

THE BUCHAREST ACADEMY OF ECONOMIC  
STUDIES

MSc. DOFIN

---

**Estimation of an open economy  
DSGE model for Romania.  
Do nominal and real frictions  
matter?**

---

*Author:*

Veaceslav GRIGORAȘ

*Supervisor:*

Prof. Moisă ALTĂR

Bucharest  
2010

## **Abstract**

In this paper I<sup>1</sup> will use a medium scale open economy DSGE model developed by Adolfson et al. (2005). Besides authors' observables I will include also one extra observable series (CPI) in the model. Some of the parameters will be calibrated as to match sample's mean or common values found in literature and others will be estimated on Romania's data with the help of Bayesian techniques. Next, I will specify some alternative scenarios where nominal or real rigidities will be "turned off" and I will assess their importance for the data generating process (with the help of marginal log likelihood).

---

<sup>1</sup> Author would like to thank Mihai Copaciu for his comments, suggestions and support, and Jesper Lindé for his help in writing measurement equations.

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Literature review</b>	<b>4</b>
<b>3</b>	<b>The model</b>	<b>6</b>
3.1	Firms . . . . .	6
3.1.1	Domestic producers . . . . .	6
3.1.2	Importers . . . . .	11
3.1.3	Exporters . . . . .	14
3.2	Households . . . . .	15
3.3	Sticky Wages . . . . .	22
3.4	Employment . . . . .	24
3.5	Government . . . . .	24
3.6	Monetary Policy . . . . .	25
3.7	Market equilibrium . . . . .	25
3.8	Foreign variables . . . . .	26
<b>4</b>	<b>Estimation</b>	<b>26</b>
4.1	Data . . . . .	26
4.2	Calibrated Parameters . . . . .	27
4.3	Prior distributions . . . . .	29
4.4	Estimation Procedure . . . . .	30
<b>5</b>	<b>Results</b>	<b>31</b>
<b>6</b>	<b>Conclusion</b>	<b>33</b>
	<b>Appendices</b>	<b>39</b>
<b>A</b>	<b>Data</b>	<b>39</b>
<b>B</b>	<b>Prior distributions</b>	<b>42</b>
<b>C</b>	<b>Estimation results</b>	<b>44</b>

## List of Figures

1	Observed Data . . . . .	40
2	Observed Data . . . . .	41
3	Posterior distributions . . . . .	47
4	Posterior distributions . . . . .	48
5	Posterior distributions . . . . .	49
6	Posterior distributions . . . . .	50
7	Smoothed observed variables . . . . .	51
8	Smoothed observed variables . . . . .	52

## List of Tables

1	Prior distribution of Shocks . . . . .	42
2	Prior distributions of Parameters . . . . .	43
3	Baseline model . . . . .	44
4	Scenarios . . . . .	46

# 1 Introduction

Over the last 20 years, Dynamic Stochastic General Equilibrium (DSGE) models became the cornerstone of policy analysis and forecast. Today, central banks all over the world adopt the unified and coherent framework of DSGE in their working process. As Tovar (2008) says: *"DSGE models can help to identify sources of fluctuations; answer questions about structural changes; forecast and predict the effect of policy changes; and perform counterfactual experiments"*.

An advantage of DSGE models lies in their microeconomic foundations, their ability to model agents' behaviour, fact that doesn't make them subject to Lucas' critique. Another advantage lies in the fact that DSGE models are able to identify deep structural parameters and their link to reduced form estimated parameters. In their paper Christiano et al. (2005) were first to show that a DSGE including nominal and real rigidities could account successfully for the effects of a monetary policy shock.

Although the potential benefits of using DSGE models as a framework for policy analysis are promising, they still do not play the main role in the central bank's decision making process. Given the novelty and complexity of modeling, technical and computing aspects, some central bankers consider it hard to communicate DSGE's results to the public.

Some economists like Sims (2006) consider that DSGE models are only a tool to tell stories and understand how economy works. They argue that there is no aggregate consumption or investment good, and that real economy consists of many financial markets which were not yet included in a consistent way in the framework of DSGE.

Given the importance and usefulness of DSGE models, I have decided to estimate a DSGE for Romania's economy. I selected the DSGE model described in Adolfson et al. (2005), because their model has some features that makes it useful for Romania's case, a small open economy with incomplete pass-through. The model incorporates some important aspects that are used in generating persistence as observed in data: variable capital utilization rate, working capital channel for firms, investment adjustment cost, sticky prices and wages, habit in consumption.

I will use Bayesian techniques to estimate deep structural parameters, analyse

the importance of frictions: determine price adjustment frequency, whether there is a habit involved in consumption making decision, or wage contracts are sticky. Next, I will specify alternative scenarios that will lack some of the introduced frictions and I will analyse their importance by confrontations with data and by evaluation of marginal log-likelihood.

The rest of the paper is organised as follows: section 2 reviews the recent working papers that appeared in the field of DSGE, section 3 describes the DSGE model used in Adolfson et al. (2005) and Adolfson et al. (2007), section 4 describes the data and estimation techniques, section 5 presents the results and section 6 concludes.

## 2 Literature review

Dynamic Stochastic General Equilibrium models have their origins in the Real Business Cycle (RBC) theory of Kydland and Prescott (1982). Their model substitutes aggregate behavioral equations describing macroeconomic relationships with first order conditions of firms and households. However, their model doesn't leave any role for money and monetary policy, and assumes that the source of all aggregate fluctuations is technology shocks.

Latter, New Keynesian Economists<sup>2</sup> added some extensions to the classical RBC model: monopolistic competition that implies price stickyness (Calvo, 1983) (without any monopolistical power, firms that aren't able to adjust their prices will lose their sales); following sticky price framework, Erceg et al. (2000) introduce wages stickiness; Christiano et al. (2005) introduce the concept of variable capacity utilization rate (variable capital utilization) and investment adjustment costs.

However, all these models were developed for closed economies, and could not account for all the shocks that matter in an open economy (given the fact that, in practice, monetary policy is conducted in open economies). This lack has encouraged the work of New Open Economy Macroeconomist (NOEM), who extend closed economy DSGE models to incorporate open economy features, like Galí and Monacelli (2002). But their model has one limitation, assumption of

---

<sup>2</sup>see Clarida et al. (1999) for a synthesis

complete exchange rate pass-through to import prices, in contrast to empirical evidence on incomplete exchange rate pass-through. In Monacelli (2003), incomplete exchange rate pass-through to import prices is added, by assuming failure in the law of one price or local currency price stickiness.

In the domain of parameter identification, two main approaches were developed. The first one involves identification of parameters by matching the impulse response of a shock to monetary policy of DSGE and a VAR (Monacelli (2003), Christiano et al. (2005)). The second approach, takes the advantages of unified DSGE framework and uses Bayesian techniques. Bayesian estimation has some advantages over Maximum Likelihood estimation: by specification of priors we restrict our analysis only in the space where model is identified, they act as weights and allow avoidance of the domains where likelihood is flat. Another advantage of this approach lies also in the description of uncertainty of parameter estimation through posterior distribution. In fact, Bayesian estimation joins two main approaches in macroeconomic modeling: calibration (inherited through the specification of priors) and estimation (developed through maximization of the likelihood function).

McCandless (2008) and Gali (2008) are very good introductory references in the field of DSGE, also a complete review of DSGE solving and estimation is done by Fernández-Villaverde (2009).

Paper of Adolfson et al. (2005) incorporates all the features of new Keynesian open economy macroeconomics. Authors adopt the model of Christiano et al. (2005) adding some open economy features: incomplete exchange rate pass-through to import prices and presence of exports due to foreign economy demand for domestic produced goods; sticky wages as in Erceg et al. (2000); a stochastic unit root technology process that induces a common trend in all real variables, allowing for estimation on unfiltered data.

All of the above mentioned features and the ability to estimate parameters via a Bayesian approach made me adopt this model for my thesis.

## 3 The model

### 3.1 Firms

There are three types of firms that operate in the economy: domestic, importing and exporting. The domestic firms class includes: an intermediate good producer which produces a differentiated good, and uses capital and labor as inputs; the intermediate good is sold to the final good producer, who transforms a continuum of these goods into a final good. The importing firms buy a homogenous good on foreign market and transform it into a differentiated good, which is sold directly to the households. Importing firms can sell consumption or investment goods. The exporting firms buy domestic good and transform it into a differentiated export good which is sold on foreign market, which leads to the exporting firms being the monopoly supplier of differentiated goods.

#### 3.1.1 Domestic producers

Domestic production sector consists of three firms. First one hires differentiated labor from households and aggregates it into homogenous labor good, which is used as input by a continuum of intermediate good producing firms along with capital and technology. Intermediate good producing firms sell their goods to the final good producing firm. Separation of production sector into two parts is done in order to give firms some market power that can be exploited to change prices higher than their marginal cost (see McCandless (2008, p. 258)). The final good producing firm takes intermediate good prices  $P_{j,t}$  and final good price  $P_t$  as given.

The final good is produced from a continuum of intermediate goods according to the following technology:

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{1}{\lambda_{d,t}}} dj \right]^{\lambda_{d,t}} \quad (3.1)$$

where  $1 \leq \lambda_{d,t} < \infty$  is the markup in the domestic goods market. Note that the markup is time-varying. Considering time-varying markups will lead to the shocks on the Phillips curve to be in the fact shocks on markups. The markup follows a stochastic process as a mean between a steady state value  $\lambda_d$  and its past

values:

$$\lambda_{d,t} = (1 - \rho_{\lambda_d}) \lambda_d + \rho_{\lambda_d} \lambda_{d,t-1} + \varepsilon_{\lambda_d,t} \quad (3.2)$$

Since the final good producing firm takes its input and output prices as given (the prices are beyond its control), it operates on a perfect competition market.

Profit maximization problem is:

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \quad (3.3)$$

Subject to (3.1). Solving profit maximization problem yields demand for each domestic differentiated good.

$$Y_{j,t} = \left( \frac{P_t}{P_{j,t}} \right)^{\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} Y_t \quad (3.4)$$

Integrating individual demand (3.4) and imposing restriction (3.1), a relationship between the prices of intermediate goods and the price of final good is obtained:

$$P_t = \left[ \int_0^1 P_{j,t}^{\frac{1}{1-\lambda_{d,t}}} dj \right]^{(1-\lambda_{d,t})} \quad (3.5)$$

Any intermediate good producing firm  $j$  ( $j \in (0, 1)$ ), uses technology, capital and labor as inputs to produce an intermediate good. Being the only supplier of differentiated good  $Y_j$ , the firm acts on a market with monopolistic competition. The Cobb Douglas production function for intermediate good producing firm is:

$$Y_{j,t} = z_t^{1-\alpha} \epsilon_t K_{j,t}^\alpha H_{j,t}^{1-\alpha} - z_t \phi \quad (3.6)$$

where  $0 < \alpha < 1$  is the share of capital in the production function,  $K_{j,t}$  are capital services at time  $t$  used by the firm (notice that capital services can be different from capital stock, since the model assumes variable capital utilization rate),  $H_{j,t}$  is labor hired by the firm at time  $t$ ,  $z_t$  is a permanent technology shock,  $\epsilon_t$  is a domestic production stationary technology shock,  $\phi$  is fixed costs. Fixed costs grow with technology rate in order to ensure that profits are zero at steady state, and do not become systematically positive because of the presence of monopolistical power. Costs of exit or entry on the production market of intermediate good  $j$  are

considered to be zero.

Permanent technology level  $z_t$  follows a unit root process (with  $\mu_{z,t} > 1$ ):

$$z_t = \mu_{z,t} z_{t-1} \quad (3.7)$$

while technology growth rate follows a stochastic process as a mean between steady state value and past value:

$$\mu_{z,t} = (1 - \rho_{\mu_z}) \mu_z + \rho_{\mu_z} \mu_{z,t-1} + \varepsilon_{\mu_z,t} \quad (3.8)$$

Domestic stationary technology shock  $\varepsilon_t$  is assumed to have expected value 1 (note that the model will be written in log linear form and at steady state the shock will be zero since  $\ln 1 = 0$ )

Any intermediate good producing firm  $j$  faces a cost minimization problem (assume that  $P_{j,t}$  is given, the firm is constrained to produce  $Y_{j,t}$ ):

$$\min_{K_{j,t}, H_{j,t}} W_t R_t^f H_{j,t} + R_t^k K_{j,t} \quad (3.9)$$

subject to production function (3.6). Where  $W_t$  is the nominal wage,  $R_t^f$  is gross rate paid by the firm,  $R_t^k$  is rental rate of capital. A working capital channel is introduced by assuming that a fraction of firms  $\nu_t$  borrow money to finance their wage bill in advance. If the gross nominal economy wide interest rate is  $R_t$  then the rental rate paid by the firms is:

$$R_t^f = \nu_t R_{t-1} + (1 - \nu_t) \quad (3.10)$$

In terms of Lagrange multiplier ( $\lambda_t P_{j,t}$ ) the cost minimization problem can be written (note that  $\lambda_t P_{j,t}$  is nominal marginal cost, whereas  $\lambda_t$  is real marginal cost):

$$\min_{K_{j,t}, H_{j,t}} W_t R_t^f H_{j,t} + R_t^k K_{j,t} + \lambda_t P_{j,t} (Y_{j,t} - z_t^{1-\alpha} \varepsilon_t K_{j,t}^\alpha H_{j,t}^{1-\alpha} + z_t \phi) \quad (3.11)$$

First order condition with respect to  $H_{j,t}$  is:

$$W_t R_t^f = (1 - \alpha) \lambda_t P_{j,t} z_t^{1-\alpha} \epsilon_t K_{j,t}^\alpha H_{j,t}^{-\alpha} \quad (3.12)$$

and with respect to  $K_{j,t}$ :

$$R_t^k = \alpha \lambda_t P_{j,t} z_t^{1-\alpha} \epsilon_t K_{j,t}^{\alpha-1} H_{j,t}^{1-\alpha}. \quad (3.13)$$

Using (3.12) and (3.13) can be shown that the real marginal cost is:

$$MC_t^d = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^\alpha (R_t^k)^\alpha (W_t R_t^f)^{1-\alpha} \left( \frac{1}{z_t} \right)^{1-\alpha} \frac{1}{\epsilon_t} \quad (3.14)$$

The problem of price setting faced by the intermediate firm is similar to the one in Calvo (1983). In any period, each intermediate firm faces a random probability (an exogenous Poisson process) of  $1 - \xi_d$  that it is permitted to reoptimize its price (independent of last price adjustment). The average price duration (expected time between price adjustment) is  $\frac{1}{1 - \xi_d}$  (see Walsh (2010, pg. 241)).

