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Value at Risk: A Comparative Analysis

- Dissertation Paper -

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Contents

Abstract

This study develops a comparative analysis concerning Value at Risk measure for a portfolio consisting of three stocks traded at Bucharest Stock Exchange. The analysis set out from 1-day, 1% VaR and has been extended in two directions: the volatility models and the distributions which are used when computing VaR. Thus, the historical volatility, the EWMA volatility model, GARCH-type models for the volatility of the stocks and of the portfolio and a dynamic conditional correlation (DCC) model were considered while VaR was computed using, apart from the standard normal distribution, different approaches for taking into account the non-normality of the returns (such as the Cornish-Fisher approximation, the modeling of the empirical distribution of the standardized returns and the Extreme Value Theory approach).

The results indicate that using conditional volatility models and distributional tools that account for the non-normality of the returns leads to a better VaR-based risk management. For the considered portfolio VaR computed on the basis of a GARCH (1,1) model for the volatility of the portfolio returns where the standardized returns are modeled using the generalized hyperbolic distribution seems to be the best compromise between precision, capital coverage levels and the required amount of calculations. Moreover, the Expected Shortfall risk measure offers very good precision results in all approaches, but at the cost of rather high capital coverage levels.

1. Introduction

Value at Risk (VaR) is a simple risk measure which tries to represent through a single number the total risk of a portfolio consisting of financial assets. It was introduced by J. P. Morgan in 1994 and now it is largely used not only by financial institutions, but also by companies and investment funds. Moreover, The Basle Committee on Banking Supervision of the Bank for International Settlements uses it in order to determine the capital requirements for banks[1].

VaR is the estimated loss of a fixed portfolio over a fixed period of time and it answers the following question: "What loss level is such that it only will be exceeded p% of the time in the next T trading days?". This may be written as follows:

[1] See Codirlasu (2007)

$$\Pr\left(R_{1,T} < -VaR_T^p\right) = p$$

where $R_{1,T}$ is the total return over the next T trading days, p is the probability level and the minus sign in front of VaR term shows that VaR itself is a positive number corresponding to a loss. The interpretation is thus that the VaR gives a number such that is a p% chance of loosing more than it.

It is not unreasonable to say that VaR has become the benchmark for risk measuring. This is due to the fact that it captures an important aspect of risk: how bad things can get with a certain probability p. Furthermore, it is easily communicated and understood.

However, VaR has certain shortcomings[2]. Perhaps the most important one is the fact that extreme losses may be ignored. VaR tells only that p% of the time the portfolio return will be below the reported number, but it says nothing about the size of the losses in those p% worst cases. Moreover, VaR assumes that the portfolio is constant across the next T days which is unrealistic in many cases when T is larger than a day or a week. Finally, the parameters T and p are arbitrarily chosen[3].

I consider that the VaR subject is very relevant and challenging for certain reasons such as: it raises a great interest from both financial researchers and practitioners, being a risk measure built on financial and statistical theory but with vast applications in real business life; although is a number easily communicated and understood it may include rather complex features from finance and statistics; although it represents the benchmark for risk calculation it is generally considered not to be a satisfactory risk measure due to certain shortcomings, some of them being mentioned in the paragraph above.

Therefore, in this study, I developed a comparative analysis between different approaches to VaR. This analysis does not refer to the process of choosing the two parameters of VaR which were considered to be p = 1% and T = 1 day. The most important reason for setting T to 1 day is that, as I have already mentioned, the assumption that the portfolio will remain constant across the next T days (T > 1) is not realistic. Instead, this study focuses on the volatility models and on the distributions which can be used in order to obtain the VaR number, the comparative analysis being developed on a simple portfolio consisting of three stocks traded at Bucharest Stock Exchange.

Section 2 gives a concise literature review regarding the subject of this study. Section 3 presents a few theoretical aspects regarding the methodology used for computing VaR, section 4

[2] See Christoffersen (2002)

[3] For example, The Basle Accord for Capital Adequacy sets T = 10 (2 weeks in terms of trading days) and p = 1%

offers a presentation of the primary data for the portfolio on which the analysis was developed while sections 5 to 9 show the results obtained for the five volatility approaches to VaR: historical volatility, EWMA model for the volatility of the stocks, GARCH model for the volatility of the portfolio, GARCH models for the volatility of the stocks, this approach being than considered together with a dynamic conditional correlation model. Finally, section 10 presents the conclusions of this study.

2. A Brief Literature Review

The analysis developed in this study is mainly based on Christoffersen (2002). His book on financial risk management presents the great majority of the volatility and distributional approaches to VaR that I used here.

In what concerns the volatility models, Christoffersen discusses the following ones that are used in this study: the EWMA model introduced by JP Morgan's RiskMetrics system in 1996, along with its limitations; the basic GARCH model introduced in Engle (1982) and Bollerslev (1986) with some of its extensions that include the leverage effect, as it is the TARCH model which was developed independently by Zakoian (1990) and Glosten, Jagannathan and Runkle (1993); the Quasi Maximum Likelihood estimation of GARCH models based on Bollerslev and Wooldridge (1998); and the simple dynamic conditional correlation model used in this study based on Engle (2000) and Engle and Sheppard (2001).

Moreover, in Christoffersen is discussed the process of modeling the distribution of the standardized returns with regard to the following results of the previous studies: the empirical properties of asset returns as found in Cont (2001), especially the non-normality of the returns (even when they are standardized using conditional volatility models) and the leverage effect; modeling the conditional distribution of the returns using t-Student is considered in Bollerslev (1987); applications of Extreme Value Theory in financial risk management are discussed in McNeil (2000), while the choice of threshold under the EVT approach is discussed in McNeil and Frey (2000); in what concerns the Expected Shortfall measure Artzner, Delbaen, Eber and Heath (1999) showed that it has nicer theoretical properties than VaR, while Basak and Shapiro (2000) found that portfolio management based on ES leads to lower losses than that based on VaR. Most of the models and approaches that are presented here, and that are also used in my study,, were taken and implemented from Christoffersen (2002). However, I included in the references of this study the papers of the originators of these models and approaches.

Also, of great interest are the results of Codirlasu (2007) who performed a comparable analysis developed on a portfolio consisting of four stocks traded at Bucharest Stock Exchange. His analysis takes into account four of the volatility approaches used in my study but computes VaR only under the assumption that the standardized returns follow a standard normal distribution. His results indicate that the EWMA model performed best followed by the GARCH models which also have the advantage of a reduced level of capital coverage.

These are only a few results from the fast growing literature on risk management, the literature review given here being focused especially on the aspects that are considered in my study.

3. The Methodology

I set out from the analytical 1-day, 1% VaR formula

$$VaR_{1\%} = -(Q_{1\%}\sigma + \mu)S$$

where μ denotes the mean of the daily returns of the portfolio, σ denotes the volatility of the daily returns of the portfolio, $Q_{1\%}$ denotes the 1% quantile of the distribution considered for the daily standardized returns and S denotes the value of the portfolio at the moment when VaR is computed. The analysis developed in this study is focused on two directions: the usage of different volatility models when determining VaR and the modeling of the distribution of the daily standardized returns. So, the architecture of this study is constructed around these two terms from the VaR formula: $Q_{1\%}$ and σ.

I will begin by presenting the five volatility approaches to VaR that were considered in my study:

• **Historical volatility**. This is the simplest volatility approach when computing VaR and represents the first stage of the comparative analysis developed here (see section 5). In this case VaR was determined on the basis of 750 days rolling volatility of the data series of the portfolio returns.

• **EWMA volatility model**. In this stage it was considered one of the simplest conditional volatility models, namely the Exponentially Weighted Moving Average (EWMA) model proposed by JP Morgan's RiskMetrics system in 1996[4]. The model is written as

$$\sigma_{t+1}^2 = \lambda \sigma_t^2 + (1-\lambda) R_t^2$$

where $\lambda < 1$, σ denotes the volatility and R the return. The key issue in determining tomorrow's volatility is represented by the value chosen for λ. Here, it was considered that it equals 0.94 which represents the value used by RiskMetrics for daily returns. Using the standard deviation of the first 750 daily returns for each stock as starting value I computed the EWMA volatility series for all the three stocks. Then, on the basis of historical correlation coefficients the volatility series of the portfolio was determined. Therefore, VaR was computed using the EWMA volatility of the portfolio (see section 6).

• **GARCH volatility model for the portfolio returns**. The analysis was extended by considering more elaborate models such as GARCH[5] (Generalized AutoRegressive Conditional Heteroskedasticity) models. In this case a simple GARCH (1,1) model for the volatility of the portfolio returns was considered, the model being written as:

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta \sigma_t^2$$

where $\alpha + \beta < 1$, the sum of these two terms denoting the persistence of the model. The parameters α, β and ω were estimated through MLE in Eviews and based on their estimated values I computed the volatility series of the portfolio returns using as starting value the standard deviation of the first 750 daily returns. Thus, VaR was computed using the GARCH volatility of the portfolio (see section 7).

• **GARCH volatility model for the stock returns**. During this stage GARCH type volatility models were considered for the returns of the three stocks, namely two GARCH (1,1) models and a TARCH[6] (1,1) model. The specification of a TARCH (1,1) model is as follows:

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \lambda R_t^2 d_t + \beta \sigma_t^2$$

where d_t is a binary variable which equals 1 if $R_t < 0$, and equals 0 if $R_t > 0$. TARCH is an asymmetric model because a negative return has a different impact on the volatility of the portfolio

[4] See JP Morgan (1996)
[5] See Engle (1982), Bollerslev (1986), Engle and Patton (2001)
[6] Threshold ARCH. See Glosten, Jagannathan and Runkle (1993)

than the one of a positive return. The usage of asymmetric volatility models is based on the fact that it has been argued that a negative return increases volatility by more than a positive return of the same magnitude[7]. The parameters of the three conditional volatility models were estimated through MLE in Eviews. On the basis of the estimation results I computed the volatility series for the three stocks considering that the starting values for each stock are the standard deviations of the first 750 daily returns. Then, using the historical correlation coefficients, the volatility series for the portfolio was computed leading to the determination of VaR (see section 8).

• **GARCH DCC**. The same approach as in the previous stage was considered but now in the context of a dynamic conditional correlation (DCC) model[8]. The conditional correlations are modeled using GARCH (1,1) type specifications with correlation targeting, the model being written as:

$$q_{ij,t+1} = E\lfloor z_{i,t} z_{j,t} \rfloor + \alpha\left(z_{i,t} z_{j,t} - E\lfloor z_{i,t} z_{j,t} \rfloor\right) + \beta\left(q_{ij,t} - E\lfloor z_{i,t} z_{j,t} \rfloor\right)$$

$$\rho_{ij,t+1} = \frac{q_{ij,t+1}}{\sqrt{q_{ii,t+1} q_{jj,t+1}}}$$

where z denotes the standardized returns. The important thing about this model is that the persistence parameters α and β are common across *i* and *j*. They were estimated through MLE in Excel using the following log likelihood function:

$$L = -\frac{1}{2}\sum_t \left(\ln|\Gamma_t| + z_t^{'}\Gamma_t^{-1} z_t\right)$$

where Γ_t is the correlation matrix and $|\Gamma_t|$ denotes its determinant. On the basis of the estimation results, the dynamic correlations between the three stocks were computed. Then, using the GARCH volatility of the three stocks (already determined in the previous stage), I obtained the volatility series for the portfolio. Finally, VaR was calculated on the basis of the GARCH DCC volatility of the portfolio (see section 9).

This is a brief presentation of the first direction taken into account in my comparative analysis on VaR measure. Now, for each of the five volatility models described above several distributional approaches were considered when modeling the series of the standardized returns of the portfolio[9], as follows:

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[7] For example, see Cont (2001). This fact is also known as the leverage effect
[8] See Engle (2000), Engle and Sheppard (2001)
[9] For each volatility approach the returns of the portfolio were standardized using the corresponding volatility model

• **The standard normal distribution**. This is probably the most comfortable situation, in which it is considered that the standardized returns of the portfolio are following a N (0,1). However, as the empirical evidence would suggest[10], the distributions of daily returns tend to be rather skewed and to have fat tails (they have excess kurtosis), so using the 1% quantile of N (0,1) for computing VaR may result in underestimating the risk of the portfolio. In this study the non-normality of the returns is verified using the Jarque-Bera test in Eviews and it is also visualized using the QQ plot, the probability density function (PDF) graph and the log PDF graph, all against the standard normal distribution. In order to deal with the non-normality of the returns, several other approaches were taken into account.

• **The historical quantile**. This is a very simple approach used for computing VaR. Instead of modeling the empirical distributions of the standardized returns it is considered that the series of past returns could offer enough information for predicting the future extreme losses. However, the results are very sensible to the length of the considered past period and to the pattern of returns in that specific period. In this analysis, it was used the 1% quantile of the last 750 daily standardized returns.

• **The t-Student, Normal Inverse Gaussian (NIG) and Generalized Hyperbolic (GH) distributions**. Another way to deal with the non-normality of the data is to model the portfolio returns series using other distributions. One of the first possibilities is to use the t-Student distribution[11] which is a relatively simple distribution and has two advantages: it has only one parameter to estimate, namely the degrees of freedom and it allows for fat tails (may have excess kurtosis) which makes it a better choice than the standard normal distribution. However, a shortcoming of this distribution is represented by the fact that it does not allow for asymmetry, having a skewness of 0. In this context I considered also the NIG and GH distributions which allow for both skewness and excess kurtosis. The main reason for choosing these specific distributions (although the number of parameters to be estimated is larger: 4, respectively 5 parameters) is that in a previous study developed by the author they performed well at modeling the distribution of the daily standardized returns[12]. The parameters for the considered distributions were estimated through MLE using the Rmetrics package of the soft R and the selection between the distributions was

[10] For example, see Cont (2001)
[11] See Bollerslev (1987)
[12] In that study I modeled the daily returns of the Russell 3000 index for the period 2000-2007

made, in each case, by means of the informational criteria Akaike and Schwarz. Thus, VaR was computed using the 1% quantile of the estimated NIG and GH distributions.

• **The Cornish-Fisher (CF) approximation**. Another simple way to take into account the non-normality of the standardized returns of the portfolio is the CF approximation. It does allow for skewness and excess kurtosis by constructing an approximation to quantiles from estimates of skewness and kurtosis on the basis of the following formula[13]:

$$CF_p = Z_p + \frac{S}{6}\left(Z_p^2 - 1\right) + \frac{K}{24}\left(Z_p^3 - 3Z_p\right) - \frac{S^2}{36}\left(2Z_p^3 - 5Z_p\right)$$

where S is the skewness and K the excess kurtosis of the standardized returns, p is the chosen probability level and Z_p is the quantile of the standard normal distribution corresponding to p. Thus the 1% CF quantile in this study was computed on the basis of the skewness and excess kurtosis of the series of the daily standardized returns of the portfolio.

• **Extreme Value Theory[14] (EVT)**. Because, generally, the biggest risk to a portfolio is the sudden incidence of a single large negative return it may be more appropriate to model the tail of the standardized returns distribution. The central result in EVT is that the extreme tail of a wide range of distributions can approximately be described by the Generalized Pareto distribution (GPD) which allows for the presence of fat tails (the parameter ξ of GPD characterizes the tail of the distribution: $\xi < 0$ means thin tails, $\xi = 0$ means the kurtosis is 3 as for the standard normal distribution while $\xi > 0$ means fat tails, which is the case of interest in this study). Because almost all results in EVT assume that the returns are iid, the analysis was developed on the standardized returns which, in many cases, could be reasonably assumed to be iid. Using GPD, EVT models only the right tail of the distribution (i.e. the standardized returns in excess of a threshold) and because we are interested in extreme negative returns the EVT analysis is developed on the negative of the returns. However, the choice of the threshold is in some ways arbitrary and in this study it was considered to be the 95 percentile of the data set[15]. Now, under the EVT approach another risk measure was considered, namely the Expected Shortfall (ES) or the TailVaR as it is sometimes called[16]. ES tries to answer to the principal shortcoming of VaR: VaR number is only concerned with the number of losses exceeding it, not with the magnitude of these losses. The most complete measure of large losses is

[13] See Christoffersen (2002)
[14] See McNeil(2000)
[15] See McNeil and Frey (2000)
[16] For theoretical and practical properties of ES, see Artzner, Delbaen, Eber and Heath (1999), Basak and Shapiro (2000)

the entire shape of the tail of the distribution of losses beyond the VaR. Therefore, ES measure tries to keep the simplicity of the VaR while giving information about the shape of the tail. ES is defined as:

$$ES_{t+1}^{p} = E_t\left[R_{t+1} \middle| R_{t+1} < -VaR_{t+1}^{p}\right]$$

So, where VaR tells the loss that only 1% of the potential losses will be worse, the ES tells the expected loss given that the portfolio actually incurred a loss from the 1% tail. For fat tailed distributions ($\xi > 0$) the ES will be larger than the VaR regardless of the probability p that is considered. The parameters of the GPD were estimated through MLE and they were determined along with the quantiles for VaR and ES using Rmetrics. It has to be mentioned that, although this analysis focuses on 1-day, 1% risk measures, under the EVT approach smaller quantiles than 1% were also considered, basically for two reasons: they allow us to see the significantly different risk profiles that may hide under close 1% quantiles (of the EVT distribution and of the normal distribution for example) and in order to see what happens if portfolio holders would choose a smaller probability level.

In conclusion, this study was developed on the two directions that are detailed in this section. The obtained results are presented in the sections 5-9 and the comparative analysis between the computed risk measures was done on the basis of the following criteria: their precision, their level of capital requirements and the amount of calculations required for computing them.

4. The Data

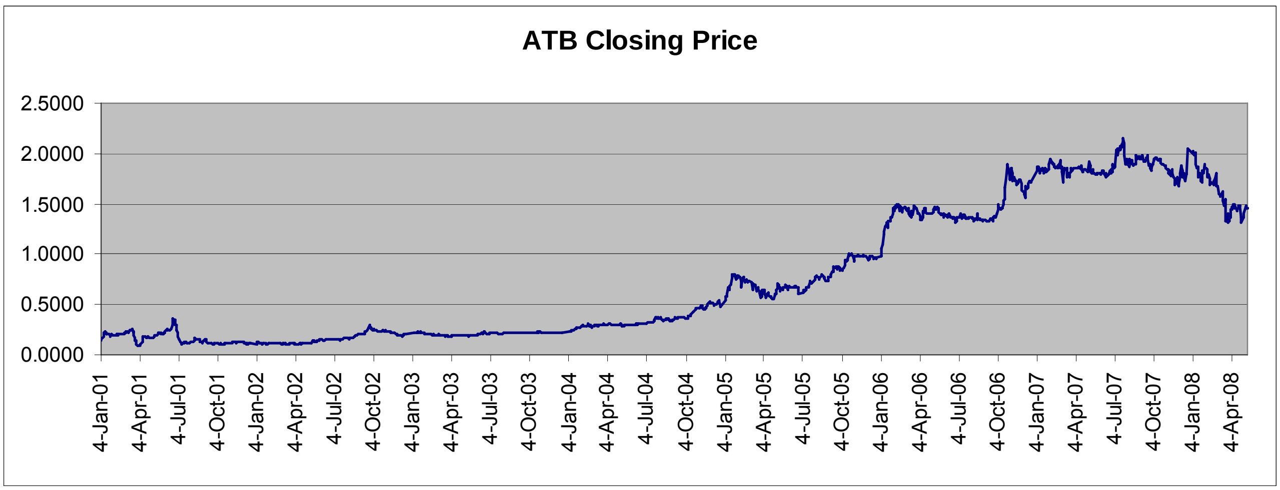
I considered a simple portfolio which has a value of 1 RON and consists of three stocks traded at Bucharest Stock Exchange (BSE): Antibiotice Iasi (ATB), Azomures Tg. Mures (AZO) and Banca Transilvania (TLV). The stocks have equal weights in the portfolio and they were selected according to the following criteria:

- They all belong to the first transaction category of BSE. Moreover, ATB and TLV are included in BET index, meaning they are among the most attractive and liquid stocks in the market;
- The companies represent different industrial areas (ATB – pharmacy, AZO – fertilizers, TLV – banking, insurance and financial services) leading to a better risk diversification in the portfolio.

Daily closing prices for the three stocks, covering the period 4 Jan 2001 – 9 May 2008, were obtained from www.ktd.ro (financial information site of KTD Invest SA). The evolution of the closing prices for the stocks is shown in Fig. 1.

Then, logarithmic daily returns were computed for the three stocks and the portfolio, each data series consisting of 1814 observations. The evolution of the daily returns for each stock and for the portfolio during the analyzed period is shown in Fig. 2, while the descriptive statistics of the returns are presented in Table 1.

Fig. 1. The evolution of the closing prices for ATB, AZO and TLV



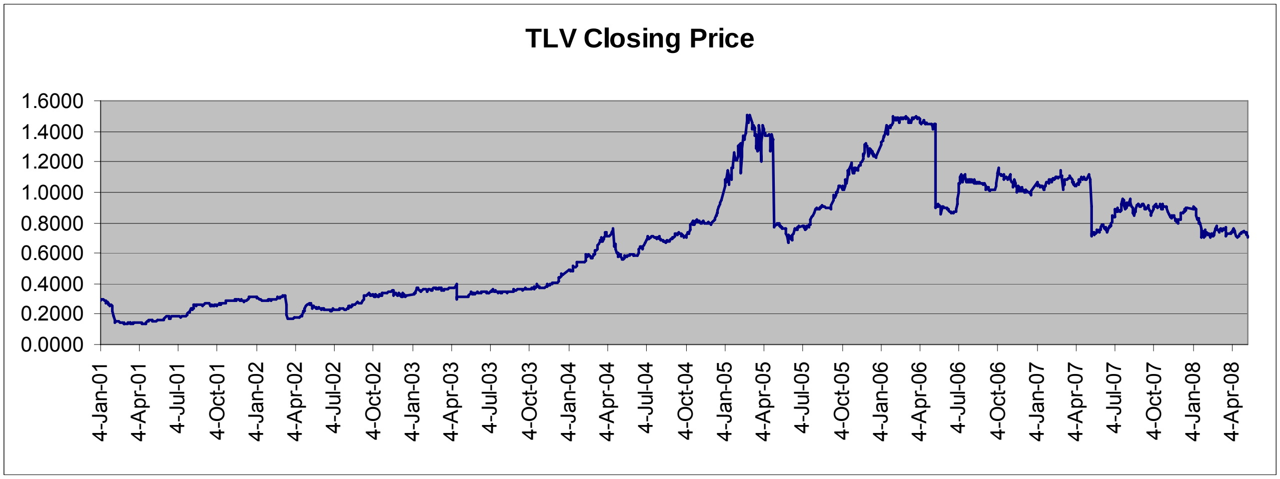


Fig. 2. The evolution of daily returns for ATB, AZO, TLV and the portfolio

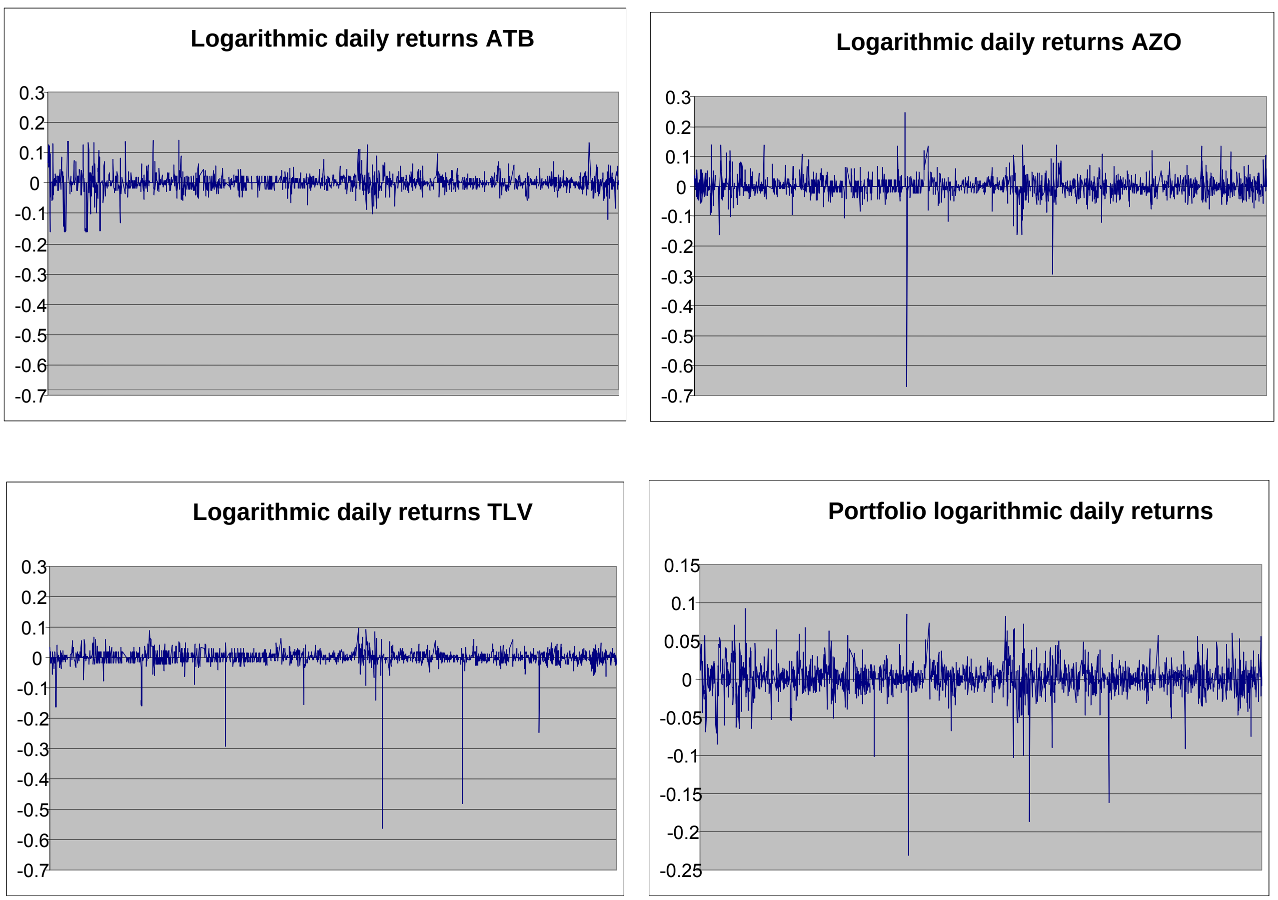


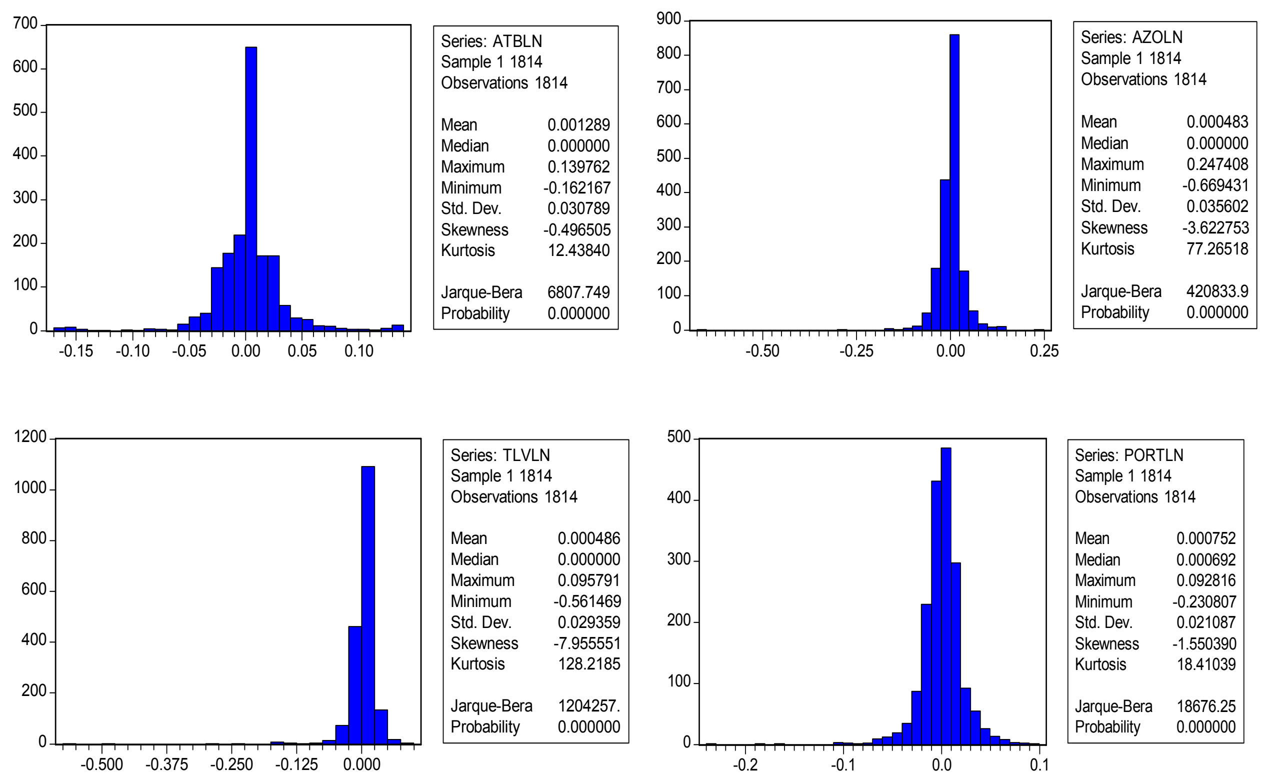
Table 1. Descriptive statistics of daily returns

	ATB	AZO	TLV	PORTFOLIO

Mean	0.001288686	0.000482618	0.000485777	0.00075236
Median	0	0	0	0.000692199
Maximum	0.139761942	0.247408173	0.095791065	0.092815783
Minimum	-0.162166755	-0.669430654	-0.561469357	-0.230806724
St. Dev.	0.030788687	0.035601895	0.029359217	0.021086889
Skewness	-0.496916313	-3.625751852	-7.962136836	-1.551673862
Kurtosis	12.46778119	77.47359745	128.567659	18.4562625

Skewness and kurtosis values of the series suggest that the stocks returns and also the portfolio returns aren't normally distributed (the empirical distributions are asymmetric to the left due to the negative skewness and they have a prominent peak due to the excess kurtosis). In order to verify the non-normality of the empirical distributions of the four returns series I used the Jarque-Bera test performed by Eviews. The test results, along with the histograms of the data series, are shown in Fig. 3 and indicate that the returns of the stocks and of the portfolio aren't normally distributed.

