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Exploring Dual Long Memory in Returns and Volatility Across Central and Eastern Europe Stock Markets

Dissertation Paper

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ABSTRACT

We investigate the presence of long memory in emerging CEE stock markets using the nonparametric, semiparametric and parametric approaches.We consider the methodology of Bai and Peron to test for structural breaks in the return series and we perform tests of fractionally integrated process on subsamples in order to identify potential evidence of spurious long memory. We test for long memory in both conditional mean and conditional variance by combining a fractionally integrated regression function and a fractionally integrated skedastic function.We estimate ARFIMA-GARCH and ARFIMA-FIGARCH models under two proposed distributions. The the skewed Student-t distribution is found to better describe the data comparing to Gaussian distribution.

We conclude that the Romanian, Hungarian and Czech Republic capital markets show evidence of dual long memory in returns and volatility, while the Bulgarian and Poland markets show strong features of long memory in volatility, but no long memory in return series.

Table of Contents

I. INTRODUCTION

The long memory property has been widely studied in economic series and its implications for economic theory have been extensively discusssed .The most considerable economical implications is the contradiction of the weak-form of market efficiency - Fama (1970) by allowing investors and portfolio managers to make prediction and to construct speculative strategies. The price of an asset determined in an efficient market should follow a martingale process in which each price change is unaffected by its predecessor and has no memory. Consequently, pricing derivative securities with martingale methods may not be appropriate if the underlying continuous stochastic processes exhibit long memory. Therefore, exploring long memory property is appealing for derivative market participants, risk managers and asset allocation decisions makers, whose interest is to reasonably forecast stock market movements.These are only few reasons explaining the high researchers' interest, and the impressive number of papers written on this subject.

This paper re-examines evidence of long-memory in the conditional mean and volatility of six stock indices, representing five emerging capital markets in Central and Eastern Europe: Romania, Bulgaria, Hungary, Poland and Czech.

Many of these papers focus on the developed financial markets, while less attention has been accorded to emerging securities markets. Emerging markets are generally characterized by low dimensions and liquidity and in the same time by a higher volatility than developed financial markets. These different features may contribute to a different dynamics underlying returns and volatility, making attractive a further investigation in these markets.

In the latest period, there is a great interest in discerning the reasons and underlying causes for the widespread a selection of relevant

literature on the issues at hand. empirical finding of long memory. A number of authors have attempted to develop methodologies to distinguish between true long memory and other types of processes displaying statistical long memory, this subject being intensively investigated in recently performed studies.

This paper is organized as follows: Section II presents a selection of relevant literature related to this papers' subject. Section III introduces the concepts and models used in the empirical analysis, while Section IV describes the data, the implementation of the models and discuss of the obtained results. Section V includes the conclusions.

II. LITERATURE REVIEW

A remarkable research has been performed concerning long memory property and its implications in various science fields from physics, climatology and hydrology to applications on stock markets, exchange rates and macroeconomic indicators. This section presents the studies supposed to be most relevant for the subject analyzed in this paper.

Long memory modeling has been studied in econometrics and finance since Mandelbrot (1969) introduced long memory specifications for price processes. Fractionally integrated models started to become usually used in the 1980s when Geweke and Porter-Hudak (1983) developed the log periodogram regression estimator for the order of integration parameter *d* in the arfima model of Granger and Joyeux (1980) and Hosking (1981).

Lo (1991) found little evidence of long-term memory in historical U.S. stock market returns while Cheung and Lai (1995) investigate the presence of long memory in stock returns for 18 indices using a modified R/S statistic and the GPH test. R/S statistic shows mostly negative results, while GPH test confirm long memory in stock returns for only five indices.

Ding, Granger and Engle (1993) found that there is substantially more correlation between absolute returns than returns themselves and consider that both ARCH type models based on squared returns and those based on absolute return can produce the property of long memory in volatility. In the same idea of spurious long memory, Ding and Granger (1996) pointed out that many other generating mechanisms can produce processes with the same features as long memory, and show that at least time-varying parameter models could be considered in this class. Lobato and Savin (1998) found no evidence of long memory in daily stock returns, but strong evidence of long memory in squared returns while. Willinger, Taqqu and Teverovsky (1999) found empirical evidence of long-range dependence in stock price returns, but the evidence was not absolutely conclusive. Granger and Hyung (1999) shows for S&P 500 that absolute stock returns tend to show the long memory property due to the presence of structural breaks in the series rather than than due to a true I(d) process, concluding that linear process with breaks can imitate autocorrelations, as well as other properties of fractionally integrated processes.Using the spectral regression method, Barkoulas, Baum and Travlos (2000) found significant and robust evidence of positive long-term persistence in the Greek stock market. Henry (2002) investigated long range dependence in nine international stock index returns and found evidence of long memory in four of them, the German, Japanese, South Korean and Taiwanese markets, but not for the markets of the UK, USA, Hong Kong, Singapore and Australia.

Chen (2000) calculated the classical rescaled range statistic of Hurst for seven Asia-Pacific countries' stock indices and concluded that all the index returns have long memory. Diebold and Inoue (2000), however, conclude upon the strong connection between long memory and regime switching, showing that that stochastic regime switching can be easily confused with long memory.

Sadique and Silvapulle (2001) examined the presence of long memory in weekly stock returns of seven countries: Japan, Korea, New Zealand, Malaysia, Singapore, the USA and Australia. They found evidence for long-term dependence in four countries: Korea, Malaysia, Singapore and New Zealand.

Cajueiro and Tabak (2005) state that the presence of long-range dependence in asset returns seems to be a stylized fact. They studied the individual stocks in the Brazilian stock market and found evidence that firm-specific variables can explain, at least partially, the long-range dependence phenomena. From the same point of view, Perron and Qu (2006) analytically show how a stationary short memory process with level shifts can generate spurious long memory.

Granger and Hyung (2004) research came to support the conclusions of Diebold and Inoue and show that occasional breaks generate slowly decaying autocorrelations and other properties of I(d) processes, and that is not easy to distinguish between the two type of processes. They demonstrated that at least a part of the long memory may be caused by the presence of neglected breaks in the series and suggest that their finding permit improvements of volatility prediction by combining I(d) model and occasional-break model.

The idea of dual long memory process was first introduced by Teyssiere (1997) which show through Monte Carlo simulations that ignoring long memory in the conditional mean of a dual long memory process leads to significant biases in the estimation of the conditional volatility process. Consequently, in order to asses the robustness of the FIGARCH model the possibility of a fractional root in the conditional mean is introduced. They conclude that the ARFIMA-FIGARCH model capture more or less the dynamics of daily exchange rates, due to the fact that the fractional parameter in the mean equation was found to be quite low, confirming the presence of long memory only in the conditional volatility. Other authors which investigated long memory using the technique proposed by Teyssiere(1997) are Beine and Sebastien (1999) which also estimate FIGARCH model for modeling daily exchange rates and conclude that allowing for a fractional root in the conditional mean appear to be pertinent but does not lead to other parameter estimates compared with the volatility sides.

Yoon and Kang(2007) investigate dual long memory in the returns and volatility of Korean stock market starting from a slight different approach from that proposed by Teyssiere(1997), given that they first estimate ARFIMA model for the conditional mean, and depending on the obtained results they further analyze dual long memory applying the joint ARFIMA-FIGARCH model. They found evidence of long memory in both moments and conclude that the dual long memory model provide a better explanation for long memory dynamics in both the conditional mean and variance.

Similar result were obtained Kasman and Torun (2008) which investigate the presence of dual long memory in eight CEE emerging stock markets and found that strong evidence of long memory in both conditional mean and variance and that the ARFIMA-FIGARCH model outperforms ARFIMA-GARCH and ARFIMA-HYGARCH models in terms of out-of-sample forecast.

Finally, more recent articles of high interest are those of Baillie and Morana (2007) and Baillie and Morana (2009) respectively, although these approaches are not followed on the present paper. Baillie and Morana (2007) propose a new model for long memory in volatility , designed to account for both long memory features and structural change in the conditional variance process. The model, named A-FIGARCH (Adaptive Fractionally Integrated GARCH) considers a time-varying intercept which allows for breaks, cycles and changes in drift. Baillie and Morana (2009) propose a similar model to A-FIGARCH for the conditional mean, named Adaptive ARFIMA for which they conclude that appears to be capable of successfully dealing with various forms of breaks and discontinuities in the conditional mean of a time series. The model was proposed for investigating inflation dynamics but can also be applied to other economic time series data. They also propose a generalization of these models, the so called A_2(Adaptive)-ARFIMA-FIGARCH , which allow for long memory and structural breaks in the conditional mean as well as for long memory and structural breaks in the conditional variance.

III. DETECTING LONG MEMORY IN TIME SERIES

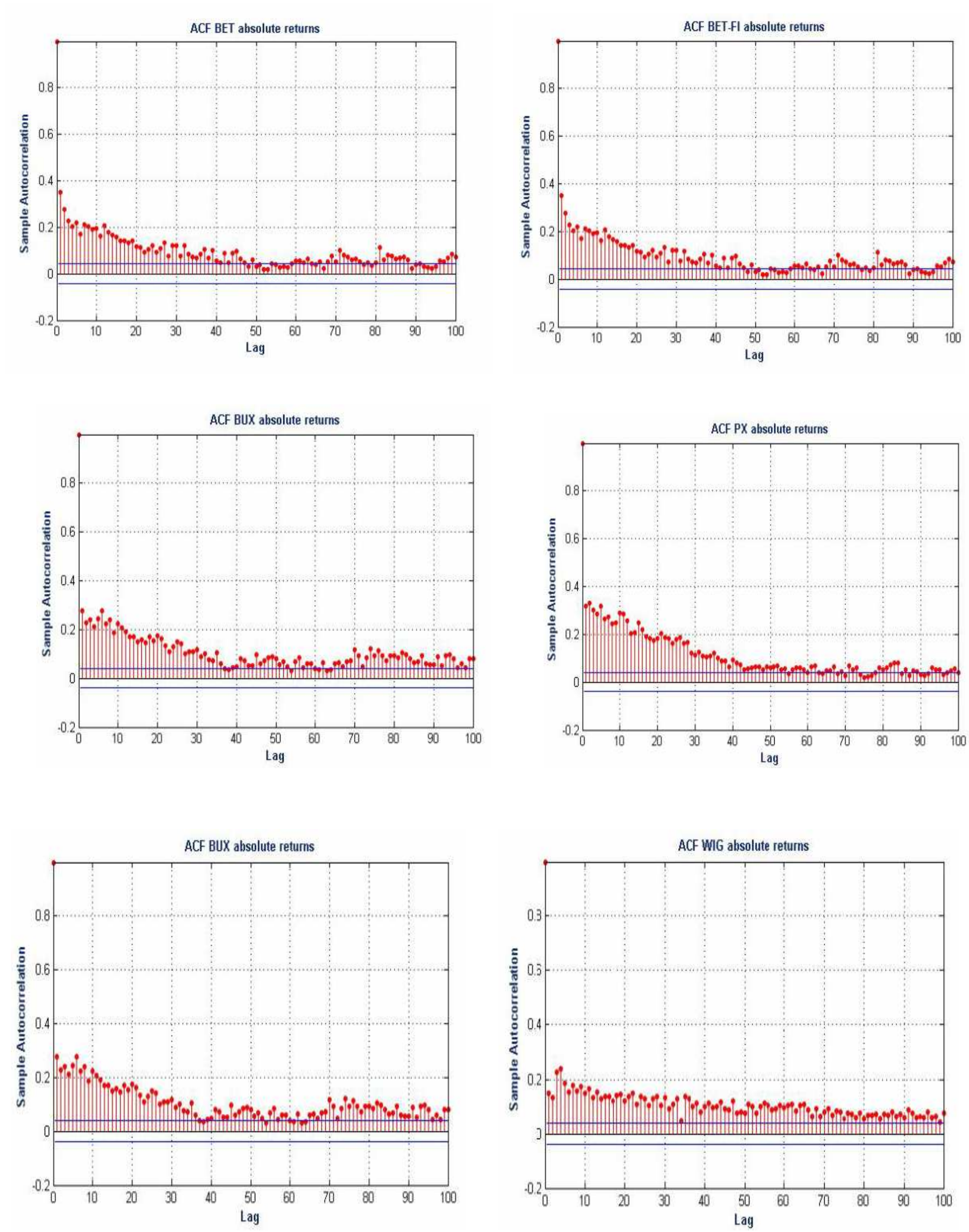
There are various definitions of long memory processes. Especially, long memory could be expressed either in the time domain or in the frequency domain. In the time domain, a stationary discrete time series is said to be long memory if its autocorrelation function decays to zero like a power function.