Let the reoptimized price be  $P_t^{new}$ . Since all reoptimizing firms at time  $t$  face the same problem, they will choose the same price level (see Walsh (2010, pg. 334)). If a firm is not allowed to optimize its price it will update the price using a rule of thumb, price will be updated by the one-period lagged realized gross inflation rate<sup>3</sup> (where  $\pi_t = \frac{P_t}{P_{t-1}}$ ):  $P_{t+1} = \pi_t P_t$ , therefore<sup>4</sup> if the firm is not allowed to change its price for  $s$  periods ahead the updated price will be  $P_{t+s} = \pi_t \pi_{t+1} \dots \pi_{t+s-1} P_t^{new}$ . Profit maximization problem faced by the firm is:

$$\max_{P_t^{new}} E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s v_{t+s} [(\pi_t \pi_{t+1} \dots \pi_{t+s-1} P_t^{new} Y_{i,t+s} - MC_{i,t+s}^d (Y_{i,t+s} + \phi z_{t+s}))] \quad (3.15)$$

where stochastic discount factor  $(\beta \xi_d)^s v_{t+s}$  used is conditional upon utility and price adjustment parameter. Using the demand schedule (3.4), the first order

<sup>3</sup>lagged inflation is used in order to allow for lagged inflation in Phillips curve

<sup>4</sup>in (Adolfson et al., 2007) price indexation is an average between lagged inflation and leading target, I choose instead a more common rule used in literature, see (Holmberg, 2006)

condition of the problem above can be written as follows:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s v_{t+s} \left( \frac{\pi_t}{\pi_{t+s}} \right)^{-\frac{\lambda_{d,t+s}}{\lambda_{d,t+s}-1}} Y_{t+s} P_{t+s} \left( \frac{\pi_t}{\pi_{t+s}} \frac{P_t^{new}}{P_t} - \frac{\lambda_{d,t} MC_{i,t+s}}{P_{t+s}} \right) = 0. \quad (3.16)$$

Using aggregate price index (3.5) an equation for price (as average between optimized and update price) can be obtained:

$$P_t = \left[ \xi_d (P_{t-1} \pi_{t-1})^{\frac{1}{1-\lambda_{d,t}}} + (1 - \xi_d) (P_t^{new})^{\frac{1}{1-\lambda_{d,t}}} \right]^{1-\lambda_{d,t}} \quad (3.17)$$

Log-linearization<sup>5</sup> and combination of (3.16) and (3.17) yield an aggregate Phillips curve<sup>6</sup>:

$$\begin{aligned} \hat{\pi}_t - \hat{\pi}_t^c &= \frac{\beta}{1 + \beta} (E_t \hat{\pi}_{t+1} - \rho_\pi \hat{\pi}_t^c) + \frac{1}{1 + \beta} (\hat{\pi}_{t-1} - \hat{\pi}_t^c) \\ &\quad - \frac{\beta(1 - \rho_\pi)}{1 + \beta} \hat{\pi}_t^c + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d(1 + \beta)} (\widehat{m}c_t + \hat{\lambda}_{d,t}) \end{aligned} \quad (3.18)$$

where hat variables mean log-linearized variables:  $\widehat{m}c_t$  means log-linearized real marginal cost and  $\hat{\lambda}_{d,t}$  is log-linearized markup. Log-linearized real marginal cost can be obtained by stationarizing real marginal cost equation (3.14) (stationarized variables will be denoted by small letters)<sup>7</sup>:

$$\begin{aligned} \widehat{m}c_t &= \alpha \hat{r}_t^k + (1 - \alpha) (\hat{w}_t + \hat{R}_t^f) - \hat{e}_t \\ &= \alpha (\hat{\mu}_{z,t} + \hat{H}_t - \hat{k}_t) + \hat{w}_t + \hat{R}_t^f - \hat{e}_t \end{aligned} \quad (3.19)$$

where the second relation is obtained by substituting log-linear equation of rental rate of capital in the first relationship:

$$\hat{r}_t^k = \hat{\mu}_{z,t} + \hat{w}_t + \hat{R}_t^f - \hat{H}_t - \hat{k}_t. \quad (3.20)$$

<sup>5</sup> see Uhlig (1999) for log-linearization procedure

<sup>6</sup> for detailed steps please see McCandless (2008, pg. 261-279)

<sup>7</sup> the nominal wage is stationarized with price and technology  $w_t = \frac{W_t}{P_t z_t}$ ; gross rental rate of capital is stationarized with price level  $r_t^k = \frac{R_t^k}{P_t}$ ; and capital is stationarized with lagged technology for convenience  $k_{t+1} = \frac{K_{t+1}}{z_t}$

Where  $\hat{R}_t^f$  is given by log-linearization of (3.10):

$$\hat{R}_t^f = \frac{\nu R}{\nu R + 1 - \nu} \hat{R}_{t-1} + \frac{\nu(R-1)}{\nu R + 1 - \nu} \hat{\nu}_t. \quad (3.21)$$

Log-linearizing markup (3.2) and technology growth rate (3.7):

$$\hat{\lambda}_{d,t} = \rho_{\lambda_d} \hat{\lambda}_{d,t-1} + \varepsilon_{\lambda_d,t} \quad (3.22)$$

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \varepsilon_{z,t} \quad (3.23)$$

### 3.1.2 Importers

Importing sector is divided into two: some firms import consumption goods  $C^m$  and others investment goods  $I^m$ . These firms buy a homogenous good from the world market and transform it into a differentiated consumption or investment good. There is a continuum of these firms in each category. Homogenous imported good is bought at foreign price  $P^*$ .

The final imported consumption good  $C_t^m$  is a Dixit-Stiglitz aggregate of a continuum  $j \in (0, 1)$  of imported consumption goods:

$$C_t^m = \left[ \int_0^1 (C_{j,t}^m)^{\frac{1}{\lambda_t^{m,c}}} dj \right]^{\lambda_t^{m,c}} \quad (3.24)$$

also the final investment good:

$$I_t^m = \left[ \int_0^1 (I_{j,t}^m)^{\frac{1}{\lambda_t^{m,i}}} dj \right]^{\lambda_t^{m,i}} \quad (3.25)$$

where  $\lambda_t^{m,c}, \lambda_t^{m,i} \in [1, \infty)$  are the markups and follow a process similar to (3.2). The first order condition of the cost minimization problem leads to the following demand for individual imported consumption good:

$$C_{j,t}^m = \left( \frac{P_{j,t}^{m,c}}{P_t^{m,c}} \right)^{-\frac{\lambda_t^{m,c}}{\lambda_t^{m,c}-1}} C_t^m \quad (3.26)$$

and individual demand for imported investment goods:

$$I_{j,t}^m = \left( \frac{P_{j,t}^{m,i}}{P_t^{m,i}} \right)^{-\frac{\lambda_t^{m,i}}{\lambda_t^{m,i}-1}} I_t^m. \quad (3.27)$$

Following Monacelli (2003), in order to allow for incomplete pass-through a local currency pricing is assumed. Calvo type pricing is assumed for local markets. An importing consumption good firm faces a random probability of  $1 - \xi_{m,c}$  that it can reoptimize its price, the same for imported investment goods ( $1 - \xi_{m,i}$ ). If an importing firm is not allowed to reoptimize its price, then it will update it by a rule of thumb similar to domestic producers  $P_{t+1}^{m,c} = \pi_t^{m,c} P_t^{m,c}$  for imported consumption goods and  $P_{t+1}^{m,i} = \pi_t^{m,i} P_t^{m,i}$  for imported investment goods. Nominal marginal cost of an importing firm is given by foreign price times the exchange rate ( $S_t P_t^*$ ), whereas real marginal cost is given by  $\frac{S_t P_t^*}{P_t^{m,c}}$  for importing consumption goods firm and  $\frac{S_t P_t^*}{P_t^{m,i}}$  for importing investment goods firm. When an importing consumption good firm  $j$  is allowed to reoptimize its price it faces the following problem:

$$\max_{P_{new,t}^{m,c}} E_t \sum_{s=0}^{\infty} (\beta \xi_{m,c})^s v_{t+s} \left[ \pi_t^{m,c} \dots \pi_{t+s-1}^{m,c} P_{new,t}^{m,c} C_{j,t+s}^m - S_{t+s} P_{t+s}^* (C_{j,t+s}^m + z_{t+s} \phi^{m,c}) \right] \quad (3.28)$$

and the same for investment good importing firm:

$$\max_{P_{new,t}^{m,i}} E_t \sum_{s=0}^{\infty} (\beta \xi_{m,i})^s v_{t+s} \left[ \pi_t^{m,i} \dots \pi_{t+s-1}^{m,i} P_{new,t}^{m,i} I_{j,t+s}^m - S_{t+s} P_{t+s}^* (I_{j,t+s}^m + z_{t+s} \phi^{m,i}) \right]. \quad (3.29)$$

Inserting (3.26) into (3.28), the following first order condition is obtained:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_{m,c})^s v_{t+s} \left( \frac{\pi_t^{m,c}}{\pi_{t+s}^{m,c}} \right)^{-\frac{\lambda_t^{m,c}}{\lambda_t^{m,c}-1}} C_{t+s}^m P_{t+s}^{m,c} \left[ \frac{\pi_t^{m,c} P_{new,t}^{m,c}}{\pi_{t+s}^{m,c} P_t^{m,c}} - \frac{\lambda_{t+s}^{m,c} S_{t+s} P_{t+s}^*}{P_t^{m,c}} \right] = 0. \quad (3.30)$$

For investment importing firm, inserting (3.27) into (3.29) leads to a similar first

order condition:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_{m,i})^s v_{t+s} \left( \frac{\pi_t^{m,i}}{\pi_{t+s}^{m,i}} \right)^{-\frac{\lambda_t^{m,i}}{\lambda_t^{m,i}-1}} I_{t+s}^m P_{t+s}^{m,i} \left[ \frac{\pi_t^{m,i}}{\pi_{t+s}^{m,i}} \frac{P_{new,t}^{m,i}}{P_t^{m,i}} - \frac{\lambda_{t+s}^{m,i} S_{t+s} P_{t+s}^*}{P_t^{m,i}} \right] = 0. \quad (3.31)$$

Similarly as in (3.17), an imported consumption price index can be derived:

$$P_t^{m,c} = \left[ \xi_{m,c} (P_{t-1}^{m,c} \pi_{t-1}^{m,c})^{\frac{1}{1-\lambda_t^{m,c}}} + (1 - \xi_{m,c}) (P_{new,t}^{m,c})^{\frac{1}{1-\lambda_t^{m,c}}} \right]^{1-\lambda_t^{m,c}} \quad (3.32)$$

the same for investment importing good firm:

$$P_t^{m,i} = \left[ \xi_{m,i} (P_{t-1}^{m,i} \pi_{t-1}^{m,i})^{\frac{1}{1-\lambda_t^{m,i}}} + (1 - \xi_{m,i}) (P_{new,t}^{m,i})^{\frac{1}{1-\lambda_t^{m,i}}} \right]^{1-\lambda_t^{m,i}}. \quad (3.33)$$

Log linearizing and combining equations (3.30) and (3.32) a Phillips curve, as in (3.18), for consumption importing goods firm is obtained:

$$\begin{aligned} \hat{\pi}_t^{m,c} - \hat{\pi}_t^c &= \frac{\beta}{1+\beta} (E_t \hat{\pi}_{t+1}^{m,c} - \rho_\pi \hat{\pi}_t^c) + \frac{1}{1+\beta} (\hat{\pi}_{t-1}^{m,c} - \hat{\pi}_t^c) \\ &\quad - \frac{\beta(1-\rho_\pi)}{1+\beta} \hat{\pi}_t^c + \frac{(1-\xi_{m,c})(1-\beta\xi_{m,c})}{\xi_{m,c}(1+\beta)} (\widehat{m}c_t^{m,c} + \hat{\lambda}_t^{m,c}) \end{aligned} \quad (3.34)$$

the same can be done for investment good importing firms, log linearizing and combining equations (3.31) and (3.33), investment importing good firm has the following Phillips curve:

$$\begin{aligned} \hat{\pi}_t^{m,i} - \hat{\pi}_t^c &= \frac{\beta}{1+\beta} (E_t \hat{\pi}_{t+1}^{m,i} - \rho_\pi \hat{\pi}_t^c) + \frac{1}{1+\beta} (\hat{\pi}_{t-1}^{m,i} - \hat{\pi}_t^c) \\ &\quad - \frac{\beta(1-\rho_\pi)}{1+\beta} \hat{\pi}_t^c + \frac{(1-\xi_{m,i})(1-\beta\xi_{m,i})}{\xi_{m,i}(1+\beta)} (\widehat{m}c_t^{m,i} + \hat{\lambda}_t^{m,i}). \end{aligned} \quad (3.35)$$

Log linearizing of real marginal costs yields:

$$\widehat{m}c_t^{m,c} = \hat{p}_t^* + \hat{s}_t - \hat{p}_t^{m,c} \quad (3.36)$$

$$\widehat{m}c_t^{m,i} = \hat{p}_t^* + \hat{s}_t - \hat{p}_t^{m,i}. \quad (3.37)$$

Log linearizing markup process equations similar to (3.22) is obtained:

$$\hat{\lambda}_t^{m,c} = \rho_{\lambda^{m,c}} \hat{\lambda}_{t-1}^{m,c} + \varepsilon_{\lambda^{m,c},t} \quad (3.38)$$

$$\hat{\lambda}_t^{m,i} = \rho_{\lambda^{m,i}} \hat{\lambda}_{t-1}^{m,i} + \varepsilon_{\lambda^{m,i},t}. \quad (3.39)$$

### 3.1.3 Exporters

Consider a continuum ( $j \in (0, 1)$ ) of exporting firms that buys a homogenous good on domestic market and transforms it into a differentiated good to be sold on foreign market. The marginal cost of an exporting firm is the price paid for domestic good ( $P_t$ ). Since our country is considered a small open economy, it plays a minor role in determining aggregate foreign consumption. Assuming that the aggregate foreign consumption and investment follow a CES function (assuming a continuum  $l \in (0, 1)$  of countries):

$$C_t^* = \left[ \int_0^1 C_{l,t}^{\frac{\eta_f-1}{\eta_f}} dl \right]^{\frac{\eta_f}{\eta_f-1}} \quad (3.40)$$

and<sup>8</sup>

$$I_t^* = \left[ \int_0^1 I_{l,t}^{\frac{\eta_f-1}{\eta_f}} dl \right]^{\frac{\eta_f}{\eta_f-1}}. \quad (3.41)$$

Cost minimization problem of foreign market yields foreign consumption or investment demand for domestic good:

$$C_t^x = \left[ \frac{P_t^x}{P_t^*} \right]^{-\eta_f} C_t^* \quad (3.42)$$

and

$$I_t^x = \left[ \frac{P_t^x}{P_t^*} \right]^{-\eta_f} I_t^* \quad (3.43)$$

Similar to importing firms each exporting firm  $j$  faces a demand for its prod-