Fig. 3. Jarque-Bera test results and histograms for ATB, AZO, TLV and the portfolio



The four data series were also tested for the presence of the unit root using the Augmented Dickey-Fuller (ADF) performed in Eviews. The results are shown in Table 2 and, taking into account that ADF test critical value for the 1% level is -3.433757, they lead to the conclusion that none of the data series has the unit root. The full results of the ADF test along with the test equations are given in the Appendix 1. Then, the correlation matrix of the raw returns was computed in Eviews and is shown in Table 3.

Table 2. ADF test results for daily returns of ATB, AZO, TLV and the portfolio

Return series	Value of t-statistic	P value
ATB	-24.32559	0 %
AZO	-42.60324	0 %
TLV	-37.72185	0 %
Portfolio	-36.55895	0 %

Table 3. The correlation matrix of the raw returns

	ATB	AZO	TLV
ATB	1	0.176804	0.150407
AZO	0.176804	1	0.126762
TLV	0.150407	0.126762	1

The descriptive analysis of the primary data developed in this section leads to the following conclusions:

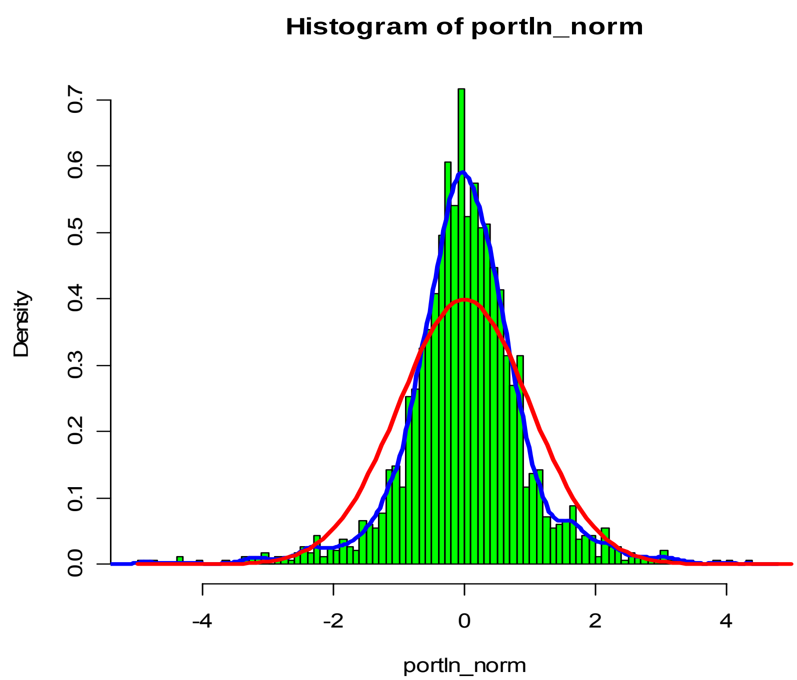
- The graphs of the returns show the presence of volatility clustering. This fact suggests that it would be better to compute VaR using conditional variance models, such as EWMA and GARCH models;
- The non-normality of the empirical distributions of the raw returns suggests that computing VaR with the quantiles of the standard normal distribution will tend to underestimate risk;
- The results of the ADF test allow us to consider that the mean and the variance of the data series are constant in time;
- The correlation coefficients, although positive, haven't high values so that the diversification effect is strong enough. Moreover, taking into account that former studies[17] have documented a high average correlation inside emerging markets and the presence of a strong market factor, we may consider that the correlation coefficients in our case are very advantageous.

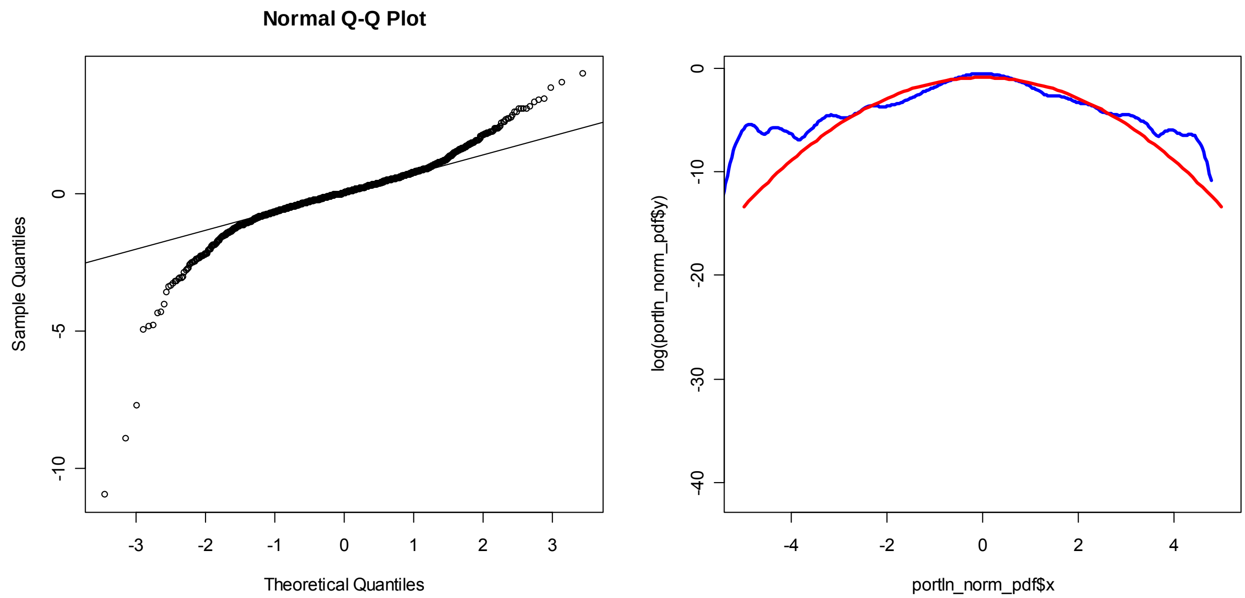
5. VaR using the historical volatility

[17] See, for example, Divecha, Drach and Stefek (1992)

The portfolio returns were standardized using the historical mean and volatility of the sample. Because, generally, VaR is computed using the quantiles of the normal distribution I constructed three graphs (see Fig. 4) in order to visualize the non-normality of the standardized returns: the probability density function (PDF) of the empirical distribution, the QQ plot and the log PDF graph, all plotted against the standard normal distribution. Of course, from the previous section we already know that the portfolio returns aren't normally distributed and the situation doesn't change when they are standardized using their historical mean and volatility. So, the problem of non-normality is still to be dealt with.

Fig. 4. PDF of the empirical distribution, QQ plot and log PDF against the standard normal distribution





In order to bypass this issue I estimated the parameters for three other distributions: t-Student (which allows for excess kurtosis), NIG and GH (which allow for skewness and excess kurtosis). The parameters were estimated through MLE using the Rmetrics package of the soft R. The results, along with the values for the informational criteria Akaike and Schwarz, are given in Table 4.

Table 4. Estimation results for t-Student, NIG and GH

t-Student		Normal Inverse Gaussian Distribution		General Hyperbolic Distribution	
degrees of freedom	8.724664	alpha	0.72873077	alpha	0.24382855
		beta	-0.0394378	beta	-0.03873113
		delta	0.67966102	delta	0.96331143
		mu	0.03683542	mu	0.03825432
				lambda	-1.29956291
MLE	-2453.487	MLE	-2306.181	MLE	-2302.68
AIC	2.7066	AIC	2.5471	AIC	2.5443
SIC	2.7092	SIC	2.5592	SIC	2.5595

Because the AIC and SIC values for NIG and GH are very close and the two criteria lead to different conclusions, both distributions were taken into account. Fig. 5 and Fig. 6 show the PDF of the empirical distribution and log PDF against the estimated distributions NIG and GH (while the same graphs for the estimated t-Student distribution are given in Appendix 2).

Fig. 5. PDF of the empirical distribution and log PDF against the estimated NIG distribution

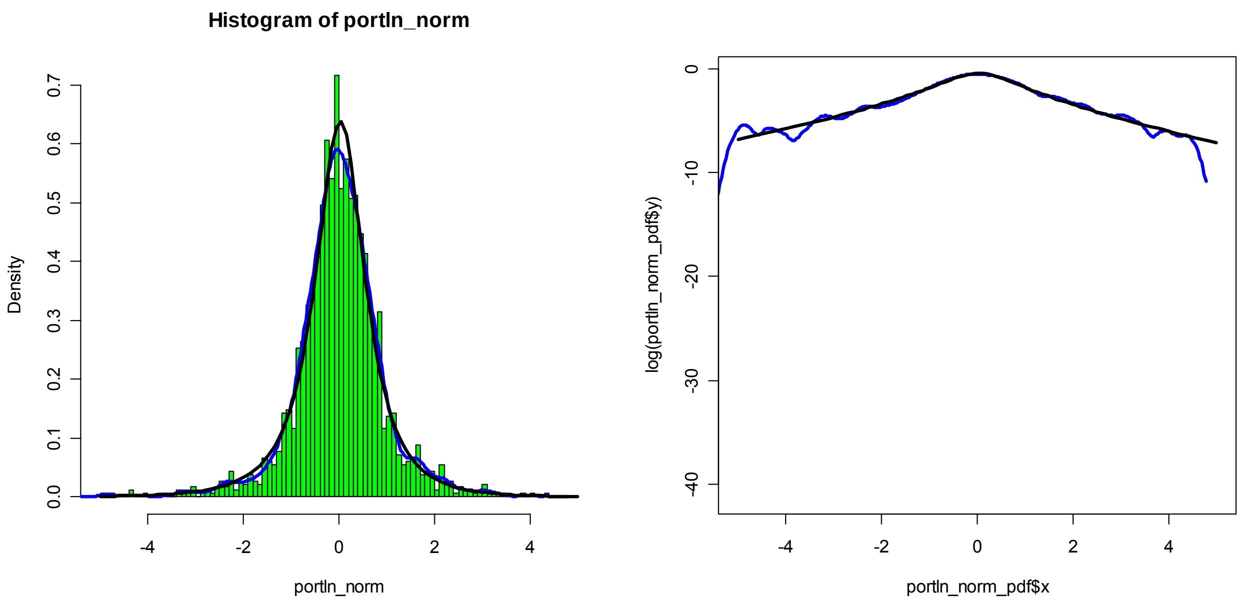
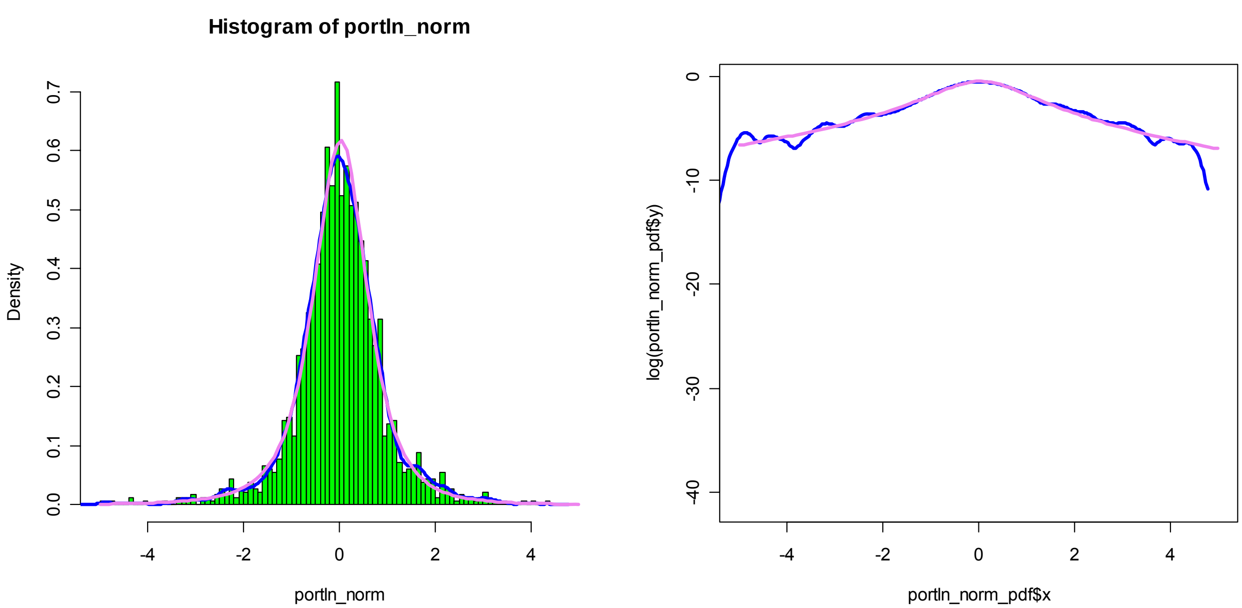


Fig. 6. PDF of the empirical distribution and log PDF against the estimated GH distribution



Analyzing these graphs it is clearly that NIG and GH fit much better the empirical distribution than does the standard normal distribution. Therefore, it would be reasonable to compute VaR using the quantiles of the NIG and GH distributions instead the ones of the standard normal distribution. The 1% quantiles for these distributions are given in Table 5.

Table 5. 1% quantiles for the standard normal distribution and estimated NIG and GH distributions

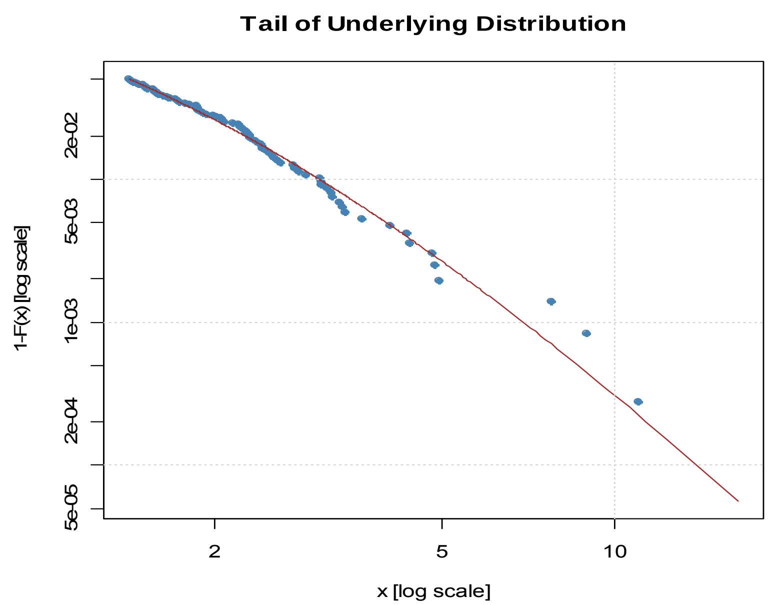
N Std.	-2.326347	NIG	-2.858291	GH	-2.898633

Another way of dealing with the non-normality of the standardized returns is the Extreme Value Theory (EVT). Using the Generalized Pareto Distribution (GPD), EVT models only the right tail of the distribution (the standardized returns in excess of a threshold) and because we are interested in extreme negative returns the EVT analysis is developed on the negative of returns. The threshold was set to be the 95 percentile of the sample of the standardized returns. The parameters of GPD were estimated through MLE and then the quantiles for VaR and Expected Shortfall (ES) were computed using the Rmetrics package of the soft R. The results are given in Table 6, while Fig. 7 shows the tail of the distribution along with the estimated GPD.

Table 6. Estimated parameters for GPD and the quantiles for VaR and ES according to EVT

Parameter Estimates for GPD			
xi	0.2575924	beta	0.8137205
Quantiles for VaR and ES for different probability levels according to EVT and N Std			
Probability	VaR EVT	ES EVT	N Std
1%	3.041225	4.701774	2.326347
0.5%	3.976777	5.961933	2.575831
0.1%	6.916062	9.921058	3.090253
0.05%	8.609074	12.20149	3.290560
0.01%	13.92812	19.36609	3.719090

Fig. 7. The tail of the distribution along with the estimated GDP



The reason I included smaller quantiles than 1% and the standard normal distribution quantiles is that the normal and EVT distribution may lead to close values for 1% VaRs but very different 0.1% VaRs - for example - due to the different tail shapes. In such situations the risk profiles are very different and this is our case too. So, ES could be a better risk measure for our portfolio because it takes into consideration the shape of the tail.

At last, I also considered the Cornish-Fisher approach. It constructs approximation to quantiles using the skewness and excess kurtosis of the standardized returns. The result obtained for our data set is presented in Table 7.

Table 7. Cornish-Fisher approximation

Skewness	-1.551673862
Excess Kurtosis	15.4562625
1% Q N Std	-2.326347
1% CF	-6.17468952

After modeling the distribution of the standardized returns and computing all the necessary quantiles, it was proceeded to the determination of VaR and ES. Using 750 days[18] rolling mean and volatility I computed 1-day, 1% VaR for all the approaches considered above (except for the EVT approach where VaR and ES were computed for all of the specified quantiles) for the period 29 Jan

[18] Approximately 3 years of data

2004 – 9 May 2008[19]. Also, historical VaR was computed as the 1% quantile of the series of the past 750 daily returns of the portfolio. The results obtained for VaR along with the negative of the returns of the portfolio (because VaR is a positive number) are shown in Fig. 8 and 9.

Fig. 8. 1-day, 1% VaR comparative graph

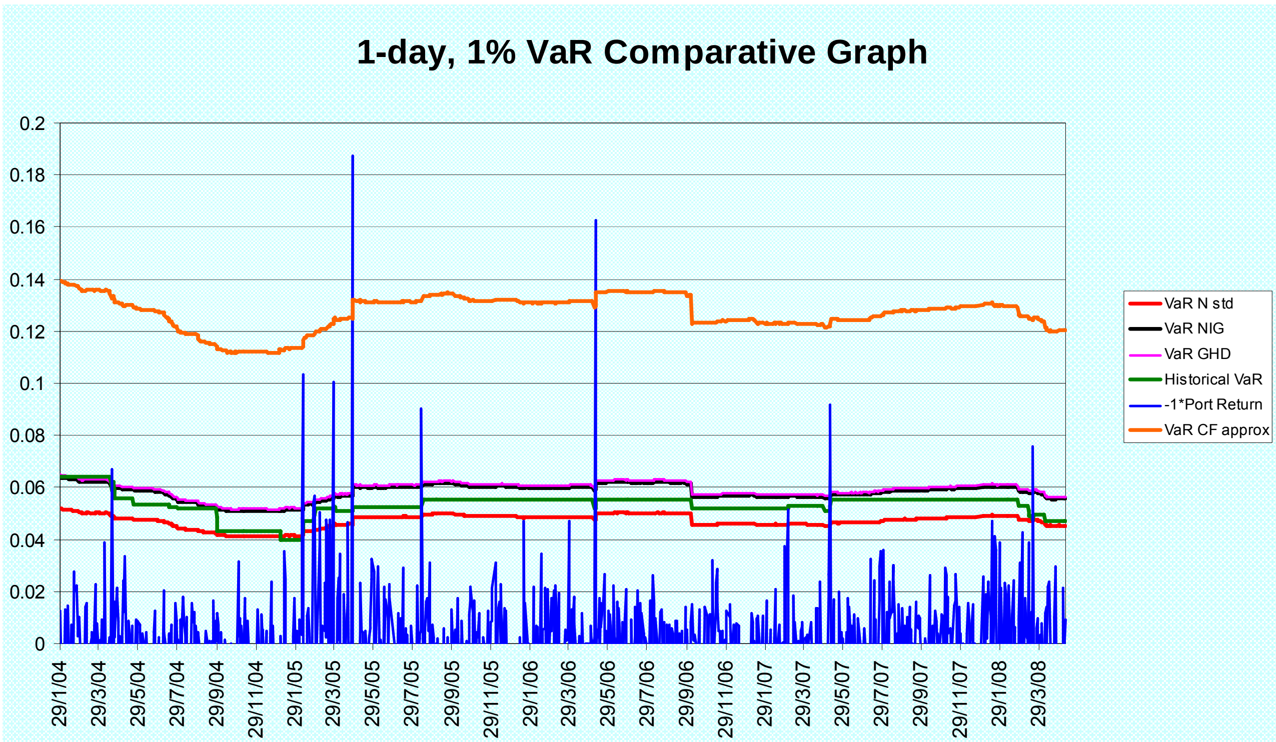
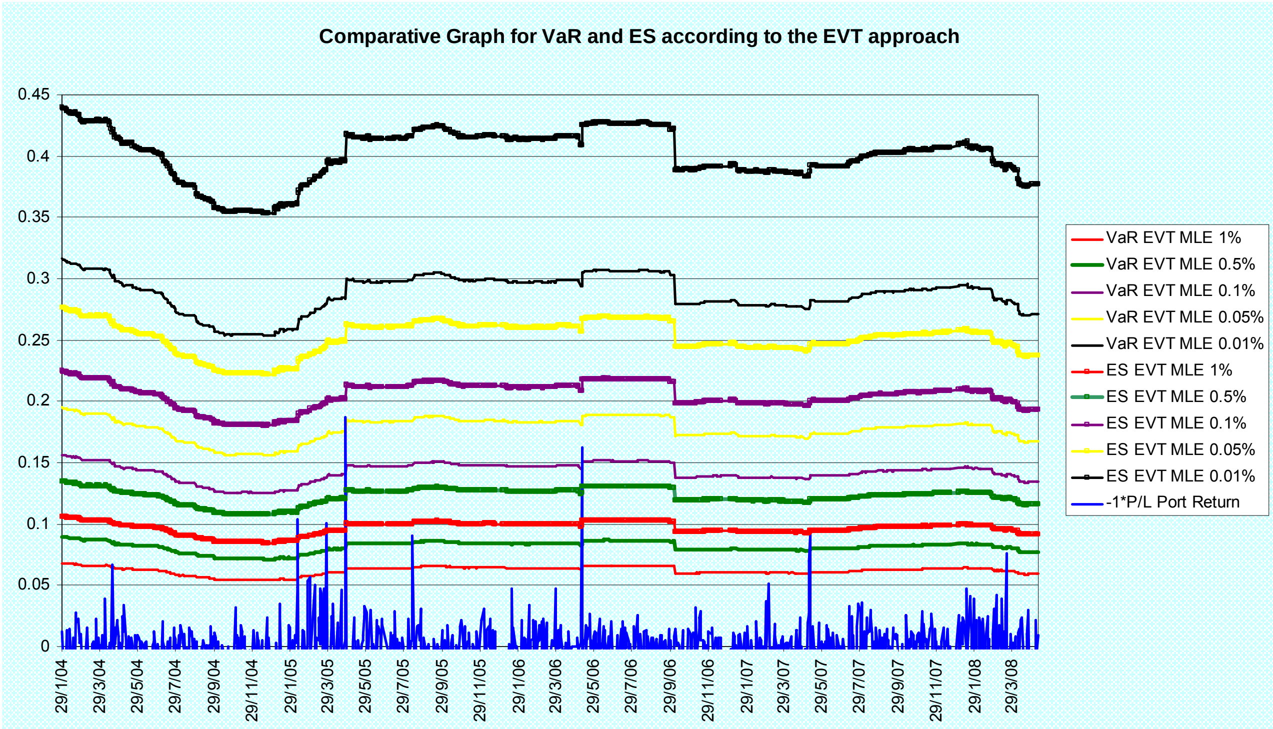


Fig. 9. 1-day VaR and ES for the EVT approach

[19] The remaining period after subtracting 750 days



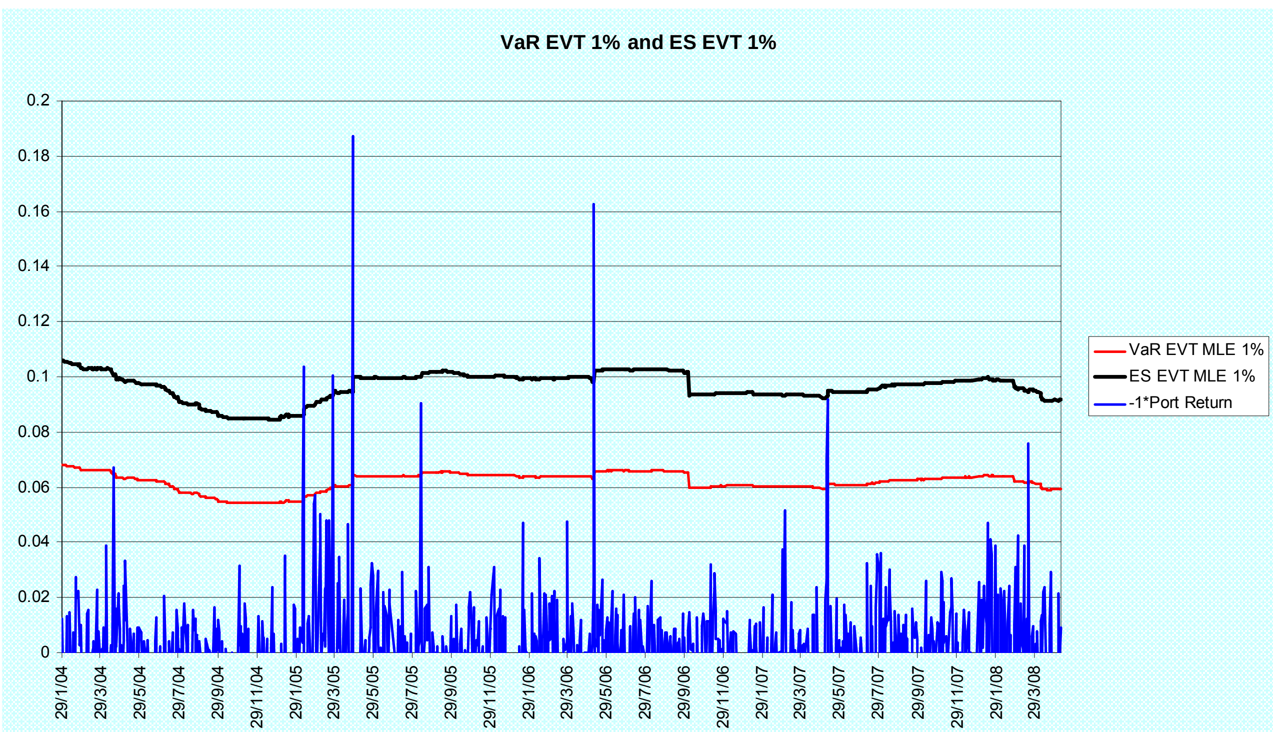


Table 8 shows if the computed risk measures match their respective critical levels (the term "errors %" denotes the percentage of the cases in which the portfolio loss exceeded VaR or ES) and if the average ES indicators were exceeded by the average losses bigger than the corresponding VaR (marked with red if ES was exceeded).

Table 8. The precision of computed VaR and ES indicators

VaR Type	**VaR N Std**	**VaR NIG**	**VaR GHD**	**VaR CF**	**Historical VaR**
Errors (%)	**1.60%**	**1.03%**	**0.94%**	**0.19%**	**1.13%**
VaR Type	**VaR EVT 1%**	**VaR EVT 0.5%**	**VaR EVT 0.1%**	**VaR EVT 0.05%**	**VaR EVT 0.01%**
Errors (%)	**0.85%**	**0.56%**	**0.19%**	**0.09%**	**0.00%**
ES Type	**ES EVT 1%**	**ES EVT 0.5%**	**ES EVT 0.1%**	**ES EVT 0.05%**	**ES EVT 0.01%**
Errors (%)	**0.38%**	**0.19%**	**0.00%**	**0.00%**	**0.00%**
Avg ES	**0.096504267**	**0.122663027**	**0.204847737**	**0.252185694**	**0.400910516**
Avg Loss (> VaR)	**0.105368373**	**0.122588525**	**0.174782102**	**0.187156452**	**0**

The results reported here account for the following conclusions:

- VaR N Std, Historical VaR and VaR NIG don't match the critical level of 1% (although VaR NIG is very close) while VaR GHD and VaR CF pass the precision test (VaR CF obtaining a very good score, only 0.19% errors);
- Under the EVT approach only two VaRs, at 1% and 0.01%, match their critical levels, while all ES indicators pass the precision test (although ES 1% has a problem: on average it has a smaller value than the average of the losses exceeding VaR 1%).

Table 9. Capital requirements for VaRs 1% and ES 1% that passed the precision test

	VaR GHD	**VaR CF**	**VaR EVT**	**ES EVT**
Average value	0.059074122	0.127079491	0.06203409	0.096504267
Maximum	0.051504545	0.111509139	0.054116275	0.084531064
Minimum	0.064807013	0.139454722	0.068056092	0.1058931

If we are to choose between the 1% risk measures that passed the precision test VaR GHD would be the most appropriate because it provides the lowest level of capital coverage (see Table 9). The capital requirement is about 5.91% (average level) of the portfolio value (which is 1 RON in our case), while VaR EVT requires about 6.2%. VaR CF, although provides the best protection (only 0.19% errors), has the highest level of capital requirements being more expensive even than ES 1%.