This meaning involves that the dependence between successive observations decays slowly as the number of lags tends to infinity. On the other hand, in the frequency domain, a stationary discrete time series is supposed to be long memory if the spectral density is unbounded at low frequencies.

ACF and fractional integration

A standard approach to examining long memory within time series is through an examination of the sample autocorrelation function. In particular, if the sample autocorrelations take a long time to decline to zero, then the process is said to exhibit long memory. That is, if the autocorrelations decay very slowly as the lag length increases then current values of the series are related to own distant values. Absolute returns and squared returns show most obvious these patterns.

Fig.1 Autocorrelation function for absolute returns



Testing stationarity

It is well known that deciding whether data are fractionally integrated or not based on the ADF tests may be inadequate, due to the fact that this type of unit root tests has low power to distinguish between the I(1) null hypothesis and the I(d) alternative. (Diebold and Rudebusch (1991) and Hassler and Wolters(1994).

Originally designed to test an I(0) null hypothesis versus an I(1) alternative, the KPSS test proposed by Lee and Schmidt (1996) proved to perform well as a test for the null stationarity against the alternative of fractional integration.

Therefore, by testing the both ADF and KPSS tests, one can distinguish between the three type of series: unit root, stationary and fractionally integrated.

As noted in Baillie, Chung and Tieslau (1996) the combined use of ADF, PP and KPSS test leads to the following possible results:

- rejection by the ADF and PP and failure to reject by the KPSS is considered a strong evidence of a stationary I(0) process;
- failure to reject by the ADF and PP and rejection by the KPSS statistic indicates a unit root I(1) process;
- failure to reject by all ADF, PP and KPSS is probably a consequence of data being insufficiently informative for the long-run characteristics of the process;
- rejection by all ADF, PP and KPSS indicates that the process is described by neither I(0) nor I(1) processes and therefore it is probable better described by the fractional integrated alternative.

In this paper we perform only ADF and KPSS tests but the interpretations remain the same.

The combined evidence based on the ADF and KPSS test results indicates that for the index returns series neither an I(1) nor an I(0) process is a good representation of the data process, which suggests that a fractionally differenced process may be an appropriate representation for these series.

Determining the degree of fractional integration

There are currently a significant number of estimation methods for and tests of long memory models. Probably one of the reasons for this large collection of tools for estimation and testing is the fact that good estimation techniques remain elusive, and many of the tests used for long memory have been shown through finite sample experiments to perform quite poorly.
Therefore, we consider some of the most widely used estimators and tests: R/S statistic, wavelet based estimator, GPH estimator -as nonparametric and semiparametric approaches- and ARFIMA/FIGARCH model as parametric approach.

- **Nonparametric approach: The Rescaled Range Statistic**

The original statistical measurement of long memory introduced by Hurst(1951), and subsequently used by Mandelbrot(1975) is the R/S statistic (rescaled range statistic).
The classical rescaled range statistic is defined as:

$$R/S(n) = \frac{1}{s_n}\left[Max\sum_{j=1}^{k}(X_j - \overline{X_n}) - Min\sum_{j=1}^{k}(X_j - \overline{X_n})\right],\ 1 \le k \le n \qquad (1)$$

Where s_n the sample standard deviation:

$$s_n = \left[\frac{1}{n}\sum_j (X_j - \overline{X_n})^2\right]^{\frac{1}{2}}$$

The first term represents the maximum of the partial sums of the first k deviations of X_j from the sample mean. Given that the sum of all n deviations of the X_j's from their mean is zero, this term is always nonnegative. In the same way, the second term is always nonpositive and hence the difference between the two terms, known as the range , is always nonnegative.
Madelbrot and Wallis (1969) use the R/S statistic to detect long memory patterns using the following rationale: for a random process there is scaling relationship between the rescaled range and the number of observations n of the form:

$$R/S(n) \sim nH \qquad (2)$$

where H is the Hurst exponent. For a white noise process H = 0.5, while for a persistent, long memory process H > 0.5 The difference d = (H-0.5) represents the degree of fractional integration in the process.

Mandelbrot and Wallis suggest estimating the Hurst coefficient by plotting the logarithm of R/S(n) against log(n). For large n, the slope of such a plot should provide an estimate of H.

A major limitation of the rescaled range is its sensitivity to short-range dependence. Any departure from the predicted behavior of the R/S statistic under the null hypothesis need not be the result of long-range dependence, but may simply be an indication of short-term memory. Lo (1991) show that this result from the limiting distribution of the rescaled range:

$$\frac{1}{\sqrt{n}} R/S(n) \Rightarrow V \qquad (3)$$

where V is the range of a Brownian bridge on the unit interval.

To differentiate between long-range and short-term dependence, Lo proposes a modification of the R/S statistic to ensure that its statistical behavior is invariant over a general class of short memory processes, but deviates for long memory processes. His version of the R/S test statistic differs only in the denominator. Rather than using the sample standard deviation, Lo's formula applies the standard deviation of the partial sum, which includes not only the sums of squares of deviations for X_j, but also the weighted autocovariances (up to lag q):

$$\hat{\sigma}_n^2(q) = \frac{1}{n}\sum_{j=1}^{n}(X_j - \overline{X_n})^2 + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j \qquad \omega_j(q) = 1 - \frac{j}{q+1} \qquad q < n \qquad (4)$$

where the γ_j are the usual autocovariance estimators.

If q = 0, Lo's statistic reduces to Hurst's R/S statistic. This statistic is highly sensitive to the order of truncation q but there is no a statistical criteria for choosing q in the framework of this statistic. If q is too small, the statistic does not account for the autocorrelation of the process, while if q is too large, it accounts for any form of autocorrelation and the power of this test tends to its size.

Therefore, while in principle this adjustment to the R/S statistic ensures its robustness in the presence of short-term dependency, selecting an appropriate lag order q still remains a problem. Moreover, Teverovsky, Taqqu and Willinger (1999) proved that Lo's modification of R/S statistic is too strict. They show that Lo's method has a strong preference for accepting the null hypothesis of no long-range dependence, irrespective of whether long-range dependence is present in the data or not. They also conclude that an acceptance of the null hypothesis of no long-range dependence based on the modified R/S statistic should never be viewed as conclusive but should always be accompanied and supported by further analysis of the data.

- **Semiparametric approach**

a. Geweke-Porter-Hudak estimator

Geweke and Porter-Hudak (1983) suggested a semi-parametric procedure to obtain an estimate of the fractional differencing parameter d based on the slope of the spectral density function around the angular frequency x = 0. More specifically, let I(x) be the periodogram of y at frequency x defined by:

$$I(\xi) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\xi}(y_t - \bar{y})\right|^2 \qquad (5)$$

Then the spectral regression is defined by:

$$\ln\{I(\xi_\lambda)\} = \beta_0 + \beta_1 \ln\left\{4\sin^2\left(\frac{\xi_\lambda}{2}\right)\right\} + \eta_\lambda \qquad (6)$$

Where $\xi_\lambda = \frac{2\pi\lambda}{T}(\lambda = 0,...,T-1)$ denotes the Fourier frequencies of the sample, T is the number of observations, and n = g(T) << T is the number of Fourier frequencies included in the spectral regression.

Assuming that $\lim_{T\to\infty} g(T) = \infty, \lim_{T\to\infty}\{g(T)/T\} = 0, \lim_{T\to\infty} \ln(T)^2 / g(T) = 0$ the negative of the slope coefficient in the spectral regression provides an estimate of d. Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality for d < 0, while Robinson (1990) proves consistency for d $\in$ (0, 0.5).

The spectral regression estimator is not $T^{1/2}$ consistent as it converges at a slower rate. The theoretical variance of the error term in the spectral regression is known to be $\frac{\pi^2}{6}$.

A choice must be made with respect to the number of low-frequency periodogram ordinates used in the spectral regression. Improper inclusion of medium or high-frequency periodogram ordinates will contaminate the estimate of d; at the same time too small a regression sample will lead to imprecise estimates.

We report fractional differencing parameter for $T^{0.45}$, $T^{0.50}$, $T^{0.55}$ and $T^{0.60}$ to investigate the sensitivity of our results to the choice of the sample size of the spectral regression.

b. Wavelet based estimator

Wavelet based estimator estimation of fractionally integration parameter allows for analysis in both the time and frequency domain and is based on discrete wavelet transform.

For a chosen bandwidth, the discrete wavelet transform decomposes the data series in a sum of elementary contributions called wavelets which are well localized both in time and frequency domain.

The Discrete Wavelet Transform is computed, the squares of the coefficients transform are averaged, and subsequently a linear regression on the logarithm of the average, versus the log of the scale parameter of the transform is performed. The result is directly proportional to H providing an estimate for the Hurst exponent.

- **Parametric approach: ARFIMA model**

All the estimation techniques presented above, are included in the two-step estimation procedure (this distinction was first made by Sowell (1991).