---

<sup>8</sup>note that by choosing the same elasticity of substitution  $\eta_f$  between investment or consumption goods on foreign market allows us to consider foreign output as the only demand variable and we don't need to track whether exported goods are used for consumption or investment, see Adolfson et al. (2007)

uct:

$$X_{j,t} = \left[ \frac{P_{j,t}^x}{P_t^x} \right]^{-\frac{\lambda_{x,t}}{\lambda_{x,t}-1}} X_t \quad (3.44)$$

where  $\lambda_{x,t}$  is the time-varying markup of exporting firms and follows a stochastic process similar to (3.2), or, in log-linearized form:

$$\hat{\lambda}_{x,t} = \rho_{\lambda_x} \hat{\lambda}_{x,t-1} + \varepsilon_{\lambda_{x,t}}. \quad (3.45)$$

Export prices are assumed to be sticky in the foreign currency, in order to allow for incomplete exchange rate pass-through on the export market. Calvo type pricing is assumed. In any given period, an exporter can reoptimize its price with a given probability  $1 - \xi_x$ , with probability  $\xi_x$  prices won't be optimized, but will be updated with a rule of thumb:  $P_{t+1}^x = \pi_t^x P_t^x$ . Profit maximization (taking into account the probability of being able to optimize price) problem is:

$$\max_{P_{new,t}^x} E_t \sum_{s=0}^{\infty} (\beta \xi_x)^s v_{t+s} \left[ \pi_t^x \dots \pi_{t+s-1}^x P_{new,t}^x X_{j,t+s}^m - \frac{P_{t+s}}{S_{t+s}} (X_{j,t+s} + z_{t+s} \phi^x) \right] \quad (3.46)$$

subject to (3.44). Log-linearization of the FOC yields a Phillips curve for inflation of export prices:

$$\begin{aligned} \hat{\pi}_t^x - \hat{\pi}_t^c &= \frac{\beta}{1+\beta} (E_t \hat{\pi}_{t+1}^x - \rho_{\pi} \hat{\pi}_t^c) + \frac{1}{1+\beta} (\hat{\pi}_{t-1}^x - \hat{\pi}_t^c) \\ &\quad - \frac{\beta(1-\rho_{\pi})}{1+\beta} \hat{\pi}_t^c + \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\beta)} (\widehat{mc}_t^x + \hat{\lambda}_{x,t}) \end{aligned} \quad (3.47)$$

where  $\widehat{mc}_t^x$  is log linearized real marginal cost and follows a process similar to (3.38):

$$\widehat{mc}_t^x = \hat{p}_t - \hat{s}_t - \hat{p}_t^x. \quad (3.48)$$

## 3.2 Households

A continuum  $j \in (0, 1)$  of households, that maximizes utility gain from consumption, leisure and cash balances (non interest bearing form), subject to a budget constraint, is considered. When maximizing their utility, households decide on:

current consumption, cash holdings, labor supply, domestic and foreign bond holdings, investment, capital utilization rate and capital stock. Any household  $j$  has the following single period utility function:

$$\zeta_t^c \ln(C_{j,t} - bC_{j,t-1}) - \zeta_t^h A_L \frac{h_{j,t}^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t}\right)^{1-\sigma_q}}{1-\sigma_q} \quad (3.49)$$

where  $C_{j,t}$  is the current level of consumption (internal habit in consumption is introduced via the lagged consumption term in the utility  $bC_{j,t-1}$ <sup>9</sup>),  $A_L$  is the labor disutility constant,  $h_{j,t}$  is labor supplied by the household,  $\sigma_L$  is labor supply elasticity, and  $\sigma_q$  is the curvature parameter related to money demand,  $A_q$  is the constant related to non interest bearing assets' ( $Q_{j,t}$ ) utility, these assets are stationarized by rendering them real (divide by  $P_t$ ) and taking out the common trend induced by  $z_t$ . Finally  $\zeta_t^c$  and  $\zeta_t^h$  are consumption preference and labor supply shocks that have steady state value of 1. Following Monacelli (2003) household's consumption is a bundle of domestic and imported consumption goods:

$$C_t = \left[ (1 - \omega_c)^{\frac{1}{\eta_c}} (C_t^d)^{\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} (C_t^m)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}} \quad (3.50)$$

where  $\eta_c$  is the elasticity of substitution between domestic and imported consumption goods and  $\omega_c$  is the share of imported consumption goods in total consumption. Besides deciding how much to consume, households must divide their consumption expenditure between two types of goods. The first order condition of the cost minimization problem, subject to aggregated consumption bundle (3.50), yields the following demands for domestic and imported consumption goods:

$$C_t^d = (1 - \omega_c) \left( \frac{P_t}{P_t^c} \right)^{-\eta_c} C_t \quad (3.51)$$

and

$$C_t^m = \omega_c \left( \frac{P_t^{m,c}}{P_t^c} \right)^{-\eta_c} C_t \quad (3.52)$$

---

<sup>9</sup>habit in consumption is introduced to match empirical evidence of consumption persistence

where  $P_t^c$  is the CPI price index and can be obtained by inserting individual demands (3.51) and (3.52) into expenditure relationship  $P_t^c C_t = P_t C_t^d + P_t^{m,c} C_t^m$  and taking into account the relationship (3.50):

$$P_t^c = \left[ (1 - \omega_c) (P_t)^{1-\eta_c} + \omega_c (P_t^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \quad (3.53)$$

In order to increase their capital stock, households must purchase investment goods. As in the case of consumption, investment is a bundle between domestically produced and imported investment goods:

$$I_t = \left[ (1 - \omega_i)^{\frac{1}{\eta_i}} (I_t^d)^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} (I_t^m)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}} \quad (3.54)$$

where  $\eta_i$  is the elasticity of substitution between domestic and imported investment goods and  $\omega_i$  is the share of imported investment goods in total investment. Since the domestic producer produces a homogenous good  $Y_t$  at price  $P_t$ , domestically consumption and investment goods will have the same price  $P_t$ . Cost minimization problem leads to similar individual demand functions:

$$I_t^d = (1 - \omega_i) \left( \frac{P_t}{P_t^i} \right)^{-\eta_i} I_t \quad (3.55)$$

and

$$I_t^m = \omega_i \left( \frac{P_t^{m,i}}{P_t^i} \right)^{-\eta_i} I_t \quad (3.56)$$

with the similar aggregated investment price:

$$P_t^i = \left[ (1 - \omega_i) (P_t)^{1-\eta_i} + \omega_i (P_t^{m,i})^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}}. \quad (3.57)$$

A standard RBC literature law of motion of capital is considered:

$$\bar{K}_{t+1} = (1 - \delta) \bar{K}_t + \Upsilon_t F(I_t, I_{t-1}) + \Delta_t \quad (3.58)$$

where  $\bar{K}_t$  is the physical capital stock,  $\delta$  is the depreciation rate of capital stock,  $F(I_t, I_{t-1})$  is a function that transforms investment into capital,  $\Upsilon_t$  is the invest-

ment shock (with a steady state value of 1),  $\Delta_t$  represents either newly bought capital if it is positive or sold capital if it is negative. Following Christiano et al. (2005) a specific form is adopted for  $F(I_t, I_{t-1})$

$$F(I_t, I_{t-1}) = \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \quad (3.59)$$

where  $\tilde{S}$  function has the following properties:  $\tilde{S}(\mu_z) = \tilde{S}'(\mu_z) = 0$  and  $\tilde{S}''(\mu_z) \equiv \tilde{S}'' > 0$  is the investment adjustment costs.

Households face the following budget constraint:

$$\begin{aligned} & M_{j,t+1} + S_t B_{j,t+1}^* + P_t^c C_{j,t} (1 + \tau_t^c) + P_t^i I_{j,t} + P_t (a(u_{j,t}) \bar{K}_{j,t} + P_{k',t} \Delta_t) \\ &= R_{t-1} (M_{j,t} - Q_{j,t}) + Q_{j,t} + (1 - \tau_t^k) \Pi_t + (1 - \tau_t^y) \frac{W_{j,t}}{1 + \tau_t^w} h_{j,t} \\ &+ (1 - \tau_t^k) R_t^k u_{j,t} \bar{K}_{j,t} + R_{t-1}^* \Phi\left(\frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1}\right) S_t B_{j,t}^* \\ &- \tau_t^k \left[ (R_{t-1} - 1)(M_{j,t} - Q_{j,t}) + \left( R_{t-1}^* \Phi\left(\frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1}\right) - 1 \right) S_t B_{j,t}^* \right] \\ &- \tau_t^k B_{j,t}^* (S_t - S_{t-1}) + TR_t \end{aligned} \quad (3.60)$$

where  $M_{j,t+1}$  is the total money stock,  $S_t$  is the nominal exchange rate,  $B_{j,t+1}^*$  is the foreign zero coupon bond holdings (bought at the moment  $t$  with payoff 1 at  $t + 1$ ),  $\tau_t^c$  is the tax on consumption (VAT),  $a(u_{j,t})P_t$  is the the cost paid by the households to adjust capital utilization rate  $u_t$ ,  $P_{k',t}$  is the price of capital,  $R_{t-1}$  is the gross interest rate<sup>10</sup>,  $M_{j,t} - Q_{j,t}$  are the amounts of money holded as deposits,  $\tau_t^k$  is the capital income tax,  $\tau_t^y$  is the tax on income,  $\tau_t^w$  is the tax on wages (social contributions),  $R_t^k$  is rental rate of capital,  $R_t^*$  is foreign gross interest rate,  $\Phi\left(\frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1}\right)$  is the premium paid by foreign bonds and depends on a time varying risk premium shock  $\tilde{\phi}_{t-1}$  and stationarized net foreign asset position  $\frac{A_{t-1}}{z_{t-1}}$  where  $A_t \equiv \frac{S_t B_{t+1}^*}{P_t}$ ,  $TR_t$  are the lump sum governmental transfers.

Utility maximization problem, subject to budget constraints and capital motion

---

<sup>10</sup> $R_t = 1 + r_t$

equation is<sup>11</sup>:

$$\max_{C_{j,t}, M_{j,t+1}, \Delta_t, \bar{K}_{j,t}, I_{j,t}, u_{j,t}, Q_{j,t}, B_{j,t}^*, h_{j,t}} E_0^j \sum_{t=0}^{\infty} \beta^t [U_t + v_t BC_t + \omega_t KME_{t+1}] \quad (3.61)$$

where  $\beta$  is the discount factor,  $U_t$  single period utility defined in (3.49),  $BC_t$  is the budget constraint defined in (3.60),  $KME_{t+1}$  is the capital motion law defined in (3.58),  $v_t$  and  $\omega_t$  are lagrangian multipliers. All variables are stationarized<sup>12</sup> with technology level  $z_t$ . A new lagrangian multiplier is defined as  $\psi_{z,t} = z_t \psi_t = z_t P_t v_t$ . Taking derivatives with respect to decision variables<sup>13</sup> yields the following first order condition:

Derivative with respect to  $c_t$ :

$$\frac{\zeta_t^c}{c_t + bc_{t-1} \frac{1}{\mu_{z,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1} \mu_{z,t+1} - bc_t} - \psi_{z,t} \frac{P_t^c}{P_t} (1 + \tau_t^c) = 0. \quad (3.62)$$

Derivative with respect to  $m_{t+1}$ :

$$- \psi_{z,t} + \beta E_t \left[ \frac{\psi_{z,t+1} R_t}{\mu_{z,t+1} \pi_{t+1}} - \frac{1}{\mu_{z,t+1}} \frac{\psi_{z,t+1}}{\pi_{t+1}} \tau_{t+1}^k (R_t - 1) \right] = 0. \quad (3.63)$$

Derivative with respect to  $\Delta_t$ :

$$- \psi_t P_{k',t} + \omega_t = 0. \quad (3.64)$$

Derivative with respect to  $\bar{k}_{t+1}$ :

$$- P_{k',t} \psi_{z,t} + \beta E_t \left[ \frac{\psi_{z,t+1}}{\mu_{z,t+1}} ((1 - \delta) P_{k',t+1} + (1 - \tau_{t+1}^k) r_{t+1}^k u_{t+1} - a(u_{t+1})) \right] = 0. \quad (3.65)$$

<sup>11</sup>an assumption is made that assures that household will not become heterogeneous, households are allowed to enter in an insurance market, they can insure against any type of risk by purchasing a portofolio of securities, as a result a representative agent framework is preserved and it is not needed to keep track of entire distribution of households' wealth.

<sup>12</sup>small caps denote stationarized variables

<sup>13</sup>decision problem with respect to labor supply  $h_{j,t}$  is discussed in Sticky Wages section (3.3).

Derivative with respect to  $i_t$ :

$$-\psi_{z,t} \frac{P_t^i}{P_t} + P_{k',t} \Upsilon_t F_1(i_t, i_{t-1}) \mu_{z,t} + \beta E_t \left[ P_{k',t+1} \frac{\psi_{z,t+1}}{\mu_{z,t+1}} \Upsilon_{t+1} F_2(i_{t+1}, i_t) \mu_{z,t+1} \right] = 0 \quad (3.66)$$

where  $F_1(I_t, I_{t-1}) \equiv \frac{\partial F(I_t, I_{t-1})}{\partial I_t}$  and  $F_2(I_t, I_{t-1}) \equiv \frac{\partial F(I_t, I_{t-1})}{\partial I_{t-1}}$ .

Derivative with respect to  $u_t$ :

$$\psi_{z,t} \left( (1 - \tau_t^k) r_t^k - a'(u_t) \right) = 0. \quad (3.67)$$

Derivative with respect to  $q_t$ :

$$A_q q_t^{-\sigma_q} - (1 - \tau_t^k) \psi_{z,t} (R_{t-1} - 1) = 0. \quad (3.68)$$

Derivative with respect to  $b_{t+1}^*$ :

$$\begin{aligned} & -\psi_{z,t} S_t + \beta E_t \left[ \frac{\psi_{z,t+1}}{\mu_{z,t+1} \pi_{t+1}} (S_{t+1} R_t^* \Phi(a_t, \tilde{\phi}_t) \right. \\ & \left. - \tau_{t+1}^k S_{t+1} (R_t^* \Phi(a_t, \tilde{\phi}_t) - 1) - \tau_{t+1}^k (S_{t+1} - S_t) \right] = 0. \end{aligned} \quad (3.69)$$

Combination of derivative with respect to  $m_{t+1}$  (3.63) and with respect to  $b_{t+1}^*$  (3.69), and log linearization yields an UIP condition:

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* + \tilde{\phi}_a \hat{a}_t - \hat{\phi}_t \quad (3.70)$$

where premium of foreign bonds is assumed to follow the process:  $\Phi(a_t, \tilde{\phi}_t) = e^{\tilde{\phi}_t - \tilde{\phi}_a(a_t - \bar{a})}$ .