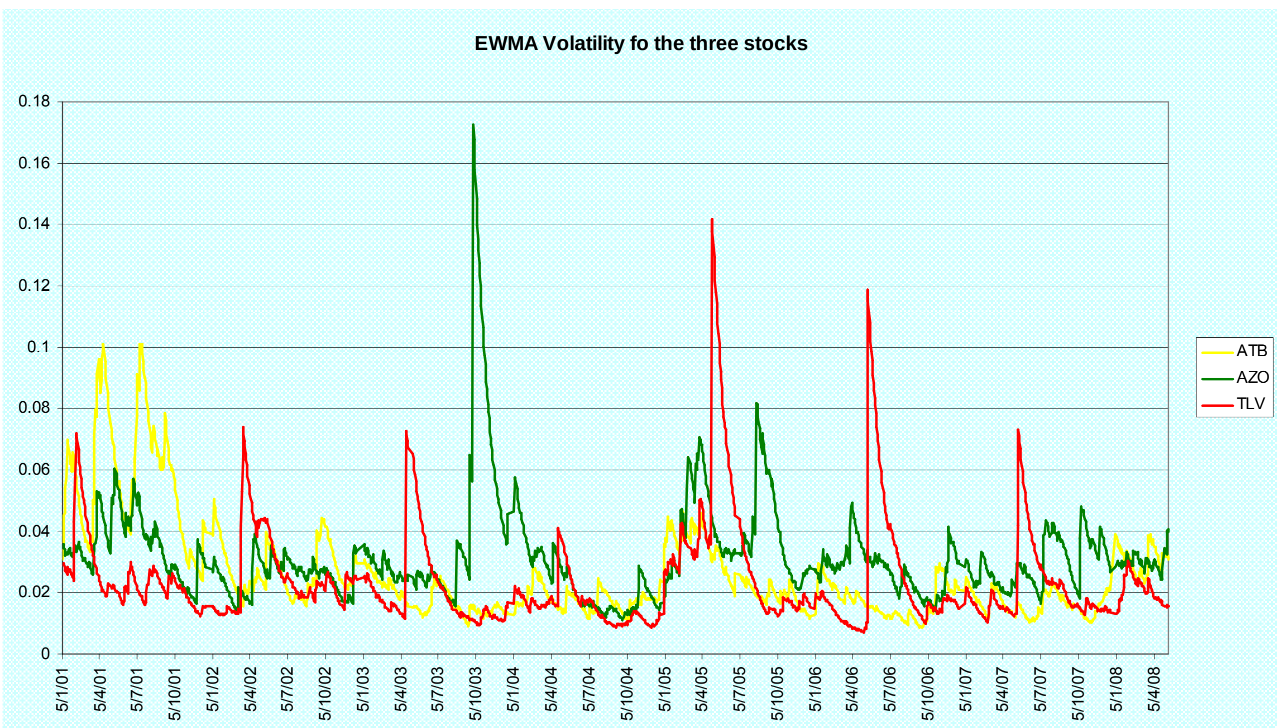
6. VaR using the EWMA volatility model

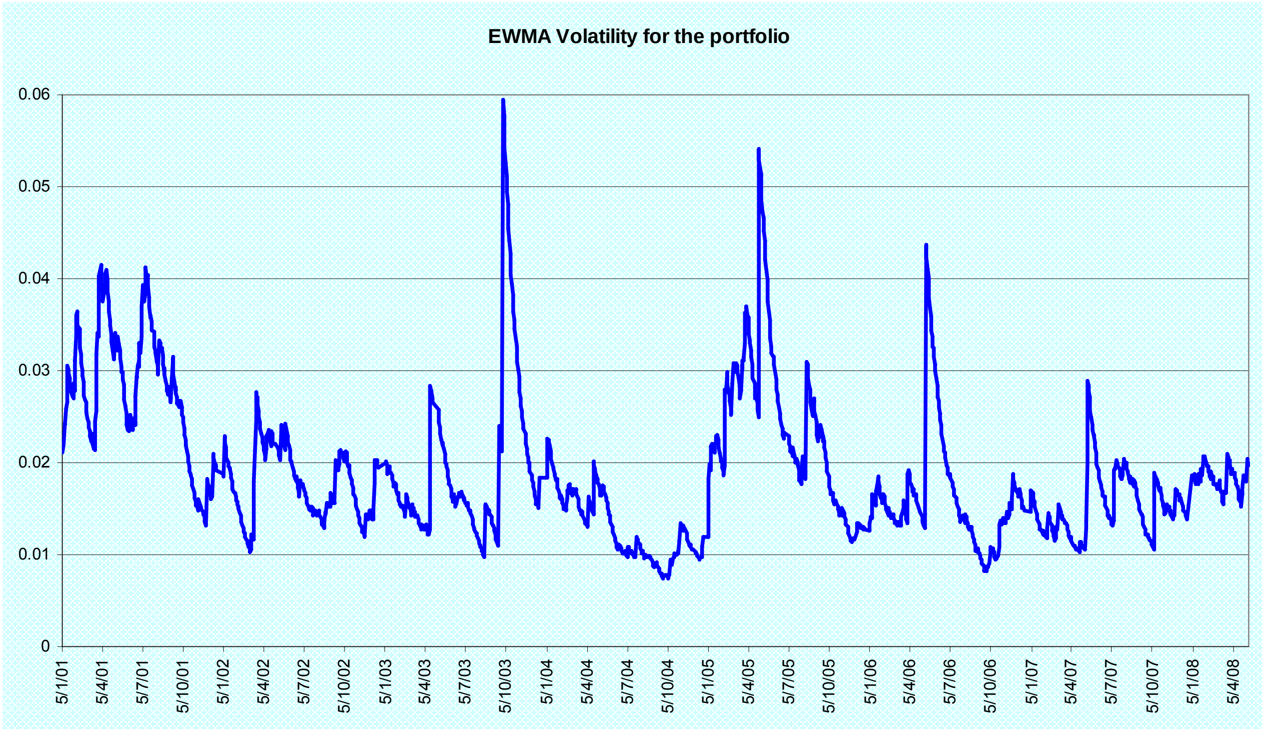
In order to enhance the analysis of the risk measures developed in this study I considered time-varying volatility models. In this section it is used one of the simplest volatility models, namely the Exponentially Weighted Moving Average (EWMA) as developed by JP Morgan's RiskMetrics system. The model is written as

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)R_t^2$$

where $\lambda < 1$, σ denotes the volatility and R the return. Using this model I computed the volatility series for the three stocks considering that $\lambda = 0.94$ (the value set by RiskMetrics) and that the starting values are the historical standard deviations of the data series. Then, using the historical correlation coefficients, I computed the volatility series for the portfolio. The volatility graphs for the three stocks and for the portfolio are shown in Fig. 10.

Fig. 10. The EWMA Volatility of the three stocks and of the portfolio





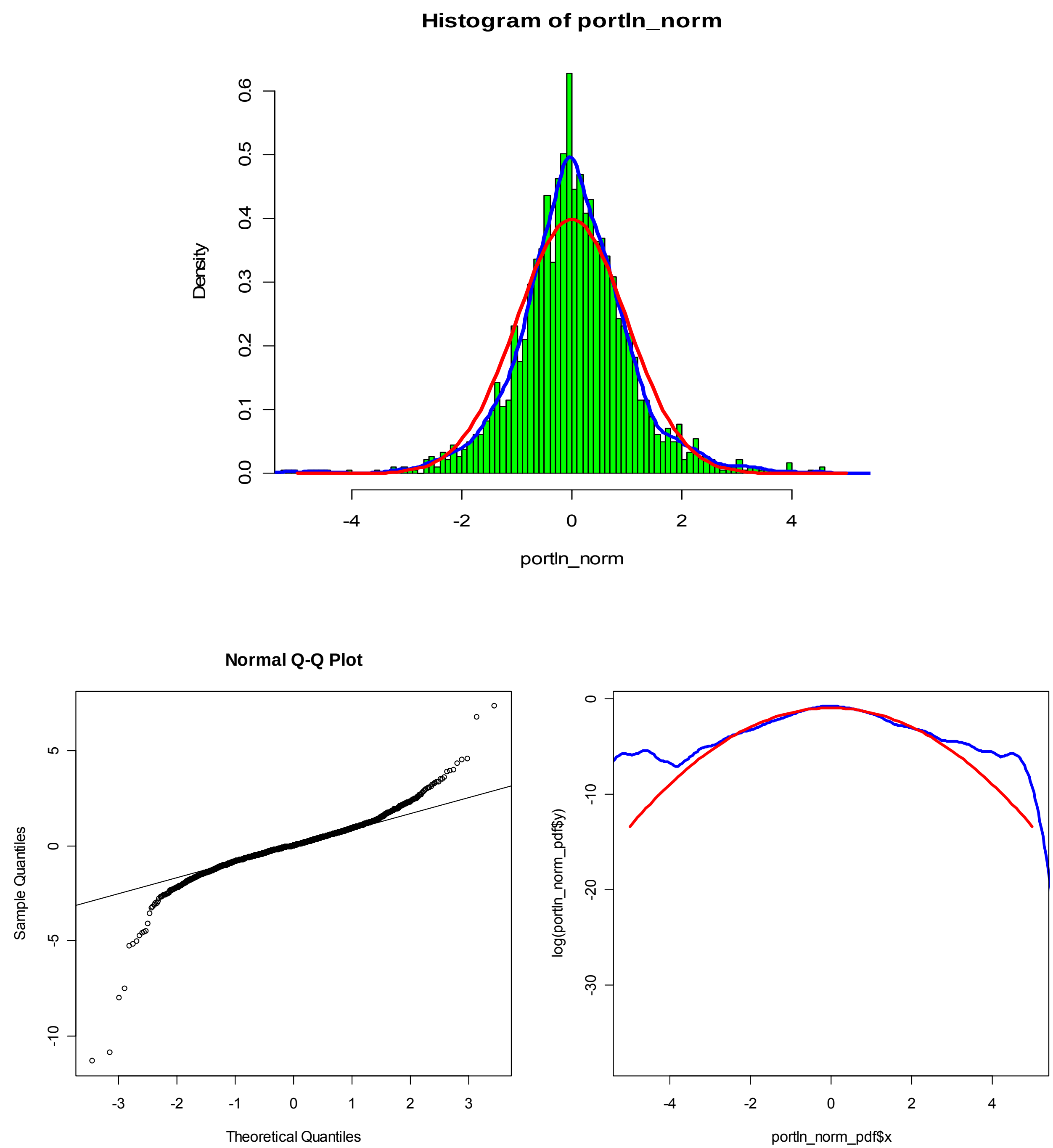
The returns of the stocks and of the portfolio were standardized using the historical mean and the computed EWMA volatility. The descriptive statistics for the series of the standardized returns are given in Table 10. The values for skewness and kurtosis indicate that we still have to face non-normality. The Jarque-Bera test applied to the standardized returns of the portfolio has a value of 15225.66 (with an associated p value of 0%) meaning they aren't normally distributed, while the ADF test confirms that the unit root is not present (the two tests are given in Appendix 3).

Table 10. Descriptive statistics of the returns standardized with EWMA volatility

	ATB	AZO	TLV	PORTFOLIO
Mean	0.022853282	0.008773423	0.01151103	0.015033927
Median	-0.062906618	-0.016700982	-0.023587187	-0.004146442
Maximum	6.66875623	10.1983607	7.288494731	7.380381514
Minimum	-5.677694158	-11.92240587	-47.57183779	-11.35463294
Std. Dev.	1.077012211	1.14691887	1.658327606	1.160841498
Skewness	0.570426272	0.223325423	-15.01802686	-1.105297369
Kurtosis	7.912768016	20.12699426	396.8349199	17.0621412

In order to visualize the non-normality of the portfolio standardized returns Fig. 11 shows three graphs: the probability density function (PDF) of the empirical distribution, the QQ plot and the log PDF graph, all plotted against the standard normal distribution.

Fig. 11. PDF of the empirical distribution, QQ plot and log PDF against the standard normal distribution



As in the previous section, the first way to deal with the non-normality of the data was to estimate the parameters for t-Student, NIG and GH distributions. The parameters were estimated through MLE using the Rmetrics package of the soft R. The results, along with the values for the informational criteria Akaike and Schwarz, are given in Table 11.

Table 11. Estimation results for t-Student, NIG and GH

t-Student		Normal Inverse Gaussian Distribution		General Hyperbolic Distribution	
degrees of freedom	6.32333	alpha	0.71864289	alpha	0.01319607
		beta	-0.01048403	beta	-0.01318189
		delta	0.89238534	delta	1.38629802
		mu	0.0280556	mu	0.03261598
				lambda	-1.72038174
MLE	-2656.422	MLE	-2611.03	MLE	-2603.462
AIC	2.9299	AIC	2.8832	AIC	2.8759
SIC	2.9329	SIC	2.8953	SIC	2.8911

Because the AIC and SIC values for NIG and GH are very close both distributions were taken into account. Fig. 12 and Fig. 13 show the PDF of the empirical distribution and log PDF against the estimated distributions NIG and GH (while the same graphs for the estimated t-Student distribution are given in Appendix 3). Analyzing these graphs it is clearly that NIG and GH fit much better the empirical distribution than does the standard normal distribution. Therefore, it would be reasonable to compute VaR using the quantiles of the NIG and GH distributions instead the ones of the standard normal distribution. The 1% quantiles for these distributions are given in Table 12.

Fig. 12. PDF of the empirical distribution and log PDF against the estimated NIG distribution

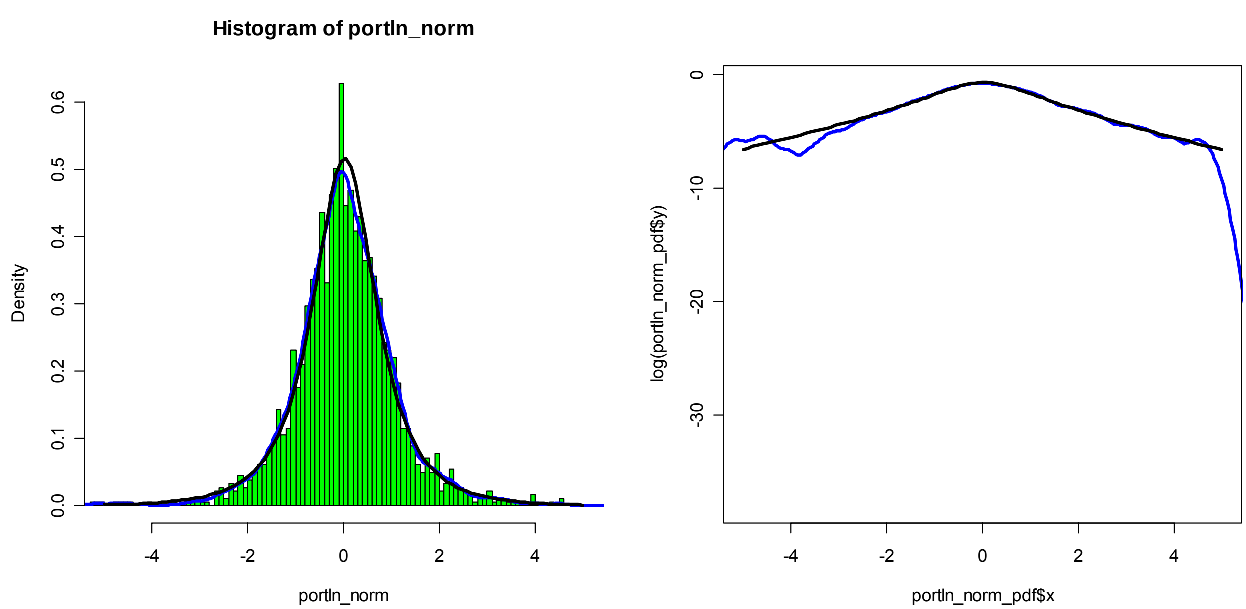


Fig. 13. PDF of the empirical distribution and log PDF against the estimated GH distribution

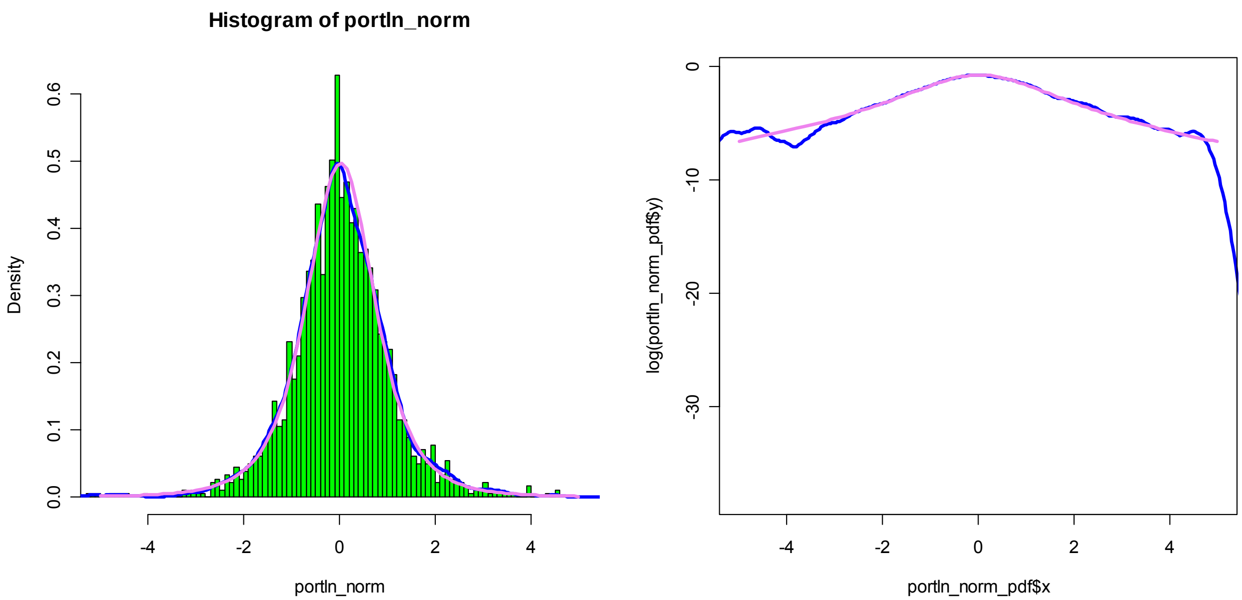


Table 12. 1% quantiles for the standard normal distribution and estimated NIG and GH distributions

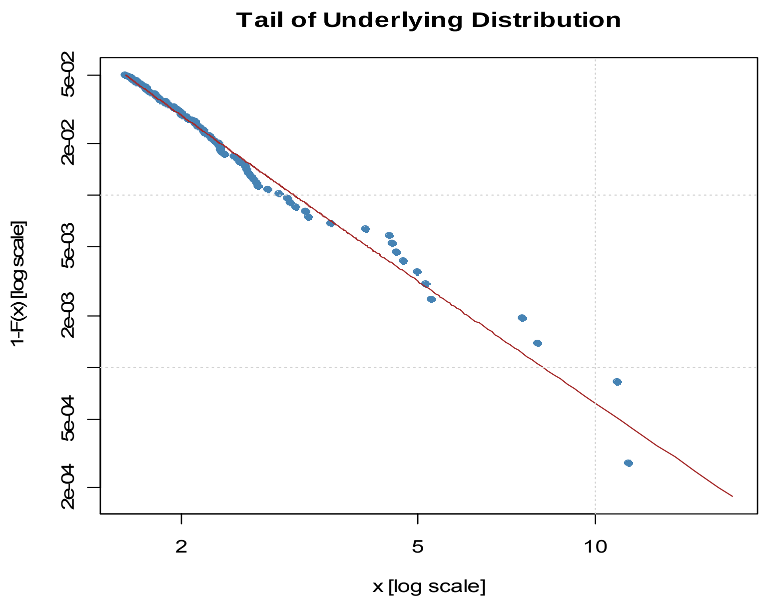
N Std.	-2.326347	NIG	-3.134162	GH	-3.098957

As in the previous section, I used also the EVT approach in order to deal with the non-normality of the standardized data. Again, the threshold was set to be the 95 percentile of the sample of the standardized returns while the parameters of GPD were estimated through MLE and then the quantiles for VaR and ES were computed using Rmetrics. The results are given in Table 13, while Fig. 14 shows the tail of the distribution along with the estimated GPD.

Table 13. Estimated parameters for GPD and the quantiles for VaR and ES according to EVT

Parameter Estimates for GPD			
xi	0.4300091	beta	0.64173
Quantiles for VaR and ES for different probability levels according to EVT and N Std			
Probability	VaR EVT	ES EVT	N Std
1%	3.100416	5.352915	2.326347
0.5%	4.137198	7.171859	2.575831
0.1%	8.151101	14.213906	3.090253
0.05%	10.941692	19.109759	3.290560
0.01%	21.745478	38.064071	3.719090

Fig. 14. The tail of the distribution along with the estimated GDP



We observe the same situation as in the previous section: the quantiles of the normal and EVT distribution are very different for probabilities below 1% due to the different tail shapes. Therefore, the risk profiles are very different and ES could be a better risk measure for our portfolio because it takes into consideration the shape of the tail.

At last, the Cornish-Fisher approach was also considered. The approximation for the 1% quantile, computed using the skewness and excess kurtosis of the returns standardized with the EWMA volatility, is given in Table 14.

Table 14. Cornish-Fisher approximation

Skewness	-1.105297369
Excess Kurtosis	14.0621412
1% Q N Std	-2.326347
1% CF	-5.966873733

After modeling the distribution of the standardized returns and computing all the necessary quantiles, it was proceeded to the determination of VaR and ES. Using the EWMA volatility I computed 1-day, 1% VaR for all the approaches considered above (except for the EVT approach where VaR and ES were computed for all of the specified quantiles) for the period 29 Jan 2004 – 9 May 2008. Also, historical VaR was computed using the 1% quantile of the series of the past 750 daily standardized returns of the portfolio. The results obtained for VaR along with the negative of the returns of the portfolio (because VaR is a positive number) are shown in Fig. 15 and 16. Table 15 shows if the computed risk measures match their respective critical levels (the term "errors %" denotes the percentage of the cases in which the portfolio loss exceeded VaR or ES) and if the average ES indicators were exceeded by the average losses bigger than the corresponding VaR (marked with red if ES was exceeded).

Table 15. The precision of computed VaR and ES indicators

VaR Type	VaR N Std	VaR NIG	VaR GHD	VaR CF	Historical VaR
Errors (%)	2.26%	1.03%	1.13%	0.19%	1.41%
VaR Type	VaR EVT 1%	VaR EVT 0.5%	VaR EVT 0.1%	VaR EVT 0.05%	VaR EVT 0.01%
Errors (%)	1.13%	0.75%	0.09%	0.09%	0.00%
ES Type	ES EVT 1%	ES EVT 0.5%	ES EVT 0.1%	ES EVT 0.05%	ES EVT 0.01%
Errors (%)	0.19%	0.09%	0.00%	0.00%	0.00%
Avg ES	0.092887278	0.124823403	0.248464206	0.334423189	0.66721369
Avg Loss (> VaR)	0.085960761	0.106005833	0.162407753	0.162407753	0

Fig. 15. 1-day, 1% VaR EWMA comparative graph

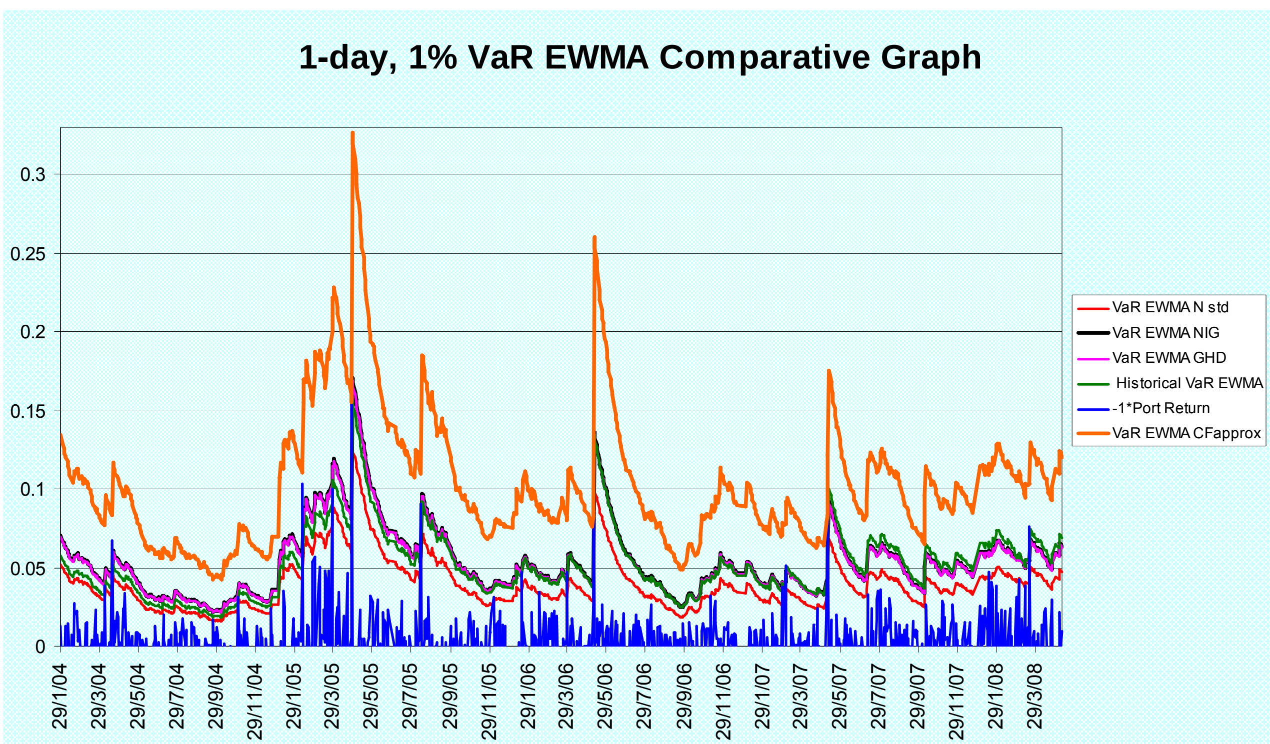
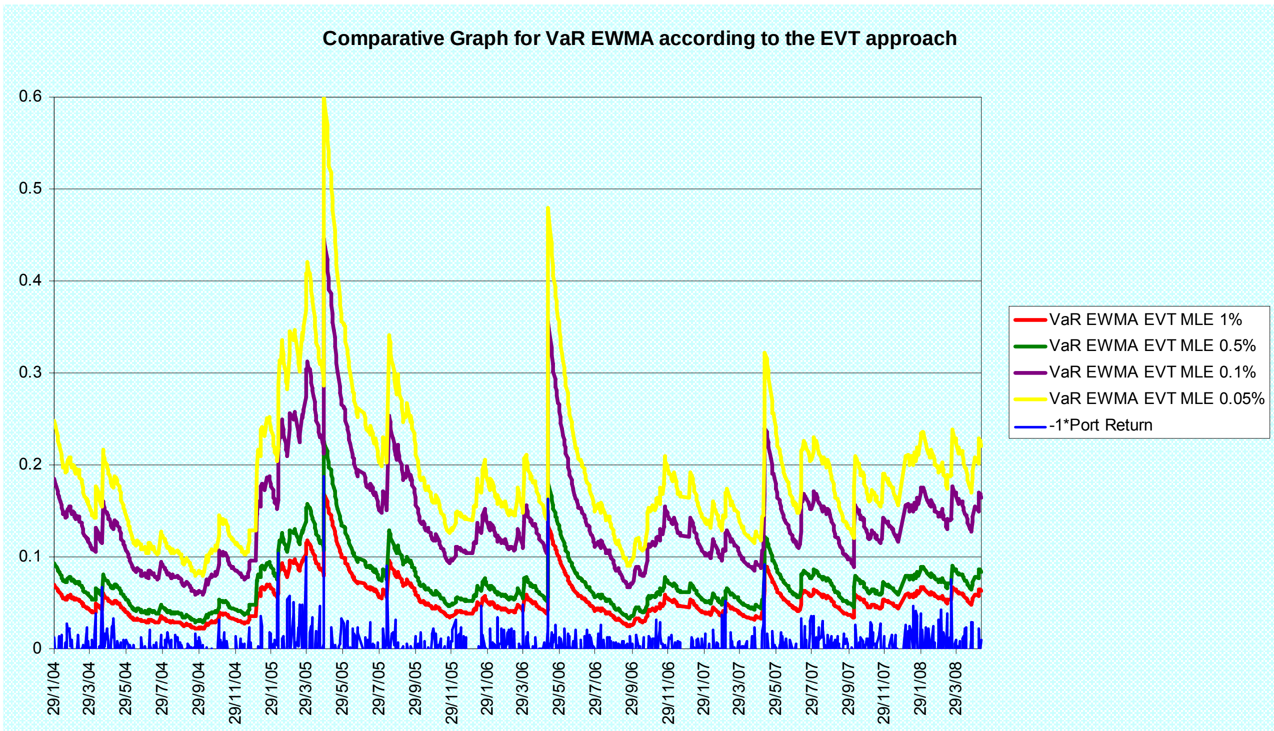
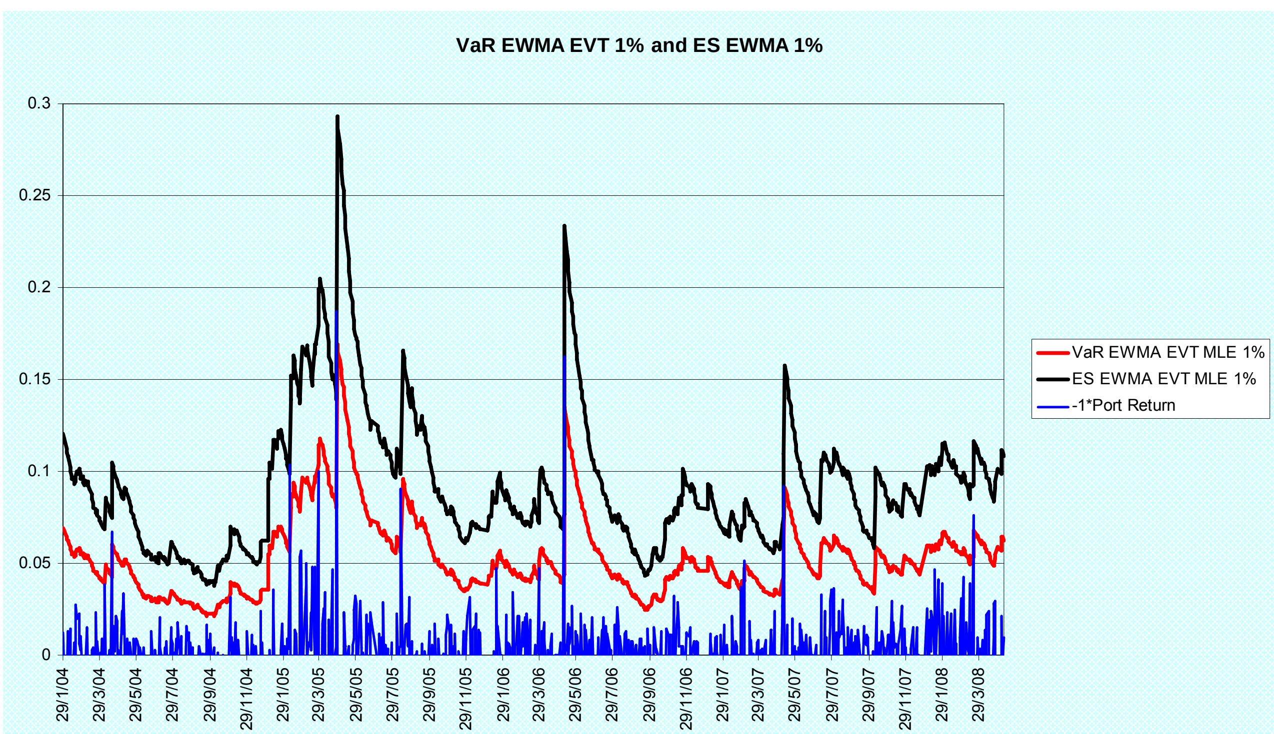


Fig. 16. 1-day VaR EWMA for the EVT approach





The results reported here account for the following conclusions:

- VaR N Std, Historical VaR, VaR NIG and VaR GHD didn't succeed in matching the critical level of 1% (although VaR NIG is very close), only VaR CF being able to pass the precision test with the same score as in the previous section;
- Under the EVT approach only two VaRs, at 0.1% and 0.01%, match their critical levels, but all ES indicators pass the precision test;
- It seems that the EWMA volatility model tends to underestimate risk for the portfolio and the period analyzed in my study, the results for the 1% risk measures being worse than in the case of using historical volatility (see the previous section).

