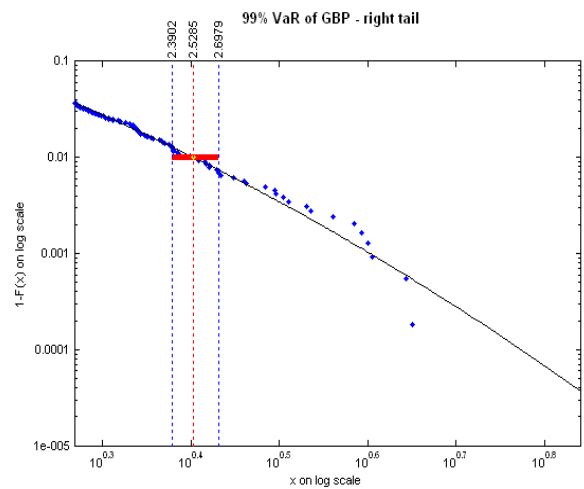
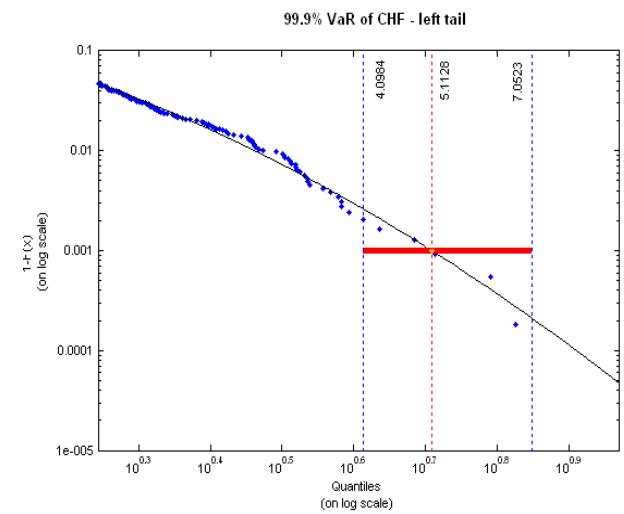
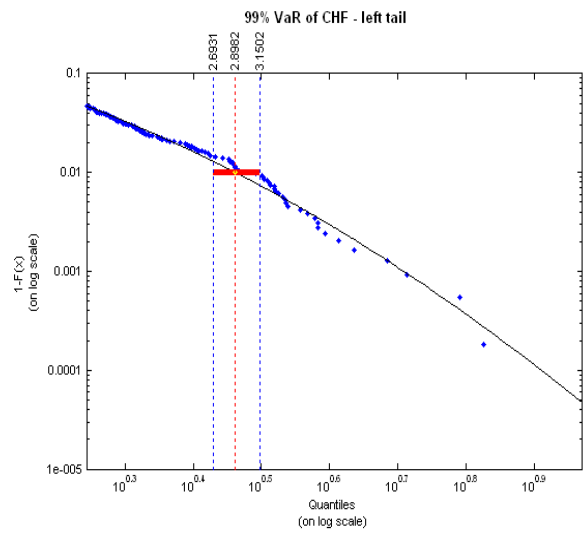
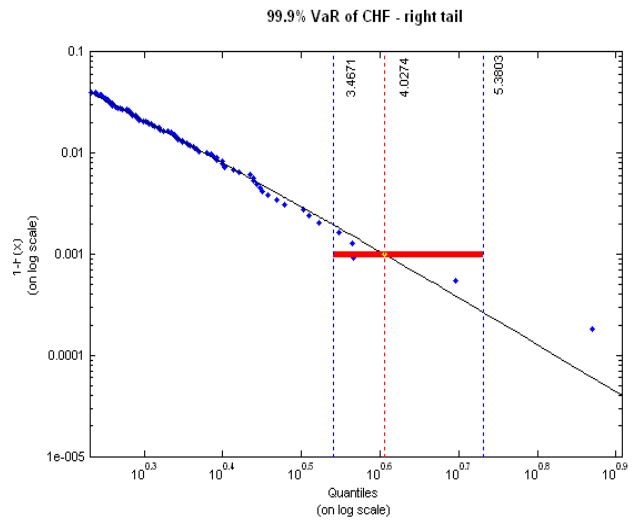
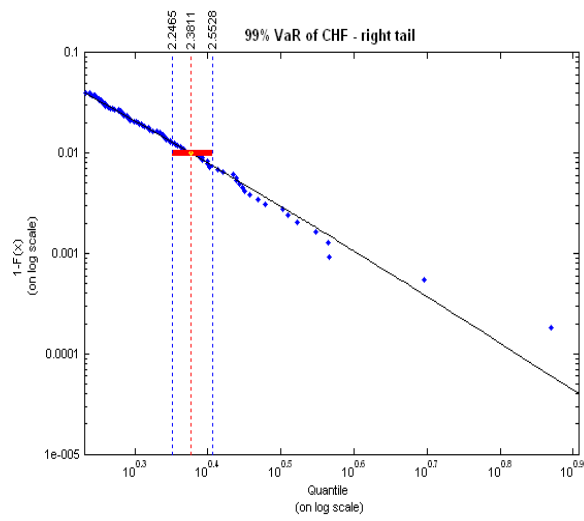


**11. GPD fit for exceedances above selected thresholds.** Threshold values are selected through graphing techniques (ME and Hill plots).