Log linearized preference shocks follow a stochastic process similar to (3.22):

$$\hat{\zeta}_t^c = \rho_{\zeta^c} \hat{\zeta}_{t-1}^c + \varepsilon_{\zeta^c,t} \quad (3.71)$$

$$\hat{\zeta}_t^h = \rho_{\zeta^h} \hat{\zeta}_{t-1}^h + \varepsilon_{\zeta^h,t}. \quad (3.72)$$

Log linearization of (3.62) yields a Euler equation for consumption:

$$\begin{aligned}
E_t[ & -b\beta\mu_z\hat{c}_{t+1} + (\mu_z^2 + b^2\beta)\hat{c}_t - b\mu_z\hat{c}_{t-1} + b\mu_z(\hat{\mu}_{z,t} - \beta\hat{\mu}_{z,t+1}) + \\
& + (\mu_z - b\beta)(\mu_z - b)\hat{\psi}_{z,t} + \frac{\tau^c}{1 + \tau^c}(\mu_z - b\beta)(\mu_z - b)\hat{\tau}_t^c + \\
& + (\mu_z - b\beta)(\mu_z - b)\hat{\gamma}_t^{c,d} - (\mu_z - b)(\mu_z\hat{\zeta}_t^c - b\beta\hat{\zeta}_{t+1}^c)] = 0 \quad (3.73)
\end{aligned}$$

where  $\hat{\gamma}_t^{c,d}$  is the log linearized price ratio (relative price) between domestic consumption price index and domestic production price index  $\gamma_t^{c,d} \equiv \frac{P_t^c}{P_t}$ .

Log linearization of (3.63) yields:

$$E_t \left[ -\mu\hat{\psi}_{z,t} + \mu\hat{\psi}_{z,t+1} - \mu\hat{\mu}_{z,t+1} + (\mu - \beta\tau^k)\hat{R}_t - \mu\hat{\pi}_{t+1} + \frac{\tau^k}{1 - \tau^k}(\beta - \mu)\hat{\tau}_{t+1}^k \right] = 0. \quad (3.74)$$

Log linearization of (3.65) yields:

$$\begin{aligned}
E_t[ & \hat{\psi}_{z,t} + \hat{\mu}_{z,t+1} - \hat{\psi}_{z,t+1} - \frac{\beta(1 - \delta)}{\mu_z}\hat{P}_{k',t+1} + \hat{P}_{k',t} \\
& - \frac{\mu_z - \beta(1 - \delta)}{\mu_z}(\hat{r}_{t+1}^k + \frac{\tau^k}{1 - \tau^k}\hat{\tau}_{t+1}^k)] = 0. \quad (3.75)
\end{aligned}$$

Log linearization of (3.66) yields:

$$E_t[\hat{P}_{k',t} + \hat{Y}_t - \hat{\gamma}_t^{i,d} - \hat{\mu}_z^2\tilde{S}''((\hat{i}_t - \hat{i}_{t-1}) - \beta(\hat{i}_{t+1} - \hat{i}_t) + \hat{\mu}_{z,t} - \beta\hat{\mu}_{z,t+1})] = 0. \quad (3.76)$$

Log linearization of (3.68) yields:

$$\hat{q}_t = \frac{1}{\sigma_q} \left[ \frac{\tau^k}{1 - \tau^k}\hat{\tau}_t^k - \hat{\psi}_{z,t} - \frac{R}{R - 1}\hat{R}_{t-1} \right]. \quad (3.77)$$

Log linear expression of capital utilization rate is:

$$\hat{u}_t = \hat{k}_t - \hat{\bar{k}}_t. \quad (3.78)$$

Log linearization of capital motion equation:

$$\hat{k}_{t+1} = (1 - \delta)\frac{1}{\mu_z}\hat{k}_t - (1 - \delta)\frac{1}{\mu_z}\hat{\mu}_{z,t} + \left(1 - (1 - \delta)\frac{1}{\mu_z}\right) (\hat{Y}_t + \hat{i}_t). \quad (3.79)$$

### 3.3 Sticky Wages

Following Erceg et al. (2000) approach, a continuum ( $j \in (0, 1)$ ) of monopolistically competitive households is assumed. Each household supplies a differentiated labor service to domestic firms. This assumption implies that households can set their wages. After setting their wages they supply labor to domestic firms. A labor aggregator (employment agency) is assumed for convenience in order to give households monopolistical power and to introduce sticky wages. Labor index aggregator  $H_t$  has the Dixit-Stiglitz form:

$$H_t = \left[ \int_0^1 h_{j,t}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} \quad (3.80)$$

where  $\lambda_w \in [1, \infty)$  is the wage markup. This employment agency takes input prices  $W_{j,t}$  and output price  $W_t$  (homogenous labor good price) as given. Similar to domestic good aggregator firm, since employment agency acts on a perfect competition market, cost minimization problem leads to individual demand for each differentiated labor service:

$$h_{j,t} = \left[ \frac{W_{j,t}}{W_t} \right]^{\frac{\lambda_w}{1-\lambda_w}} H_t \quad (3.81)$$

Wage aggregator index is given similarly to price index in domestic production sector:

$$W_t = \left[ \int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} dj \right]^{(1-\lambda_w)}. \quad (3.82)$$

Calvo type wage stickiness is introduced by assuming that, in each period, households face a random probability  $1-\xi_w$  that they can reoptimize their nominal wage. If a household is not allowed to reoptimize its nominal wage it will update it by a rule of thumb:

$$W_{j,t+1} = \pi_t^c \mu_{z,t+1} W_{j,t} \quad (3.83)$$

where  $\pi_t^c$  is the CPI inflation and  $\mu_{z,t+1}$  is the technology growth rate. Since nominal wage is considered, it must be updated with price and technology growth rates, as well.

If a household is allowed to reoptimize its wage it will face the following

problem (note that irrelevant terms were not included):

$$\max_{W_{j,t}^{new}} E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[ -\zeta_{t+s}^h A_L \frac{(h_{j,t+s}^{1+\sigma_L})}{1+\sigma_L} + v_{t+s} \frac{1-\tau_{t+s}^y}{1+\tau_{t+s}^w} \times \right. \\ \left. (\pi_t^c \dots \pi_{t+s-1}^c) (\mu_{z,t+1} \dots \mu_{z,t+s}) W_{j,t}^{new} h_{j,t+s} \right] \quad (3.84)$$

Including individual labor demand (3.81) in optimization problem (3.84) the following first order condition is obtained:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s h_{j,t+s} \left[ -\zeta_{t+s}^h A_L h_{j,t+s}^{\sigma_L} \right. \\ \left. + \frac{W_t^{new}}{z_t P_t} \frac{z_{t+s} v_{t+s} P_{t+s}}{\lambda_w} \frac{1-\tau_{t+s}^y}{1+\tau_{t+s}^w} \frac{\frac{P_{t+s}^c}{P_{t-1}^c}}{\frac{P_{t+s}^d}{P_t^d}} \right] \quad (3.85)$$

Log-linearization of (3.85) yields the following wage equation:

$$E_t [\eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_3 (\hat{\pi}_t^d - \hat{\pi}_t^c) + \eta_4 (\hat{\pi}_{t+1}^d - \rho_{\hat{\pi}^c} \hat{\pi}_t^c) \\ + \eta_5 (\hat{\pi}_{t-1}^c - \hat{\pi}_t^c) + \eta_6 (\hat{\pi}_t^c - \rho_{\hat{\pi}^c} \hat{\pi}_t^c) \\ + \eta_7 \hat{\psi}_{z,t}^\tau + \eta_8 \hat{H}_t + \eta_9 \hat{\tau}_t^y + \eta_{10} \hat{\tau}_t^w + \eta_{11} \hat{\zeta}_t^h] = 0 \quad (3.86)$$

where  $b_w = \frac{\lambda_w \sigma_L - (1-\lambda_w)}{(1-\beta \xi_w)(1-\xi_w)}$  and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \end{pmatrix} = \begin{pmatrix} b_w \xi_w \\ \sigma_L \lambda_w - b_w (1 + \beta \xi_w^2) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \xi_w \kappa_w \\ 1 - \lambda_w \\ -(1 - \lambda_w) \sigma_L \\ -(1 - \lambda_w) \frac{\tau^y}{1-\tau^y} \\ -(1 - \lambda_w) \frac{\tau^w}{1+\tau^w} \\ -(1 - \lambda_w) \end{pmatrix}$$

### 3.4 Employment

Adolfson et al. (2005) describe an employment equation linking labor supplied by households to employment because they do not have an observable series of worked hours for Euro area, although EUROSTAT supplies a series for worked hours in Romania; I can not use it since it is on yearly basis, and I conduct my estimation on quarterly data. So, I will adopt the same strategy and will specify the same equation. A "sticky employment" concept is introduced. It is assumed that domestic firms can not change their employment in every period (this technique is also adopted by Smets and Wouters (2003)), instead Calvo type adjustment of employment is introduced. In every period, a domestic firm may readjust its employment with a random probability  $1 - \xi_e$ , firms that are not allowed to adjust their employment keep employment from the previous period. Problem faced by employment adjusting firm is (trying to minimize the distance between optimal hours and effective hours of labor):

$$\min_{\tilde{E}_{i,t}^{new}} \sum_{s=0}^{\infty} (\beta \xi_e)^s (n_i \tilde{E}_{i,t}^{new} - H_{i,t+s})^2 \quad (3.87)$$

where  $n_i$  is the hours per worker, and  $n_i \tilde{E}_{i,t}^{new}$  is the total hours of labor firm hired. Or, the first order condition in log linear form is similar to a forward looking Phillips curve:

$$\Delta \hat{E}_t = \beta E_t[\Delta \hat{E}_{t+1}] + \frac{(1 - \xi_e)(1 - \beta \xi_e)}{\xi_e} (\hat{H}_t - \hat{E}_t). \quad (3.88)$$

### 3.5 Government

In the model, a balanced governmental budget with no governmental debt is assumed. All governmental earnings from taxation and seigniorage are spent on aquisition of goods and transfers to households. Although Adolfson et al. (2007) model log linearized fiscal variables and HP detrended governmental spending as a SVAR system, I will proceed to an easier approach and model each log linearized fiscal variable and governmental consumption as pure AR processes:

$$\hat{f}_t = \rho_f \hat{f}_{t-1} + \varepsilon_{f,t} \quad (3.89)$$

where  $f \in \{\tau^c, \tau^y, \tau^w, \tau^k, g\}$ .

### 3.6 Monetary Policy

Monetary policy is conducted by the central bank which is assumed to follow a Taylor type interest rate rule used in Smets and Wouters (2003). In the proposed interest rate rule, monetary policy responds to inflation deviation from the target (note that the central bank is interested in CPI inflation), output gap, real exchange rate gap (introduced by Adolfson et al. (2005) in order to check if monetary policy responds to real exchange rate deviation), but also to speed of output gap and inflation rate (not just level), if inflation rate or output gap grow faster monetary policy will respond more. Log linearized interest rate rule is given by

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (\hat{\pi}_t^c + r_\pi (\hat{\pi}^{c_{t-1}} - \hat{\pi}_t^c) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1}) + r_{\Delta\pi} \Delta \hat{\pi}_t^c + r_{\Delta y} \Delta \hat{y}_t + \varepsilon_t^R \quad (3.90)$$

Inflation target follows a mean reverting process similar to (3.2) or in log-linear form:

$$\hat{\pi}_t^c = \rho_\pi \hat{\pi}_{t-1}^c + \varepsilon_t^{\hat{\pi}^c}. \quad (3.91)$$

### 3.7 Market equilibrium

At equilibrium all markets clear. Clearing of domestic market means that demand for domestic goods (domestic consumption and domestic investment goods, governmental consumption goods, and exported goods) equal supply of domestic goods (domestic production function):

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x = z_t^{1-\alpha} K_t^\alpha H_t^{1-\alpha} \varepsilon_t - z_t \phi - a(u_t) \bar{K}_t. \quad (3.92)$$

Net foreign assets' market clears when domestic investment in foreign bonds equals the net position of export/import firms:

$$S_t B_{t+1}^* = S_t P_t^x (C_t^x + I_t^x) - S_t P_t^* (C_t^m + I_t^m) + R_{t-1}^* \Phi(a_{t-1}, \tilde{\phi}_{t-1}) S_t B_t^*. \quad (3.93)$$

The loan market clears when the firm's demand for loans is met by domestic household's supply of deposits and monetary injection of the central bank:

$$\nu_t W_t H_t = \mu_t M_t - Q(t) \quad (3.94)$$

### 3.8 Foreign variables

Adolfson et al. (2005) use a SVAR process to describe log-deviation of foreign variables, I will use instead a more simple approach and describe each individual variable's log-deviation from the steady state as an AR(1) process:

$$\hat{J}_t = \rho_J \hat{J}_{t-1} + \varepsilon_t^J \quad (3.95)$$

where  $J_t \in [\pi_t^*, y_t^*, R_t^*]$ . Since our economy is assumed to be small in comparison to the foreign economy it can not influence foreign market, so foreign variables will be considered as exogenous.

## 4 Estimation

### 4.1 Data

Following Adolfson et al. (2005), I choose to match the following set of sixteen variables<sup>14</sup> as observables:

- GDP growth rate ( $\Delta \ln Y_t$ )
- GDP deflator ( $1 + \pi^{GDP}$ )
- Consumption growth rate ( $\Delta \ln C_t$ )
- Consumption deflator ( $1 + \pi^C$ )
- Investment growth rate ( $\Delta \ln I_t$ )
- Investment deflator ( $1 + \pi^I$ )
- Exports growth rate ( $\Delta \ln X_t$ )
- Imports growth rate ( $\Delta \ln M_t$ )

---

<sup>14</sup>compared to Adolfson et al. (2005) I included one extra observable CPI inflation. For data sources and data description see appendix A

- Real wage growth rate ( $\Delta \ln \frac{W_t}{P_t^c}$ ), real wage is calculated as nominal wage deflated with CPI
- Employment as percentage deviation from its mean ( $\frac{E - \bar{E}}{\bar{E}}$ )
- Consumption Price Index (CPI) ( $1 + \pi^{CPI}$ )
- Real exchange rate as percentage deviation from its mean ( $\frac{x - \bar{x}}{\bar{x}}$ ), real exchange rate was calculated from nominal exchange rate, domestic CPI and Euro area HCPI
- ROBOR ON as quarterly gross rate ( $\sqrt[4]{1 + r}$ )
- Euro area 16 real GDP growth rate ( $\Delta \ln Y_t^*$ )
- Euro area 16 GDP deflator ( $1 + \pi^{GDP^*}$ )
- EURIBOR ON (Eonia) as quarterly gross rate ( $\sqrt[4]{1 + r}$ )

Available data set sample range is 2000:Q1 - 2010:Q1, because we use first difference of the logarithms first observation will be lost, implying a final data set of 40 observations that ranges from 2000Q2 to 2010Q1.