These procedures only estimate the differencing parameter and in the second step the estimated differencing parameter is used to transform the observed series into a series that presumably follows an ARMA(p,q) model. The limitation of these models is that they use information only at low frequencies and therefore they do not take account of the short-term properties of the series when estimating the fractional differencing parameter. This has important implications since the estimate of the long-term parameter could be contaminated by the presence of short-term components.

Therefore, a more appropriate technique is the ARFIMA process introduced by Granger and Joyeux (1980) and Hosking (1981). This is a one-step estimation procedure: the process accounts for long-term dynamic through the fractionally integration parameter *d*, while traditional AR and MA components capture the short-term dynamics of the time series. The parameters are simultaneously estimated using maximum likelihood estimation.

The ARFIMA(p,d,q) model is represented by:

$$\Phi(L)(1-L)^d (y_t - \mu) = \Theta(L)u_t \text{ where } u_t \sim i.i.d.(0,\sigma_u^2) \qquad (7)$$

where L is the lag operator, d is the fractional differencing parameter, all the roots of Φ(L) and θ(L) lie outside the unit circle, and u_t is white noise.

Granger and Joyeaux(1981) show that

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)}, \qquad (8)$$

where $\Gamma(\ )$ is the gamma function.

If -0.5<d<0.5 the process is and invertible and for such processes, the effect of shocks to t on y_t decays at the slow rate to zero; if d=0, then the process is stationary, so-called short memory, and the effect of shocks to t on y_t decays geometrically; for d=1 the process follows a unit root process; for 0<d<0.5 the process exhibits positive dependence between distant observations implying long memory and for -0.5<d<0 the process exhibits negative dependence between distant observations, so called anti-persistence.

The use of the fractional difference operator allows obtaining a continuum of possibilities between the polar cases of unit roots processes and of integrated processes of order 0.

It is well known that, for standard ARMA processes, the autocorrelation function decreases exponentially. Opposing to this processes Hosking (1981) shows that the autocorrelation function for fractionally integrated process decay “slowly”, with a hyperbolical rate:

$$\rho(\tau) \propto \tau^{2d-1} \text{ as } \tau \to \infty \qquad (9)$$

The autocorrelation of such fractionally integrated processes remain significant at long lags.

Modeling long memory in volatility: Fractionally integrated GARCH (FIGARCH)

Since the beginning of the GARCH model (Bollerslev, 1986; Engle, 1982), several researchers observed that the parameters of the GARCH model sum very close to one, indicating a high degree of volatility persistence to a shock. Following this remark, the IGARCH model has been developed (Engle & Bollerslev, 1986), in which the parameters of the GARCH model are constrained to equal one, such that shocks have impact on future volatility indefinitely.

This idea, that volatility shocks were persistent but not infinite, led Baillie, Bollerslev and Mikkelsen (1996) to apply the idea of fractionally integration introduced by Granger(1980) and Hosking(1981) for the mean, to a GARCH framework.

The FIGARCH(p,d,q) model is given by :

$$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1-\beta(L)]\nu_t \text{, where } \nu_t = \varepsilon_t^2 - \sigma_t^2 \qquad (10)$$

It is assumed that all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle.

If d=0, the FIGARCH(p,d,q) nests the classical GARCH (p,q) process, and if d=1 the process becomes an integrated GARCH process.

The equation (10) can be rearranged as:

$$[1-\beta(L)]\sigma_t^2 = \omega + [1-\beta(L)-\phi(L)(1-l)^2]\varepsilon_t^2 \qquad (11)$$

and subsequently the conditional variance of ε_t^2 is given by:

$$\sigma_t^2 = \frac{\omega}{[1-\beta(L)]} + \left[1 - \frac{\phi(L)}{[1-\beta(L)]}(1-L)^2\right]\varepsilon_t^2 \qquad (13)$$

Equation (13) can be written as $\sigma_t^2 = \frac{\omega}{[1-\beta(L)]} + \lambda(L)\varepsilon_t^2$ (14)

where $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \dots$. Since the impact of a shock on the conditional variance of FIGARCH(p,d,q) decreases at a hyperbolic rate when $0 \le d < 1$ as stated in Baillie(1996), result that the long-term dynamics of the volatility is taken into account by the fractional integration parameter *d*, while the short-term dynamics is modeled through the traditional GARCH parameters.

Model Distributions

Skewness and kurtosis are expected to be important in a number of financial market applications including the pricing of financial assets. Since the ARCH class of processes captures time varying conditional variances, they also capture time varying conditional kurtosis. However, the Gaussian density is unable to capture the fat tails present in the unconditional distributions of financial market returns. The GARCH class of processes has therefore been combined with a number of distributions with tails fatter than the normal, for example, the Student's t (Bollerslev, 1987).

The parameters of volatility models can be estimated by using non-linear optimization procedures to maximize the logarithm of the Gaussian likelihood function. Under the assumption that the random variable z_t~N(0,1) ,the log-likelihood of Gaussian or normal distribution (L_{norm}) can be expressed as:

$$L_{Norm} = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi) + \ln(\sigma_t^2) + z_t^2\right]$$

where T is number of observations. However, it is widely observed that the distribution of residuals tends to appear asymmetry and leptokurtosis. To capture excess kurtosis and skewness, the skewed Student-t distribution is considered. If z_t~ SkST(0,1,k,ν) the log-likelihood of the skewed Student-t distribution L_{SkST} is as follows:

$$L_{SkSt} = T\left\{\ln\Gamma\left(\frac{v+1}{2}\right) - \ln\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\ln[\pi(v-2)] + \ln\left(\frac{2}{k+1/k}\right) + \ln(s)\right\} - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+v)\ln\left[1 + \frac{(sz_t+m)^2}{v-2}k^{-2I_t}\right]\right]$$

Where It = 1 if zt ≥ -m/s or It = -1 if zt < -m/s, and k is an asymmetry parameter. The constants m = m(k,ν) and $s = \sqrt{s^2(k,v)}$ are the mean and standard deviation of the skewed Student-t distribution as follows:

$$m(k,v) = \frac{\Gamma((v-1)/2)\sqrt{v-2}}{\sqrt{\pi}\Gamma(v/2)}\left(k - \frac{1}{k}\right)$$

$$s^2(k,v) = \left(k^2 + \frac{1}{k^2} - 1\right) - m^2$$

The value of ln(k) can represent the degree of asymmetry of residual distribution. For example, if ln(k)>0 or ln(k)<0, the density is right (left) skewed. When k=1, the skewed Student-t distribution equals the general Student-t distribution.

The parameter ν measures the degree of freedom. The lower the value of ν, the greater the number of extreme values (i.e., the fatter are the distribution tails than the normal distribution). When ν approaches infinity, excess kurtosis becomes zero and the normal distribution results.

If the standardized residuals are not normal, assuming that the conditional mean and variance are correctly specified, GARCH estimates are consistent but asymptotically inefficient, with the degree of inefficiency increasing with the degree of departure from normality. The skewed Student's t distribution should therefore reduce the excess kurtosis and skewness in the standardized residuals and provide efficiency gains. Moreover, if the distribution exhibits excess kurtosis, the QML estimates are consistent but may be biased in finite samples.

Therefore, in order to capture skewness and kurtosis, the skewed Student's t distribution should be considered. (this distribution has been proposed by Fernandez and Steel (1998) and subsequently extended by Lambert and Laurent (2001)).

IV. EMPIRICAL DATA AND RESULTS

1. The Data

Our estimates are based on daily closing stock prices. The data source is the provider of financial news, data and analytics, Bloomberg. The sample starts at different dates for the six indices and it ends at April 30, 2009, leading to a number of observations between 2119 and 2882.

The data were transformed into continuously compounded returns $r_{i,t} = \log\left(\frac{p_{i,t}}{p_{i,t-1}}\right)$ where $p_{i,t}$ represents the value of index i at time t. In the case of a day following a nontrading day, the return is calculated using the closing price indices of the latest trading day and that day.

The estimations and tests were performed in R version 2.9.0.

For estimating ARFIMA-FIGARCH model, the Ox Console version 5.10, together with the G@rch Console 4.2 were used. Both consoles are part of Ox Econometric Software developed by Jurgen A. Doornik.

2. Preliminary analysis

After performing a preliminary analysis, the data appear extremely non-normal. Excepting BET-FI index, all of the return distributions are negatively skewed.

The data also display a high degree of excess kurtosis. Such skewness and kurtosis are common features in asset return distributions, which are repeatedly found to be leptokurtic. The null hypothesis of normality of the Bera-Jarque test is rejected for all indices. The results of the ADF unit root test indicate that all of the returns series are stationary.

The KPSS test for the null hypothesis of I (0) is conducted in two ways: based on a regression on a constant, and on a constant and time trend, respectively, and we conclude that the series cannot be characterized as I(0) processes. This is, according to Baillie, Chung and Tieslau (1996) an indication that the process is described by neither I(0) nor I(1) processes and therefore it is probable better described by the fractional integrated alternative.

3. Fractional parameter estimates

All the nonparametric and semiparametric techniques were applied to return series, as well as to absolute and squared return series which are considered the most popular proxies for volatility in financial markets.The results are reported in Table 1.

Table 1: Nonparametric and semiparametric estimates

BET	R/S H	Wavelet H	d (GPH)
Returns	0.6263291	0.4960724	0.1505057
Squared returns	0.7694657	0.5149534	0.3544141
Absolute returns	0.8325581	0.614134	0.3796489
SOFIX	**R/S H**	**Wavelet H**	**d (GPH)**
Returns	0.4805514	0.4407017	0.3027533
Squared returns	0.6632112	0.5394242	0.3865293
Absolute returns	0.7128511	0.7029066	0.4445772
WIG	**R/S H**	**Wavelet H**	**d (GPH)**
Returns	0.6368706	0.5623532	0.0201389
Squared returns	0.7688137	0.754955	0.303066
Absolute returns	0.8064734	0.7746495	0.3800708

BET-FI	R/S H	Wavelet H	d (GPH)
Returns	0.7625203	0.51634837	0.1842962
Squared returns	0.655354	0.5797175	0.4405869
Absolute returns	0.6909146	0.6645153	0.4339933
BUX	**R/S H**	**Wavelet H**	**d (GPH)**
Returns	0.5993203	0.4874384	-0.033794
Squared returns	0.7575655	0.7539982	0.3612529
Absolute returns	0.8440155	0.7708724	0.4659885
PX	**R/S H**	**Wavelet H**	**d (GPH)**
Returns	0.5964553	0.524891	0.1023029
Squared returns	0.6827839	0.738836	0.3164356
Absolute returns	0.7325632	0.7920493	0.4963583

For return series, the rescaled range statistic indicates a value of the Hurst exponent above 0.5, except for SOFIX, for which the test indicates a value of 0.48. At a first sight, the absolute return series appear to be more persistent than squared return series, since the Hurst exponent values estimated through R/S method are significantly higher for absolute than for squared returns. This fact is confirmed by the rest of test conducted: wavelet based estimator and GPH estimator. This property is known as *Taylor property*, (Taylor, 1986) namely that the time series dependencies of financial volatility as measured by the autocorrelation function of absolute returns are stronger for absolute stock returns than for the squares.