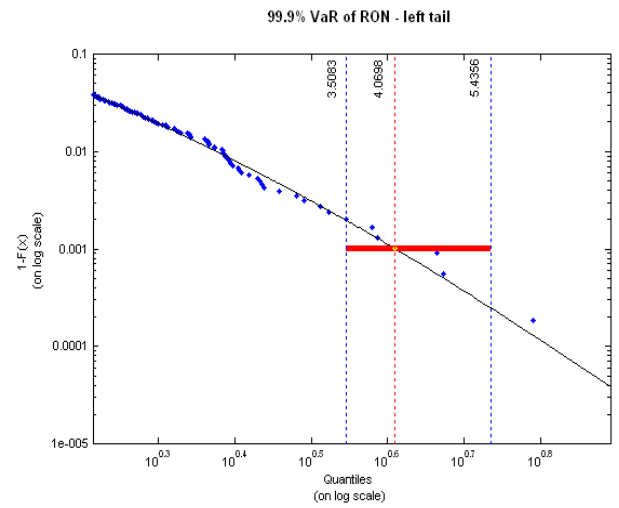
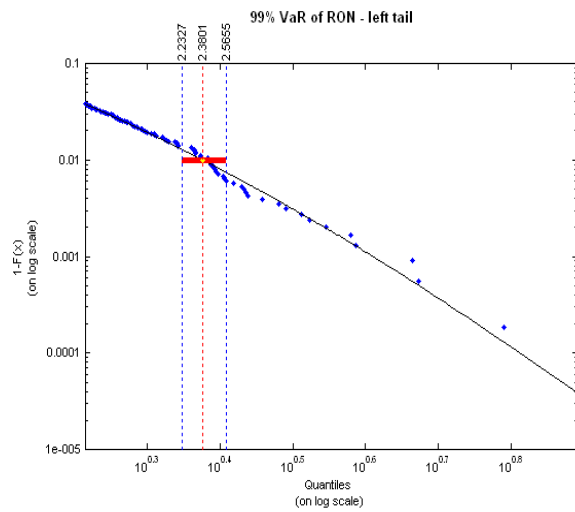
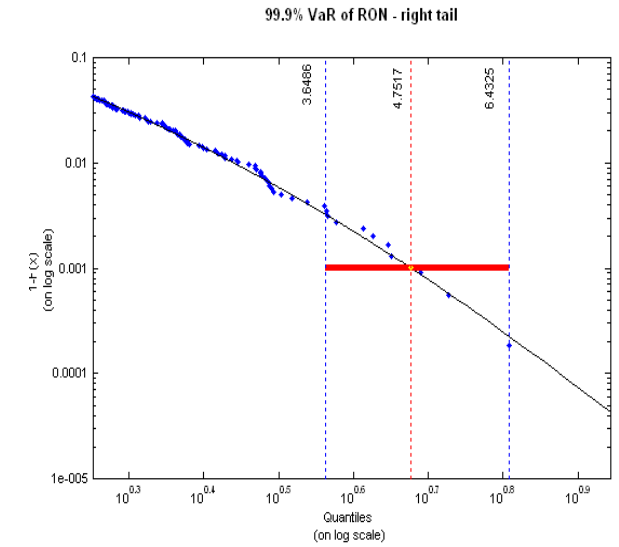
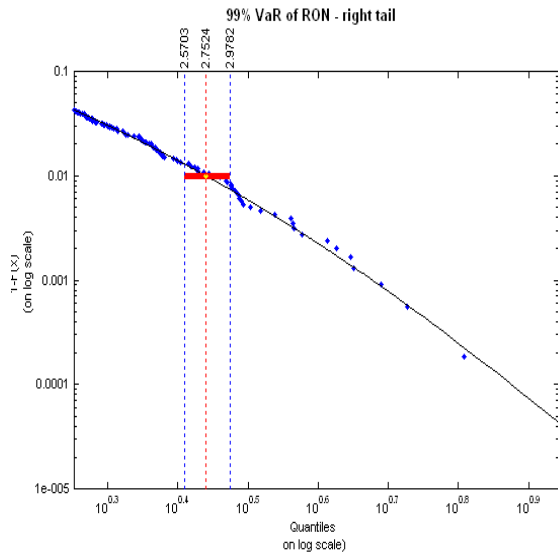
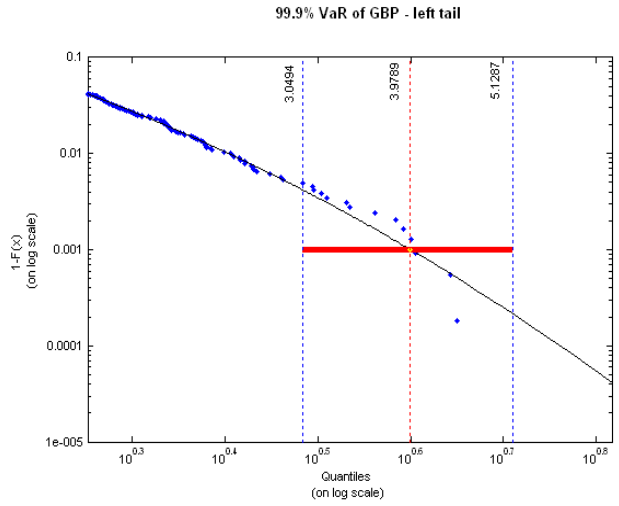
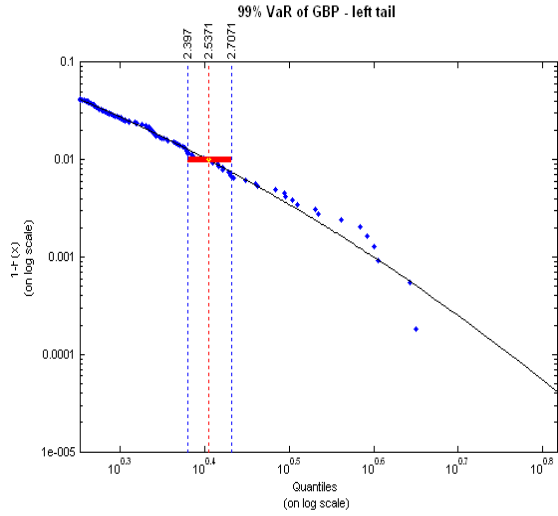