Table 16. Capital requirements for VaR CF 1% and ES 1%

	VaR CF 1%	**ES EVT 1%**
Average value	0.103666864	0.092887278
Maximum	0.042467889	0.037951806
Minimum	0.326926586	0.293116434

If we are to choose between VaR CF and ES (these being the only 1% risk measures that passed the precision test), ES 1% would be the most appropriate because it provides the lowest level of capital coverage (see Table 16). The capital requirement is about 9.29% (average level) of the portfolio value, while VaR CF requires about 10.37%. Moreover, the two risk measures offer the same accuracy, namely 0.19% errors.

7. VaR using a GARCH volatility model for portfolio returns

The analysis developed in this study was further improved by considering GARCH volatility models which overrun some of the shortcomings of EWMA model. First, I considered the simplest case: a GARCH (1,1) volatility model for the portfolio returns which has the advantage that few calculations are needed even in the case of large portfolios. The model is written as

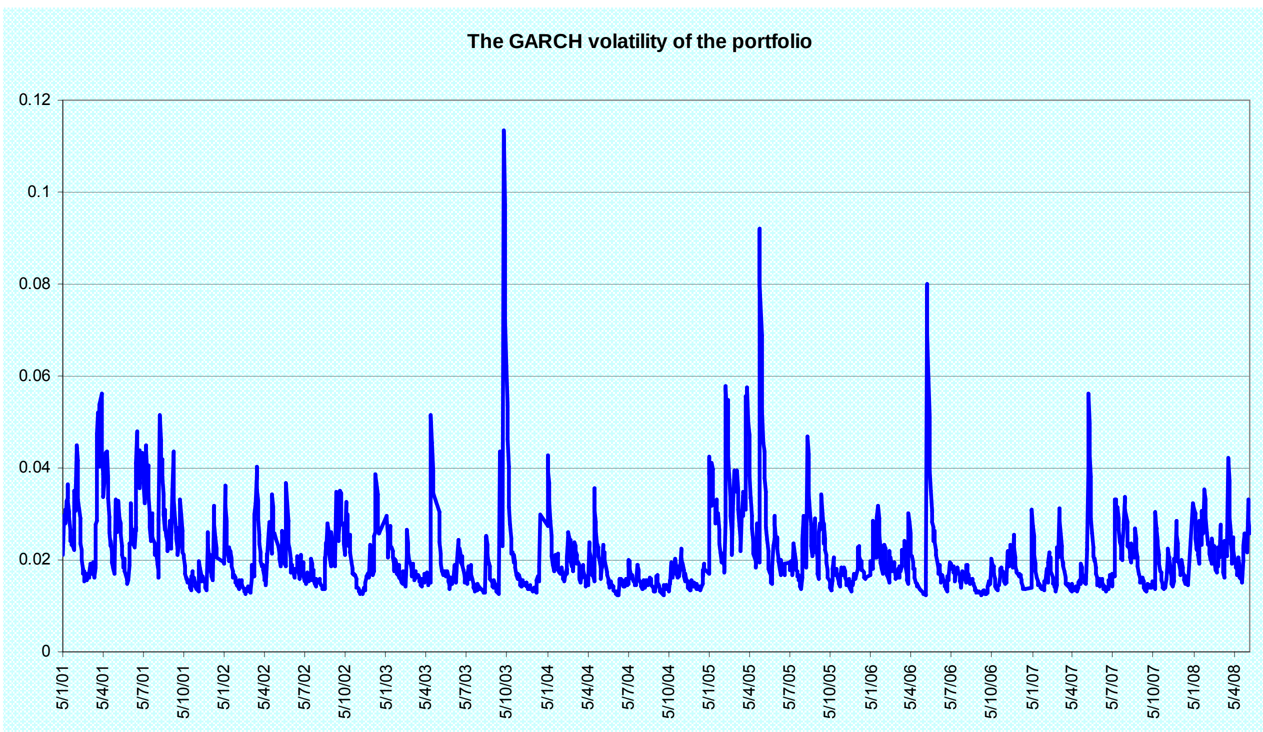
$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta \sigma_t^2$$

(where $\alpha + \beta < 1$, σ denotes the volatility, R the return) and the parameters α, β and ω were estimated through MLE in Eviews. With the estimated values of the parameters I computed the volatility series of the portfolio returns using as starting value the historical standard deviation. The results of the estimation are given in Table 17 (for the full results see Appendix 4) while Fig. 17 shows the graph of the GARCH volatility of the portfolio during the analyzed period.

Table 17. GARCH (1,1) volatility model for the portfolio returns

ω	0.00003674
α	0.23330744
β	0.73422658

Fig. 17. The GARCH Volatility of the portfolio



The returns of the portfolio were standardized using the historical mean and the computed GARCH volatility. The descriptive statistics for the series of the standardized returns are given in Table 18. The values for skewness and kurtosis indicate that we still have to face non-normality. The Jarque-Bera test applied to the standardized returns of the portfolio has a value of 25590.53 (with an associated p value of 0%) meaning they aren't normally distributed, while the ADF test confirms that the unit root is not present (the two tests are given in Appendix 4).

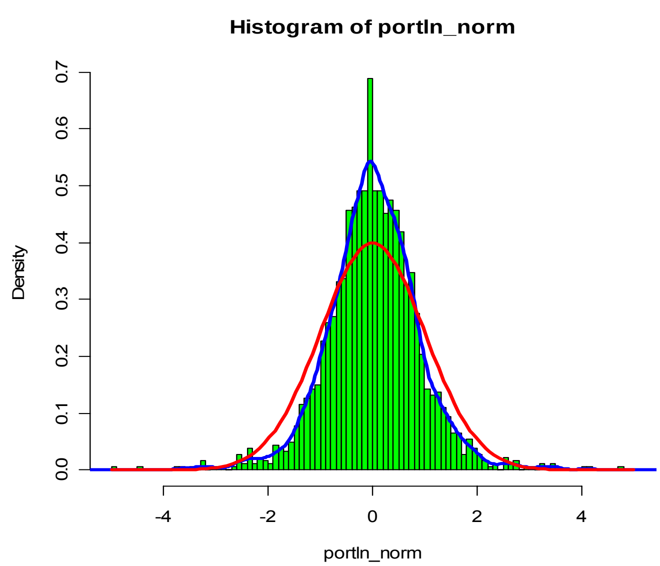
Table 18. Descriptive statistics of the portfolio returns standardized with GARCH volatility

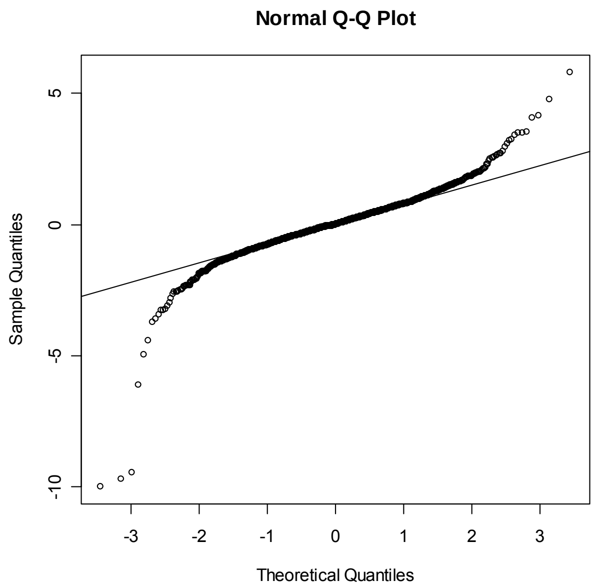
	PORTFOLIO
Mean	0.002768796
Median	-0.00332796
Maximum	5.807380535
Minimum	-10.0040107
Std. Dev.	1.001248131
Skewness	-1.513391092
Kurtosis	20.07196361

In order to visualize the non-normality of the portfolio standardized returns Fig. 18 shows three graphs: the probability density function (PDF) of the empirical distribution, the QQ plot and the log PDF graph, all plotted against the standard normal distribution.

Similarly to the previous sections, the first way to deal with the non-normality of the data was to estimate the parameters for t-Student, NIG and GH distributions. The parameters were estimated through MLE using the Rmetrics package of the soft R. The results, along with the values for the informational criteria Akaike and Schwarz, are given in Table 19.

Fig. 18. PDF of the empirical distribution, QQ plot and log PDF against the standard normal distribution





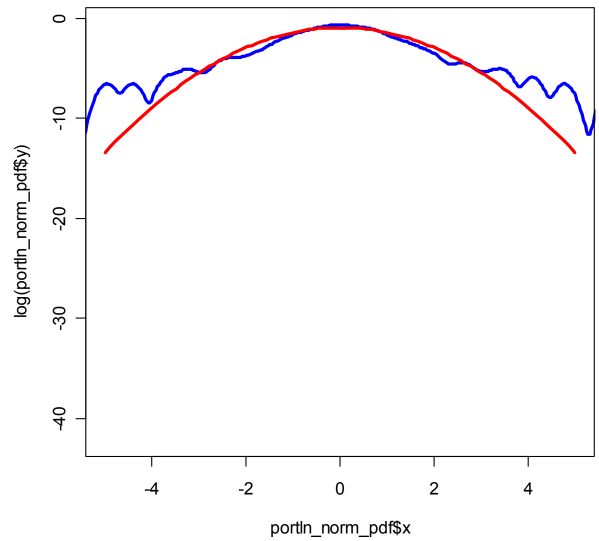


Table 19. Estimation results for t-Student, NIG and GH

t-Student		**Normal Inverse Gaussian Distribution**		**General Hyperbolic Distribution**	
degrees of freedom	9.6544	alpha	0.94278975	alpha	0.05570423
		beta	-0.05636764	beta	-0.05570417
		delta	0.84625444	delta	1.32257003
		mu	0.05345637	mu	0.054024
				lambda	-1.94888169
MLE	-2451.553	MLE	-2350.707	MLE	-2338.908
AIC	2.704	AIC	2.5961	AIC	2.5842
SIC	2.7071	SIC	2.6083	SIC	2.5994

Because the AIC and SIC values for NIG and GH are very close both distributions were taken into account. Fig. 19 and Fig. 20 show the PDF of the empirical distribution and log PDF against the estimated distributions NIG and GH (while the same graphs for the estimated t-Student distribution are given in Appendix 4).

Fig. 19. PDF of the empirical distribution and log PDF against the estimated NIG distribution

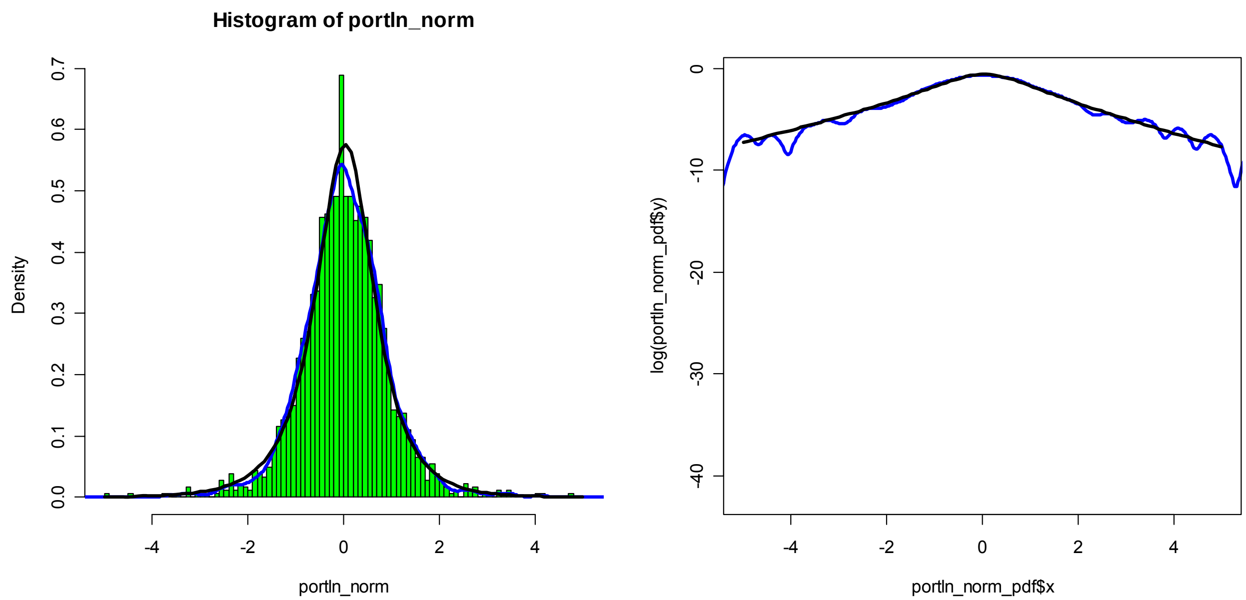
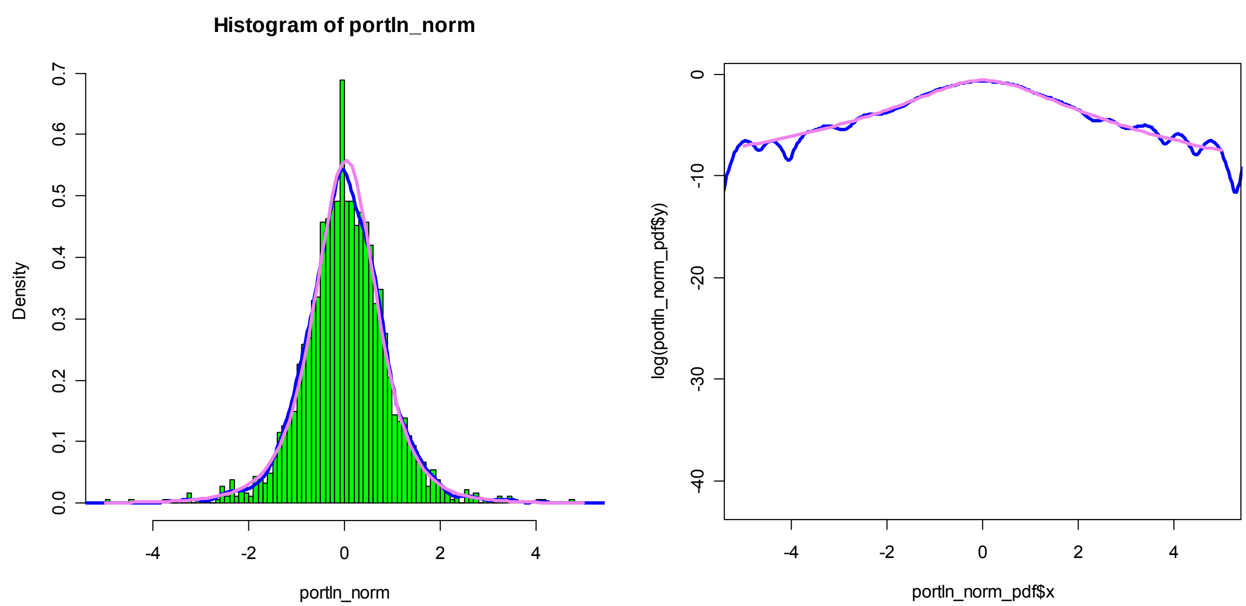


Fig. 20. PDF of the empirical distribution and log PDF against the estimated GH distribution



Analyzing these graphs it is clearly that NIG and GH fit much better the empirical distribution than does the standard normal distribution. Therefore, it would be reasonable to compute VaR using the quantiles of the NIG and GH distributions instead the ones of the standard normal distribution. The 1% quantiles for these distributions are given in Table 20.

Table 20. 1% quantiles for the standard normal distribution and estimated NIG and GH distributions

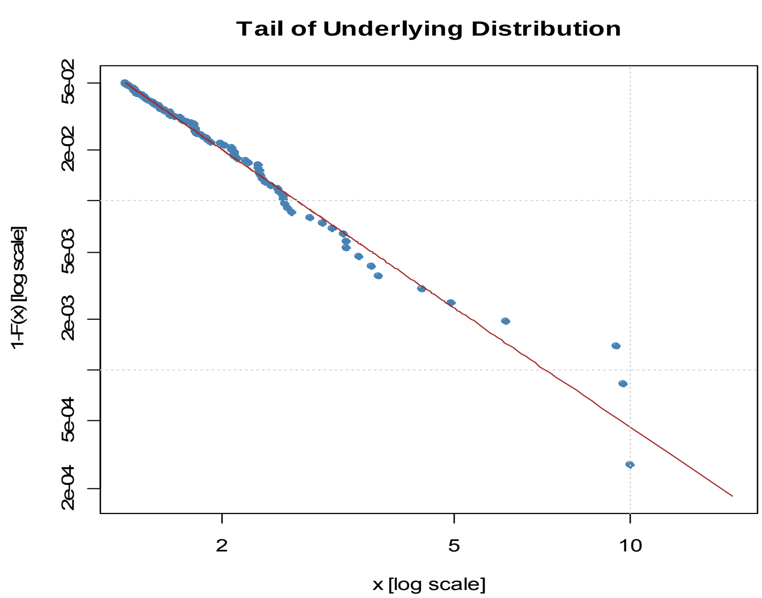
N Std.	-2.326347	NIG	-2.699688	GH	-2.658805

As in the previous sections, I used also the EVT approach in order to deal with the non-normality of the standardized data. Again, the threshold was set to be the 95 percentile of the sample of the standardized returns while the parameters of GPD were estimated through MLE and then the quantiles for VaR and ES were computed using Rmetrics. The results are given in Table 21, while Fig. 21 shows the tail of the distribution along with the estimated GPD.

Table 21. Estimated parameters for GPD and the quantiles for VaR and ES according to EVT

Parameter Estimates for GPD			
xi	0.4301966	beta	0.5696852
Quantiles for VaR and ES for different probability levels according to EVT and N Std			
Probability	VaR EVT	ES EVT	N Std
1%	2.691543	4.692425	2.326347
0.5%	3.61227	6.308292	2.575831
0.1%	7.177687	12.565568	3.090253
0.05%	9.656977	16.9167	3.290560
0.01%	19.257766	33.766002	3.719090

Fig. 21. The tail of the distribution along with the estimated GDP



We observe the same situation as in the previous sections: the quantiles of the normal and EVT distribution are very different for probabilities below 1% due to the different tail shapes. Therefore, the risk profiles are significantly different and ES could be a better risk measure for our portfolio because it takes into consideration the shape of the tail.

At last, the Cornish-Fisher approach was also considered. The approximation for the 1% quantile, computed using the skewness and excess kurtosis of the returns standardized with the GARCH volatility, is given in Table 22.

Table 22. Cornish-Fisher approximation

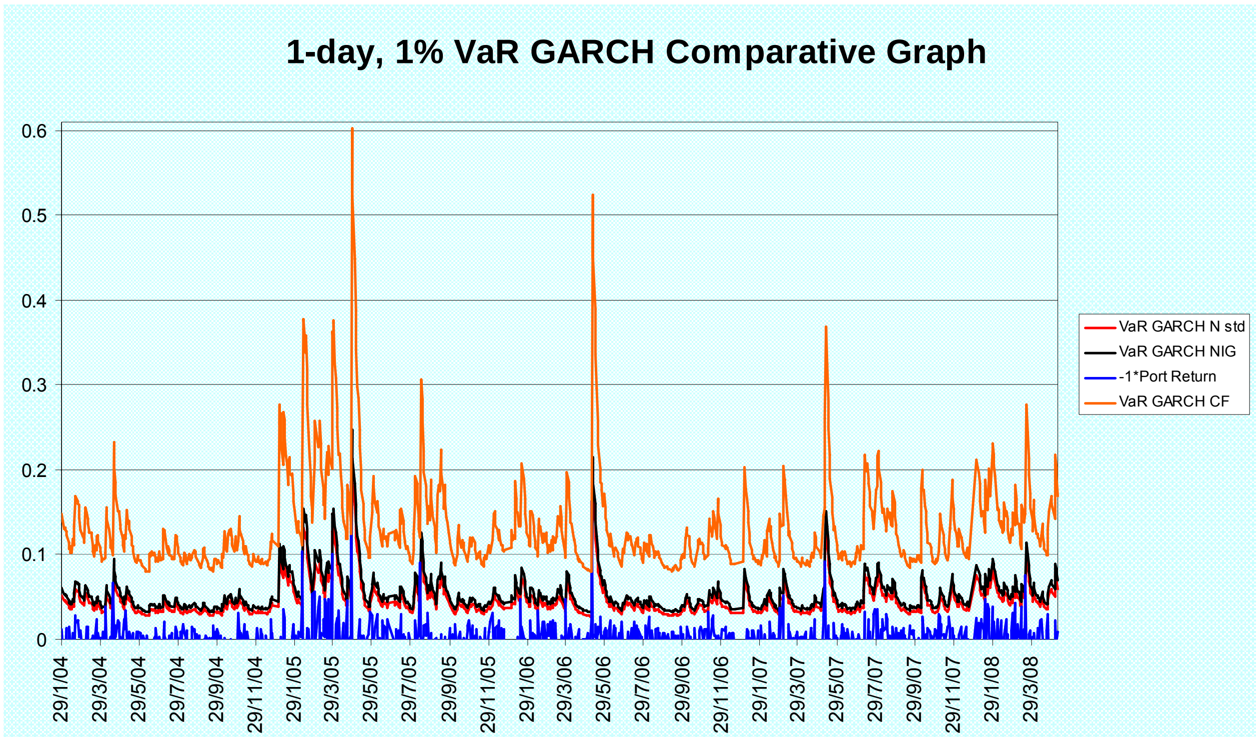
Skewness	-1.513391092
Excess Kurtosis	17.07196361
1% Q N Std	-2.326347
1% CF	-6.56842902

After modeling the distribution of the standardized returns and computing all the necessary quantiles, it was proceeded to the determination of VaR and ES. Using the GARCH volatility I computed 1-day, 1% VaR for all the approaches considered above (except for the EVT approach where VaR and ES were computed for all of the specified quantiles) for the period 29 Jan 2004 – 9 May 2008. Also, historical VaR was computed using the 1% quantile of the series of the past 750 daily standardized returns of the portfolio. The results obtained for VaR along with the negative of the returns of the portfolio (because VaR is a positive number) are shown in Fig. 22 and 23. Table 23 shows if the computed risk measures match their respective critical levels (the term "errors %" denotes the percentage of the cases in which the portfolio loss exceeded VaR or ES) and if the average ES indicators were exceeded by the average losses bigger than the corresponding VaR (marked with red if ES was exceeded).