Employment and real exchange rate are expressed as deviation from their means because in our model these are stationary variables. All other real variables are expressed as growth rate. Interest rate is expressed as gross quarterly interest rate. This helps in writing the measurement equations that link our observed data to variables from our model (for issues involved in writing measurement equations see Adolfson et al. (2005), for a more general treatment of measurement equations see Smets and Wouters (2007)).

## 4.2 Calibrated Parameters

Due to the small sample size and weak identification, in the estimation procedure some of the parameters (mostly weak identified or steady state related) were kept fixed (considered as very strict prior). Taxation rates were chosen to match their current levels, due to their relative constant values: capital income tax  $\tau^k$  and labor income tax  $\tau^y$  were set to 0.16, tax on consumption  $\tau^c$  was set to 0.19 to match Romania's VAT and labor payroll tax  $\tau^w$  was set to 0.3 which represents approximately total social contributions that are paid by the employer and employee. Discount factor  $\beta$  was set to 0.999, gross money growth rate  $\mu$  was set to 1.01 and gross technology growth rate  $\mu_z$  was calibrated to 1.005. These val-

ues were chosen from Adolfson et al. (2005), impling<sup>15</sup> a 0.5% quarterly inflation rate<sup>16</sup> and 2% annual inflation rate, value chosen for discount factor together with capital income tax implies a 1.3 % quarterly nominal interest rate<sup>17</sup> or 5.3 annual nominal interes rate, also gross techology growth rate implies a 0.5% quarterly growth rate of real variables, or 2% annual growth. Share of governamental consumption  $g$  in GDP was calibrated to match sample average of  $\frac{G}{Y}$ , impling a value of 0.13. Real balances utility coefficient was calibrated to match sample average of  $\frac{M1}{M3}$  with a value of 0.46. Share of imported consumption in total consumption  $\omega_c$  and share of imported investment in total investment  $\omega_i$  were calibrated from balance of payment data, dividing the detailed imports of goods in two categories consumption and investment and taking sample's averages. This brings us to values of 0.49 and 0.57 respectively.

Share of firms that borrow money in order to finance their wage bill  $\nu$  was set to 1, impling that all firms finance their wage bills in advance. CRRA utility parameter  $\sigma_q$  and capital utilization cost  $\sigma_a$  were set to values of 10.62 and 0.049 respectively, as in Christiano et al. (2005). Quarterly depreciation rate  $\delta$  and share of capital in production function  $\alpha$  were matched from Gălătescu et al. (2007), with values of 0.33 and 0.0123 (impling an annual depreciation rate of 5%) respectively.

Following Jakab and Világi (2008) labor disutility parameter  $A_L$  was set to 8. Value of wage markup was set to 1.5 which is implied by the elasticity of substitution of labor 3 found in Jakab and Világi (2008), also an elasticity of substitution of goods of 6 implies a markup of 1.2, so I calibrated the values of markups ( $\lambda^d, \lambda^{m,c}, \lambda^{m,i}$ ) to 1.2. Investment adjustment cost  $\tilde{S}''$  was set to 13, Calvo parameter of sticky employment  $\xi_e$  was set to 0.7 and labor supply elasticity  $\sigma_L$  was set to 1 matching values used in Jakab and Világi (2008).

Inflation target persistence  $\rho_{\pi}$  was matched to the value used in Adolfson et al. (2005) of 0.975. Autoregressive coefficients of log-deviation of foreign variables were matched to AR(1) coefficients of HP detrended foreign variables, resulting in values of 0.51 for  $\rho_{y^*}$ , 0.93 for  $\rho_{R^*}$  and 0.1 for  $\rho_{\pi^*}$ .

<sup>15</sup>for steady state relations see Appendix A in Adolfson et al. (2005)

<sup>16</sup>gross inflation at steady state is  $\pi = \frac{\mu}{\mu_z}$

<sup>17</sup>at steady state gross interest rate is  $R = \frac{\pi\mu_z - \tau^k\beta}{(1-\tau^k)\beta}$

For standard deviation of shocks I choose to calibrate two of them because estimating procedure failed to determine their variance. So I set standard deviation of shock to investment to capital production function ( $\sigma_{\varepsilon\tau}$ ) to value of 0.1, and standard deviation of technology growth rate shock ( $\sigma_{\varepsilon\mu z}$ ) to 0.2.

### 4.3 Prior distributions

In choosing the prior distributions for parameters (see Table 2 in the appendix B), I followed common distribution used in literature (see Adolfson et al. (2005), Smets and Wouters (2003) or Fernández-Villaverde (2009)). For parameters that are defined on  $(0, 1)$  range I used Beta distribution, for parameters that are always positive I used Inverse Gamma<sup>18</sup> distribution, for all other parameters I used Normal distribution.

For sticky prices parameter ( $\xi$ ) I selected a Beta distribution with mean of 0.67 and standard deviation of 0.1, implying that prices adjust every 3 quarters. For sticky wages I set the mean of the beta distribution to 0.75, which means that wages adjust once per year.

For consumption habit and all autoregressive parameters I selected a Beta distribution with mean of 0.85 and standard error of 0.05 (except for  $\rho_\tau$  for which I selected a value of 0.8).

For elasticities of substitution  $\eta$  I followed specification in Adolfson et al. (2005) and selected a Inverse Gamma distribution with mode of 1.5 and 2 degrees of freedom. In describing priors Taylor interest rule and risk premium I also followed the specification used in Adolfson et al. (2005).

For standard error of shocks I selected an Inverse Gamma distribution with 4 degrees of freedom (very loose prior to let the data determine the true value) and mode was selected depending on value of estimated shocks (see Table 1 in the appendix B), for all measurement errors I set mode to 0.02 and degrees of freedom to 6.

---

<sup>18</sup>For Inverse Gamma distribution mode and degrees of freedom are described in the table

## 4.4 Estimation Procedure

For estimation of the DSGE model I used DYNARE toolbox with MATLAB<sup>®</sup> R2010a. I preferred Bayesian estimation because it has some advantages over Maximum Likelihood estimation or Impulse Response calibration. Some of the advantages pointed out by Griffoli (2010) are:

- Bayesian estimation has the advantage of being fit to estimate the whole DSGE model, rather than GMM method that is used to estimate simple equations like Phillips curve or Euler equation;
- Using of prior distribution as weights for starting points allows better likelihood estimation and avoids points where model could not be identified, but where likelihood peaks;
- Weighting likelihood with priors allows to ensure parameter identificability and avoid problem of flat likelihood (when likelihood has the same value for different set of parameters);
- Including the shocks in the estimation procedure explicitly addresses misspecified model due to observational errors.

When Bayesian starts estimation we have prior  $p(\theta)$  and  $p(Y_t^{obs}|\theta)$  the log-likelihood, and we are interested in the posterior density  $p(\theta|Y_t^{obs})$ . Bayes theorem is used twice. First:

$$p(\theta|Y_t^{obs}) = \frac{p(\theta; Y_t^{obs})}{p(Y_t^{obs})} \quad (4.1)$$

and

$$p(Y_t^{obs}|\theta) = \frac{p(\theta; Y_t^{obs})}{p(\theta)}. \quad (4.2)$$

By combining equations (4.1) and (4.2) we obtain, conditional upon model  $M$ :

$$p(\theta_M|Y_t^{obs}, M) = \frac{p(Y_t^{obs}|\theta_M, M) p(\theta_M|M)}{p(Y_t^{obs}|M)} \quad (4.3)$$

where  $p(Y_t^{obs}|M)$  is the marginal density conditional upon model  $M$ .

First DYNARE finds model's log-likelihood  $\ln L(\theta|Y_t^{obs})$  with the help of Kalman filter, where  $\theta$  is the vector of parameters and  $Y_t^{obs}$  are observed series.

Since priors are known DYNARE can compute log posterior kernel:

$$\ln K(\theta|Y_t^{obs}) = \ln L(\theta|Y_t^{obs}) + \ln p(\theta). \quad (4.4)$$

Next step is to use a numerical optimization routine<sup>19</sup> to maximize log posterior kernel. After having maximized the posterior kernel, DYNARE uses Metropolis-Hastings algorithm to simulate posterior distributions. MH algorithm is a "rejection sampling algorithm" which generates a sequence of samples (Markov Chains) from a distribution that is unknown. To simulate posterior distribution, MH algorithm uses the fact that, under general conditions, parameters will be normally distributed. First (1) MH algorithm chooses a starting point (posterior mode), then (2) it draws a candidate value  $\theta^*$  from a jumping distribution  $J(\theta^*|\theta^{t-1}) = N(\theta^{t-1}, c\Sigma_m)$ , where  $\Sigma_m$  is the inverse of hessian at posterior mode. After (3) it computes the acceptance ratio  $r = \frac{p(\theta^*|Y_t^{obs})}{p(\theta^{t-1}|Y_t^{obs})} = \frac{K(\theta^*|Y_t^{obs})}{K(\theta^{t-1}|Y_t^{obs})}$ . After (4) it accepts the parameter value or discards it. Algorithm's steps (2)-(4) are repeated many times to simulate the posterior distributions.

For posterior distributions' simulation I used two MH chains with 10,000 draws each and tuned the scale parameter  $c = 0.26$  so as to obtain a recommended acceptance ratio of 0.25 (see Griffoli (2010)).

## 5 Results

Estimation results of baseline model are reported in table 3 in the appendix C along with prior, posterior mode and distribution.

Besides baseline model, I used six alternative scenarios in order to identify match of the data to nominal and real frictions:

- Scenario 1: There is no variable capital utilization rate  $\sigma_a = 10^6$ ;
- Scenario 2: There are no sticky wages  $\xi_w = 0.1$ ;
- Scenario 3: There are no sticky prices  $\xi_d = \xi_{m,c} = \xi_{m,i} = \xi_x = 0.1$ ;
- Scenario 4: There is no habit in consumption  $b = 0.1$ ;
- Scenario 5: There is no investment adjustment cost  $\tilde{S}'' = 0.1$ ;

<sup>19</sup>In my estimation procedure I used MATLAB's<sup>©</sup> *fmincon* that solves optimization problems with constraints.

- Scenario 6: There is no working capital channel  $\nu = 0.1$ .

Sticky wages parameter  $\xi_w$  is estimated in the baseline model to be 0.72 which leads to adjustment of the wages roughly once per year. In alternative scenarios wage stickiness parameter slightly increases, but it is not significantly different from the one estimated in the baseline model. Compared with marginal log-likelihood of baseline model of 1103.36, scenario 2 with no sticky wages yields a marginal density of 1098.6, so the data "favours" a model with sticky wages.

Parameters of price stickiness suggest that domestic price adjust once in 5 months. Although these are quite flexible prices my estimates are in line with Copaciu et al. (2010) who use survey data on Romanian firms to find price adjustment frequency less than 2 quarters. In alternative scenarios domestic price stickiness is not significantly different from the one estimated in baseline model, except scenario 1, but even here price adjustment takes place with a roughly 2 quarters frequency. Imported consumption or investment goods' prices change roughly with 4 months frequency, export prices are more sticky than import prices but still yield a frequency of 5 months. Scenario 3 with no price stickiness yields a marginal log-likelihood of 1037.85 which is lower than baseline model's marginal log-likelihood, so sticky prices assumption is preferred by the observed data.

Internal consumption habit seems to play a significant role in the dynamics of the model, although the mean of the prior was set to 0.85, estimation results are around 0.96, which indicates, along with marginal log-likelihood of scenario 4 with no habit in consumption of 1035.3, that habit in consumption can not be excluded from the model.

Estimated elasticity of substitution between domestic and foreign consumption goods  $\eta_c$  is 2.16 which means that in order to maintain the same consumption basket, if domestic consumption is reduced by 1%, consumption of foreign goods must be increased by 2.16%. Higher elasticity of substitution between consumption goods is obtained in scenarios 1, 2 and 6; scenario 5 yields a unitary elasticity of substitution, and scenario 3 yields a elasticity of substitution less than 1, which means that in the absence of price stickiness households prefer to substitute domestic consumption goods with foreign consumption goods. The elasticities of substitution in foreign market  $\eta_f$  and of investment goods  $\eta_i$  are unreasonable low, less than unity. This happens because DYNARE doesn't allow to truncate the

priors at 1, as suggested by Adolfson et al. (2005).

Risk premium parameter  $\phi_a$  has a value of 0.0055 which is quite small compared to Romania's risk premium evolution measured through CDS. In the case of scenario 5 risk premium parameter has unreasonable high value of 5.34, which yields a risk premium of 534%.

Interest rate smoothing parameter in Taylor monetary policy rule  $\rho_R$  has an estimated value of 0.74, which is able to capture quite well persistence of ROBOR ON rate (see figure C). The interest rate's response to inflation  $r_\pi$  is greater than unity and satisfies the Taylor principle. Although real exchange rate parameter  $r_x$  has the expected sign it is not significantly different from zero up to a confidence level of 10% (see table 3); response of interest to output gap has negative sign, though the expected sign was positive, but it is also not significantly different from zero. The model and the data suggest that interest rate responds stronger to the speed of adjustment of inflation than that of output gap.

The autoregressive coefficients range from 0.8 to 0.9, except for the domestic stationary productivity shock, which has an autoregressive coefficient of 0.99 which leads to a very big persistence. In alternative scenarios 1, 2, 5 and 6 autoregressive coefficient of imported consumption goods markup has a high value of 0.99.

The overall analysis of marginal log-likelihood of the baseline model and alternative scenarios suggests that data "prefers" a model with no variable capital utilization. This result is in line with the one in Adolfson et al. (2005) who use a model with no variable capital utilization as baseline model.

## 6 Conclusion

In this paper I used an open economy DSGE model developed in Adolfson et al. (2007) (the working paper version of the article is Adolfson et al. (2005)). I used a slightly simplified version of their model; first, in my model, firms update their price with past inflation, Adolfson et al. (2007) use in their price updating rule an average between past inflation and present inflation target. In authors' paper fiscal system and foreign economy are described as a SVAR, I instead have chosen to model them as independent AR processes. The model includes some common

DSGE features like: sticky prices, sticky wages, habit in consumption, variable capital utilization rate, investment adjustment costs, working capital channel. All these features are introduced to generate persistence in the observed variables. Analysis of smoothed observed variables (figures 7 and 8) reveals an acceptable "insample fit". I've also included CPI inflation over authors' observable variables. Estimation of parameters reveals that the average price adjustment time interval is between 4 and 5 months. Usually in literature price adjustment is found to take place once in 3-4 quarters, however this low estimates might be specific to Romania's economy because Copaciu et al. (2010) find, using a survey, that the average duration of prices in the Romanian economy is less than 2 quarters.