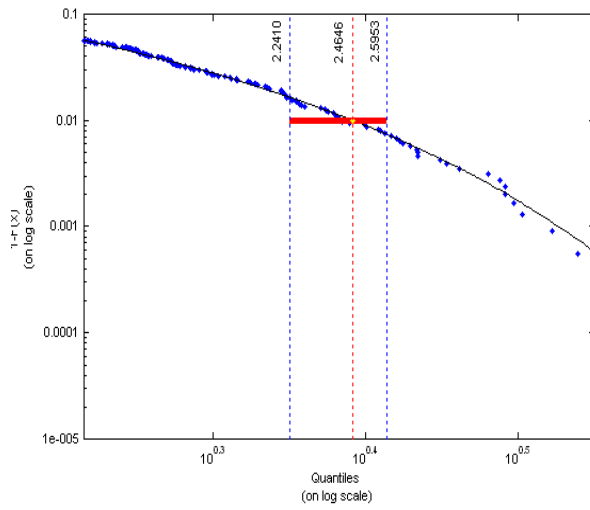
## 12. 99% and 99.9% quantiles for ML estimates (Section 5.3.2)



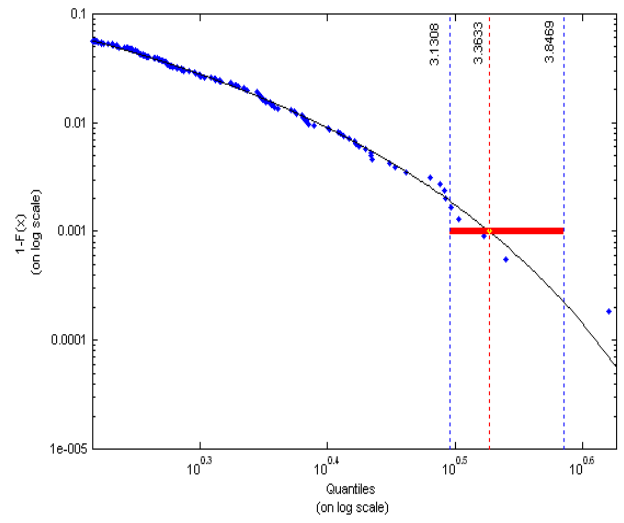




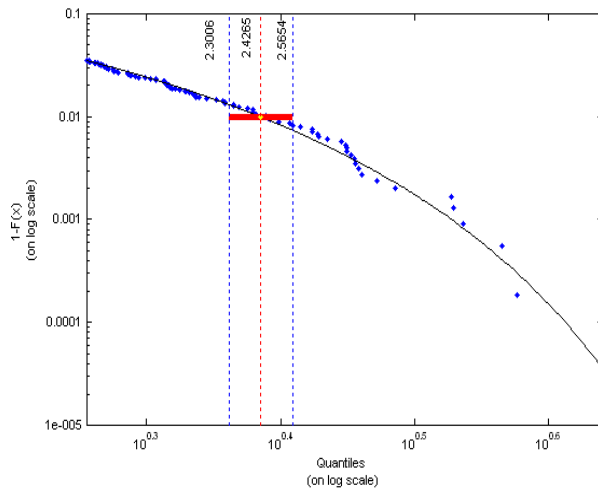
99% VaR of USD - right tail



99.9% VaR of USD - right tail



99% VaR of USD - left tail



99.9% VaR of USD - left tail