Table 23. The precision of computed VaR and ES indicators

VaR Type	**VaR N Std**	**VaR NIG**	**VaR GHD**	**VaR CF**	**Historical VaR**
Errors (%)	**1.79%**	**0.94%**	**0.94%**	**0.19%**	**1.22%**
VaR Type	**VaR EVT 1%**	**VaR EVT 0.5%**	**VaR EVT 0.1%**	**VaR EVT 0.05%**	**VaR EVT 0.01%**
Errors (%)	**0.94%**	**0.47%**	**0.19%**	**0.09%**	**0.00%**
ES Type	**ES EVT 1%**	**ES EVT 0.5%**	**ES EVT 0.1%**	**ES EVT 0.05%**	**ES EVT 0.01%**
Errors (%)	**0.28%**	**0.19%**	**0.00%**	**0.00%**	**0.00%**
Avg ES	**0.094189578**	**0.127001983**	**0.25406459**	**0.342420319**	**0.684568603**
Avg Loss (> VaR)	**0.095200041**	**0.122152479**	**0.174782102**	**0.187156452**	**0**

Fig. 22. 1-day, 1% VaR GARCH comparative graphs



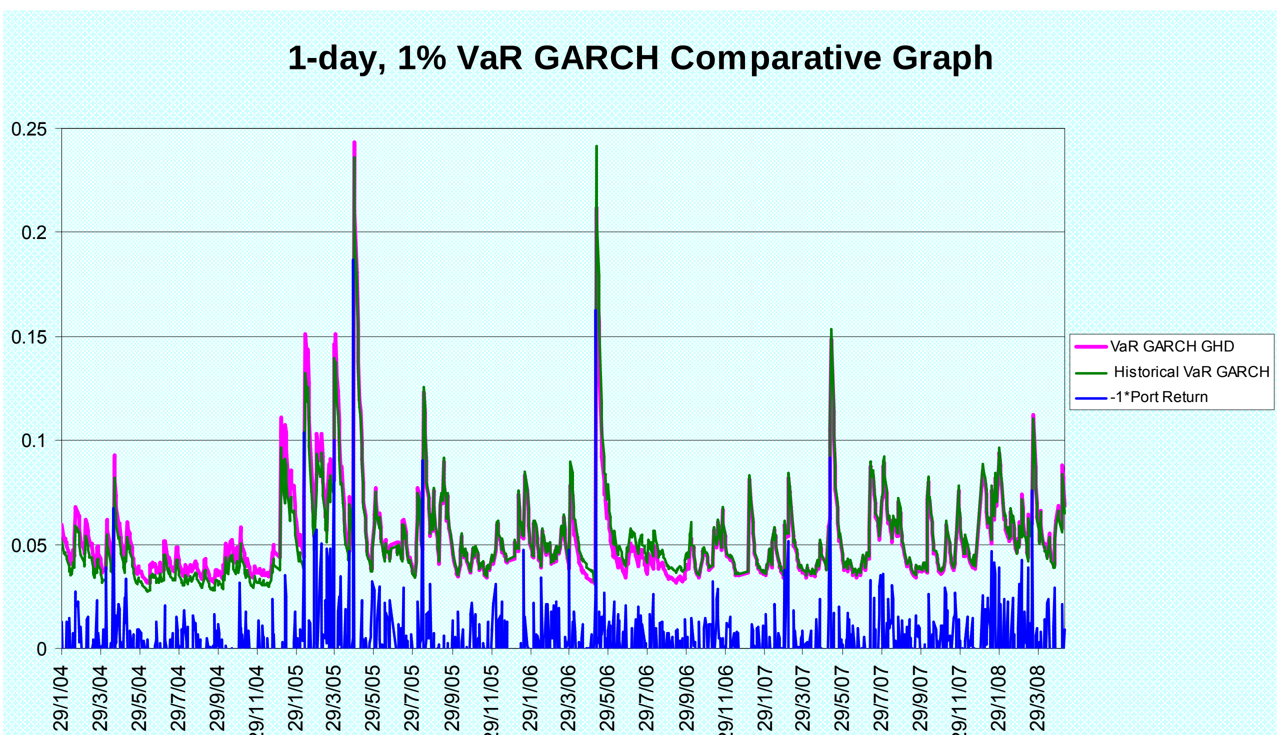
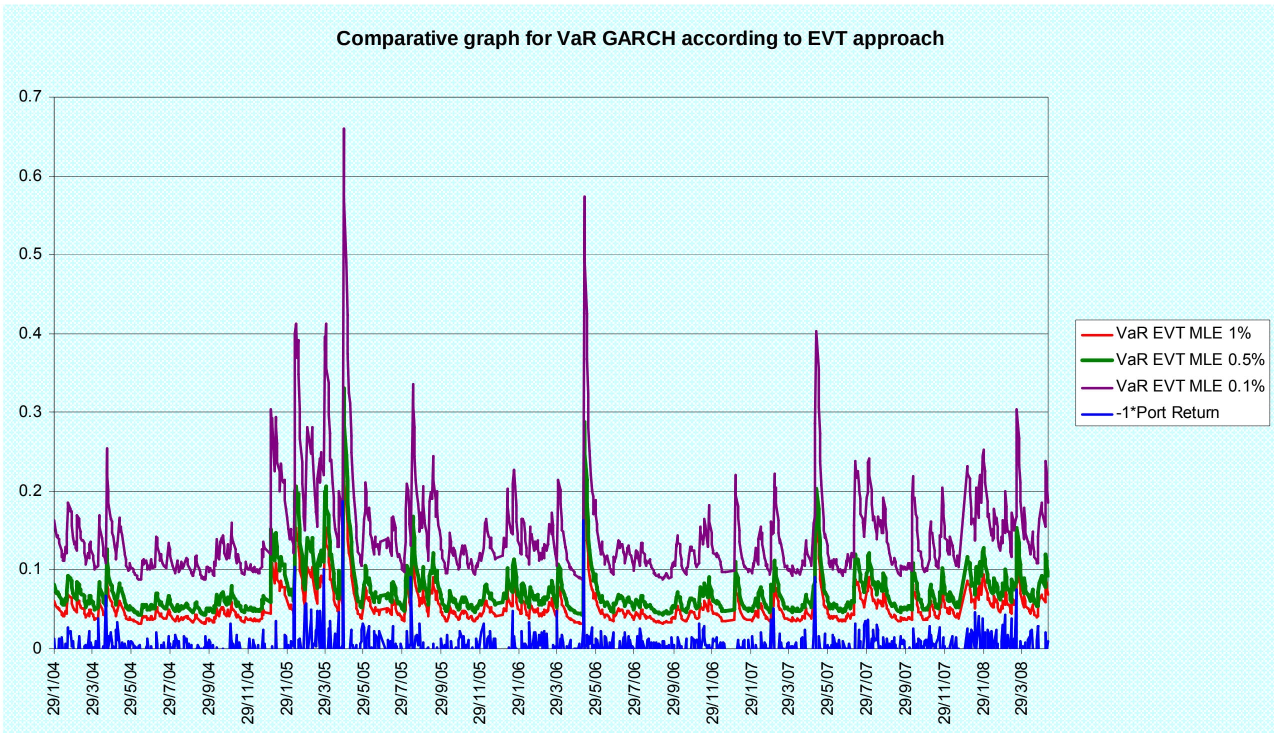
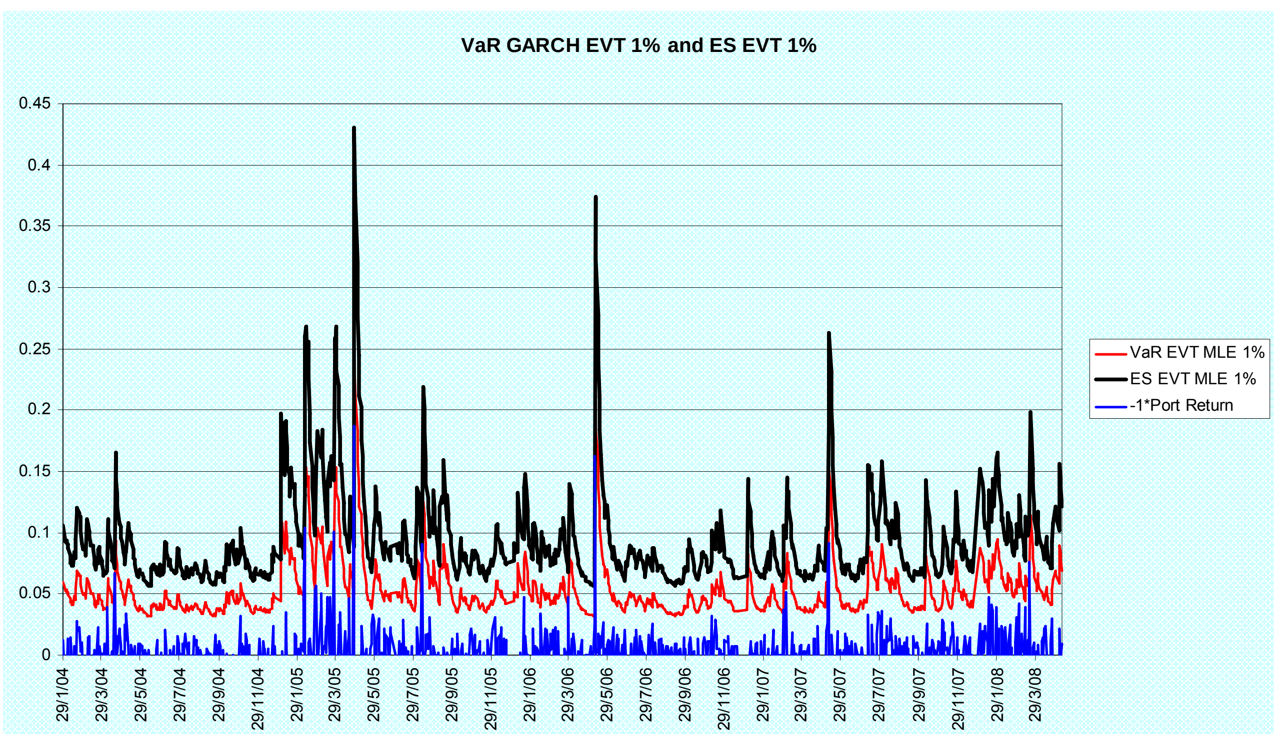


Fig. 23. 1-day VaR GARCH for the EVT approach





The results reported here account for the following conclusions:

- VaR N Std and Historical VaR didn't succeed in matching the critical level of 1%, while VaR NIG, VaR GHD and VaR CF passed the precision test, VaR CF having the best score as in the previous sections;
- Under the EVT approach three VaRs, at 1%, 0.5% and 0.01%, match their critical levels, while all ES indicators pass the precision test (although ES 1% has a problem: on average it has a smaller value than the average of the losses exceeding VaR EVT 1%);
- It seems that using the GARCH volatility model for the portfolio returns improved the risk measures analyzed in this study, these being the best results so far.

Table 24. Capital requirements for VaRs 1% and ES 1% that passed the precision test

	VaR NIG 1%	VaR GHD 1%	VaR CF 1%	VaR EVT 1%	ES EVT 1%
Average value	0.053724308	0.052894123	0.132284423	0.053558913	0.094189578
Maximum	0.031672086	0.031168008	0.079372735	0.03157166	0.056242055
Minimum	0.246993774	0.243228204	0.603328119	0.246243571	0.430536852

If we are to choose between the 1% risk measures that passed the precision test VaR GHD would be the most appropriate because it provides the lowest level of capital coverage (see Table 24). The capital requirement is about 5.29% (average level) of the portfolio value while VaR EVT requires about 5.36% and VaR NIG about 5.37%. VaR CF, as it has been already seen in the previous sections, has the highest level of capital requirement being more expensive even than ES 1%.

8. VaR using GARCH volatility models for the stock returns

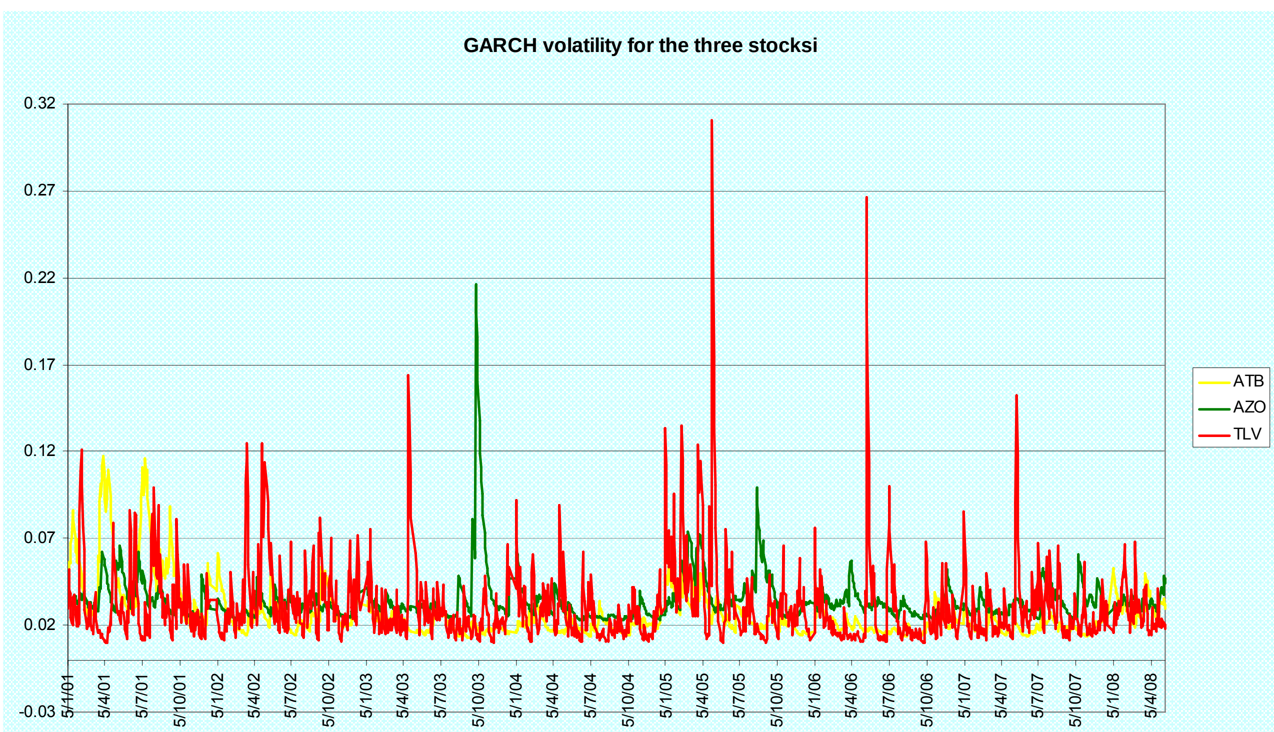
In this stage GARCH models were considered for the volatility of each stock returns. Therefore, I estimated GARCH (1,1) models for ATB and AZO, while for TLV a TARCH (1,1) was estimated. The specification of the models was established taking into account the results obtained at the autocorrelation tests of the residuals and of the squared residuals. Also, I verified that the sum of ARCH and GARCH coefficients is below 1. The parameters of the three models were estimated through MLE in Eviews. On the basis of estimation results I computed the volatility series for the three stocks considering that the starting values are the historical standard deviations of the data series. Then, using the historical correlation coefficients, I computed the volatility series for the portfolio. The results of the estimation are given in Table 25 (for the full results see Appendix 5) while Fig. 24 shows the volatility graphs for the three stocks and for the portfolio.

Table 25. Estimation results for the volatility of the stock returns

	GARCH (1,1) ATB	GARCH (1,1) AZO	TARCH (1,1) TLV
ω	0.000021415	0.000067782	0.000043996
α	0.124995782	0.097710524	1.869205806
β	0.851955326	0.857306946	0.561647124
λ	-	-	-1.566882386

The returns of the stocks and of the portfolio were standardized using the historical mean and the computed GARCH volatility. The descriptive statistics for the series of the standardized returns are given in Table 26. The values for skewness and kurtosis indicate that we still have to face non-normality. The Jarque-Bera test applied to the standardized returns of the portfolio has a value of 14846.91 (with an associated p value of 0%) meaning they aren't normally distributed, while the ADF test confirms that the unit root is not present (the two tests are given in Appendix 5).

Fig. 24. The GARCH Volatility of the three stocks and of the portfolio



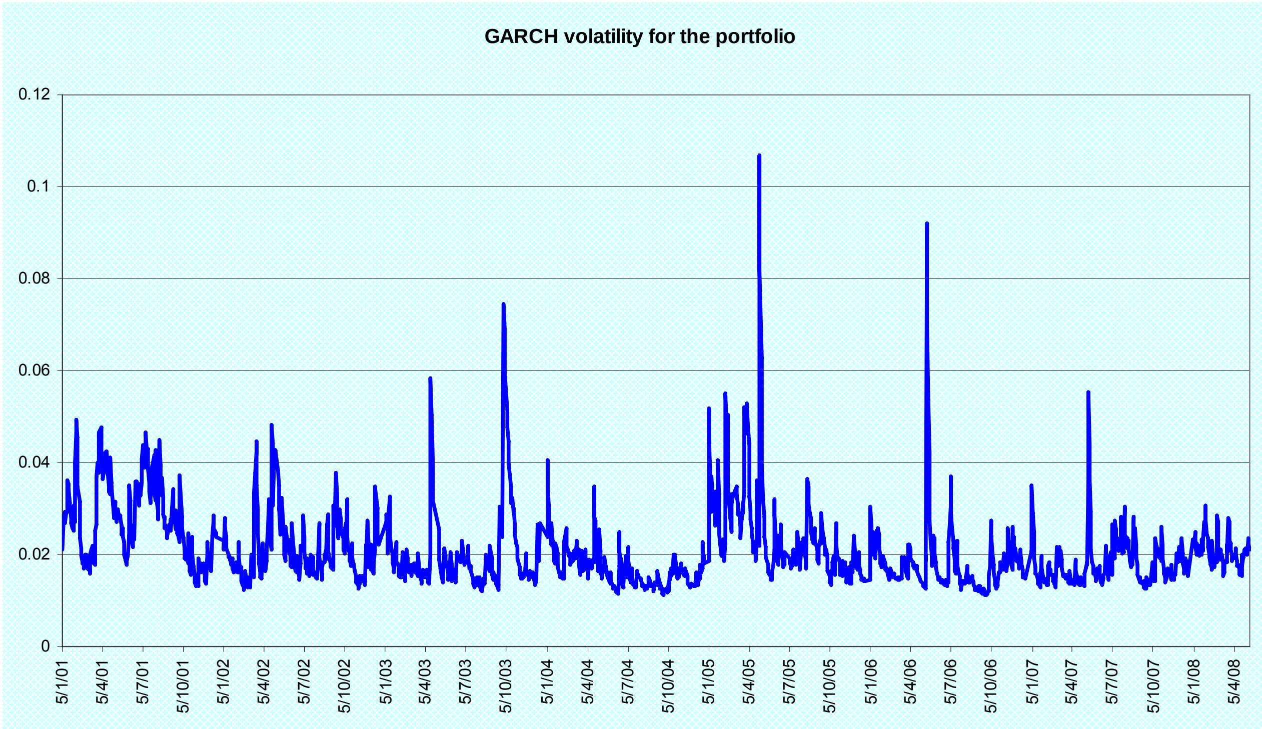
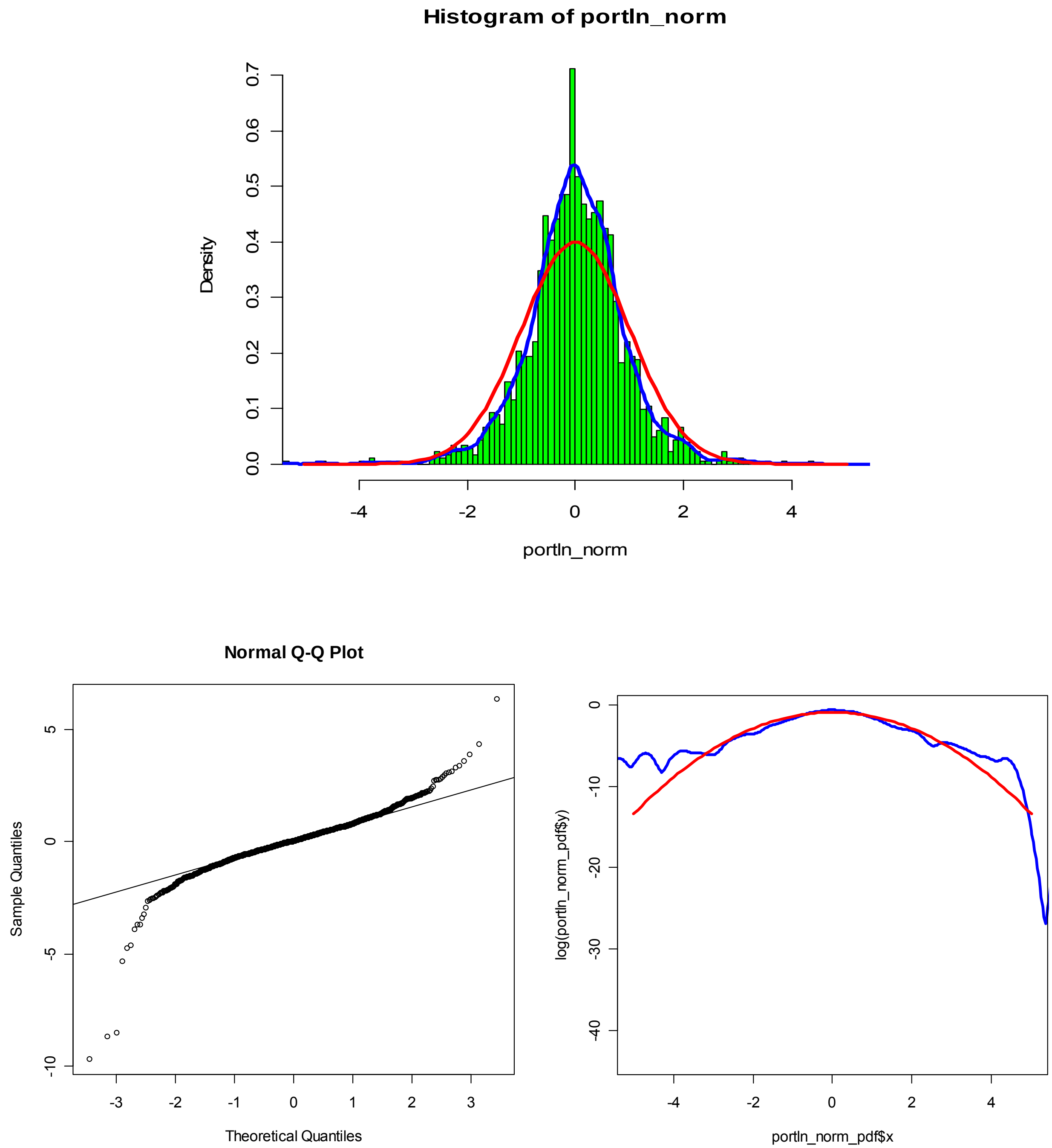


Table 26. Descriptive statistics of the returns standardized with GARCH volatility

	ATB	AZO	TLV	PORTFOLIO
Mean	0.019232517	0.001177774	0.01391282	-0.000723316
Median	-0.062499878	-0.015806861	-0.018987492	-0.003431596
Maximum	7.417686382	9.720063979	4.671654838	6.354261631
Minimum	-6.730922299	-11.5023275	-15.75192717	-9.728993413
Std. Dev.	1.004357344	0.999613558	0.951366414	0.986031755
Skewness	0.451135773	-0.339593675	-5.088694033	-1.246870005
Kurtosis	8.954522622	22.91252826	72.92644788	16.83352693

In order to visualize the non-normality of the portfolio standardized returns Fig. 25 shows three graphs: the probability density function (PDF) of the empirical distribution, the QQ plot and the log PDF graph, all plotted against the standard normal distribution.

Fig. 25. PDF of the empirical distribution, QQ plot and log PDF against the standard normal distribution



Similarly to the previous sections, the first way to deal with the non-normality of the data was to estimate the parameters for t-Student, NIG and GH distributions. The parameters were estimated through MLE using the Rmetrics package of the soft R. The results, along with the values for the informational criteria Akaike and Schwarz, are given in Table 27.

Table 27. Estimation results for t-Student, NIG and GH

t-Student		Normal Inverse Gaussian Distribution		General Hyperbolic Distribution	
degrees of freedom	10.47379	alpha	0.98043236	alpha	0.06596887
		beta	-0.06929081	beta	-0.06596871
		delta	0.87046629	delta	1.37267037
		mu	0.06095307	mu	0.05925987
				lambda	-2.03883398
MLE	-2449.394	MLE	-2353.705	MLE	-2344.757
AIC	2.7016	AIC	2.5995	AIC	2.5907
SIC	2.7047	SIC	2.6116	SIC	2.6059

Because the AIC and SIC values for NIG and GH are very close both distributions were taken into account. Fig. 26 and Fig. 27 show the PDF of the empirical distribution and log PDF against the estimated distributions NIG and GH (while the same graphs for the estimated t-Student distribution are given in Appendix 5). Analyzing these graphs it is clearly that NIG and GH fit much better the empirical distribution than does the standard normal distribution. Therefore, it would be reasonable to compute VaR using the quantiles of the NIG and GH distributions instead the ones of the standard normal distribution. The 1% quantiles for these distributions are given in Table 28.

Fig. 26. PDF of the empirical distribution and log PDF against the estimated NIG distribution

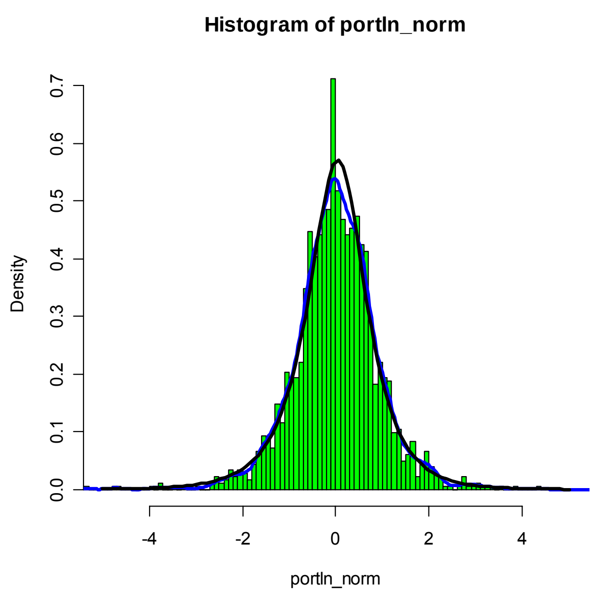
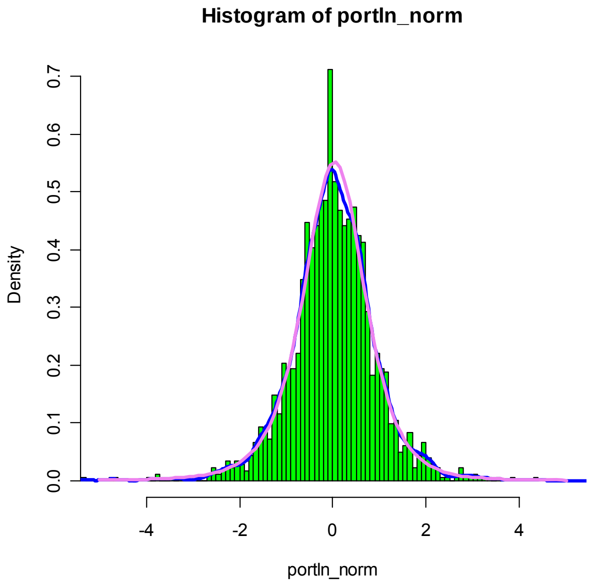


Fig. 27. PDF of the empirical distribution and log PDF against the estimated GH distribution



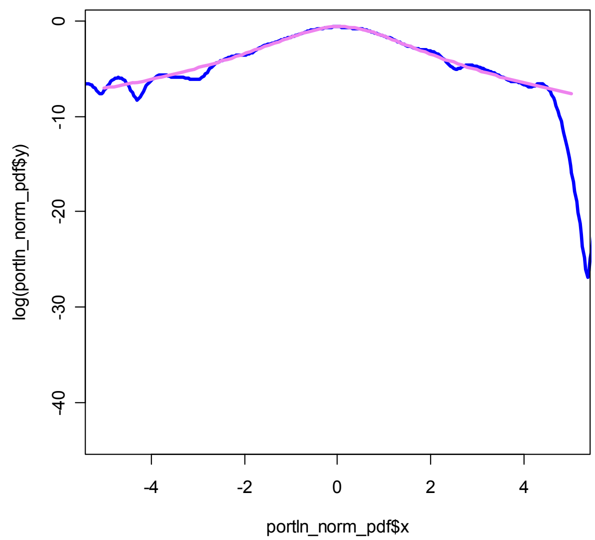


Table 28. 1% quantiles for the standard normal distribution and estimated NIG and GH distributions

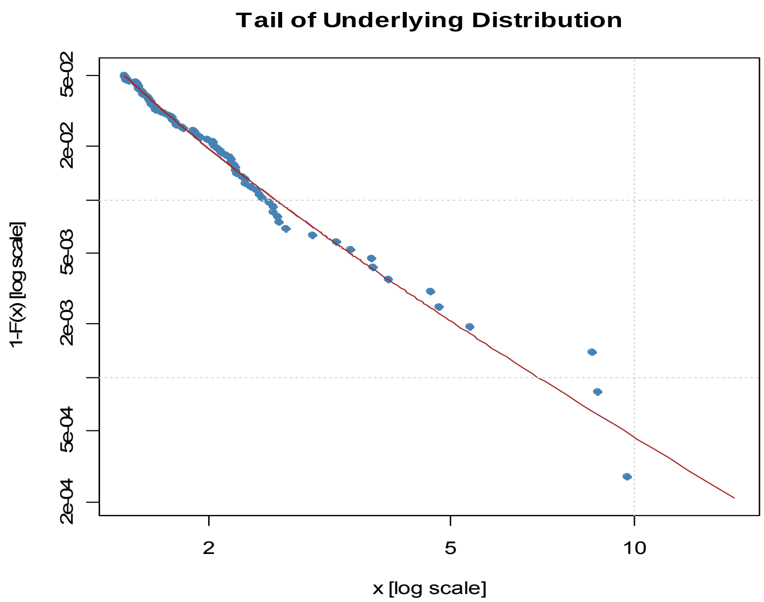
N Std.	-2.326347	NIG	-2.691367	GH	-2.647208

As in the previous sections, I used also the EVT approach in order to deal with the non-normality of the standardized data. Again, the threshold was set to be the 95 percentile of the sample of the standardized returns while the parameters of GPD were estimated through MLE and then the quantiles for VaR and ES were computed using Rmetrics. The results are given in Table 29, while Fig. 28 shows the tail of the distribution along with the estimated GPD.

Table 29. Estimated parameters for GPD and the quantiles for VaR and ES according to EVT

Parameter Estimates for GPD			
xi	0.5019587	beta	0.4505349
Quantiles for VaR and ES for different probability levels according to EVT and N Std			
Probability	VaR EVT	ES EVT	N Std
1%	2.574199	4.606729	2.326347
0.5%	3.413405	6.291742	2.575831
0.1%	6.963622	13.420102	3.090253
0.05%	9.62942	18.772667	3.290560
0.01%	20.906942	41.416415	3.719090

Fig. 28. The tail of the distribution along with the estimated GDP



We observe the same situation as in the previous sections: the quantiles of the normal and EVT distribution are very different for probabilities below 1% due to the different tail shapes. Therefore, the risk profiles are also very different and ES could be a better risk measure for our portfolio because it takes into consideration the shape of the tail.

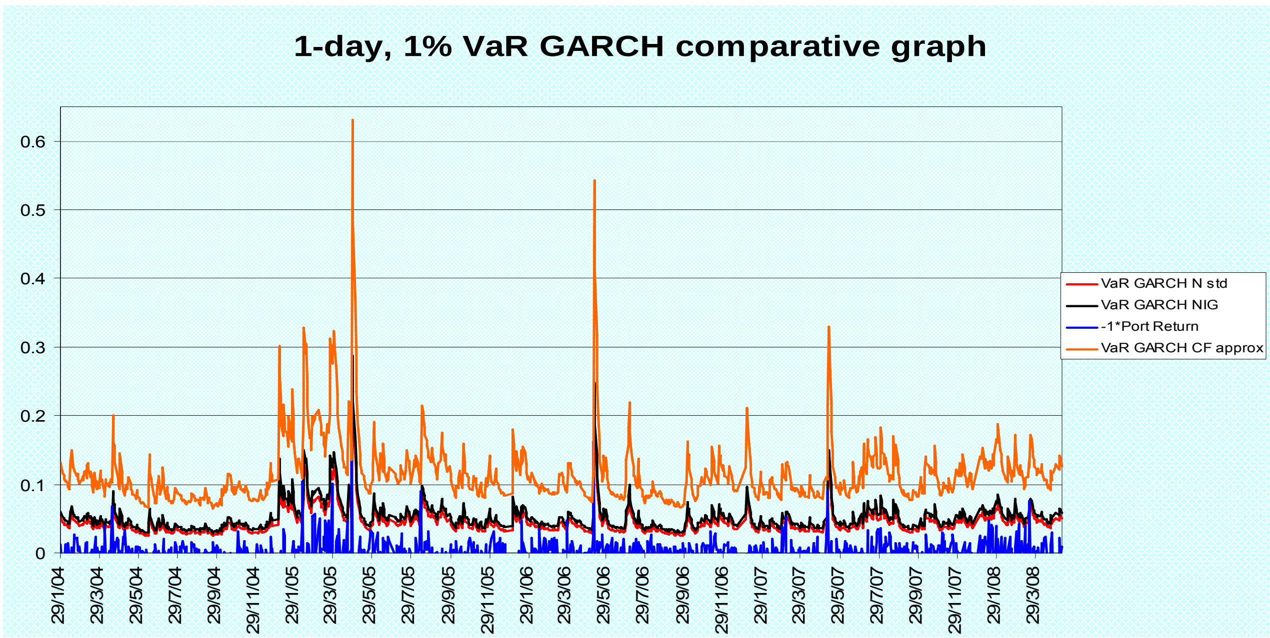
At last, the Cornish-Fisher approach was considered. The approximation for the 1% quantile, computed using the skewness and excess kurtosis of the returns standardized with the GARCH volatility, is given in Table 30.