Confronting the model with the data reveals also the importance of the hypothesis related to habit in consumption. The estimated value of the parameter is quite high, around 0.96, which suggests that households take into account their previous consumption level when deciding on their current consumption, in order to try to maintain their standard of living.

The analysis of Taylor type interest rate rule reveals that the interest rate smoothing plays a key role in monetary policy decision. Looking at the coefficients that relate central bank's response to real exchange rate and output gap, and their 10% confidence band, we can conclude that the monetary policy does not respond to these key macroeconomic variables, but more significance is played by the speed of growth of output gap rather than its level, as well as the speed of inflation growth.

From all scenarios analysis, a model with no variable capital utilization rate is selected as preferred by the data. Although several surveys on capacity utilization rate (see for example NBR or DGEFCFIN survey), our model might have suggested that variable capital utilization rate is not preferred due to the fact that I didn't include capacity utilization as an observable (due to series' short range).

As further work, this estimated model could serve in variance decomposition analysis (to see which shocks matter the most in the dynamics of observable variables). A key feature of this model would be its usefulness in impulse response analysis.

Furthermore, this model could be improved by a more rigorous selection of priors, use of longer data span and more observable series. The DSGE model

could also be used in forecasting of observables on medium term (4-8 quarters), although, if near term forecast is desirable literature suggests that simple time series approach (AR, VAR) generates a much more reliable forecast.

## References

- Adolfson, M., Laséen, S., Lindé, J. and Villani, M. (2005), ‘Bayesian estimation of an open economy DSGE model with incomplete pass-through’, *Sveriges Riksbank Working Paper Series* (179).
- Adolfson, M., Laséen, S., Lindé, J. and Villani, M. (2007), ‘Bayesian estimation of an open economy DSGE model with incomplete pass-through’, *Journal of International Economics* **72**(2), 481 – 511.
- Calvo, G. A. (1983), ‘Staggered prices in a utility-maximizing framework’, *Journal of Monetary Economics* **12**(3), 383 – 398.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), ‘Nominal rigidities and the dynamic effects of a shock to monetary policy’, *The Journal of Political Economy* **113**(1), 1–45.
- Clarida, R., Gali, J. and Gertler, M. (1999), ‘The Science of Monetary Policy: A New Keynesian Perspective’, *Journal of Economic Literature* **37**(6), 1661–1707.
- Copaciu, M., Neagu, F. and Braun-Erdei, H. (2010), ‘Survey evidence on price-setting patterns of Romanian firms’, *Managerial and Decision Economics* **31**(2-3), 235–247.
- Erceg, C. J., Henderson, D. W. and Levin, A. T. (2000), ‘Optimal monetary policy with staggered wage and price contracts’, *Journal of Monetary Economics* **46**(2), 281 – 313.
- Fernández-Villaverde, J. (2009), *The Econometrics of DSGE Models*, Working Paper 14677, National Bureau of Economic Research.
- Gali, J. (2008), *Monetary Policy, Inflation, and the Business Cycle*, first edn, Princeton University Press, Princeton and Oxford.
- Gali, J. and Monacelli, T. (2002), *Monetary Policy and Exchange Rate Volatility in a Small Open Economy*, Working Paper 8905, National Bureau of Economic Research.

- Griffoli, T. M. (2010), DYNARE v4 - User Guide, Public beta version.
- Gălăţescu, A. A., Rădulescu, B. and Copaciu, M. (2007), ‘Potential GDP Estimation for Romania’, *National Bank of Romania Occasional Papers* (6).
- Holmberg, K. (2006), ‘Derivation and Estimation of a New Keynesian Phillips Curve in a Small Open Economy’, *Sveriges Riksbank Working Paper Series* (197).
- Jakab, Z. M. and Világi, B. (2008), ‘An estimated DSGE model of the Hungarian economy’, *Magyar Nemzeti Bank Working Paper* (9).
- Kydland, F. E. and Prescott, E. C. (1982), ‘Time to Build and Aggregate Fluctuations’, *Econometrica* **50**(6), 1345–1370.
- McCandless, G. (2008), *The ABCs of RBCs: an introduction to dynamic macroeconomic models*, first edn, Harvard University Press, Cambridge, Massachusetts.
- Monacelli, T. (2003), ‘Monetary policy in a low pass-through environment’, *European Central Bank Working Paper Series* (227).
- Sims, C. A. (2006), Comment on Del Negro, Schorfheide, Smets and Wouters, Technical report.  
**URL:** <http://sims.princeton.edu/yftp/DSSW806/DSseattleComment.pdf>
- Smets, F. and Wouters, R. (2003), ‘An estimated stochastic dynamic general equilibrium model of the euro area’, *Journal of European Economic Association* **1**(5), 1123 – 1175.
- Smets, F. and Wouters, R. (2007), ‘Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach’, *American Economic Review* **97**(3), 586–606.
- Tovar, C. E. (2008), DSGE models and central banks, Technical Report 258, Bank for International Settlements.
- Uhlig, H. (1999), *A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily*, 30-61, in Marimon R., Scott, A., *Computational Methods for the Study of Dynamic Economies*, Oxford.

Walsh, C. E. (2010), *Monetary Theory and Policy*, third edn, The MIT Press, Cambridge, Massachusetts.

# Appendices

## A Data

Data used in the estimation procedure with data sources are:

- Real GDP - source of the data National Institute of Statistics (NIS)
- GDP deflator - NIS
- Consumption - NIS
- Consumption deflator - NIS
- Investmen - NIS
- Investmen deflator - NIS
- Export - NIS
- Import - NIS
- Nominal wage - NIS
- Employment - NIS
- Consumption Price Index (CPI) - NIS
- Nominal exchange rate - National Bank of Romania (NBR)
- ROBOR ON (overnight money market loan rate) - NBR
- Euro area 16 real GDP - EUROSTAT
- Euro area 16 GDP deflator - EUROSTAT
- EURIBOR ON (Eonia) - [www.euribor.org](http://www.euribor.org) official benchmark rate of the Euro money market

Transformed variables as described in Data subsection (4.1) are represented in the following figures :

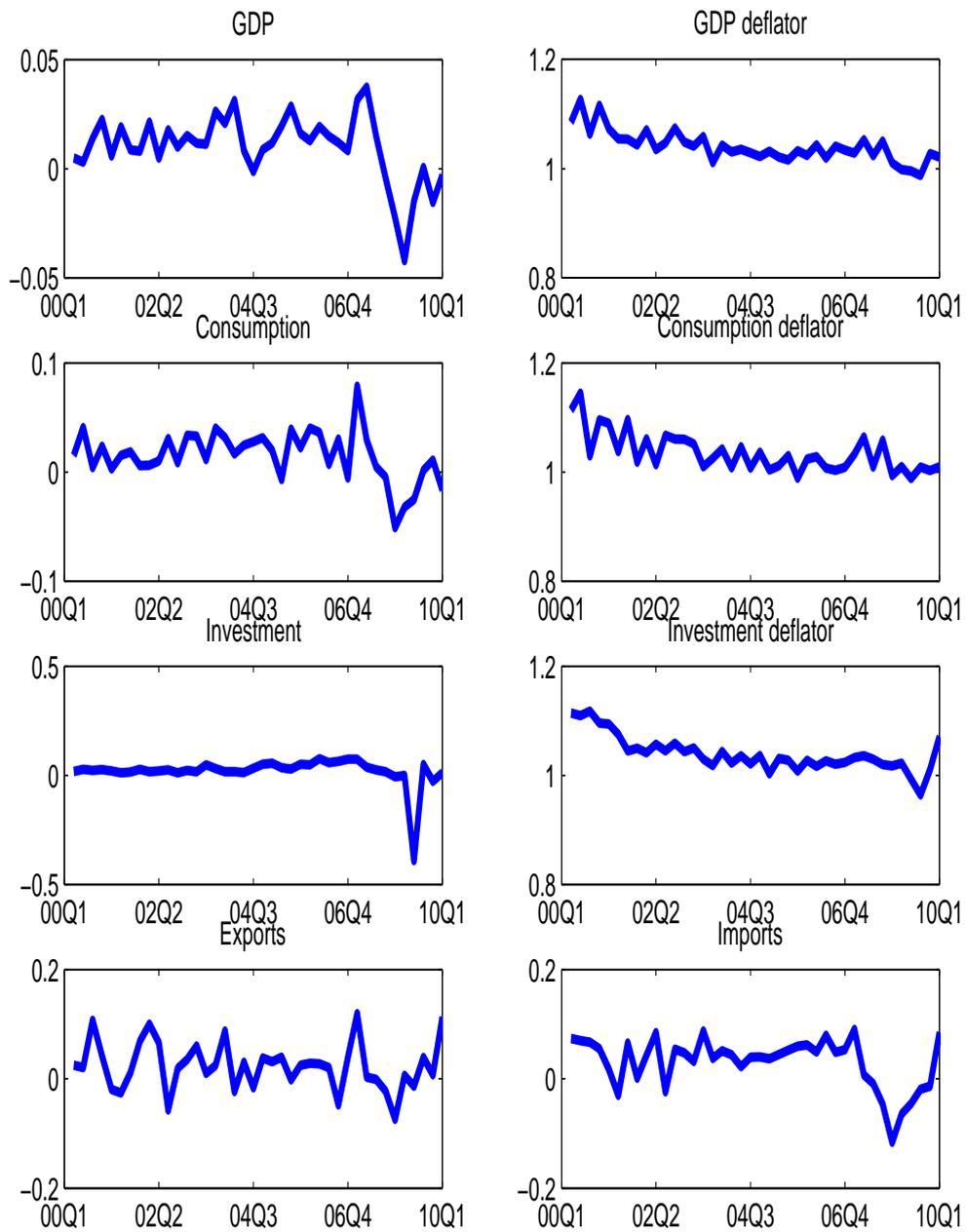


Figure 1: Observed Data

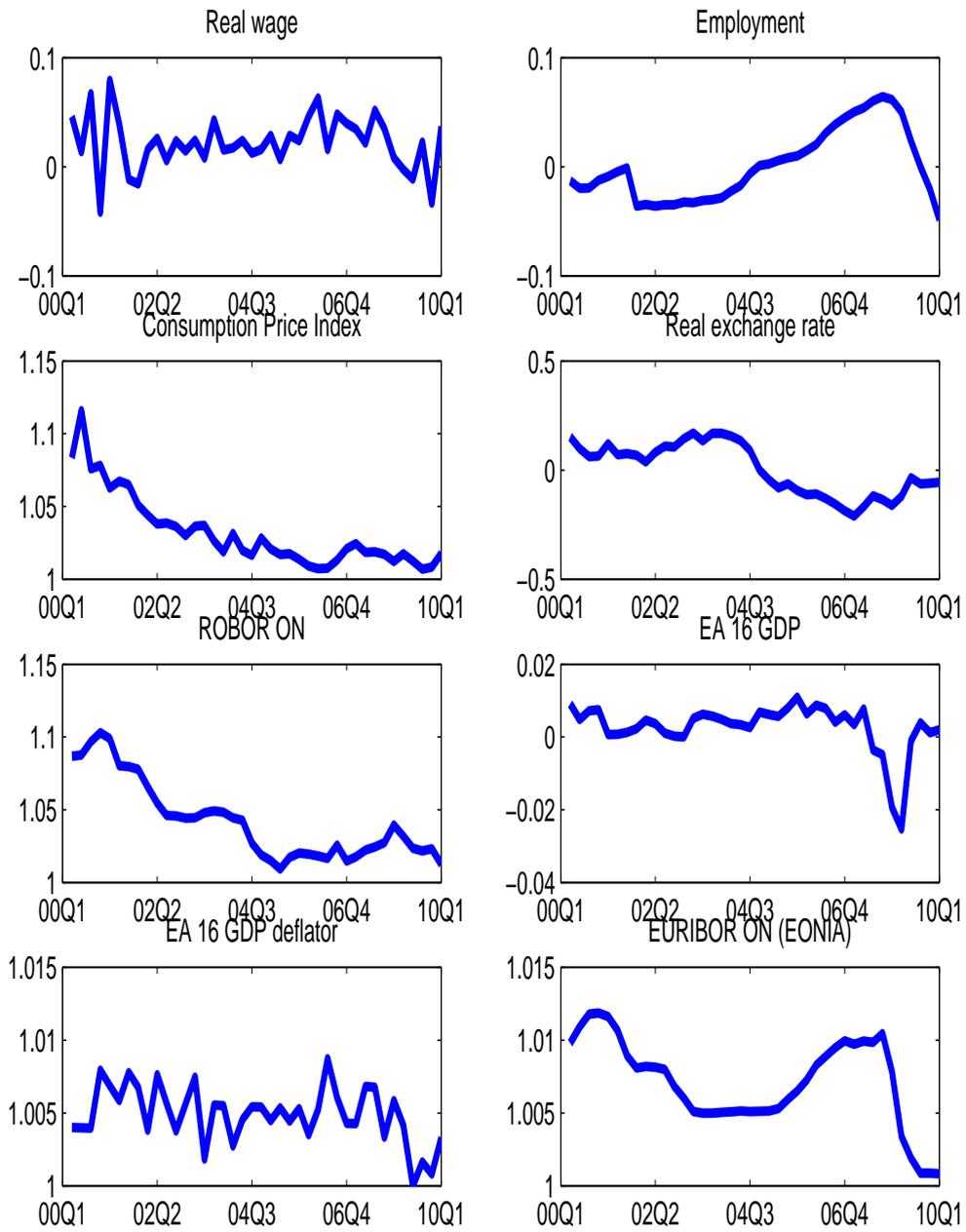


Figure 2: Observed Data

## B Prior distributions

Shock	Distribution	Mode	Degrees of freedom
$\sigma_{\varepsilon_e}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\zeta^c}}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\zeta^h}}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\tilde{\phi}}}$	Inverse Gamma	0.05	4
$\sigma_{\varepsilon_{\lambda^d}}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\lambda^{m,c}}}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\lambda^{m,i}}}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\lambda^x}}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_R}$	Inverse Gamma	0.2	4
$\sigma_{\varepsilon_{\tilde{\pi}}}$	Inverse Gamma	0.05	4
$\sigma_{\varepsilon_{\tau^k}}$	Inverse Gamma	0.01	4
$\sigma_{\varepsilon_{\tau^c}}$	Inverse Gamma	0.01	4
$\sigma_{\varepsilon_{\tau^w}}$	Inverse Gamma	0.01	4
$\sigma_{\varepsilon_{\tau^y}}$	Inverse Gamma	0.01	4
$\sigma_{\varepsilon_{z^*}}$	Inverse Gamma	0.1	4
$\sigma_{\varepsilon_{\hat{g}}}$	Inverse Gamma	0.1	4
$\sigma_{\varepsilon_{\hat{y}^*}}$	Inverse Gamma	0.1	4
$\sigma_{\varepsilon_{\hat{R}^*}}$	Inverse Gamma	0.1	4
$\sigma_{\varepsilon_{\hat{\pi}^*}}$	Inverse Gamma	0.1	4