Following Cheung(1993), for choosing the right order of ARFIMA models, different specifications of the ARFIMA (p, ξ, q) with p,q=0:2 were estimated for each return series. The Akaike's information Criterion (AIC), is used to choose the best model that describes the data. The selected orders and the estimation results are reported in Table 2.

The results indicate that the long memory parameter (ξ) is significantly different from zero for all the return series. These results appear in line with those of recent studies which state that long memory property is in general a characteristic of emerging rather than developed stock market (among the researcher which consider this, we mention Barkoulas (2000), Kang and Yoon(2007), Kasman and Torun (2008)).

Table 2. Estimation results of the ARFIMA models

	BET	**BET-FI**	**SOFIX**	**BUX**	**WIG**	**PX**
Model	**ARFIMA (0,ξ,1)**	**ARFIMA (0,ξ,0)**	**ARFIMA (1,ξ,1)**	**ARFIMA (1,ξ,2)**	**ARFIMA (0,ξ,2)**	**ARFIMA (1,ξ,2)**
Φ_1	-	-	-0.9034	0.52359	-	0.38472
			(0.0000)	(0.0014)		(0.0061)
Φ_2	-	-	-	-	-	-
ξ	**0.0461**	**0.1096**	**0.0713**	**0.0814**	**0.03135****	**0.1026**
	(0.0048)	(0.0000)	(0.0000)	(0.0000)	(0.0671)	(0.0000)
θ_1	-0.17412	-	-0.850	0.53144	-0.0663	0.40374
	(0.0000)		(0.0000)	(0.0007)	(0.0017)	(0.0035)
θ_2	-	-	-	0.06745	0.02975	0.09132
				(0.0000)	(0.0823)	(0.0000)
ln(L)	-5862	-5042	-4376	-5643	-5208	-5189
SIC	4.0735	4.8101	4.1411	4.0755	3.7312	3.7418
AIC	4.0694	4.8081	4.1331	4.0670	3.7248	3.7333
Skewness	-0.2621	0.1727	-0.5017	-0.1618	-0.2378	1.7411
kurtosis	5.7822	5.1083	25.4701	8.6850	3.1972	11.0143
Excess kurtosis	2.7822	2.1083	22.4701	5.6850	0.1972	8.0143
J-B	3915.92	2189.28	54780.01	8442.15	1177.87	14939.26
Q(20)	95.6889	26.2347	39.9162	94.0865	33.8216	67.5781

P-values are reported in the parentheses below corresponding parameter estimates; ln(L) is the value of the maximized Gaussian Likelihood; SIC/AIC is the Schwarz/Akaike information criteria. The Q(20) is the Ljung-Box statistic with 20 degrees of freedom based on the standardized residuals. The skewness and kurtosis are also based on standardized residuals. * and ** indicate significance level at the 5% and 10% respectively

As can be observed in Table 2, all of the return distributions are negatively skewed except for BET-FI and PX. The data also display excess kurtosis suggesting that residuals appear to be fat-tailed and sharply peaked about the mean when compared with the normal distribution. Such skewness and kurtosis are common features in asset return distributions, which are frequently found to be leptokurtic.

Kasman and Torun (2008) suggest that modeling only the level of returns does not offer a clear representation on the presence of long memory in the CEE countries stock markets and therefore, one should also investigate the presence of long memory in volatility.

Before investigating dual long memory in returns and volatility as suggested by Kasman and Torun (2008) we test for structural breaks in the return series, since it is well-known that estimating the long memory parameter without taking account existence of breaks in the data sets may lead to misspecification and to overestimate the true parameter.

Numerous researchers consider that in many cases long memory could be seen as an artifact of processes that exhibit structural change over time (Diebold and Inoue (2002), Granger and Hyung (2004), Teverosky and Taqqu (1997)). They observed that in the presence of structural breaks, the series reveals the same properties as a long memory process (mainly in terms of persistence) leading to a so-called "spurious long-memory".

In order to examine for potential breaks within the conditional mean we first use the Supremum F test which computes the F statistic for each potential change point and find their maximum. The asymptotic critical values of the SupF test are reported by Andrews (1993).

Subsequently, we follow the methodology of Bai and Perron (2003). The method tests for multiple breaks of unknown break dates without imposing any prior beliefs. The break tests involve regressing the variable of interest (in this case returns) on a constant and testing for breaks within that constant. First, the procedure assumes there is no break within the data against an alternate that there is up to *b* breaks in the data, where *b* is specified by the user. Furthermore, a minimum distance between breaks can also be specified. More details regarding Bai and Perron methodology can be found in Appendix 1.

The result of the tests are reported in the Table 3.

Table 3: Testing for structural breaks

	BET	BET-FI	SOFIX	BUX	WIG	PX
F statistic	15.4789	20.0107	32.4592	6.6669	9.8167	8.0598
p-value	0.001986	0.0002238	0.0000	0.12	0.02872*	0.06421**
Breakpoint at obs.no.	2440	1657	1746	-	2347	2329
Breakdate	7/23/2007	7/24/2007	10/30/2007	-	7/6/2007	7/9/2007

The supF test indicates a single break for BET, BET-FI, and SOFIX. For Hungary Stock Exchange index, the null hypothesis of no structural breaks can not be rejected. For WIG and PX, the null can be rejected at 5% and 10% significance level.

We observe that the break date for BET and BET-FI is reported in the same period, at one day distance.Both dates are directly related to the historical maximum recorded on Bucharest Stock Exchange in July 2007. On July 24, 2007, the BET index registered 10814 points, the maximum historical value, representing a growth of 34% from early 2007. That day, value of transactions in shares made on the regular market of BSE was RON 60.6 Mio.

One year letar, on July 24, 2008, BET index registered a value of 5915 points, down 45% from the peak reached in the previous year. BET-FI index dropped 62% during this period. On 24 July 2008, the trading in shares was RON 18.6 Mio - down with 69% from 24 July 2007.The descending trend is still significant for the Romanian Stock Exchange, since as of April 30, 2009 (the date of the last observation in the sample) BET index dropped by 71.8% comparing to the maximum value registered in July 2007 and BET-FI by 80.5%

The Andrews’ supremum test for SOFIX indicates as breakdate October 30, 2007, corresponding to its historical maximum value of 1,952.4 points. Since then, the index’ value decreased constantly to 358 points, the decline representing 81.66% from its historical maximum.

The Warsaw Stock Exchange index, WIG, reached its all-time high 67 568.50 points on July 6, 2007. That date is identified by the supF test as breakdate. The index is now at 57% from its historical maximum. The supF test indicates for PX index a breakpoint on July 9, 2007.

Following Shimotsu (2006), in order to asses the stability of the properties for the entire sample, we split each full sample in two distinct subsamples (considering the breakpoint previously determined).

Shimotsu (2006) show that if a time series follows an I(d) process, then each subsample of the time series also follows an I(d) process with the same value of d. He split the sample into *b* subsamples, estimate *d* for each subsample, and compare them with the estimate of *d* from the full sample .For spurious I(d) models, it turns out that the averaged estimates from the subsamples tend to differ from the full sample estimate, and their difference increases as the degree of sample splitting increases.

All the procedures (R/S, wavelet estimator, GPH, ARFIMA) were reestimated for each subsample so that to be able to make inferences about the properties of samples and subsamples.The results are reported in Table 4.

Table 4. Subsamples estimates

BET	Full sample	Before StrBreak	After StrBreak
R/S Hurst Exponent	0.6107111	0.6232049	0.590718
Wavelet Estimator for H	0.5090784	0.496072447	0.4388235
GPH estimator	0.157134	0.1518671	0.1496232
BET-FI	Full sample	Before StrBreak	After StrBreak
R/S Hurst Exponent	0.7625203	0.7571819	0.598512
Wavelet Estimator for H	0.5163484	0.6916477	0.6628132
GPH estimator	0.1842962	0.03527433	0.2535763
SOFIX	Full sample	Before StrBreak	After StrBreak
R/S Hurst Exponent	0.4805514	0.4152389	0.4356528
Wavelet Estimator for H	0.4407017	0.4328664	0.6404247
GPH estimator	0.3027533	0.1097595	0.5111778
WIG	Full sample	Before StrBreak	After StrBreak
R/S Hurst Exponent	0.6368706	0.6276843	0.7175956
Wavelet Estimator for H	0.5623532	0.5611866	0.7650836
GPH estimator	0.02013889	0.06955524	0.02715736
PX	Full sample	Before StrBreak	After StrBreak
R/S Hurst Exponent	0.5964553	0.5932412	0.7381239
Wavelet Estimator for H	0.524891	0.5214645	0.5052401
GPH estimator	0.1023029	0.104628	0.07740972

For most of the indices, the subsamples appear to keep approximately the same features as the full sample. However, SOFIX index shows values of H estimated via R/S and wavelet analysis which are below 0.5 for the full sample as well as for each subsample in part, except for the wavelet estimator for the second subsample. In addition, the GPH estimator for *d* is quite different for each subsample, suggesting that the long memory pattern showed by the full sample is based in fact only on the second subsample period which has no more than 372 observations. These findings motivate as to further investigate for a possible spurious long memory in index returns, by using the parametric approach ARFIMA(p,d,q), with the orders previously identified using AIC information criteria. The results are reported in Table 5.

Table 5. Subsamples estimates for ARFIMA model

BET ARFIMA(0,d,1)	Full sample	Before structural break	After structural break
d	0.04656	0.04479	0.003575
p-value	0.0059	(0.00758)	(0.798)
BET-FI ARFIMA(0,d,0)	Full sample	Before structural break	After structural break
d	0.1096	0.07605	0.1294
p-value	0.0000	0.0000	0.0000
SOFIX ARFIMA(1,d,1)	Full sample	Before structural break	After structural break
d	0.0713	0.00004583	0.15008
p-value	0.0000	(0.998)	(0.000692)
WIG ARFIMA(0,d,2)	Full sample	Before structural break	After structural break
d	0.03135	0.00004583	0.05557
p-value	0.0671	(0.998657)	0.0000
PX ARFIMA(1,d,2)	Full sample	Before structural break	After structural break
d	0.10255	0.05806	0.07736
p-value	0.0000	0.0000	0.0000

Estimating fractionally integrated parameter via ARFIMA(1,d,1), can be clearly observed that the first sample between October 23, 2000 and October 30,2007 , provide no evidence of long memory features in SOFIX returns. Therefore, we suggest that in the case of Bulgarian capital market, the structural break occurred in October 2007 when SOFIX registered its historical high value (and after that declined drastically) is the underlying cause of persistence in return series, rather than a true long memory process.It is questionable if in general, a single breakpoint within a series could have the power to induce persistence consistent with long memory processes. However, in our case, we applied the same structural break tests on subsamples, and we found no evidence of other breakpoints. Therefore, we will consider only the first identified breakpoint As a result, we will further investigate for long memory in SOFIX volatility using a pure FIGARCH model.