Table 30. Cornish-Fisher approximation

Skewness	-1.246870005
Excess Kurtosis	13.83352693
1% Q N Std	-2.326347
1% CF	-5.892205923

After modeling the distribution of the standardized returns and computing all the necessary quantiles, it was proceeded to the determination of VaR and ES. Using the GARCH volatility I computed 1-day, 1% VaR for all the approaches considered above (except for the EVT approach where VaR and ES were computed for all of the specified quantiles) for the period 29 Jan 2004 – 9 May 2008. Also, historical VaR was computed using the 1% quantile of the series of the past 750 daily standardized returns of the portfolio. The results obtained for VaR along with the negative of the returns of the portfolio (because VaR is a positive number) are shown in Fig. 29 and 30. Table 31 shows if the computed risk measures match their respective critical levels (the term "errors %" denotes the percentage of the cases in which the portfolio loss exceeded VaR or ES) and if the average ES indicators were exceeded by the average losses bigger than the corresponding VaR (marked with red if ES was exceeded).

Fig. 29. 1-day, 1% VaR GARCH comparative graphs



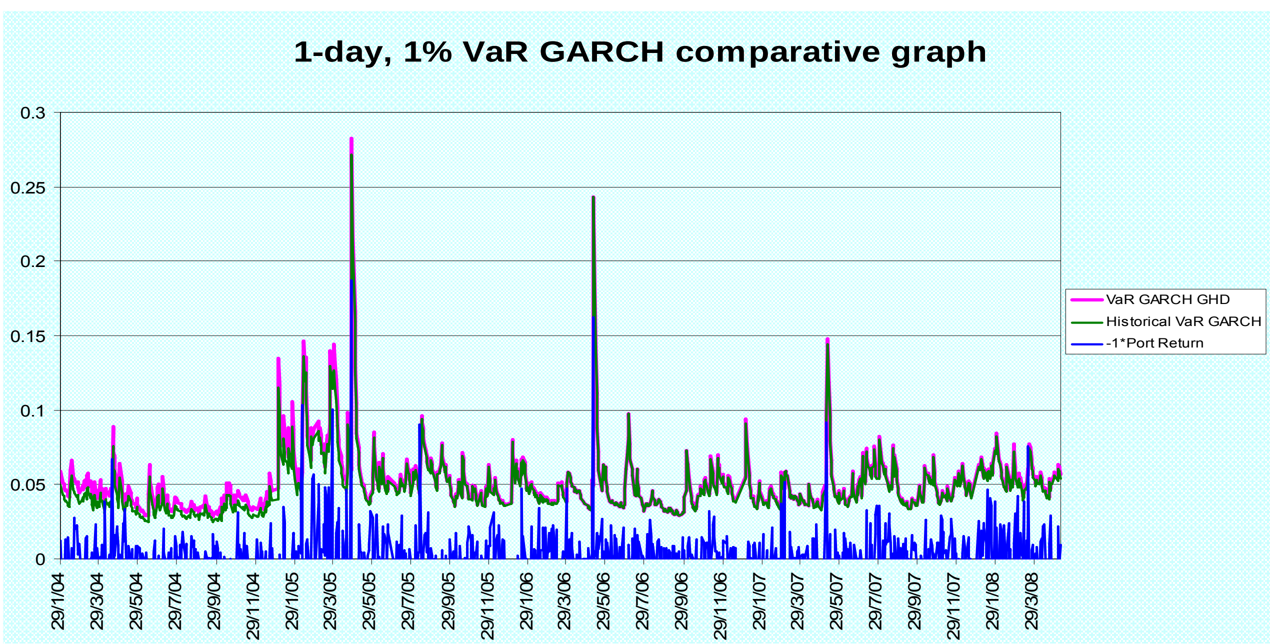
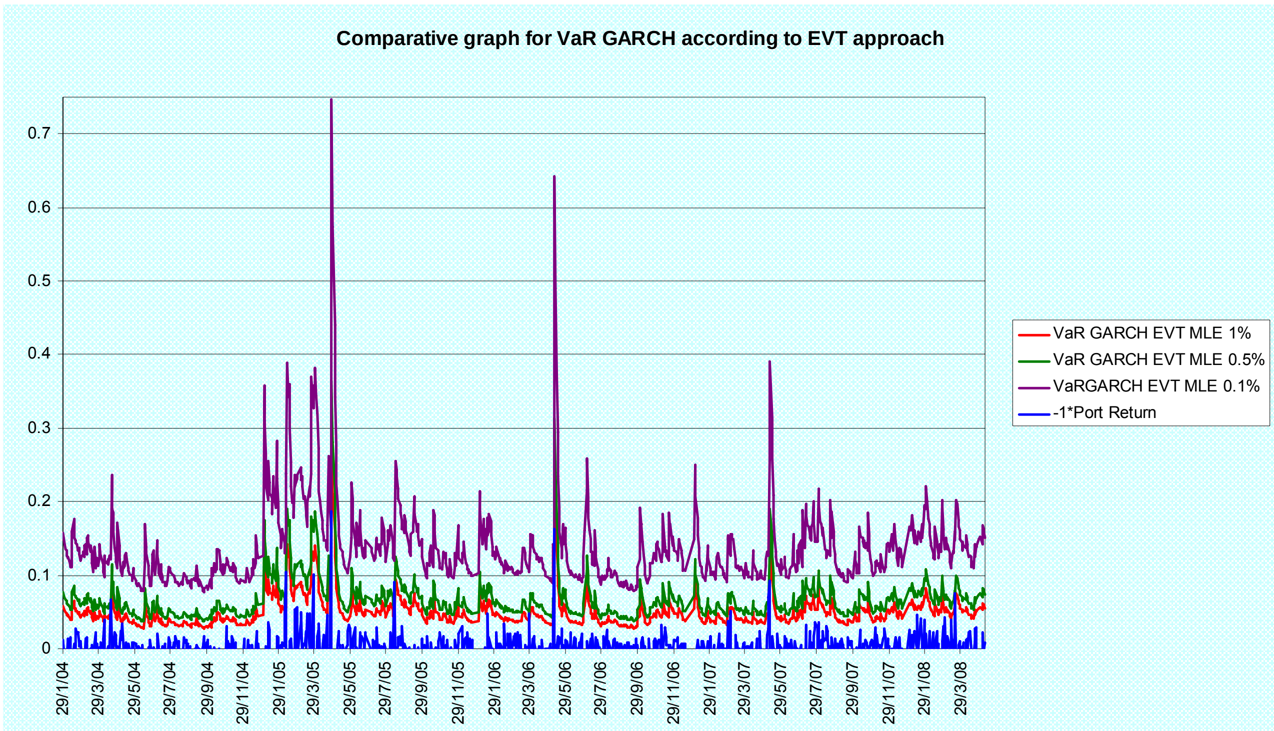


Fig. 30. 1-day VaR GARCH for the EVT approach



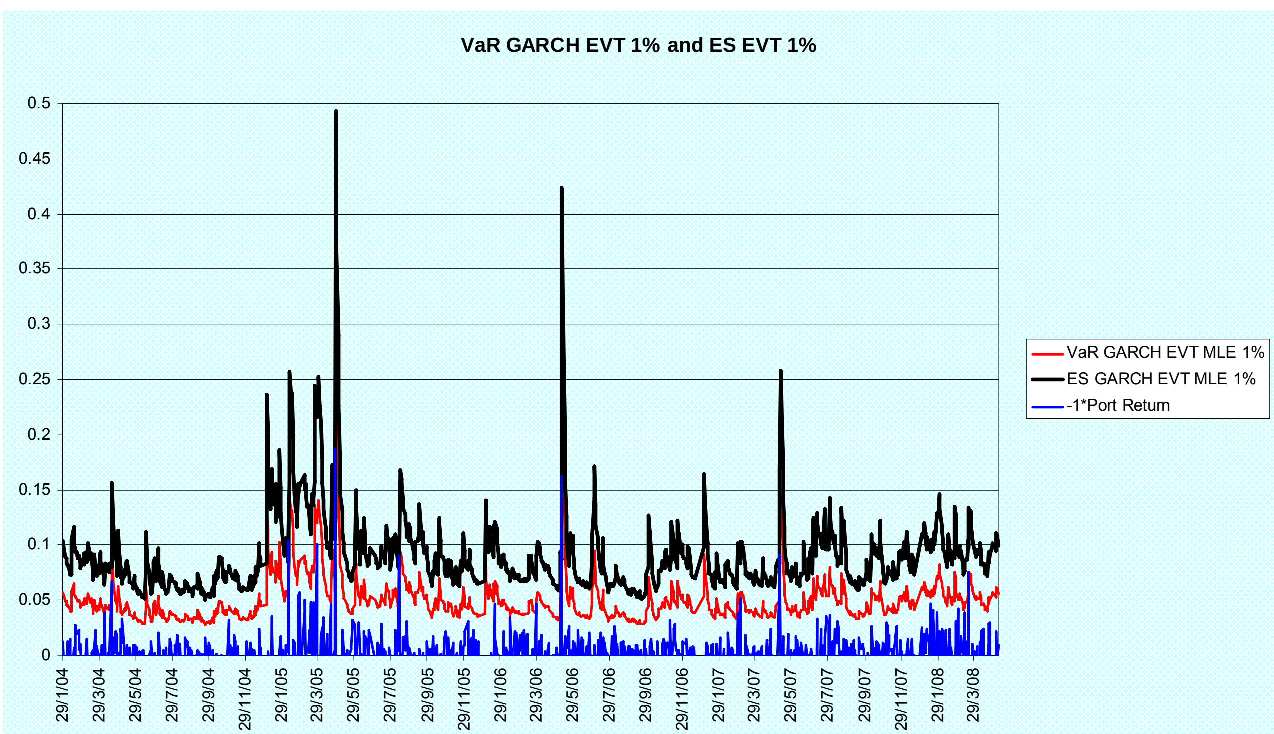


Table 31. The precision of computed VaR and ES indicators

VaR Type	**VaR N Std**	**VaR NIG**	**VaR GHD**	**VaR CF**	**Historical VaR**
Errors (%)	**1.41%**	**0.94%**	**1.03%**	**0.19%**	**1.22%**
VaR Type	**VaR EVT 1%**	**VaR EVT 0.5%**	**VaR EVT 0.1%**	**VaR EVT 0.05%**	**VaR EVT 0.01%**
Errors (%)	**1.03%**	**0.66%**	**0.19%**	**0.00%**	**0.00%**
ES Type	**ES EVT 1%**	**ES EVT 0.5%**	**ES EVT 0.1%**	**ES EVT 0.05%**	**ES EVT 0.01%**
Errors (%)	**0.38%**	**0.19%**	**0.00%**	**0.00%**	**0.00%**
Avg ES	**0.089993566**	**0.123311799**	**0.264262854**	**0.370100619**	**0.817841779**
Avg Loss (> VaR)	**0.085643689**	**0.108055871**	**0.174782102**	**0**	**0**

The results reported here account for the following conclusions:

- VaR N Std, Historical VaR, and VaR GHD didn't succeed in matching the critical level of 1% (although VaR GHD is very close), only VaR NIG and VaR CF being able to pass the precision test (VaR CF having the highest score as in the previous sections);

- Under the EVT approach only two VaRs, at 0.05% and 0.01%, match their critical levels, but all ES indicators pass the precision test;

- Computing VaR on the basis of GARCH volatility models for each stock appears to perform worse than using a GARCH volatility model for the portfolio, the risk measures determined here being less accurate than the ones from the previous section.

Table 32. Capital requirements for VaRs 1% and ES 1% that passed the precision test

	VaR NIG 1%	**VaR CF 1%**	**ES EVT 1%**
Average value	0.052120577	0.11541166	0.089993566
Maximum	0.028795451	0.064583312	0.050210683
Minimum	0.28753768	0.631484524	0.493353321

If we are to choose between the 1% risk measures that passed the precision test VaR NIG 1% would be the most appropriate because it provides the lowest level of capital coverage (see Table 32). The capital requirement is about 5.21% (average level) of the portfolio value, while ES 1% requires about 9%. VaR CF 1% is, as obtained in all the previous sections, the most expensive risk measure requiring about 11.54% of the portfolio value in this case.

9. VaR using GARCH volatility models for the stock returns and a DCC model

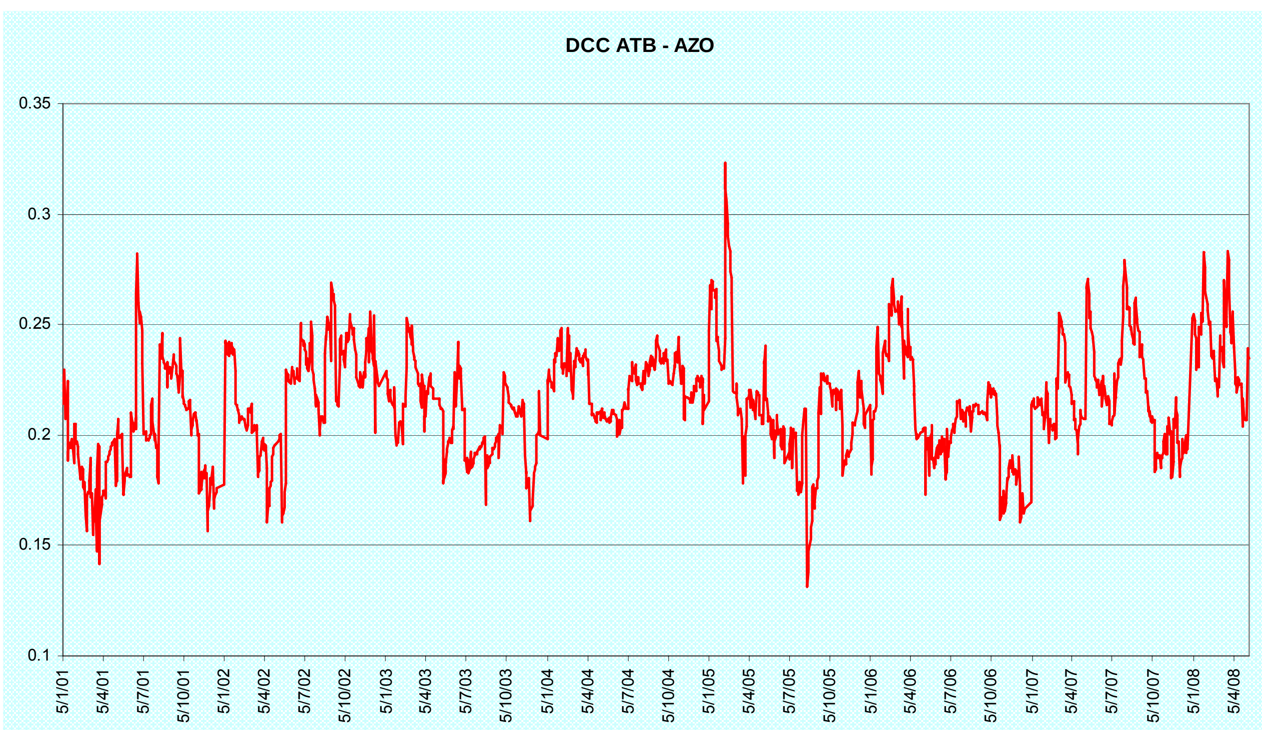
In the final stage of this study the GARCH approach from the previous section is further enhanced by considering a dynamic conditional correlation (DCC) model. Thus, the conditional correlations are modeled using GARCH (1,1) type specifications with correlation targeting, the model being written as:

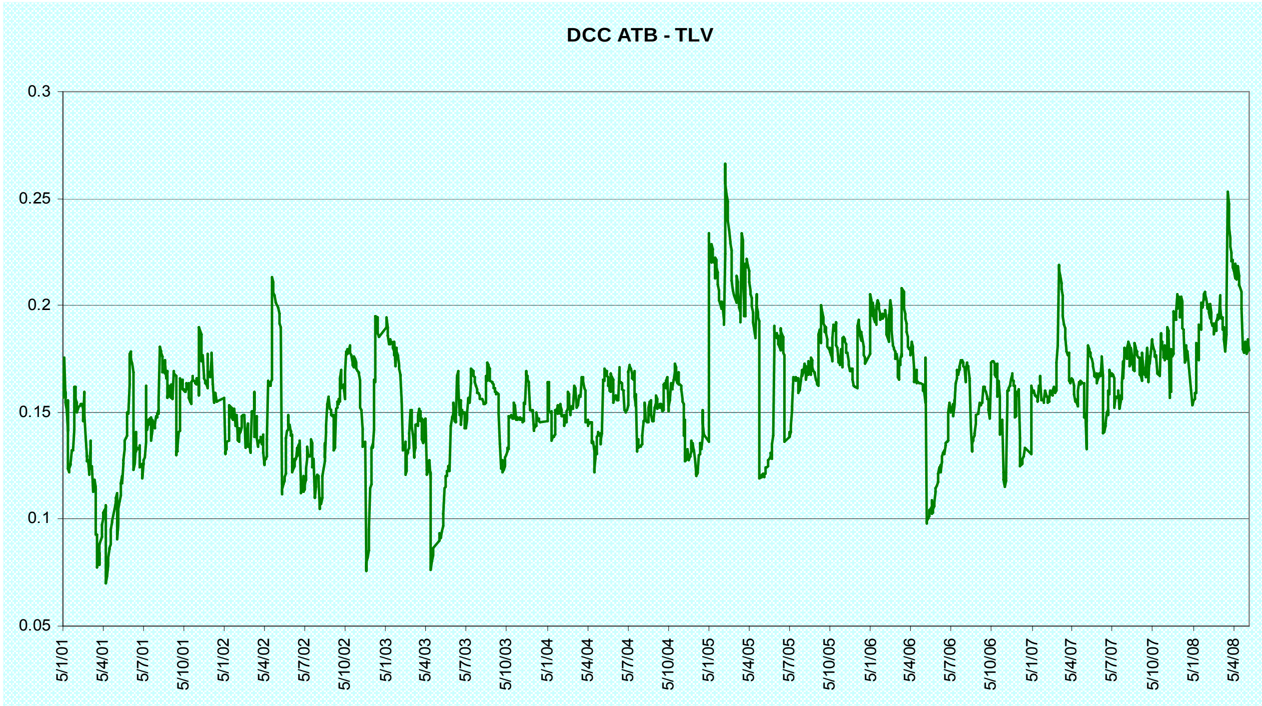
$$q_{ij,t+1} = E\lfloor z_{i,t} z_{j,t} \rfloor + \alpha\left(z_{i,t} z_{j,t} - E\lfloor z_{i,t} z_{j,t} \rfloor\right) + \beta\left(q_{ij,t} - E\lfloor z_{i,t} z_{j,t} \rfloor\right)$$

$$\rho_{ij,t+1} = \frac{q_{ij,t+1}}{\sqrt{q_{ii,t+1} q_{jj,t+1}}}$$

The important thing about this model is that the parameters α and β are common across *i* and *j* (so they are common for the three stocks). They were estimated through MLE in Excel and, on the basis of the estimation results, the dynamic correlations between ATB, AZO and TLV were computed. Then, using the GARCH volatility of the three stocks (already determined in the previous section), I obtained the volatility series for the portfolio. The results of the estimation are given in Table 33 while Fig. 31 and Fig. 32 show the graphs of the dynamic correlations and of the portfolio volatility.

Fig. 31. The dynamic conditional correlations





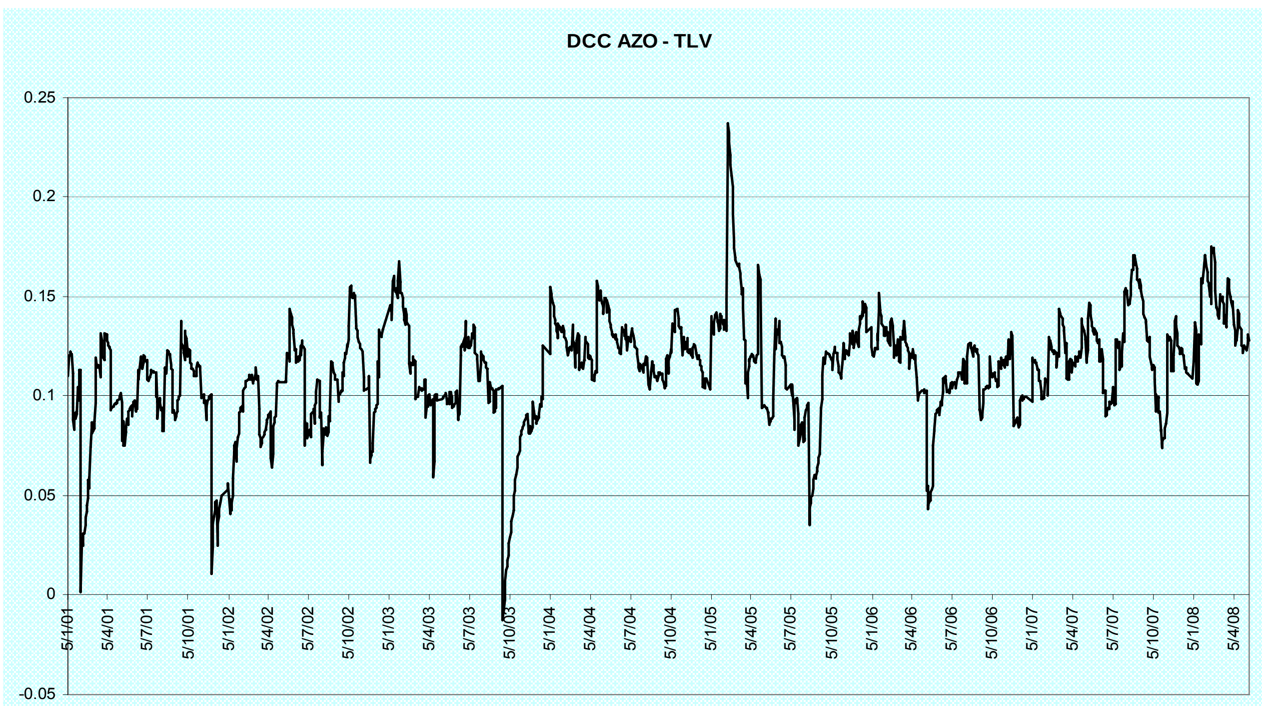
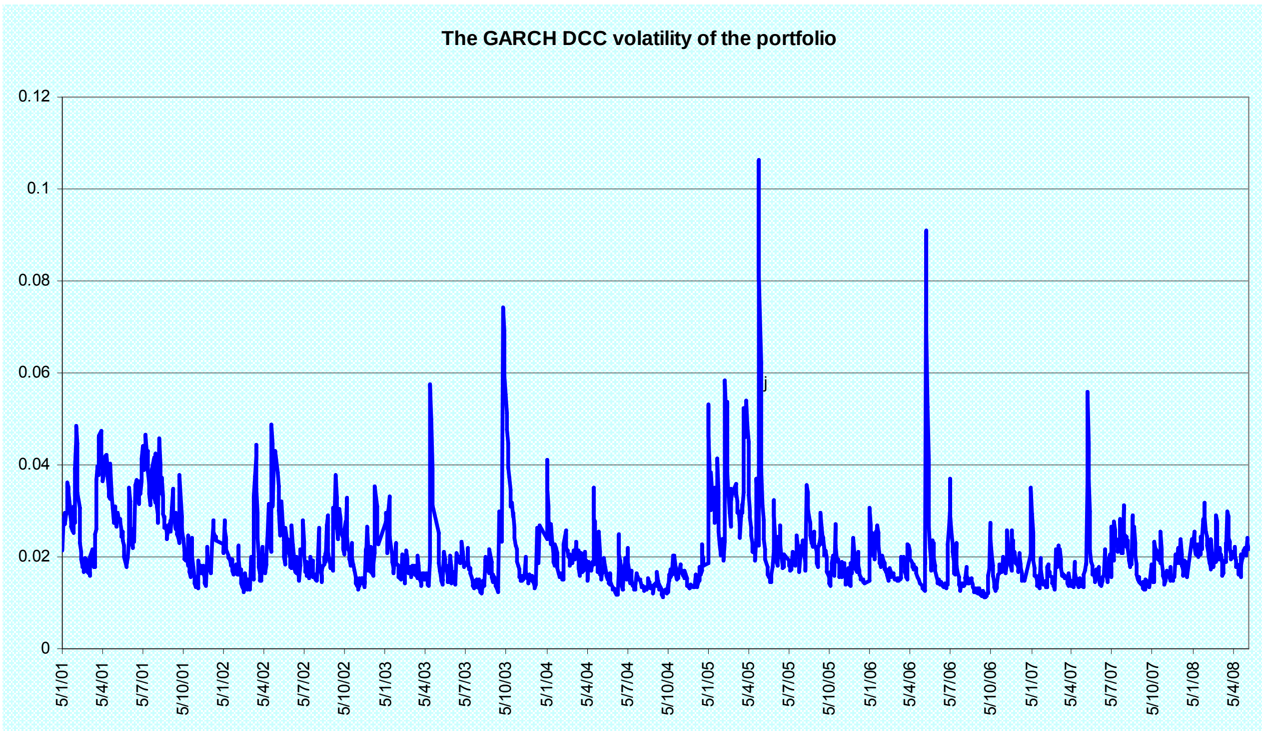


Table 33. Estimation results for the DCC model

α	0.009581808
β	0.938457995

Fig. 32. The GARCH DCC Volatility of the portfolio



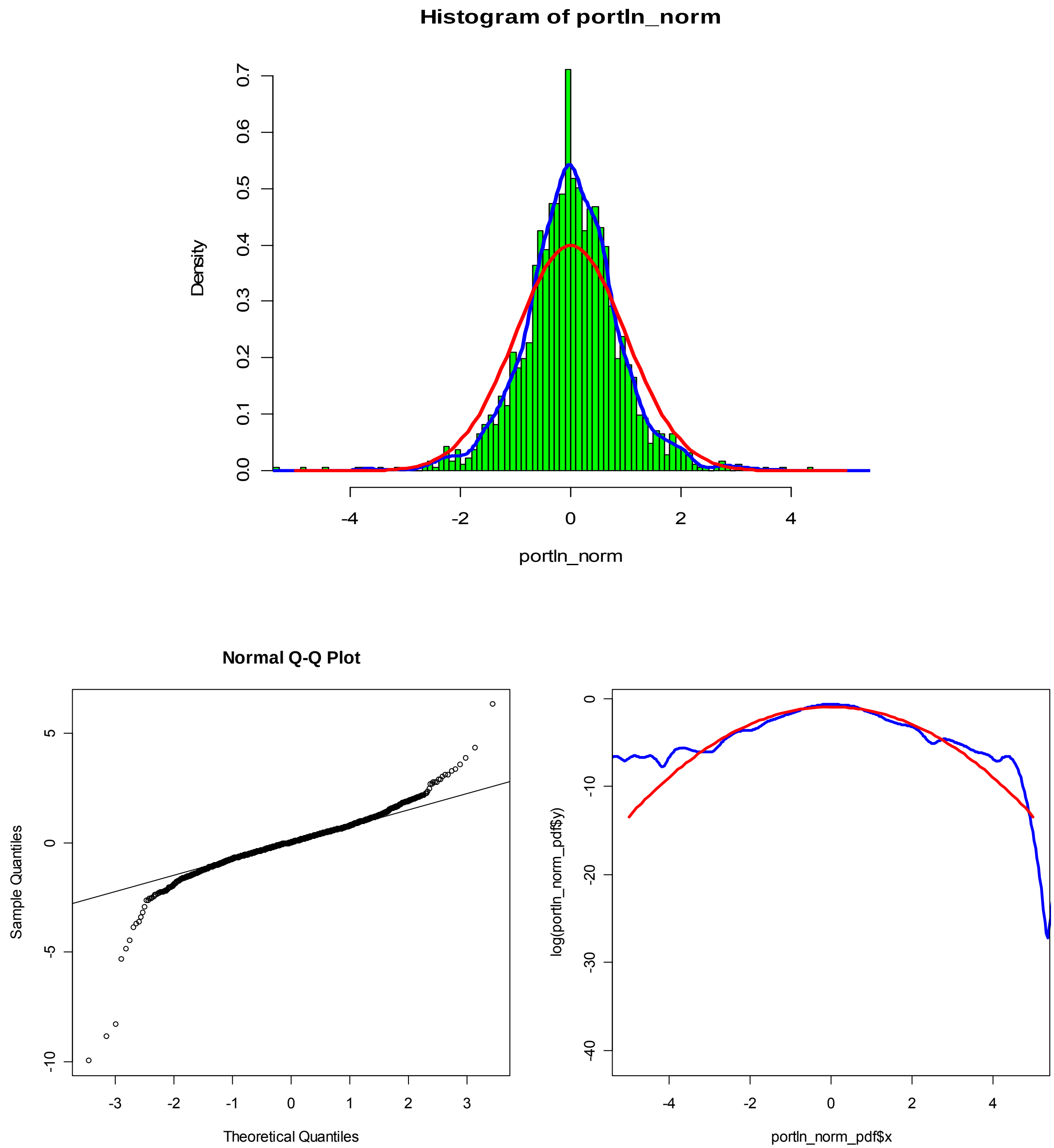
The returns of the portfolio were standardized using the historical mean and the computed GARCH DCC volatility. The descriptive statistics for the series of the standardized returns are given in Table 34. The values for skewness and kurtosis indicate that we still have to face non-normality. The Jarque-Bera test applied to the standardized returns of the portfolio has a value of 16561.46 (with an associated p value of 0%) meaning they aren't normally distributed, while the ADF test confirms that the unit root is not present (the two tests are given in Appendix 6).