Table 1: Prior distribution of Shocks

Parameter	Distribution	Mean	Std. err.
Calvo wages $\xi_w$	Beta	0.75	0.1
Calvo domestic price $\xi_d$	Beta	0.67	0.1
Calvo import consumption price $\xi_{m,c}$	Beta	0.67	0.1
Calvo import investment price $\xi_{m,i}$	Beta	0.67	0.1
Calvo export price $\xi_x$	Beta	0.67	0.1
Consumption habit $b$	Beta	0.85	0.05
Elasticity of substitution investment $\eta_i$	Inverse Gamma	1.5	2
Elasticity of substitution foreign $\eta_f$	Inverse Gamma	1.5	2
Elasticity of substitution consumption $\eta_c$	Inverse Gamma	1.5	2
Risk premium $\phi_a$	Inverse Gamma	0.01	2
Taylor interes rate smoothing $\rho_R$	Beta	0.85	0.05
Taylor inflation $\rho_\pi$	Normal	1.3	0.05
Taylor RER $\rho_x$	Normal	0.01	0.005
Taylor output gap $\rho_y$	Normal	0.2	0.05
Taylor change in inflation $\rho_{\Delta\pi}$	Normal	0.3	0.1
Taylor change in output gap $\rho_{\Delta y}$	Normal	0.0625	0.05
AR capital tax $\rho_{\tau^k}$	Beta	0.85	0.05
AR wage tax $\rho_{\tau^w}$	Beta	0.85	0.05
AR consumption tax $\rho_{\tau^c}$	Beta	0.85	0.05
AR labor income tax $\rho_{\tau^y}$	Beta	0.85	0.05
AR technology growth rate $\rho_{\mu z}$	Beta	0.85	0.05
AR stationary technology shock $\rho_\epsilon$	Beta	0.85	0.05
AR investment to capital $\rho_\Upsilon$	Beta	0.8	0.05
AR consumption preference $\rho_{\zeta^c}$	Beta	0.85	0.05
AR labor preference $\rho_{\zeta^h}$	Beta	0.85	0.05
AR assymetric technology growth $\rho_{\bar{z}^*}$	Beta	0.85	0.05
AR risk premium $\rho_{\tilde{\phi}}$	Beta	0.85	0.05
AR dometic markup $\rho_{\lambda^d}$	Beta	0.85	0.05
AR imported consumption markup $\rho_{\lambda^{m,c}}$	Beta	0.85	0.05
AR imported investment markup $\rho_{\lambda^{m,i}}$	Beta	0.85	0.05
AR export markup $\rho_{\lambda^x}$	Beta	0.85	0.05

Table 2: Prior distributions of Parameters

## C Estimation results

Param.	Post. mode	Post. Mean	Lo conf. band	Up. conf. band
$\xi_w$	0.7817	0.7249	0.5353	0.9212
$\xi_d$	0.3554	0.3578	0.2852	0.4247
$\xi_{m,c}$	0.2875	0.2828	0.2047	0.3529
$\xi_{m,i}$	0.3395	0.3329	0.2316	0.4232
$\xi_x$	0.4687	0.4097	0.285	0.5439
$b$	0.9586	0.9593	0.9577	0.961
$\eta_i$	0.7144	0.8737	0.4585	1.3181
$\eta_f$	0.5628	0.5879	0.3198	0.8795
$\eta_c$	2.2422	2.1607	1.4258	3.0272
$\phi_a$	0.0045	0.0055	0.0027	0.0084
$\rho_R$	0.7446	0.7428	0.6693	0.8029
$\rho_\pi$	1.3309	1.3312	1.2633	1.3987
$\rho_x$	0.005	0.0055	-0.001	0.0138
$\rho_y$	-0.0013	-0.0021	-0.0062	0.0014
$\rho_{\Delta\pi}$	0.3921	0.4003	0.2297	0.5743
$\rho_{\Delta y}$	0.1686	0.1668	0.1051	0.2269
$\rho_{\tau^k}$	0.8646	0.8556	0.7793	0.9359
$\rho_{\tau^w}$	0.8646	0.856	0.7698	0.9288
$\rho_{\tau^c}$	0.8646	0.8367	0.7556	0.9358
$\rho_{\tau^y}$	0.8646	0.8513	0.7863	0.9261
$\rho_{\mu_z}$	0.8732	0.8696	0.8468	0.8902
$\rho_\epsilon$	0.9808	0.9806	0.9742	0.9879
$\rho_\Upsilon$	0.7932	0.7839	0.7017	0.8844
$\rho_{\zeta^c}$	0.8665	0.8334	0.7578	0.9161
$\rho_{\zeta^h}$	0.9013	0.8689	0.8004	0.949
$\rho_{\tilde{z}^*}$	0.8833	0.8645	0.7771	0.9484
$\rho_{\tilde{\phi}}$	0.8125	0.789	0.7031	0.8643
$\rho_{\lambda^d}$	0.8883	0.8612	0.7923	0.9408
$\rho_{\lambda^{m,c}}$	0.8941	0.8748	0.8075	0.9478
$\rho_{\lambda^{m,i}}$	0.8702	0.8456	0.7608	0.9347
$\rho_{\lambda^x}$	0.9068	0.8868	0.8177	0.9433

Table 3: Baseline model

Param.	Baseline Model	S1 No variable capital utiliz.	S2 No sticky wages	S3 No sticky prices	S4 No habit in consumption	S5 No investment adj. cost	S6 No working capital channel
$\xi_w$	0.7249	0.7610	-	0.7555	0.7544	0.7661	0.7638
$\xi_d$	0.3578	0.4713	0.3150	-	0.3448	0.3077	0.2763
$\xi_{m,c}$	0.2828	0.4075	0.4420	-	0.3053	0.5500	0.4558
$\xi_{m,i}$	0.3329	0.3475	0.3554	-	0.3329	0.2165	0.3507
$\xi_x$	0.4097	0.4355	0.4343	-	0.9851	0.3528	0.4351
$b$	0.9593	0.9650	0.9678	0.9603	-	0.9702	0.9699
$\eta_i$	0.8737	0.9783	0.8816	0.6858	2.5719	0.6848	0.7477
$\eta_f$	0.5879	0.5455	0.7502	0.4430	0.8611	0.4772	0.6515
$\eta_c$	2.1607	3.1299	3.0339	0.9080	2.0194	1.0000	2.6593
$\phi_a$	0.0055	0.0049	0.0062	0.0074	0.0348	5.3463	0.0062
$\rho_R$	0.7428	0.8012	0.6918	0.7518	0.7319	0.7377	0.7268
$\rho_\pi$	1.3312	1.3203	1.3495	1.3391	1.3255	1.4016	1.3466
$\rho_x$	0.0055	0.0134	0.0099	0.0063	0.0108	0.0139	0.0091
$\rho_y$	-0.0021	-0.0042	-0.0115	-0.0018	-0.0082	0.0433	-0.0048
$\rho_{\Delta\pi}$	0.4003	0.3359	0.3734	0.4349	0.3230	0.3279	0.4205
$\rho_{\Delta y}$	0.1668	0.1898	0.1118	0.1848	0.1046	0.0981	0.0937
$\rho_{\tau^k}$	0.8556	0.8407	0.8481	0.8422	0.8552	0.8571	0.8524
$\rho_{\tau^w}$	0.856	0.8531	0.8499	0.8588	0.8449	0.8457	0.8590
$\rho_{\tau^c}$	0.8367	0.8561	0.8432	0.8563	0.8413	0.8431	0.8556
$\rho_{\tau^y}$	0.8513	0.8458	0.8475	0.8471	0.8503	0.8461	0.8451
$\rho_{\mu z}$	0.8696	0.8812	0.8558	0.8725	0.8006	0.8757	0.8921
$\rho_\epsilon$	0.9806	0.7931	0.8439	0.9782	0.8556	0.8495	0.8551
$\rho_\Upsilon$	0.7839	0.7969	0.7929	0.8001	0.7809	0.8812	0.7864
$\rho_{\zeta^c}$	0.8334	0.8927	0.8520	0.8475	0.8367	0.8509	0.8586
$\rho_{\zeta^h}$	0.8689	0.8708	0.8697	0.8600	0.8767	0.8513	0.8620
$\rho_{\tilde{z}^*}$	0.8645	0.8566	0.8641	0.8731	0.8774	0.8654	0.8588
$\rho_{\tilde{\phi}}$	0.7890	0.7803	0.7628	0.7661	0.8518	0.8645	0.7828
$\rho_{\lambda^d}$	0.8612	0.8095	0.8745	0.8641	0.9087	0.8552	0.8949
$\rho_{\lambda^{m,c}}$	0.8748	0.9950	0.9955	0.8860	0.8740	0.9874	0.9954
$\rho_{\lambda^{m,i}}$	0.8456	0.8572	0.8524	0.8534	0.8731	0.8782	0.8586
$\rho_{\lambda^x}$	0.8868	0.8567	0.8919	0.8561	0.6118	0.8508	0.8751
Log data density	1103.36	1121.95	1098.60	1037.85	1035.3	1085.94	1027.59