The results in Table 5 suggest that BET-FI and PX display the long memory feature in returns regardless if structural break occur. The fractional parameter differs significantly from zero in each subsample, confirming the result obtained for the full sample. BET shows long memory in the first subsample, while for the second, *d* does not differ significantly from zero. However, considering the relevance of the first subsample in terms of number of observation included (2439 vs. 442), and that usually the presence of structural breaks influence the subsamples in the opposite way, we will consider as substantial the finding of long memory in the full sample.

Finally, WIG return series appears to be significantly affected by the structural break occurred in July 2007, since for the first subsample the null hypothesis that the parameter equals zero cannot be rejected, while for the second subsample, *d* differs significantly from zero. As well as for SOFIX, this could be considered evidence of “spurious” long memory and therefore we will further investigate long memory in WIG volatility via FIGARCH model.

We were unable to follow the same subsamples technique to test for structural breaks in absolute return series due to insufficient number of observations in the second subsamples.

More specifically, unlike the finite-lag representation for the classical GARCH(p,q), the approximate maximum likelihood technique (QMLE) for FIGARCH(p,d,q) necessitates the truncation of the infinite distributed lags. Since the fractional differencing parameter is designed to capture the long-memory features, truncating at too low a lag may destroy important long-run dependencies, as shown in Bollerslev and Mikkelsen(1996) who fix the truncation lag at 1000 after performing Monte Carlo simulations.

Since we have around 400 observations for each index in the second subsample, the FIGARCH model cannot be estimated due tot the fact that the truncation order must be less than the sample size.

4. Estimating ARFIMA-FIGARCH model

An important matter in the ARFIMA-FIGARCH framework is the selection of appropriate lags ARFIMA (n,s)-FIGARCH(p,q). Following Yoon(2007) and Kasman(2008) we use for ARFIMA estimation the lag orders previously selected using AIC information criteria (n*,s*), and we estimate all the specification models ARFIMA(n*,s*)-FIGARCH(p,q) for p,q=0:2. The model which has the lowest AIC and passes Q-test simultaneously is used.

In order to perform a comparison between classical GARCH and the fractionally integrated version and to make also inferences regarding the most appropriate distribution which describe the data, we estimate for each index ARFIMA-GARCH and ARFIMA-FIGARCH models, under both the normal and skewed Student-t distribution. The fractionally integrated parameters, together with Pearson goodness-of-fit tests are reported in Table 6.

Table 6: Estimation results of the ARFIMA-FIGARCH models

BET	ARFIMA(0,ξ,1)-GARCH(1,d,1)		ARFIMA(0,ξ,1)-FIGARCH(1,d,1)	
	Normal	Skewed Student t	Normal	Skewed Student t
ξ	**0.038559****	**0.049464***	**0.0322**	**0.049475***
	(0.0902)	(0.0220)	(0.1665)	(0.0262)
d	-	-	**0.519482**	**0.371215**
			(0.0000)	(0.0000)
P(60)	**145.5949**	**44.7966**	**131.3915**	**39.5901**
	(0.0000)	(0.9143)	(0.0000)	(0.9755)

PX	ARFIMA(1,ξ,2)-GARCH(1,1)		ARFIMA(1,ξ,2)-FIGARCH(1,1)	
	Normal	Skewed Student t	Normal	Skewed Student t
ξ	**0.1609**	**0.1753**	**0.1554**	**0.1759**
	(0.0095)	(0.0016)	(0.0143)	(0.0013)
d	-	-	**0.702972**	**0.60617**
			(0.0000)	(0.0000)
P(60)	**61.7239**	**58.7045**	**88.5967**	**56.1596**
	(0.1675)	(0.1866)	(0.0009)	(0.2244)

BET-FI	ARFIMA(0,ξ,0)-GARCH(1,1)		ARFIMA(0,ξ,0)-FIGARCH(1,1)	
	Normal	Skewed Student t	Normal	Skewed Student t
ξ	**0.1006**	**0.0852**	**0.0983**	**0.0828**
	(0.0000)	(0.0001)	(0.0000)	(0.0002)
d	**0**	**0**	**0.755778**	**0.595596**
			(0.0006)	(0.0000)
P(60)	**136.9056**	**53.5866**	**139.5951**	**55.4750**
	(0.0000)	(0.6746)	(0.0000)	(0.6062)

BUX	ARFIMA(1,ξ,2)-GARCH(1,1)		ARFIMA(1,ξ,2)-FIGARCH(1,d,1)	
	Normal	Skewed Student t	Normal	Skewed Student t
ξ	**-0.0283**	**0.0640**	**0.0600**	**0.067143****
	(0.0251)	(0.0589)	(0.0659)	(0.0589)
d	-	**0**	**0.462322**	**0.455978**
			(0.0000)	(0.0000)
P(60)	**90.3605**	**66.0320**	**65.7296**	**64.1739**
	(0.0006)	(0.0527)	(0.0671)	(0.0592)

P-values are reported in the parentheses below corresponding parameter estimates. P(60) is the Pearson goodness-of-fit statistic for 60 cells; in breackets p-value(g-k-1).* and ** indicate significance level at the 5% and 10% respectively

As seen in Table 6, for BET, BET-FI, PX and BUX, both long memory parameters ξ and d are significantly different from zero, indicating the presence of dual long memory property in return and volatility of the Romanian stock market representative indices.

For SOFIX and WIG pure FIGARCH models were estimated and the results reported in Table 7 show strong evidence of long memory in volatility.

Table 7: Estimation results of the FIGARCH models

SOFIX	FIGARCH(1,d,1)	
	Normal	Skewed Student t
d	**0.5410**	**0.5686**
	0.0000	0.0000
P(60)	**285.5021**	**74.4403**
	0.0000	0.0848

WIG	FIGARCH(2,d,1)	
	Normal	Skewed Student t
d	**0.4505**	**0.4707**
	0.0000	0.0000
P(60)	**95.7241**	**63.1294**
	0.0003	0.1186

Analyzing the parameter estimates of the joint ARFIMA-FIGARCH model, we can make some inferences related to the most appropriated model and the distribution which best describes the series. All the estimated models are detailed reported in Appendix 3.

First, it can be observed that the sum of the estimates of α1 and β1 in the ARFIMA–GARCH model is very close to one, indicating that the volatility process is highly persistent. The sum of these parameters decreases when we use the ARFIMA-FIGARCH specification for modeling the series, in case of all indices. Moreover, the results indicate that the estimates of β1 in the GARCH model are very high, suggesting a strong autoregressive component in the conditional variance process and that the β1 estimates are lower in the FIGARCH than those of in the GARCH model.

Also, according to the AIC, the FIGARCH models fit the return series better than the GARCH models. Unsurprisingly, the skewed Student-t distribution is found to outperform the normal distribution returns, since the t-statistics of the parameter ν is highly significant in all the returns series. The lower values of P(60) test statistics reconfirm the relevance of skewed Student-t distribution for all returns. Hence, the skewed Student-t distribution can be used to capture the tendency of stock return distribution referring to leptokurtosis. It should be noted that in all cases the FIGARCH coefficients satisfy the necessary and sufficient conditions for the nonnegativitiy of the conditional variances. (derived by Baillie(1996)).

Similar results were obtained by Kang and Yoon (2007) in their search for long memory patterns in return and volatility of the Korean stock market. Their findings indicate that long memory dynamics in the return and volatility can be adequately estimated by the joint ARFIMA-FIGARCH model and that the skewed Student-t distribution is appropriate for incorporating the tendency of asymmetric leptokurtosis in a return distribution.

Kasman and Torun (2008) perform a research over eight CEE emerging capital markets with the purpose to investigate dual long memory property. Overall, they conclude that dual long memory is present in five from eight countries, and performing an out-of-sample forecast they found that ARFIMA-FIGARCH model provides better forecast comparing to ARFIMA-GARCH and ARFIMA-HYGARCH models, also developed in their paper. For all the data series in their research, the sample period ends in January 2007. They also found evidence of long memory in returns and volatilities of Hungary and Czech Republic stock markets, while for Poland and Bulgaria they conclude upon the presence of long memory only in volatility. This is in line with our results, since for WIG and SOFIX, we found no long memory in returns on the first subsample (which ends in July and October 2007 respectively) and strong evidence of long memory on the second subsample.

Our results appear to confirm once more the idea that due to their different characteristic from the developed markets, emerging markets are more likely to be described by long memory processes, and therefore, this feature should be more investigated and explored in order conclude upon the reliability of these findings, and their direct implications in the economy.

V. CONCLUSIONS

We have used non and semipramateric techniques, as well as the parametric ARFIMA model proposed by Granger and Joyeux(1980), in our search for long memory features in the Romanian capital market and other four emerging stock markets in the region. We also use the approach first proposed by Teyssiere(1997) consisting in the joint estimate of the ARFIMA-FIGARCH model. Our results are similar to those obtained by Kasman and Torun (2008) who investigate the dual long memory property in eight emerging CEE capital markets, without including the Romanian market.

We have considered the methodology of Bai and Peron for testing for structural breaks in the return series and we have reestimated the non and semiparametric techniques, for each subsample in order to identify potential evidence of spurious long memory. We have subsequently decided upon the distribution which best describes the data, comparing the performance of Gaussian distribution with skewed Student-t distribution.

We have investigated for long memory in both conditional mean and conditional variance by combining a fractionally integrated regression function and a fractionally integrated skedastic function.More specifically, we have estimated ARFIMA-GARCH and ARFIMA-FIGARCH models using both proposed distribution, and we assessed the results using the Pearson goodness-of-fit test.

The results strongly support the idea of dual long memory in Romanian capital market, as well as in the Hungarian and Czeck stock markets. For Bulgarian and Poland's markets, strong features of long memory in volatility were identified, while concerning the long memory in these return series, we suggested that the apparent long memory features may represent a consequence of structural breaks presence in the return series.

However, at least for the Romanian capital market (for which, to the authors' knowledge, the joint ARFIMA-FIGARCH model has not been estimated in previous papers) further research should be performed in order to make inferences regarding the consistency of our findings.

The main limitation of the ARFIMA-FIGARCH model is related to the fact that it does not take into account for structural breaks in both the conditional mean and conditional variance.