Table 34. Descriptive statistics of the returns standardized with GARCH DCC volatility

	PORTFOLIO
Mean	-0.000637837
Median	-0.003443953
Maximum	6.353614668
Minimum	-9.979407553
Std. Dev.	0.980753023
Skewness	-1.293801451
Kurtosis	17.61856705

In order to visualize the non-normality of the portfolio standardized returns Fig. 33 shows three graphs: the probability density function (PDF) of the empirical distribution, the QQ plot and the log PDF graph, all plotted against the standard normal distribution.

Fig. 33. PDF of the empirical distribution, QQ plot and log PDF against the standard normal distribution



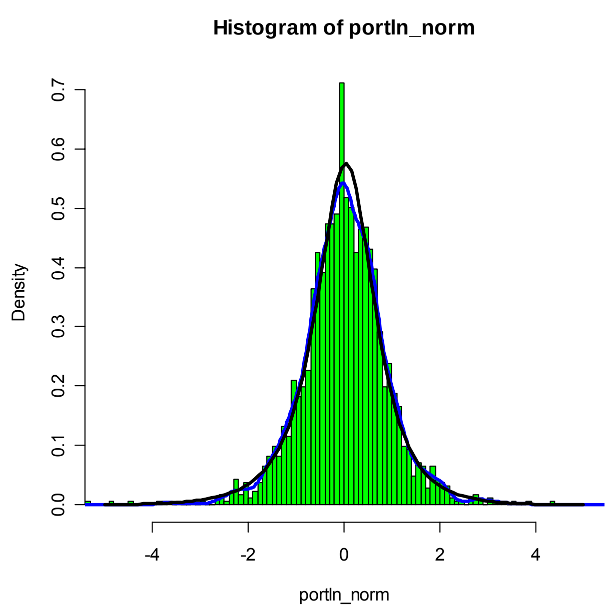
Similarly to the previous sections, the first way to deal with the non-normality of the data was to estimate the parameters for t-Student, NIG and GH distributions. The parameters were estimated through MLE using the Rmetrics package of the soft R. The results, along with the values for the informational criteria Akaike and Schwarz, are given in Table 35.

Table 35. Estimation results for t-Student, NIG and GH

t-Student		**Normal Inverse Gaussian Distribution**		**General Hyperbolic Distribution**	
degrees of freedom	10.61822	alpha	0.98485968	alpha	0.06326102
		beta	-0.06831799	beta	-0.06326024
		delta	0.86204843	delta	1.351071
		mu	0.05930329	mu	0.05644313
				lambda	-2.01591331
MLE	-2439.812	MLE	-2339.684	MLE	-2330.34
AIC	2.6912	AIC	2.5840	AIC	2.5748
SIC	2.6941	SIC	2.5961	SIC	2.590

Because the AIC and SIC values for NIG and GH are very close both distributions were taken into account. Fig. 34 and Fig. 35 show the PDF of the empirical distribution and log PDF against the estimated distributions NIG and GH (while the same graphs for the estimated t-Student distribution are given in Appendix 6).

Fig. 34. PDF of the empirical distribution and log PDF against the estimated NIG distribution



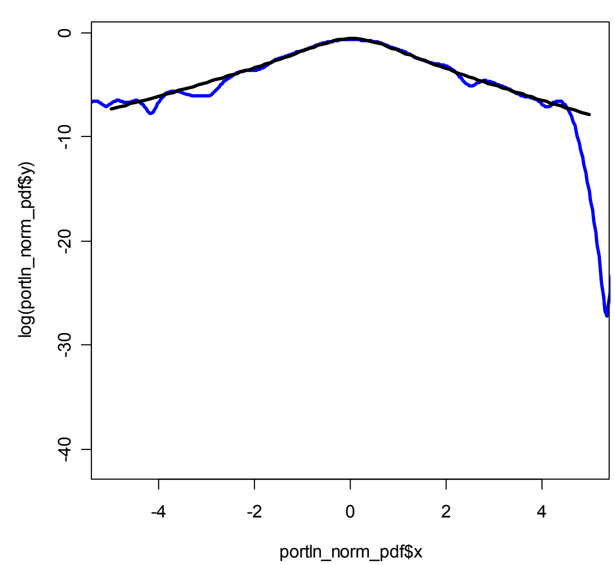
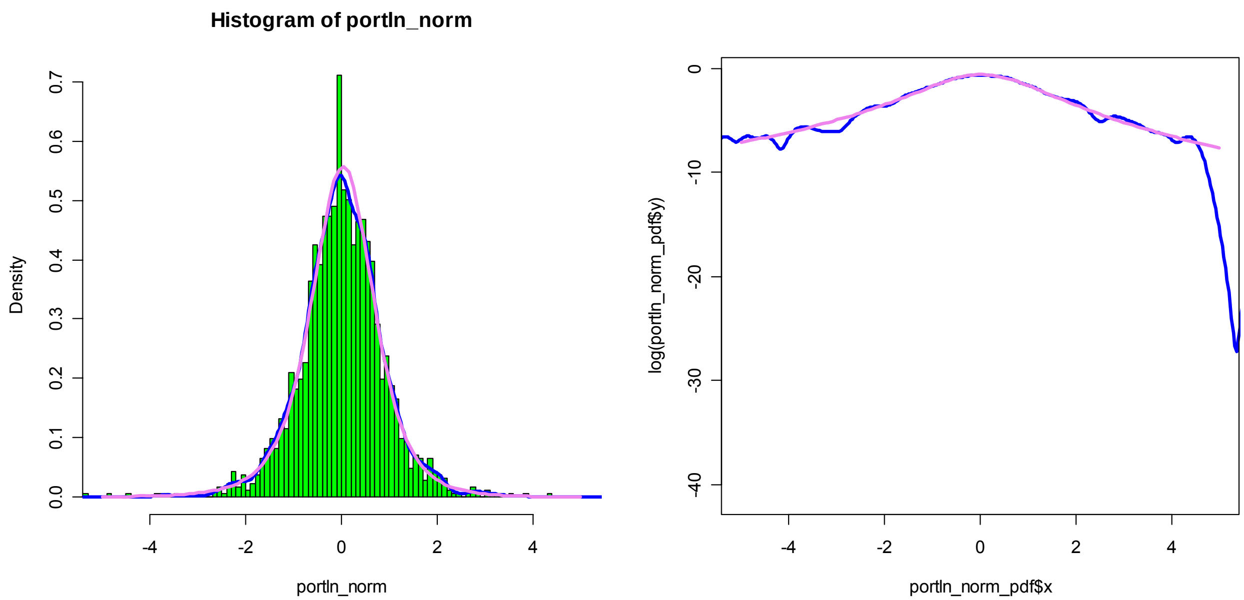


Fig. 35. PDF of the empirical distribution and log PDF against the estimated GH distribution



Analyzing these graphs it is clearly that NIG and GH fit much better the empirical distribution than does the standard normal distribution. Therefore, it would be reasonable to compute VaR using the quantiles of the NIG and GH distributions instead the ones of the standard normal distribution. The 1% quantiles for these distributions are given in Table 36.

Table 36. 1% quantiles for the standard normal distribution and estimated NIG and GH distributions

N Std.	-2.326347	NIG	-2.6712	GH	-2.632146

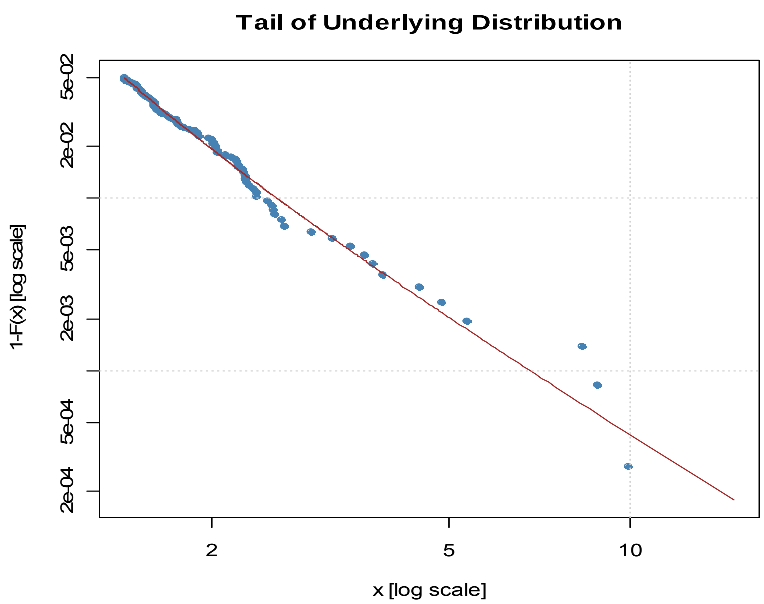
As in the previous sections, the EVT approach was also used in order to deal with the non-normality of the standardized data. Again, the threshold was set to be the 95 percentile of the sample of the standardized returns while the parameters of GPD were estimated through MLE and then the quantiles for VaR and ES were computed using Rmetrics. The results are given in Table 37, while Fig. 36 shows the tail of the distribution along with the estimated GPD.

The same situation as in the previous sections is to be observed: the quantiles of the normal and EVT distribution are very different for probabilities below 1% due to the different tail shapes. Therefore, the risk profiles are also very different, and ES could be a better risk measure for our portfolio because it takes into account the shape of the tail.

Table 37. Estimated parameters for GPD and the quantiles for VaR and ES according to EVT

Parameter Estimates for GPD			
xi	0.475801	beta	0.4708492
Quantiles for VaR and ES for different probability levels according to EVT and N Std			
Probability	VaR EVT	ES EVT	N Std
1%	2.570969	4.505781	2.326347
0.5%	3.403771	6.094495	2.575831
0.1%	6.814795	12.601612	3.090253
0.05%	9.305619	17.353288	3.290560
0.01%	19.507631	36.815387	3.719090

Fig. 36. The tail of the distribution along with the estimated GDP



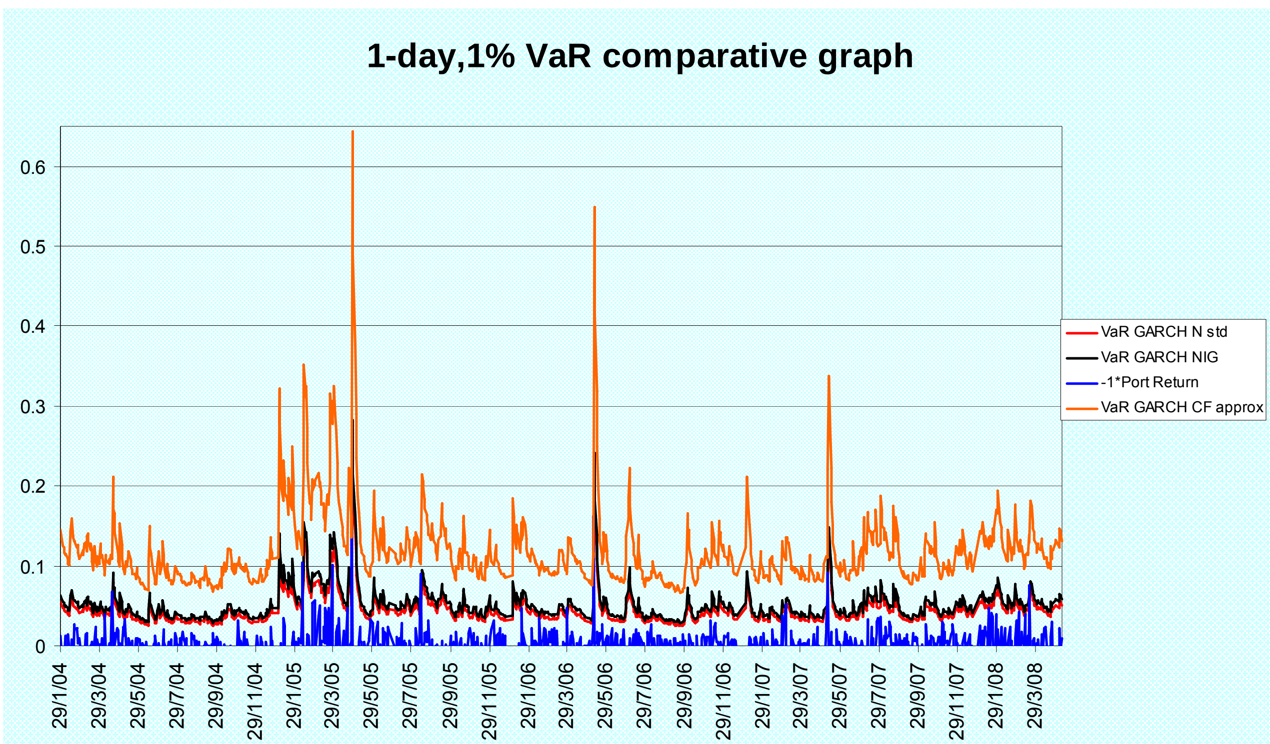
At last, the Cornish-Fisher approach was considered. The approximation for the 1% quantile, computed using the skewness and excess kurtosis of the returns standardized with the GARCH DCC volatility, is given in Table 38.

Table 38. Cornish-Fisher approximation

Skewness	-1.293801451
Excess Kurtosis	14.61856705
1% Q N Std	-2.326347
1% CF	-6.065374241

After modeling the distribution of the standardized returns and computing all the necessary quantiles, it was proceeded to the determination of VaR and ES. Using the GARCH DCC volatility I computed 1-day, 1% VaR for all the approaches considered above (except for the EVT approach where VaR and ES were computed for all of the specified quantiles) for the period 29 Jan 2004 – 9 May 2008. Also, historical VaR was computed using the 1% quantile of the series of the past 750 standardized returns of the portfolio. The results obtained for VaR along with the negative of the returns of the portfolio (because VaR is a positive number) are shown in Fig. 37 and 38. Table 39 shows if the computed risk measures match their respective critical levels (the term "errors %" denotes the percentage of the cases in which the portfolio loss exceeded VaR or ES) and if the average ES indicators were exceeded by the average losses bigger than the corresponding VaR (marked with red if ES was exceeded).

Fig. 37. 1-day, 1% VaR GARCH DCC comparative graphs



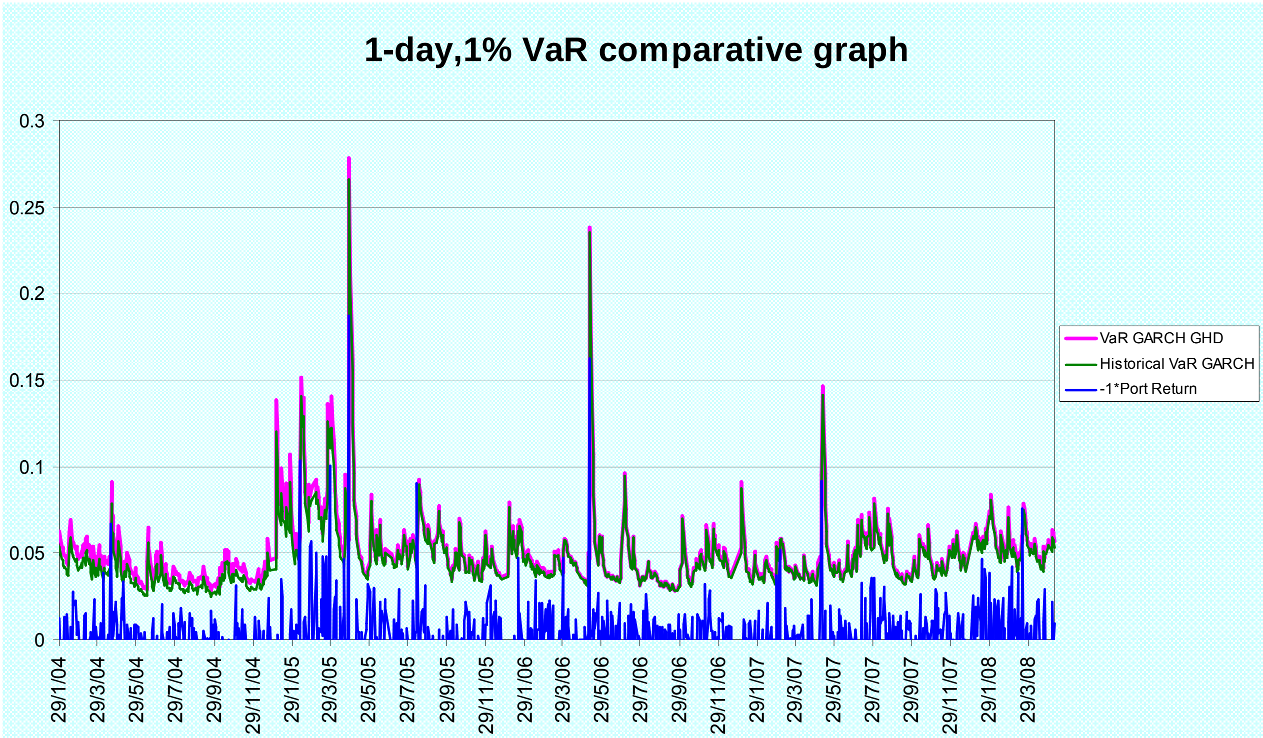
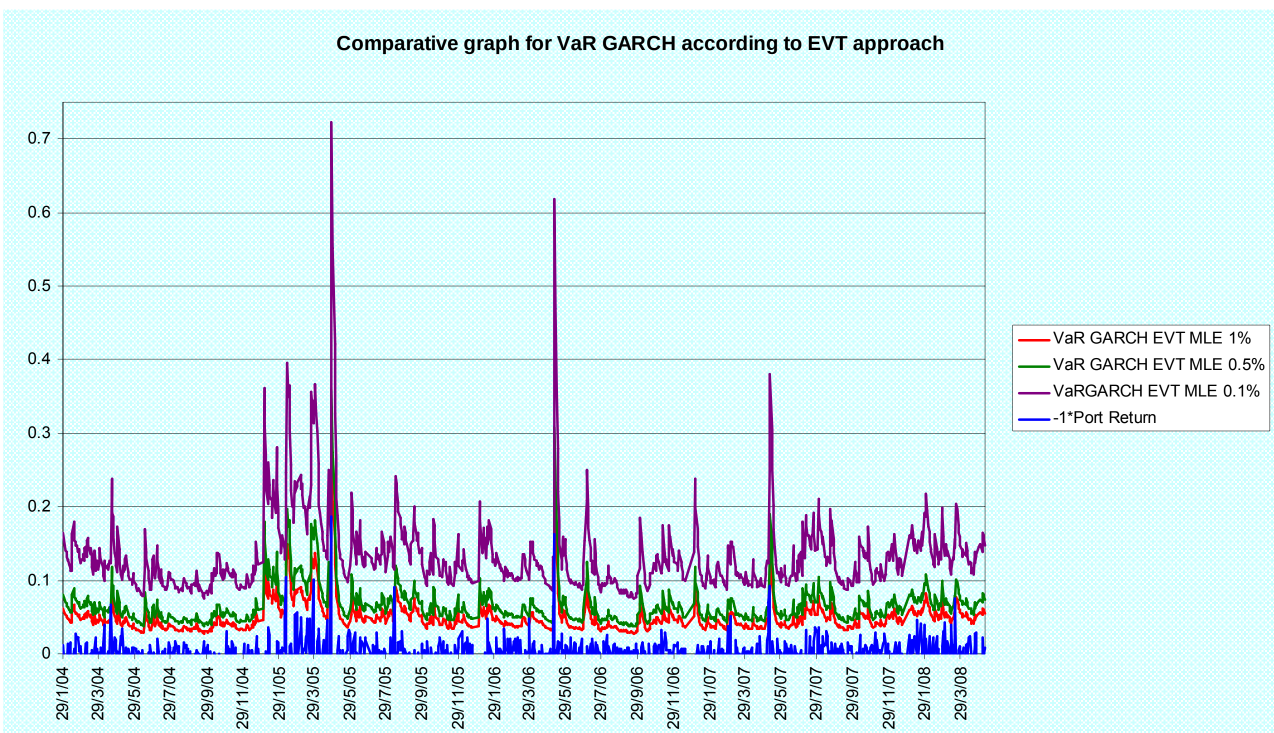


Fig. 38. 1-day VaR GARCH DCC for the EVT approach



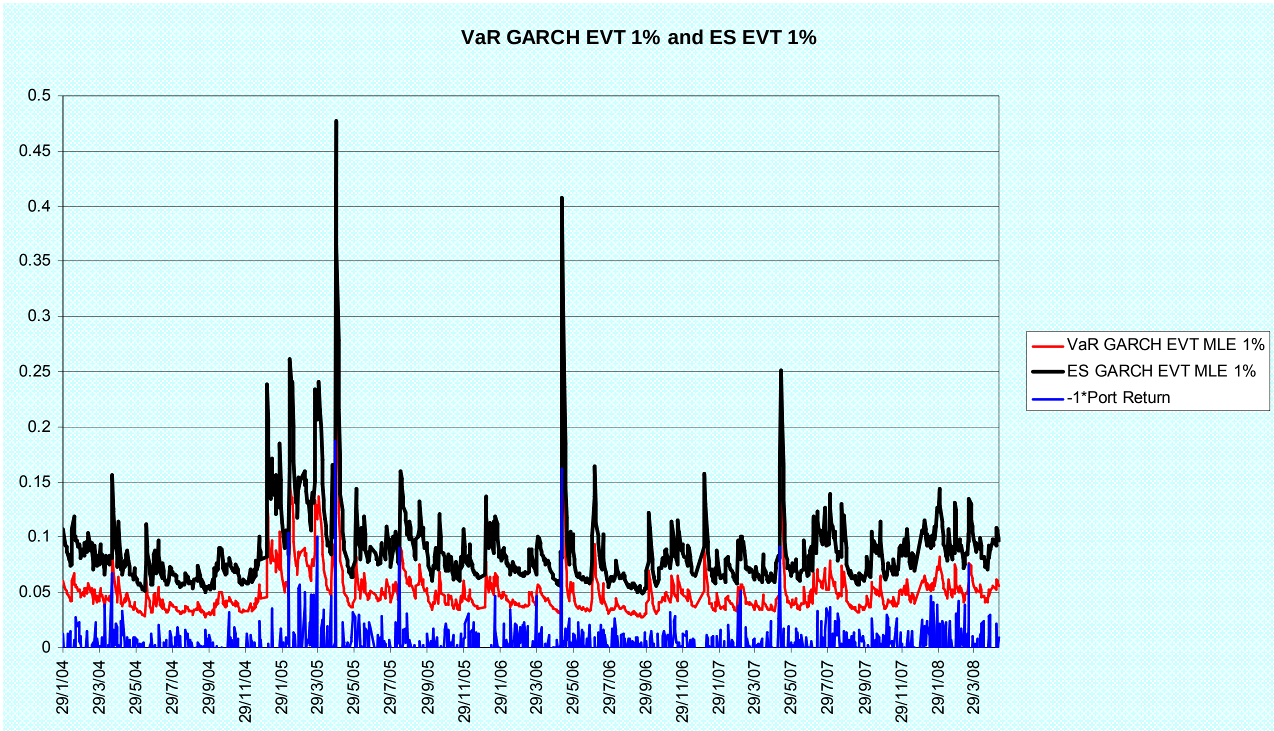


Table 39. The precision of computed VaR and ES indicators

VaR Type	**VaR N Std**	**VaR NIG**	**VaR GHD**	**VaR CF**	**Historical VaR**
Errors (%)	**1.41%**	**0.94%**	**0.94%**	**0.19%**	**1.32%**
VaR Type	**VaR EVT 1%**	**VaR EVT 0.5%**	**VaR EVT 0.1%**	**VaR EVT 0.05%**	**VaR EVT 0.01%**
Errors (%)	**1.03%**	**0.66%**	**0.19%**	**0.00%**	**0.00%**
ES Type	**ES EVT 1%**	**ES EVT 0.5%**	**ES EVT 0.1%**	**ES EVT 0.05%**	**ES EVT 0.01%**
Errors (%)	**0.38%**	**0.19%**	**0.00%**	**0.00%**	**0.00%**
Avg ES	**0.08759016**	**0.118860623**	**0.24693941**	**0.340466067**	**0.723536177**
Avg Loss (> VaR)	**0.085643689**	**0.108055871**	**0.174782102**	**0**	**0**

The results reported here account for the following conclusions:

- VaR N Std and Historical VaR didn't succeed in matching the critical level of 1%, but VaR NIG, VaR GHD and VaR CF were able to pass the precision test (VaR CF having the highest score, as in the previous sections);

- Under the EVT approach only two VaRs, at 0.05% and 0.01%, match their critical levels, but all ES indicators pass the precision test. This result is similar to the one obtained in the previous section where the DCC model was not considered;

- Overall, the results were improved by including GARCH DCC in our analysis (VaR GHD being now able to pass the precision test).

Table 40. Capital requirements for VaRs 1% and ES 1% that passed the precision test

	VaR NIG 1%	VaR GHD 1%	VaR CF 1%	ES EVT 1%
Average value	0.051480327	0.050711632	0.118287443	0.08759016
Maximum	0.028935082	0.028496749	0.067030449	0.049525963
Minimum	0.282240644	0.278089849	0.642985243	0.477226269

If we are to choose between the 1% risk measures that passed the precision test VaR GHD 1% would be the most appropriate because it provides the lowest level of capital coverage (see Table 40). The capital requirement is about 5.07% (average level) of the portfolio value, while VaR NIG 1% requires about 5.15% and ES 1% about 8.76%. VaR CF 1% is, as obtained in all the previous sections, the most expensive risk measure requiring about 11.83% of the portfolio value in this case. VaR GARCH DCC led to the lowest levels of capital requirements so, perhaps, this is the most important effect of using the DCC model.

10. Conclusions

In this study I developed a comparative analysis regarding Value at Risk by the means of a simple portfolio consisting of three stocks traded at Bucharest Stock Exchange. In all cases 1-day, 1% VaR was computed (except for the EVT approach where smaller quantiles were also considered), so the analysis does not refer to the number of days or to the probability level that are taken into account when determining VaR. Instead I focused on the following two directions: the volatility models and the distributions that form the premises for this risk measure. Thus, VaR was calculated using the historical volatility, the EWMA volatility model for the three stocks, a GARCH model for the volatility of the portfolio, GARCH models for the volatility of the three stocks and this stage was further improved by introducing a DCC model. In each case, the following distributional approaches were considered: the standard normal distribution, the historical distribution, the Cornish-Fisher approximation, distribution modeling using t-Student, NIG and GHD and the EVT approach which models the tail of the distribution using GPD. The results obtained in each stage of the analysis were presented in sections 4-8.

One of the first conclusions that can be drawn out from this study is that enhanced (and at some extent difficult) approaches to VaR are now more pervious to the practitioners. Even when a dedicated software is not available Eviews, Excel and Rmetrics could do the job, the latter having also the advantage that is a free software. For example, not quite usual distributions are available with only one command in Rmetrics and this study is an example itself of the idea mentioned above. But, even if more complicated VaR approaches are available and the practitioners are able to tailor these approaches to their specific needs, are the results worth the effort? Are the obtained risk measures worth the greater amount of calculations required to achieve them? Of course, a lot of subjective elements are taken into account when answering such questions. However, for the portfolio considered in this study I will try to see if the enhanced approaches that were used led to better results.

Table 41 presents the aggregate results for the five volatility approaches to VaR considered in this study. By aggregate results it is meant the number of 1 % VaR and ES measures that passed the precision test in each case.

Table 41. Aggregate results for the volatility approaches

VaR historical volatility	VaR EWMA	VaR GARCH (portfolio)	VaR GARCH (stocks)	VaR GARCH DCC
4	2	5	3	4

So, if we are to rank them, VaR using a GARCH model for the volatility of the portfolio performed best followed by VaR GARCH DCC (which, if compared to VaR based on historical volatility, has the advantage that its average 1% ES is not exceeded by the average of the losses bigger then the corresponding VaR). VaR GARCH DCC has also the advantage that it has the lowest average capital requirements while VaR GARCH (portfolio) requires a much smaller amount of calculations. On the other hand, as it was presumed in section 6, the EWMA volatility model fits poorly our portfolio leading to the worst results from all the volatility approaches.

Table 42 presents the aggregate results for the distributional approaches to VaR considered in this study. By aggregate results it is meant the number of 1 % VaR and ES measures that passed the precision test for each approach.