Table 4: Scenarios

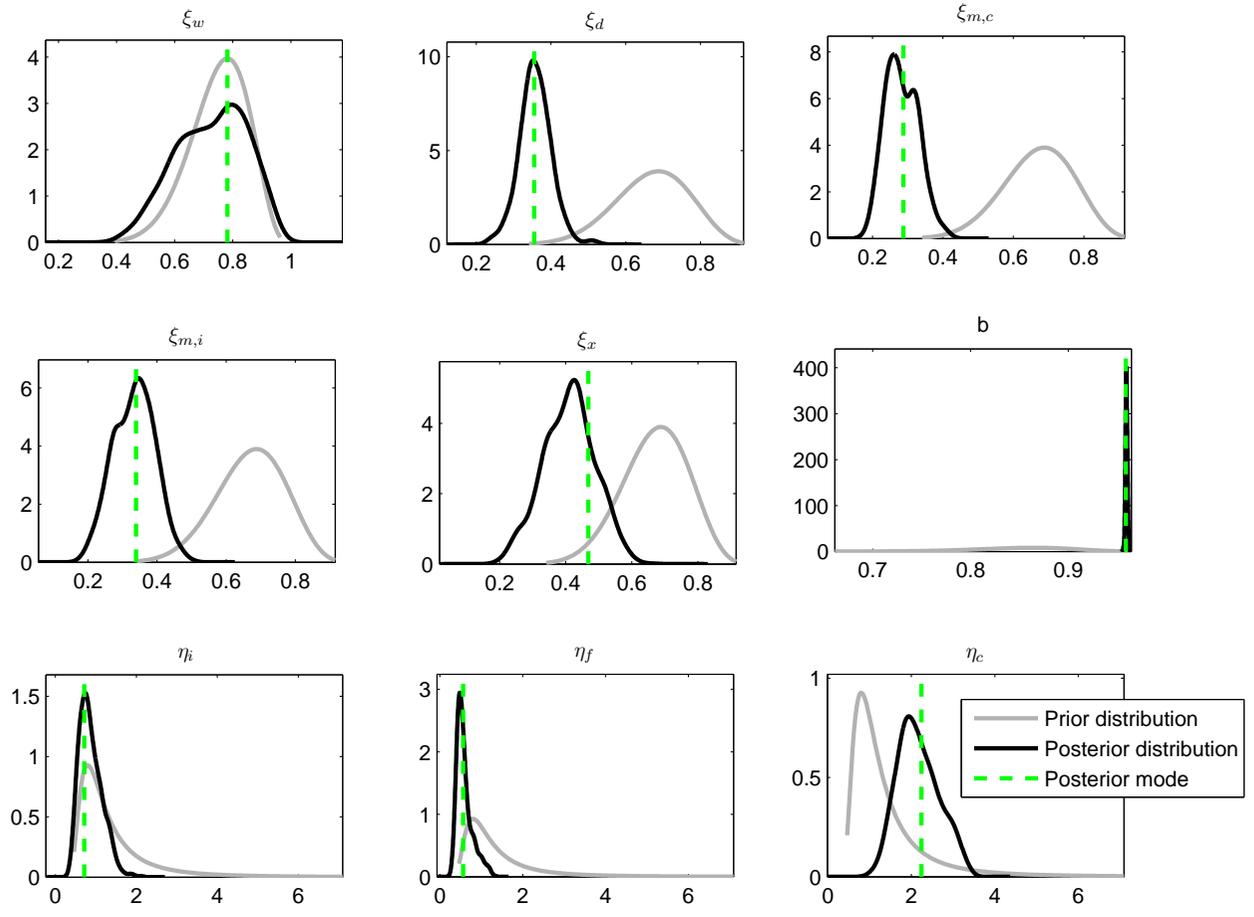


Figure 3: Posterior distributions

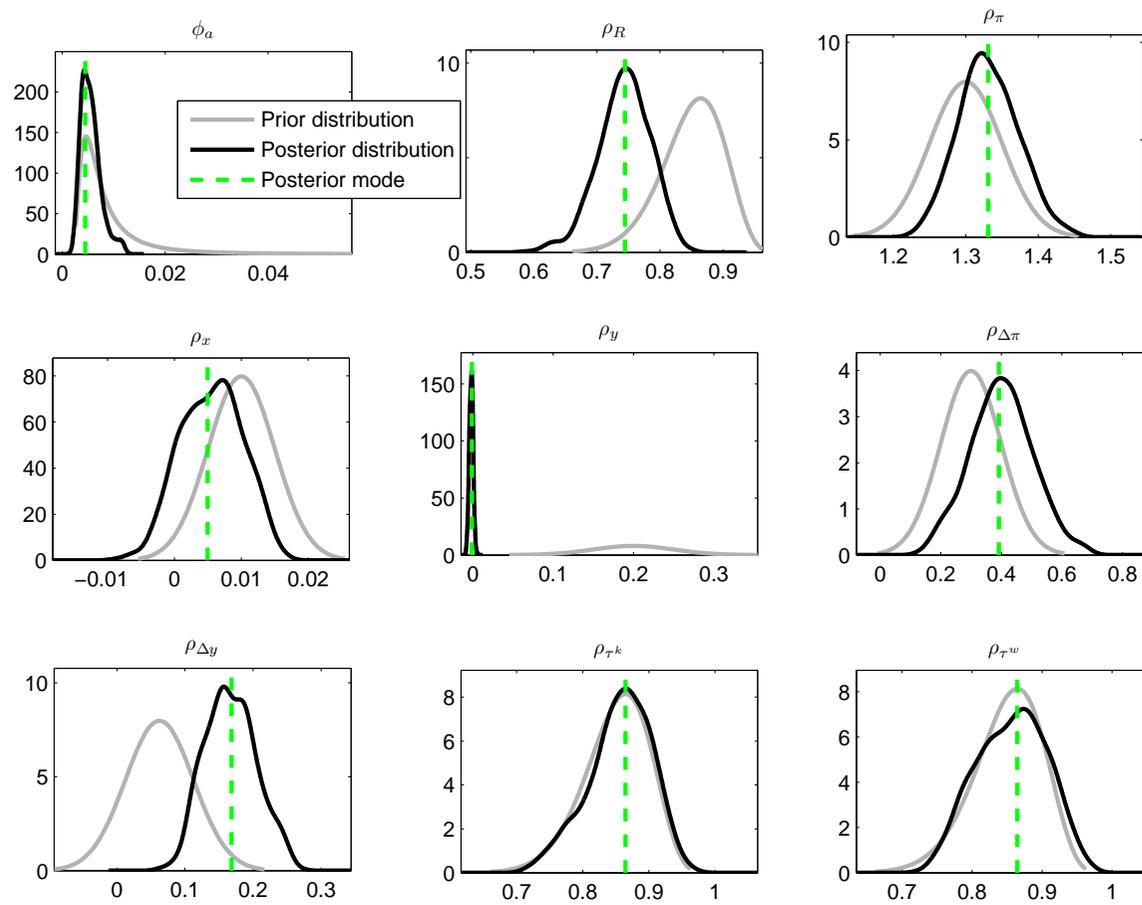


Figure 4: Posterior distributions

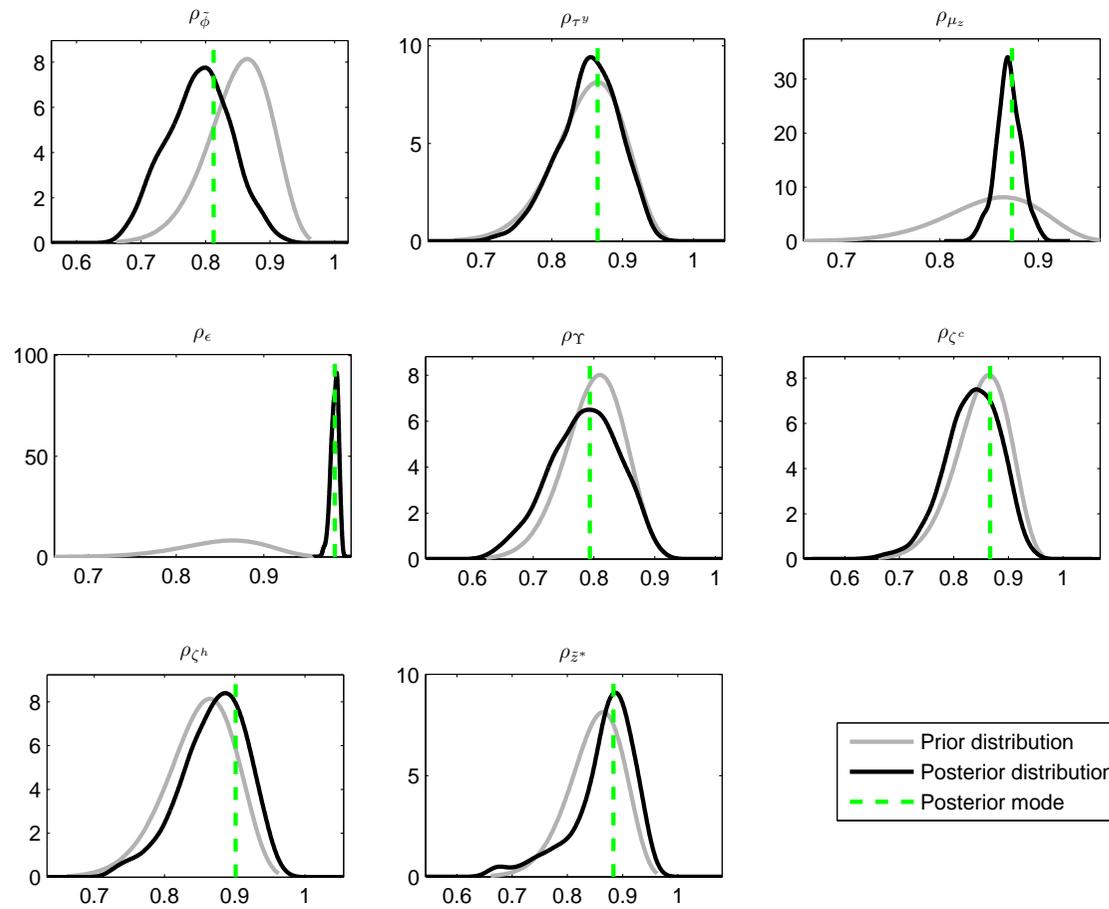


Figure 5: Posterior distributions

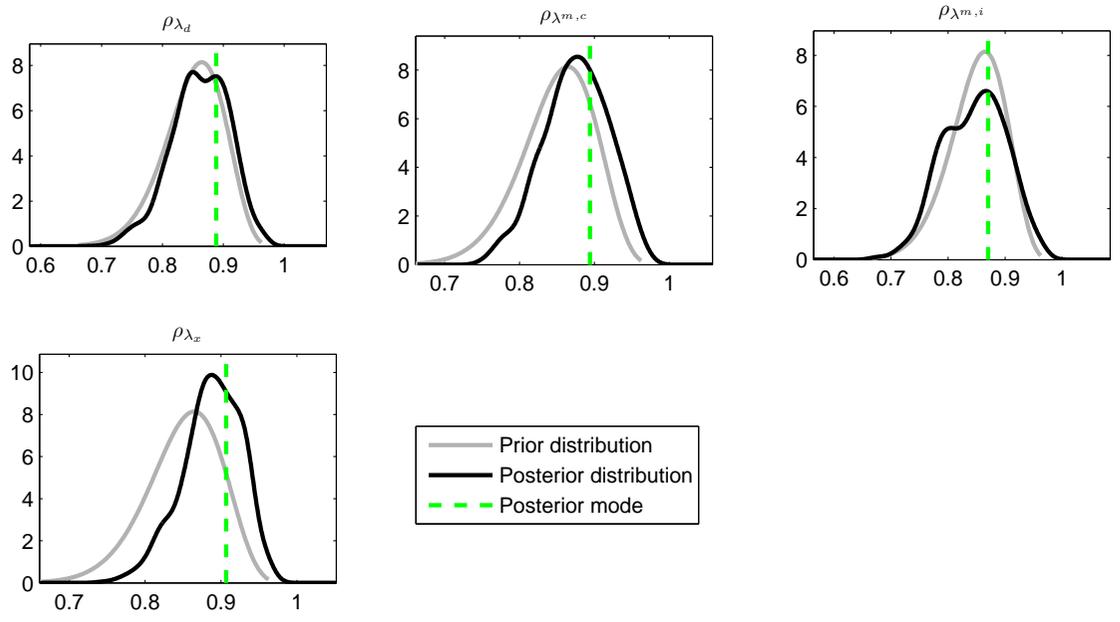


Figure 6: Posterior distributions

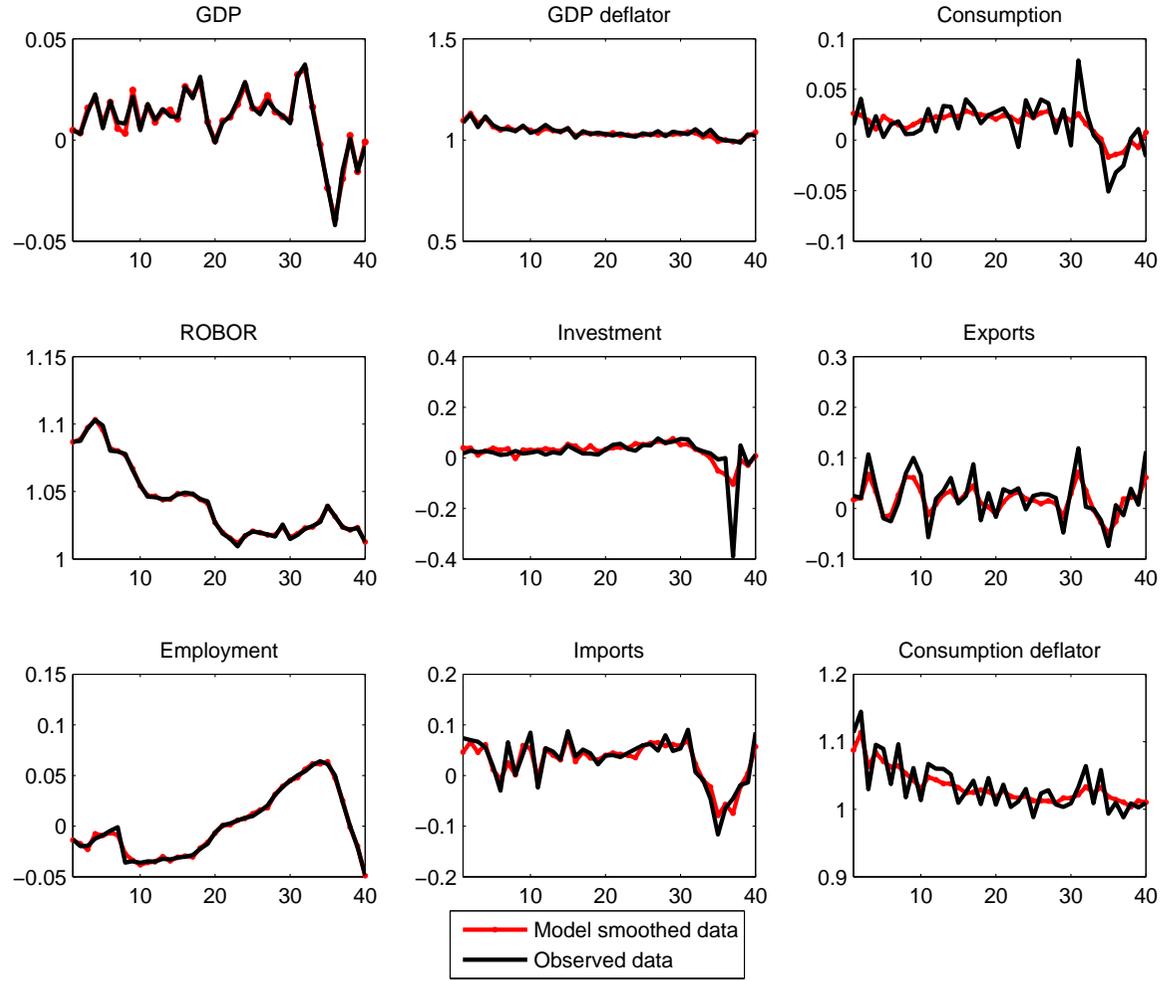


Figure 7: Smoothed observed variables

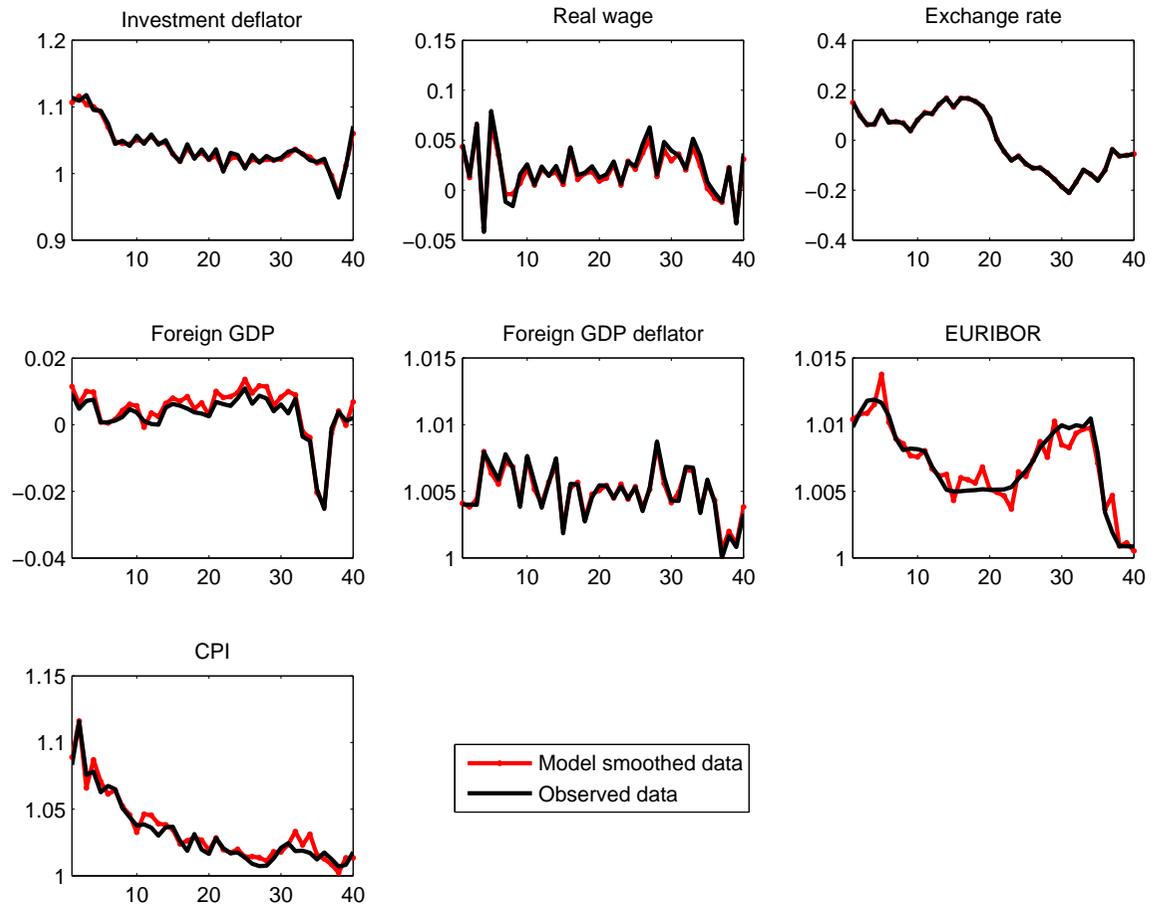


Figure 8: Smoothed observed variables