In this respect, one could further explore the long memory patterns in the Romanian capital market (and the analysis could be extended as to include also other stock markets) by using one of the most recently models developed for modeling both long memory processes and structural breaks, namely the A_2 (Adaptive)ARFIMA-FIGARCH model proposed by Baillie and Morana (2009). They first proposed the A-FIGARCH model for the conditional volatility (2007) and subsequently they developed a similar model for the conditional mean, which considers a time-varying intercept allowing for breaks, cycles and changes in drift. The generalization of these models, the A_2-ARFIMA-FIGARCH, allows for long memory and structural breaks simultaneously in the conditional mean and conditional variance.

To conclude, due to long memory implications to risk management, asset allocation decision, pricing derivatives or constructing speculative strategies, our results suggest that it is worth exploring long memory in emerging markets which are more likely to show significant evidence of such features. However, the researcher should first perform a thorough investigation of models and techniques available for long memory testing due to the well-known sensitivity of results to the selected technique.

Moreover, it cannot be overstated how important is to use such a technique which takes into account for other processes which can induce similar properties with long memory processes, in order to ensure the genuine character of the long memory phenomenon.

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Appendix 1: Non-negativity constraints

When estimating a FIGARCH model, the parameters have to fulfill some restrictions to ensure the positivity of conditional variances. Baillie et al. (1996) derived a group of two sets of inequalities. For a FIGARCH(1,d,1), the positivity constraints are:

$$\beta_1 - d \leq \phi_1 \leq \frac{2-d}{3}$$

$$d\left(\phi_1 - \frac{1-d}{2}\right) \leq \beta_1 \left(d + \alpha_1\right)$$

where $\phi_1 = \alpha_1 + \beta_1$

Restrictions for lower order models can be derived directly from the previously presented while for higher order models parameters restrictions cannot be so easily represented.

However, Caporini(2003) mentions that in practical applications one will rarely have to make use of a specification with p>2. (where p is the GARCH term).

Appendix 2: Bai and Perron methodology for structural breaks

The Bai-Perron (BP) methodology considers the following multiple structural break model, with m breaks (m+1 regimes)

$$y_t = x_t^{'}\beta + z_t^{'}\delta_1 + u_1,\ t = 1,.....T_1$$

$$y_t = x_t^{'}\beta + z_t^{'}\delta_2 + u_1,\ t = T_1 + 1,....T_2$$

……………………………………….

$$y_t = x_t^{'}\beta + z_t^{'}\delta_{m+1} + u_1,\ t = T_m + 1,....T$$

Where y_t is the observed dependant variable at time t.The break points $\left(T_1,....T_m\right)$ are treated as unknown, and are estimated together with the unknown coefficients when T observations are available. In the terminology of Bai and Perron, this is a partial structural change model, in the sense that β does not change, and is effectively estimated over the entire sample. If β=0, this becomes a pure structural change model where all coefficients are subject to change.

Appendix 3: Estimation of ARFIMA-FIGARCH models

Table 1: ARFIMA-FIGARCH for BET

BET	ARFIMA(0,ξ,1)-GARCH(1,d,1)		ARFIMA(0,ξ,1)-FIGARCH(1,d,1)	
	Normal	Skewed Student t	Normal	Skewed Student t
μ	**0.120116**	**0.08669***	**0.103417**	**0.076813**
	(0.002)	(0.0387)	(0.0041)	(0.0559)
$Φ_1$	-	-	-	-
$Φ_2$	-	-	-	-
ξ	**0.038559****	**0.049464***	**0.0322**	**0.049475***
	(0.0902)	(0.022)	(0.1665)	(0.0262)
$θ_1$	**0.150723**	**0.13304**	**0.156281**	**0.134828**
	0.0000	0.0000	0.0000	0.0000
$θ_2$	-		-	-
ώ	**0.138226**	**0.173226**	**0.095609**	**0.300451**
	(0.0051)	(0.0033)	(0.0402)	0.0000
α1	**0.227037**	**0.269633**	**0.466026**	**0.42907**
	0.00000	0.0000	0.0003	(0.0984)
$α_2$	-	-	-	-
$β_1$	**0.752619**	**0.713049**	**0.645352**	**0.54033**
	0.00000	0.0000	0.00000	(0.044)
$β_2$	-	-	-	-
d	-	-	**0.519482**	**0.371215**
			0.0000	0.0000
v	-	**5.16013**	-	**5.59537**
		0.0000		0.0000
ln(k)	-	**0.025942**	-	**0.029179**
		(0.3042)		(0.2343)
ln(L)	**-5390.6**	**-5291.51**	**-5364.9**	**-5272.30**
AIC	**3.746309**	**3.678938**	**3.729169**	**3.666299**
Q(20)	**32.6493****	**33.4769****	**34.4892****	**32.0277 ****
	(0.0263764)	(0.021166)	(0.0160801)	(0.0310313)
$Q_s(20)$	**23.2455**	**33.4769**	**13.914**	**15.616**
	(0.1813325)	(0.021166)	(0.7346793)	(0.6193279)
ARCH(5)	**2.0222***	**1.5052**	**0.33442**	**0.33497**
	(0.0725)	(0.1847)	(0.8923)	(0.892)
RBD(10)	**13.69**	**10.00**	**4.59**	**3.53**
	(0.1875974)	(0.4401308)	(0.9168096)	(0.9659432)
P(60)	**145.5949**	**44.7966**	**131.3915**	**39.5901**
	0.0000	(0.91426)	(0.000001)	(0.975501)
$Σα_i+Σβ_i$	0.979656	0.982682	1.111378	**0.9694**
Σβi	0.752619	0.713049	0.645352	**0.54033**

Table 2: ARFIMA-FIGARCH for BET-FI

BET-FI	ARFIMA(0,ξ,0)-GARCH(1,1)		ARFIMA(0,ξ,0)-FIGARCH(1,1)	
	Normal	**Skewed Student t**	**Normal**	**Skewed Student t**
μ	**0.1170**	**0.1310**	**0.1135**	**0.1315**
	(0.1462)	(0.0715)	(0.154)	(0.0677)
Φ_1	**-**	**-**	**-**	**-**
Φ_2	**-**	**-**	**-**	**-**
ξ	**0.1006**	**0.0852**	**0.0983**	**0.0828**
	0.0000	0.0001	0	0.0002
θ_1	**-**	**-**	-	-
θ_2	**-**	**-**	**-**	**-**
ώ	**0.148166**	**0.191158**	**0.179273**	**0.240682**
	(0.0073)	(0.0086)	(0.0137)	0.0288
α1	**0.177186**	**0.217455**	**0.161053**	**0.227043**
	0.00000	0.0000	0.1388	(0.0548)
α_2	**-**	**-**	**-**	**-**
β_1	**0.81906**	**0.785537**	**0.708688**	**0.562969**
	0.00000	0.0000	0.00000	(0.0006)
β_2	**-**	**-**	**-**	**-**
d	**0**	**0**	**0.755778**	**0.595596**
			0.0006	0.0000
v	**-**	**5.205205**	**-**	**5.536404**
		0.0000		0.0000
ln(k)	**-**	**0.077417**	**-**	**0.081156**
		(0.0052)		(0.0033)
ln(L)	**-4688.2**	**-4618.66**	**-4686.0**	**-4613.20**
AIC	**4.476064**	**4.411697**	**4.474999**	**4.407442**
Q(20)	**24.8211**	**31.6462****	**25.1964**	**32.2579****
	[0.2083615]	[0.0472168]	[0.1940193]	[0.0406264]
Q_s(20)	**10.622**	**12.7388**	**9.19695**	**9.98524**
	[0.9097031]	[0.8068488]	[0.9550042]	[0.9323872]
ARCH(5)	**0.55115**	**0.76162**	**0.38078**	**0.51003**
	[0.7376]	[0.5775]	[0.8622]	[0.7689]
RBD(10)	**3.46**	**4.35**	**2.43**	**1.93**
	[0.9683062]	[0.9303439]	[0.9918248]	[0.9968829]
P(60)	**136.9056**	**53.5866**	**139.5951**	**55.4750**
	0.0000	(0.674567)	(0.000001)	(0.606217)
$\Sigma\alpha_i+\Sigma\beta_i$	0.996246	1.002992	0.869741	**0.790012**
Σβi	0.81906	0.785537	0.708688	**0.562969**

Table 3: ARFIMA-FIGARCH for BUX

BUX	ARFIMA(1,ξ,2)-GARCH(1,1)		ARFIMA(1,ξ,2)-FIGARCH(1,d,1)	
	Normal	Skewed Student t	Normal	Skewed Student t
μ	**0.081444**	**0.070431**	**0.09509**	**0.071909**
	0.0003	0.0428	0.0074	0.0428
Φ_1	**-0.866006**	**0.556353**	**0.659877**	**0.571698**
	0.00000	0.0019	0.0000	0.0005
Φ_2	-	-	-	-
ξ	**-0.0283**	**0.063991**	**0.0600**	**0.0671**
	0.0250660	0.0588720	0.0659170	0.0588750
θ_1	**0.954776**	**-0.586381**	**-0.665399**	**-0.605014**
	0.0000	0.0034	0.0000	0.0009
θ_2	**0.083519**	**-0.047753**	**-0.052083**	**-0.045309**
	0.0079	0.0704	0.0347	0.0868
ώ	**0.07013**	**0.081663**	**0.091313**	**0.112467**
	0.0029	0.0001	0.0066	0.007
α1	**0.106261**	**0.103916**	**0.2145**	**0.207993**
	0.0000	0.0000	0.0025	0.0053
α_2	-	-	-	-
β_1	**0.871877**	**0.868124**	**0.564758**	**0.545716**
	0.0000	0.0000	0.0000	0.0000
β_2	-	-	-	-
d	-	-	**0.462322**	**0.455978**
			0.0000	0.0000
v	-	**7.535435**	-	**7.478346**
		0.0000		0.0000
ln(k)	-	**-0.014351**	-	**-0.013331**
		0.5998		0.6287
ln(L)	**-5163**	**-5108**	**-5155**	**-5106**
AIC	**3.724107**	**3.68608**	**3.719145**	**3.68563**
Q(20)	**36.4234**	**33.9567**	**33.0919**	**34.405**
	0.0040253	0.0085047	0.010973	0.0074407
$Q_s(20)$	**22.0843**	**21.9771**	**18.2669**	**18.887**
	0.2282653	0.2330027	0.4382021	0.3988139
ARCH(5)	**0.37304**	**0.32292**	**0.14252**	**0.15631**
	0.8674	0.8994	0.9823	0.9782
RBD(10)	**9.58**	**10.63**	**9.57**	**10.27**
	0.4784	0.3873	0.4787	0.4169
P(60)	**90.3605**	**66.0320**	**65.7296**	**64.1739**
	0.0006	0.0527	0.0671	0.0592
$\Sigma\alpha_i+\Sigma\beta$	**0.9781**	**0.9720**	**0.7793**	**0.7537**
$\Sigma\beta i$	**0.8719**	**0.8681**	**0.5648**	**0.5457**