Table 42. Aggregate results for the distributional approaches

VaR N Std	Historical VaR	VaR NIG	VaR GHD	VaR CF	VaR EVT	ES EVT
0	0	3	3	5	2	5

The results are concordant with the non-normality of the standardized returns, VaR based on the standard normal distribution obtaining (together with historical VaR) the lowest score possible. Therefore, the distributional approaches intended to address non-normality are reasonable, VaR N Std leading to the underestimation of the risk. On the other side, although both VaR CF and ES EVT obtained the highest score possible, it should be taken into account that these measures have also the highest average levels of capital requirement. So, even if they didn't perform so well, it seems that VaR NIG and VaR GHD could be more attractive because their average levels of capital requirements are much smaller than the ones for VaR CF and ES EVT.

Although the main concern of this study is represented by the 1% risk measures, in the EVT approach I also considered quantiles smaller than 1%. Two reasons stand for that: the smaller quantiles allow us to see the significantly different risk profiles that may hide under very close 1% quantiles (and this is the case for our portfolio) and in order to see what happens if portfolio holders would choose a smaller probability level. The aggregate results for smaller quantiles under the EVT approach are given in Table 43.

Table 43. Aggregate results for quantiles smaller than 1% under the EVT approach

0.5%		0.1%		0.05%		0.01%	
VaR	ES	VaR	ES	VaR	ES	VaR	ES
1	5	1	5	2	5	5	5

The obtained results show that even at lower probability levels VaR EVT didn't perform very well. Moreover, the smaller the quantile the highest the capital requirement, leading to expensive portfolio management. The results for ES, although very good in terms of precision, should be taken with the reserve that ES itself is higher than the corresponding VaR, leading also to high capital requirements.

Finally, let us consider the five 1% risk measures (one for each volatility approach) that were selected in this study using the criteria of the precision tests and the level of capital requirements. Table 44 puts together these measures along with their error percentage and their average levels.

Table 44. Aggregate results for the 1% risk measures that performed best (one per volatility approach)

Volatility approach	**Historical volatility**	**EWMA**	**GARCH (portfolio)**	**GARCH (stocks)**	**GARCH DCC**
Selected risk measure	**VaR GHD**	**ES EVT**	**VaR GHD**	**VaR NIG**	**VaR GHD**
Error %	0.94%	0.19%	0.94%	0.94%	0.94%
Average value	0.0591	0.0929	0.0529	0.0521	0.0507

Once again it is possible to select what it could be the "best" risk measure for our portfolio during the considered period. And, taking into account that all of them are below the critical level of 1% VaR GHD under GARCH DCC approach would "win", considering that it has the lowest level of capital requirements. However, the answer may be not that simple considering the fact that GARCH DCC requires a great amount of calculations and it is difficult to implement in large portfolios (indeed, this is one of the reasons for which the portfolio considered in this study consists of only three stocks). Is it the calculations effort worth the decrease in capital requirement from 0.0529, or from 0.0521 to 0.0507? Certainly, is difficult to give an answer because the selection depends also on the experience of the practitioners, on the value of the portfolio and on the hardware and software available to the one measuring the risk.

Still, I consider that a few general conclusions (with a certain degree of subjectivity) may be formulated taking into account the results obtained in this study:

- Using more improved volatility models tends to offer better results, especially in terms of capital requirements. In my opinion, VaR based on a GARCH model for the volatility of the portfolio realizes perhaps the most appropriate compromise between the capital requirements and the required amount of calculations;
- It is recommended to consider other distributions than the standard normal distribution when computing VaR. However, the CF approximation tends to get overestimated 1% quantiles leading to high capital requirements (the highest in this study), while modeling with GHD and NIG leads to nice results. In my opinion, for the specific portfolio considered in this study, VaR GHD using a GARCH model for portfolio volatility is perhaps the most appropriate choice in terms of precision, capital requirements and the necessary amount of calculations;
- It seems that VaR computed under the EVT approach does not perform very well. It generally leads to lower capital requirements and so the precision results are affected. On the contrary, ES

performs very well in terms of precision and the risk profile of the tail indicates that it could be a better risk measure. But, along with the precision comes a higher level of capital requirements, not as high as VaR CF, but substantially different from the other computed VaRs.

Of course, it could be argued that the results of this study apply strictly to the considered portfolio and, at some extent, it may be true. However, I do believe that the comparative analysis done in this study could be of a more general use, especially to practitioners.

11. References

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APPENDIX 1

Full ADF test results for the daily returns of ATB, AZO, TLV and the portfolio

Null Hypothesis: ATBLN has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-24.32559	0.0000
Test critical values:	1% level	-3.433757	
	5% level	-2.862932	
	10% level	-2.567558	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(ATBLN)
Method: Least Squares
Date: 05/15/08 Time: 21:12
Sample(adjusted): 3 1814
Included observations: 1812 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ATBLN(-1)	-0.701029	0.028819	-24.32559	0.0000
D(ATBLN(-1))	-0.088011	0.023304	-3.776587	0.0002
C	0.000804	0.000698	1.152047	0.2495

R-squared	0.391148	Mean dependent var	-7.29E-05
Adjusted R-squared	0.390475	S.D. dependent var	0.038022
S.E. of regression	0.029684	Akaike info criterion	-4.194742
Sum squared resid	1.594016	Schwarz criterion	-4.185632
Log likelihood	3803.436	F-statistic	581.0820
Durbin-Watson stat	2.007280	Prob(F-statistic)	0.000000

Null Hypothesis: AZOLN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-42.60324	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(AZOLN)
Method: Least Squares
Date: 05/15/08 Time: 21:15
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AZOLN(-1)	-1.000826	0.023492	-42.60324	0.0000
C	0.000463	0.000836	0.553517	0.5800

R-squared	0.500557	Mean dependent var	-2.03E-05
Adjusted R-squared	0.500281	S.D. dependent var	0.050376
S.E. of regression	0.035611	Akaike info criterion	-3.831205
Sum squared resid	2.296646	Schwarz criterion	-3.825135
Log likelihood	3474.987	F-statistic	1815.036
Durbin-Watson stat	1.999550	Prob(F-statistic)	0.000000

Null Hypothesis: TLVLN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-37.72185	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(TLVLN)
Method: Least Squares
Date: 05/15/08 Time: 21:21
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TLVLN(-1)	-0.879796	0.023323	-37.72185	0.0000
C	0.000408	0.000685	0.595132	0.5518

R-squared	0.440002	Mean dependent var	-3.04E-05
Adjusted R-squared	0.439692	S.D. dependent var	0.038945
S.E. of regression	0.029152	Akaike info criterion	-4.231490
Sum squared resid	1.539050	Schwarz criterion	-4.225419
Log likelihood	3837.845	F-statistic	1422.938
Durbin-Watson stat	2.000186	Prob(F-statistic)	0.000000

Null Hypothesis: PORTLN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-36.55895	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

*MacKinnon (1996) one-sided p-values.

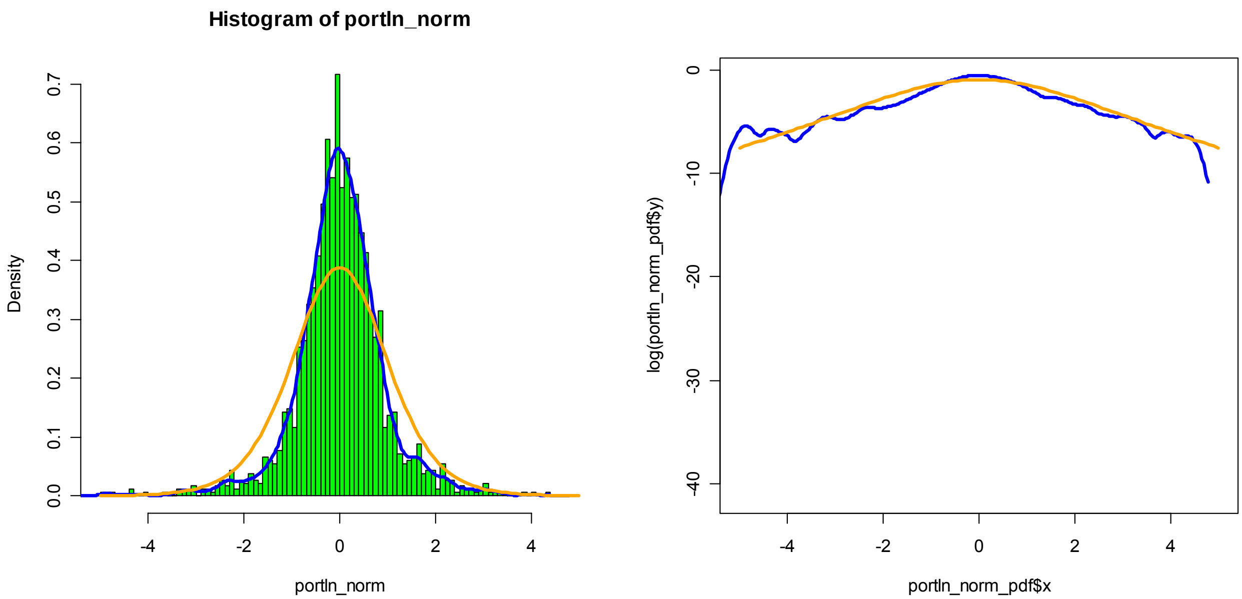
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PORTLN)
Method: Least Squares
Date: 06/15/08 Time: 11:33
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PORTLN(-1)	-0.848025	0.023196	-36.55895	0.0000
C	0.000612	0.000489	1.250262	0.2114

R-squared	0.424633	Mean dependent var	-3.08E-05
Adjusted R-squared	0.424315	S.D. dependent var	0.027448
S.E. of regression	0.020826	Akaike info criterion	-4.904159
Sum squared resid	0.785445	Schwarz criterion	-4.898089
Log likelihood	4447.620	F-statistic	1336.557
Durbin-Watson stat	2.013358	Prob(F-statistic)	0.000000

APPENDIX 2

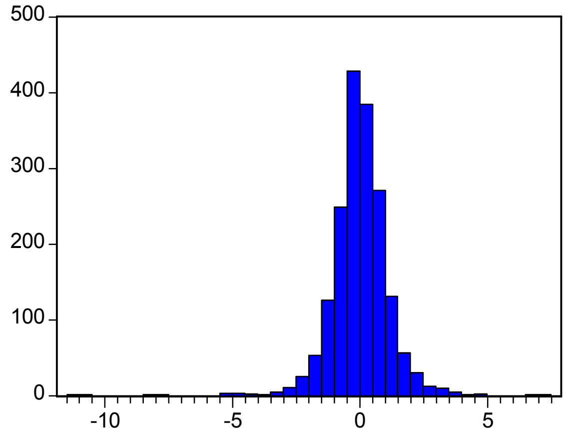
PDF of the empirical distributions and log PDF graph of the standardized returns[20] of the portfolio against the estimated t-Student distribution



[20] The returns have been standardized using their historical mean and volatility

APPENDIX 3

The histogram and the Jarque-Bera test applied to the standardized returns of the portfolio[21]



Series: PORTLN_EWMA
Sample 1 1814
Observations 1814

Mean	0.015034
Median	-0.004146
Maximum	7.380382
Minimum	-11.35463
Std. Dev.	1.160841
Skewness	-1.104383
Kurtosis	17.02011
Jarque-Bera	15225.66
Probability	0.000000

The ADF test applied to the standardized returns of the portfolio

Null Hypothesis: PORTLN_EWMA has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-37.99145	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

*MacKinnon (1996) one-sided p-values.

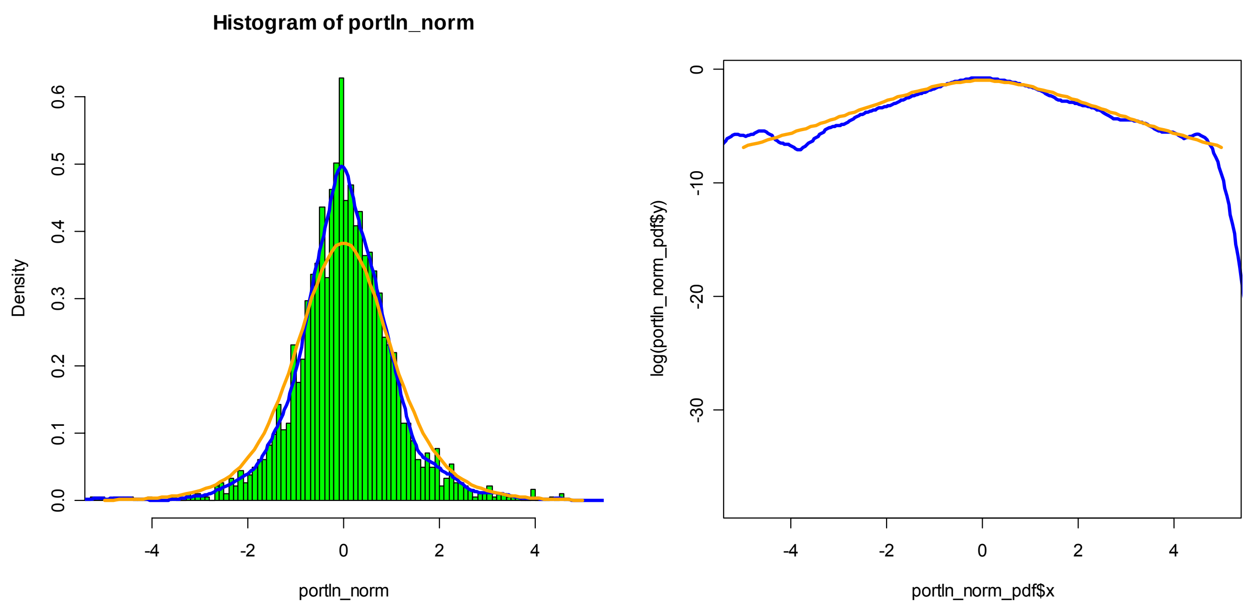
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PORTLN_EWMA)
Method: Least Squares
Date: 06/15/08 Time: 16:30
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PORTLN_EWMA(-1)	-0.886131	0.023324	-37.99145	0.0000
C	0.012099	0.027077	0.446832	0.6551

R-squared	0.443514	Mean dependent var	-0.001481
Adjusted R-squared	0.443207	S.D. dependent var	1.544942
S.E. of regression	1.152813	Akaike info criterion	3.123391
Sum squared resid	2406.781	Schwarz criterion	3.129461
Log likelihood	-2829.354	F-statistic	1443.351
Durbin-Watson stat	2.010906	Prob(F-statistic)	0.000000

[21] The information displayed in this appendix concerns the portfolio returns standardized using the EWMA volatility

PDF of the empirical distributions and log PDF graph of the standardized returns of the portfolio against the estimated t-Student distribution



APPENDIX 4

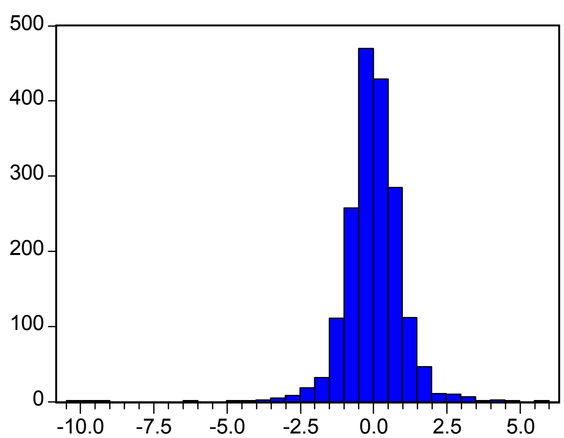
GARCH (1,1) estimated equation for the volatility of the portfolio returns

Dependent Variable: PORTLN
Method: ML - ARCH (Marquardt)
Date: 06/21/08 Time: 13:52
Sample: 1 1814
Included observations: 1814
Convergence achieved after 27 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001145	0.000383	2.986488	0.0028
		Variance Equation		
C	3.67E-05	2.45E-06	14.99823	0.0000
ARCH(1)	0.233307	0.015075	15.47647	0.0000
GARCH(1)	0.734227	0.008139	90.21299	0.0000

R-squared	-0.000346	Mean dependent var	0.000752
Adjusted R-squared	-0.002005	S.D. dependent var	0.021087
S.E. of regression	0.021108	Akaike info criterion	-5.002785
Sum squared resid	0.806442	Schwarz criterion	-4.990650
Log likelihood	4541.526	Durbin-Watson stat	1.692772

The histogram and the Jarque-Bera test applied to the standardized returns of the portfolio[22]



Series: PORTLN_GARCH	
Sample 1 1814	
Observations 1814	
Mean	0.002769
Median	-0.003328
Maximum	5.807381
Minimum	-10.00401
Std. Dev.	1.001248
Skewness	-1.512139
Kurtosis	20.02164
Jarque-Bera	22590.53
Probability	0.000000

The ADF test applied to the standardized returns of the portfolio

Null Hypothesis: PORTLN_GARCH has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

[22] The information displayed in this appendix concerns the portfolio returns standardized using the GARCH volatility

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-38.01752	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

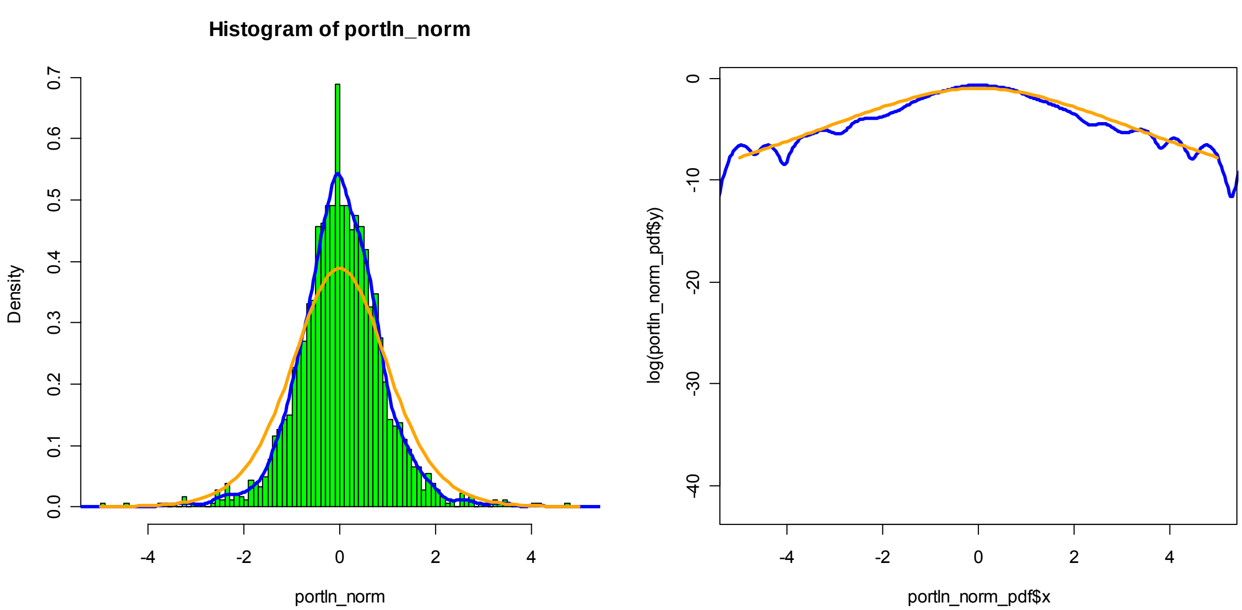
*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PORTLN_GARCH)
Method: Least Squares
Date: 06/21/08 Time: 14:35
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PORTLN_GARCH(-1)	-0.886451	0.023317	-38.01752	0.0000
C	0.001233	0.023345	0.052803	0.9579

R-squared	0.443853	Mean dependent var	-0.001415
Adjusted R-squared	0.443546	S.D. dependent var	1.332533
S.E. of regression	0.994014	Akaike info criterion	2.826972
Sum squared resid	1789.385	Schwarz criterion	2.833043
Log likelihood	-2560.650	F-statistic	1445.332
Durbin-Watson stat	2.007919	Prob(F-statistic)	0.000000

PDF of the empirical distributions and log PDF graph of the standardized returns of the portfolio against the estimated t-Student distribution



APPENDIX 5

Estimated GARCH equations for the volatility of the three stocks

Dependent Variable: ATBLN
Method: ML - ARCH (Marquardt)
Date: 06/22/08 Time: 12:34
Sample: 1 1814
Included observations: 1814
Convergence achieved after 14 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001452	0.000486	2.986079	0.0028
	Variance Equation			
C	2.14E-05	2.46E-06	8.706563	0.0000
ARCH(1)	0.124996	0.008175	15.28975	0.0000
GARCH(1)	0.851955	0.007247	117.5653	0.0000

R-squared	-0.000028	Mean dependent var	0.001289
Adjusted R-squared	-0.001686	S.D. dependent var	0.030789
S.E. of regression	0.030815	Akaike info criterion	-4.619317
Sum squared resid	1.718670	Schwarz criterion	-4.607182
Log likelihood	4193.720	Durbin-Watson stat	1.525151

Dependent Variable: AZOLN
Method: ML - ARCH (Marquardt)
Date: 06/22/08 Time: 12:53
Sample: 1 1814
Included observations: 1814
Convergence achieved after 53 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000356	0.000769	0.463350	0.6431
	Variance Equation			
C	6.78E-05	6.11E-06	11.09809	0.0000
ARCH(1)	0.097711	0.004702	20.78116	0.0000
GARCH(1)	0.857307	0.007828	109.5134	0.0000

R-squared	-0.000013	Mean dependent var	0.000483
Adjusted R-squared	-0.001670	S.D. dependent var	0.035602
S.E. of regression	0.035632	Akaike info criterion	-4.009225
Sum squared resid	2.297997	Schwarz criterion	-3.997090
Log likelihood	3640.367	Durbin-Watson stat	2.001052

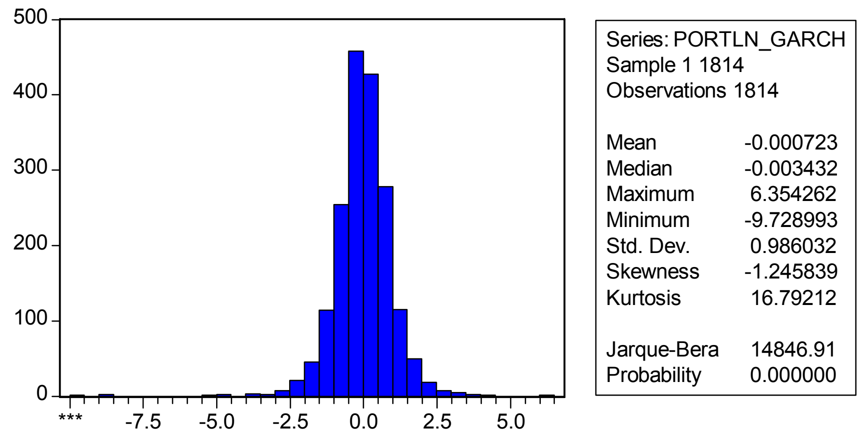
Dependent Variable: TLVLN
Method: ML - ARCH (Marquardt)
Date: 06/22/08 Time: 14:31
Sample: 1 1814
Included observations: 1814
Convergence achieved after 120 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002483	0.000380	6.531685	0.0000

Variance Equation				
C	4.40E-05	2.20E-06	20.01196	0.0000
ARCH(1)	1.869206	0.071401	26.17891	0.0000
(RESID<0)*ARCH(1)	-1.566882	0.071943	-21.77950	0.0000
GARCH(1)	0.561647	0.005775	97.25708	0.0000

R-squared	-0.004632	Mean dependent var	0.000486
Adjusted R-squared	-0.006854	S.D. dependent var	0.029359
S.E. of regression	0.029460	Akaike info criterion	-4.591038
Sum squared resid	1.569979	Schwarz criterion	-4.575869
Log likelihood	4169.071	Durbin-Watson stat	1.750541

The histogram and the Jarque-Bera test applied to the standardized returns of the portfolio[23]



The ADF test applied to the standardized returns of the portfolio

Null Hypothesis: PORTLN_GARCH has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-38.42488	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

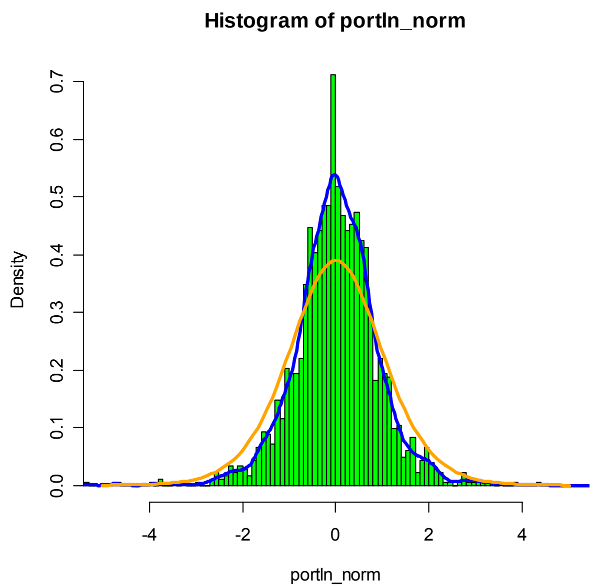
*MacKinnon (1996) one-sided p-values.

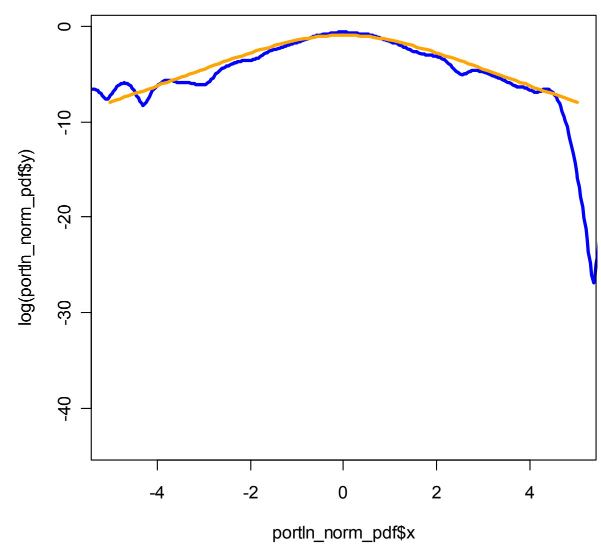
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PORTLN_GARCH)
Method: Least Squares
Date: 06/23/08 Time: 21:27
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

[23] The information displayed in this appendix concerns the portfolio returns standardized using the GARCH volatility

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PORTLN_GARCH(-1)	-0.896964	0.023343	-38.42488	0.0000
C	-0.001875	0.023016	-0.081458	0.9351
R-squared	0.449121	Mean dependent var		-0.001463
Adjusted R-squared	0.448817	S.D. dependent var		1.320006
S.E. of regression	0.979995	Akaike info criterion		2.798564
Sum squared resid	1739.267	Schwarz criterion		2.804634
Log likelihood	-2534.898	F-statistic		1476.472
Durbin-Watson stat	2.006469	Prob(F-statistic)		0.000000

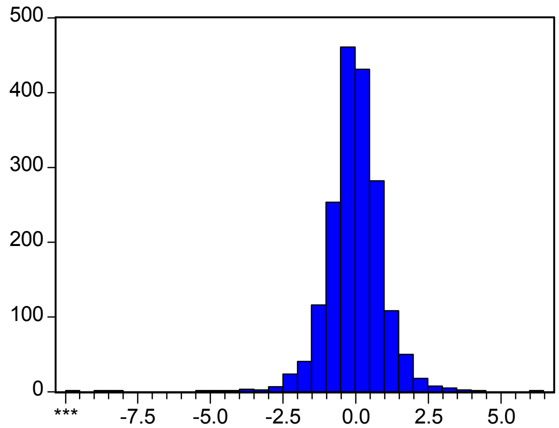
PDF of the empirical distributions and log PDF graph of the standardized returns of the portfolio against the estimated t-Student distribution





APPENDIX 6

The histogram and the Jarque-Bera test applied to the standardized returns of the portfolio[24]



Series: PORTLN_GARCHDCC
Sample 1 1814
Observations 1814

Mean	-0.000638
Median	-0.003444
Maximum	6.353615
Minimum	-9.979408
Std. Dev.	0.980753
Skewness	-1.292731
Kurtosis	17.57500
Jarque-Bera	16561.46
Probability	0.000000

The ADF test applied to the standardized returns of the portfolio

Null Hypothesis: PORTLN_GARCHDCC has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=24)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-38.40774	0.0000
Test critical values:	1% level	-3.433755	
	5% level	-2.862931	
	10% level	-2.567557	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PORTLN_GARCHDCC)
Method: Least Squares
Date: 06/30/08 Time: 22:52
Sample(adjusted): 2 1814
Included observations: 1813 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PORTLN_GARCHDCC(-1)	-0.896525	0.023342	-38.40774	0.0000
C	-0.001789	0.022892	-0.078130	0.9377

R-squared	0.448900	Mean dependent var	-0.001448
Adjusted R-squared	0.448596	S.D. dependent var	1.312623
S.E. of regression	0.974709	Akaike info criterion	2.787748
Sum squared resid	1720.555	Schwarz criterion	2.793818
Log likelihood	-2525.093	F-statistic	1475.155
Durbin-Watson stat	2.006671	Prob(F-statistic)	0.000000

[24] The information displayed in this appendix concerns the portfolio returns standardized using the GARCH DCC volatility

PDF of the empirical distributions and log PDF graph of the standardized returns of the portfolio against the estimated t-Student distribution