Table 4: ARFIMA-FIGARCH for PX

PX	ARFIMA(1,ξ,2)-GARCH(1,1)		ARFIMA(1,ξ,2)-FIGARCH(1,1)	
	Normal	**Skewed Student t**	**Normal**	**Skewed Student t**
Φ_1	**0.639916**	**0.560244**	**0.651441**	**0.559181**
	0.0000	0.0000	0.0000	0.0000
Φ_2		**-**	**-**	**-**
ξ	**0.1609**	**0.1753**	**0.1554**	**0.1759**
	(0.0095)	(0.0016)	(0.0143)	(0.0013)
θ_1	**-0.71838**	**-0.671191**	**-0.726863**	**-0.670871**
	0.0000	0.0000	0.0000	0.0000
θ_2	**-0.02519**	**-0.026412**	**-0.022742**	**-0.027224**
	(0.4105)	(0.4372)	(0.4551)	(0.4327)
$\dot{\omega}$	**0.062633**	**0.052873**	**0.06788**	**0.064888**
	0.0000	0.0000	(0.0002)	(0.0008)
α1	**0.137459**	**0.128792**	**0.068355**	**0.11511**
	0.0000	0.0000	(0.3497)	(0.0523)
α_2	**-**	**-**		**-**
β_1	**0.837682**	**0.851172**	**0.670748**	**0.620867**
	0.0000	0.0000	0.0000	0.0000
β_2	**-**	**-**	**-**	**-**
d	**-**	-	**0.702972**	**0.60617**
			0.0000	0.0000
v	**-**	**7.901925**	**-**	**7.563628**
		0.0000		0.0000
ln(k)	**-**	**-0.045633**	**-**	**-0.043358**
		(0.1173)		(0.1477)
ln(L)	**-4662**	**-4613**	**-4661**	**-4611**
AIC	**3.356238**	**3.323018**	**3.356776**	**3.32182**
Q(20)	**20.2115**	**20.8214**	**19.9535**	**20.2973**
	(0.2635526)	(0.2343822)	(0.2766138)	(0.2593065)
Q_s(20)	**27.9482**	**27.0261**	**21.8433**	**22.1935**
	(0.0628485)	(0.0785073)	(0.2390143)	(0.2235076)
ARCH(5)	**2.3281**	**2.1888***	**1.6832**	**1.7511**
	(0.0403)	(0.0528)	0.1351	(0.1196)
RBD(10)	**16.19**	**15.27**	**12.92**	**12.38**
	(0.0943828)	(0.1224334)	(0.2281323)	(0.260621)
P(60)	**61.7239**	**58.7045**	**88.5967**	**56.1596**
	(0.16745)	(0.186643)	(0.000863)	(0.224418)
$\Sigma\alpha_i+\Sigma\beta_i$	0.9751	0.9800	0.7391	**0.7360**
$\Sigma\beta i$	0.8377	0.8512	0.6707	**0.6209**

Table 5: FIGARCH estimation for SOFIX

SOFIX	FIGARCH(1,d,1)	
	Normal	**Skewed Student t**
ώ	**0.010318**	**0.058635**
	(0.2915)	(0.1741)
α1	**0.622053**	**0.481553**
	0.0000	(0.0417)
α_2	**-**	**-**
β_1	**0.795544**	**0.621756**
	0.0000	(0.0044)
β_2	**-**	**-**
d	**0.541028**	**0.568614**
	0.0000	0.0000
ν	**-**	**3.647303**
		0.0000
ln(k)	**-**	**0.020172**
		(0.3795)
ln(L)	**-3656**	**-3455**
AIC	**3.455042**	**3.267697**
$Q_s(20)$	**19.9213**	**22.8513**
	(0.3373)	(0.19636)
ARCH(5)	**0.71708**	**0.9729**
	(0.6106)	(0.4329)
RBD(10)	**8.49**	**6.97**
	(0.5809)	(0.72833)
P(60)	**285.5021**	**74.4403***
	0.0000	(0.084777)

Table 6: FIGARCH estimation for WIG

WIG	FIGARCH(1,d,1)	
	Normal	**Skewed Student t**
ω	**0.060636**	**0.055881**
	(0.0257)	(0.0151)
α1	**0.165798**	**0.156037**
	(0.0001)	(0.0001)
α2	**-**	**-**
β1	**0.62756**	**0.636777**
	0.0000	0.0000
β2	**-**	**-**
d	**0.47143**	**0.493638**
	0.0000	0.0000
ν	**-**	**7.3008**
		0.0000
ln(k)	**-**	**-0.0030**
		(0.9022)
ln(L)	**-4930.167**	**-4880.571**
AIC	**3.527639**	**3.493617**
Qs(20)	**14.3419**	**14.4898**
	(0.706547)	(0.6966464)
ARCH(5)	**0.67035**	**0.62153**
	(0.646)	(0.6834)
RBD(10)	**7.15881**	**6.8078**
	(0.7103718)	(0.743457)
P(60)	**104.4732**	**68.619***
	(0.000046)	(0.061)

Appendix 4: Estimation results of the GPH tests

	λ=0.45	λ=0.5	λ=0.55	λ=0.6	λ=0.65
BET	**0.1223**	**0.1505**	**0.1498**	**0.1131**	**0.1618**
	(0.1064)	(0.0958)	(0.0735)	(0.0550)	(0.0510)
BET-FI	**0.0802**	**0.1843**	**0.2549**	**0.2384**	**0.1593**
	(0.0899)	(0.0883)	(0.0817)	(0.0623)	(0.0537)
SOFIX	**0.3029**	**0.3028**	**0.3060**	**0.2348**	**0.1638**
	(0.1048)	(0.0894)	(0.0820)	(0.0662)	(0.0541)
BUX	**0.0209**	**-0.0338**	**0.0721**	**0.1558**	**0.1038**
	(0.1256)	(0.0895)	(0.0700)	(0.0573)	(0.0508)
WIG	**0.1470**	**0.0201**	**0.0222**	**0.0475**	**0.0773**
	(0.1496)	(0.1080)	(0.0850)	(0.0638)	(0.0507)
PX	**0.1296**	**0.1023**	**0.1578**	**0.1488**	**0.1073**
	(0.1109)	(0.0914)	(0.0750)	(0.0575)	(0.0463)

*the standard error deviations are reported in parenthesis.

Appendix 5: Index Graphs

Fig.1 BET Index

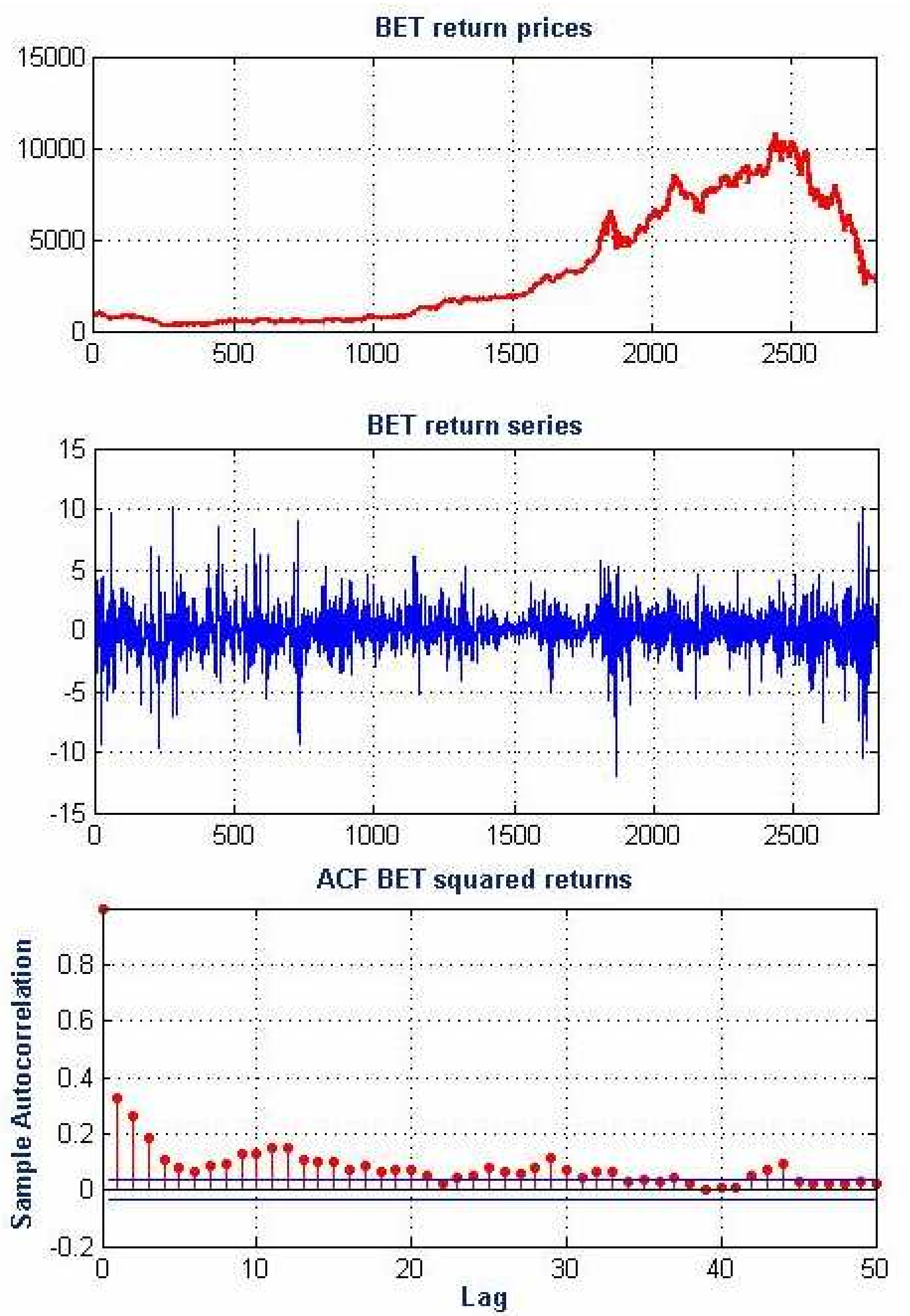

Fig.2 BET-FI Index

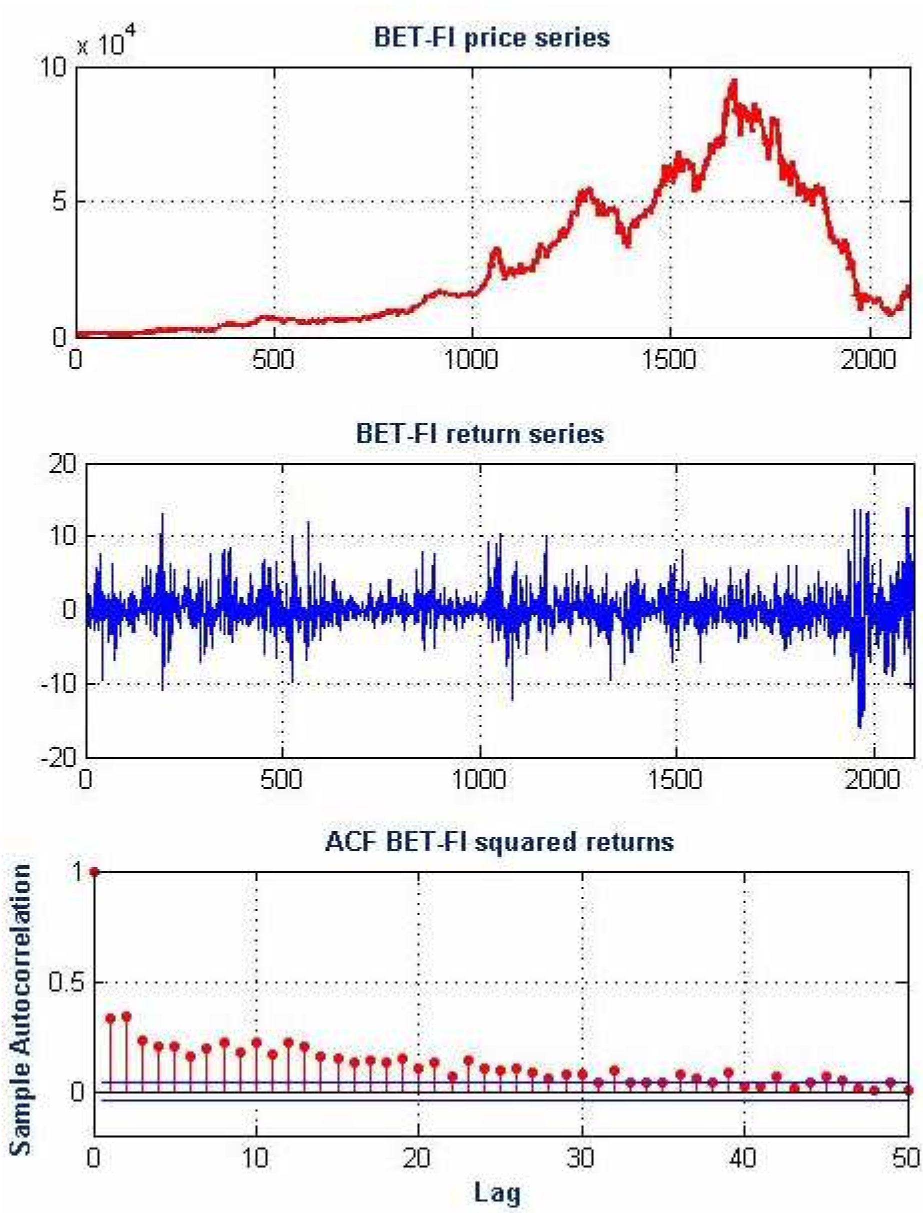

Fig.3 BUX Index

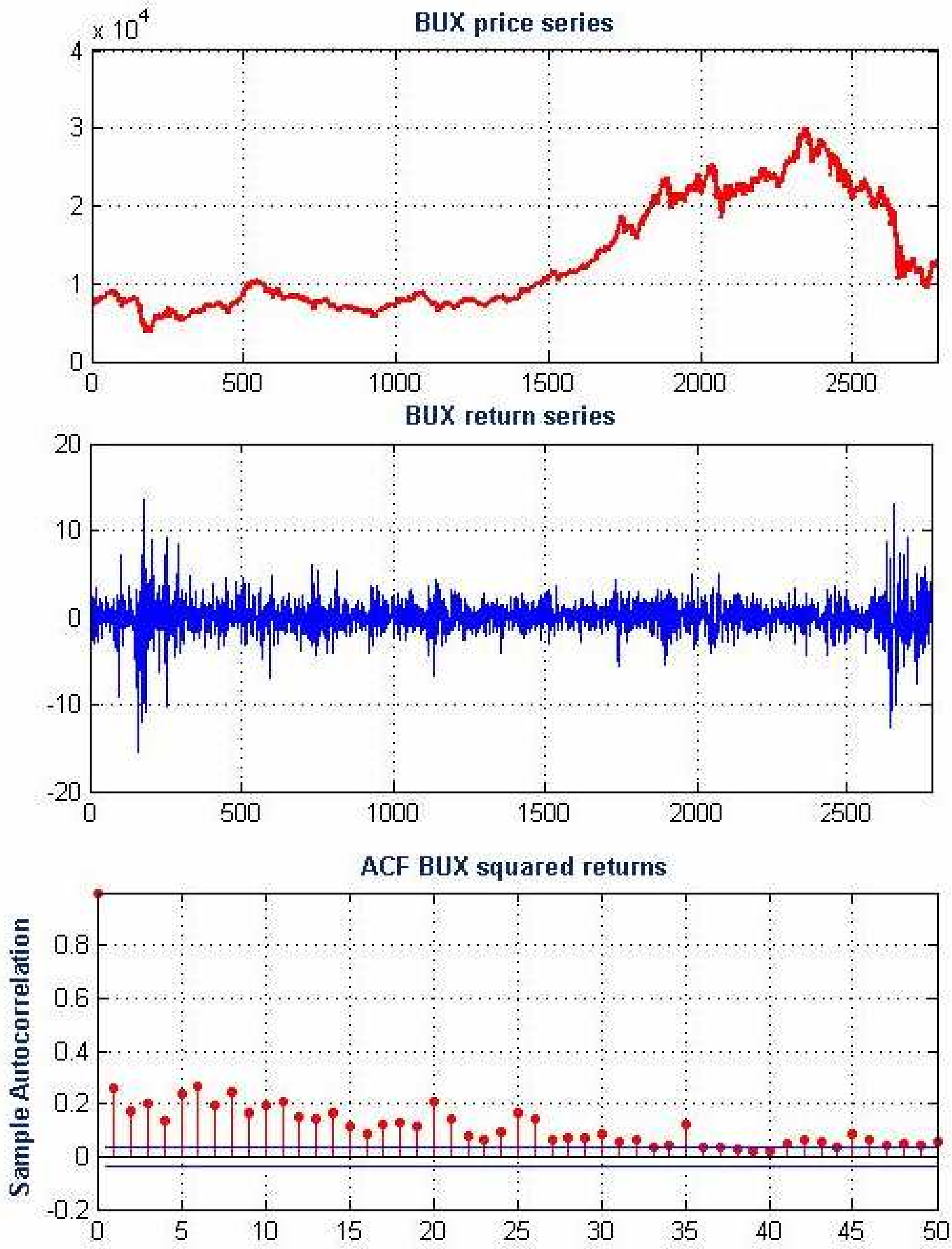

Fig.4 PX Index

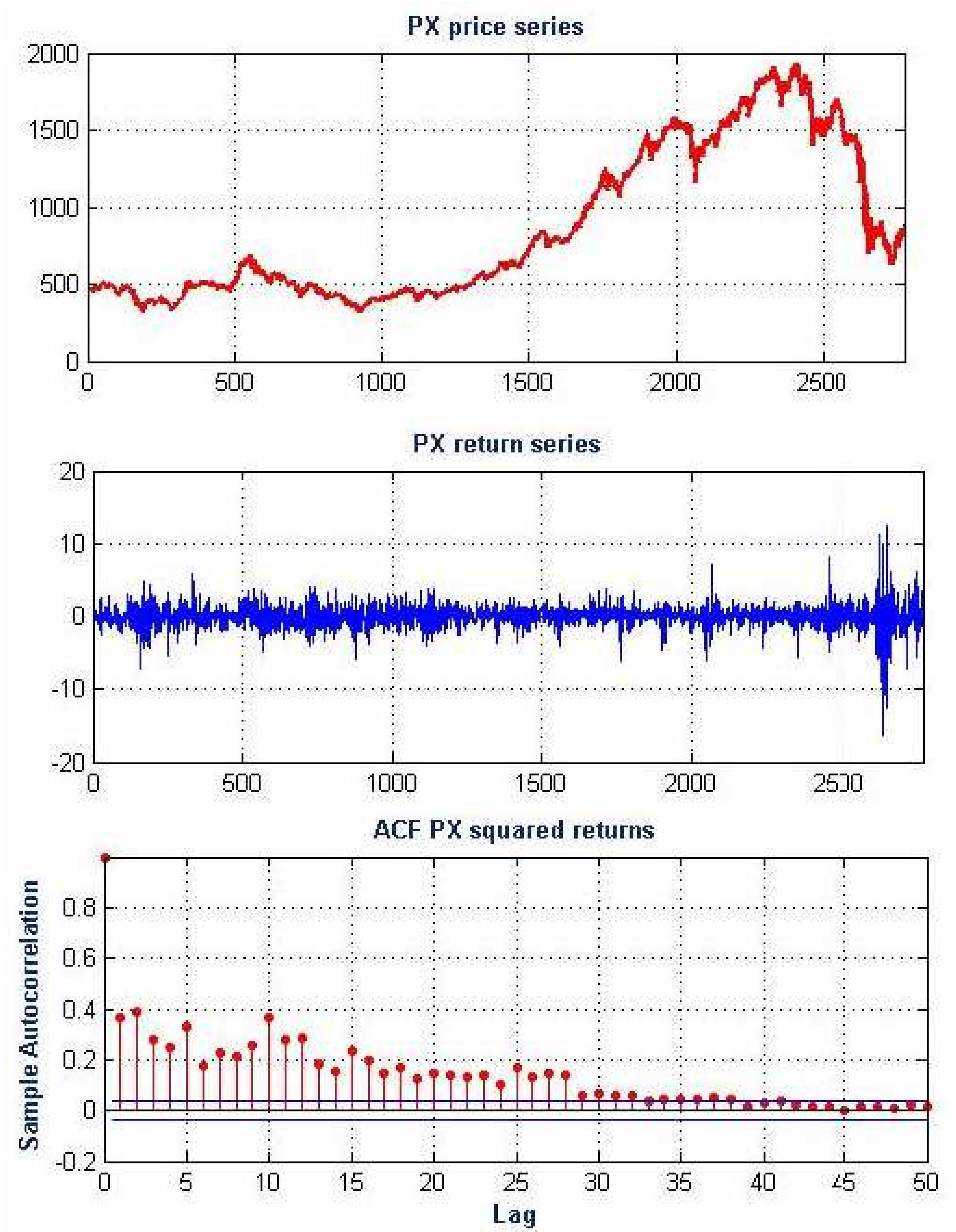

Fig.5 SOFIX Index

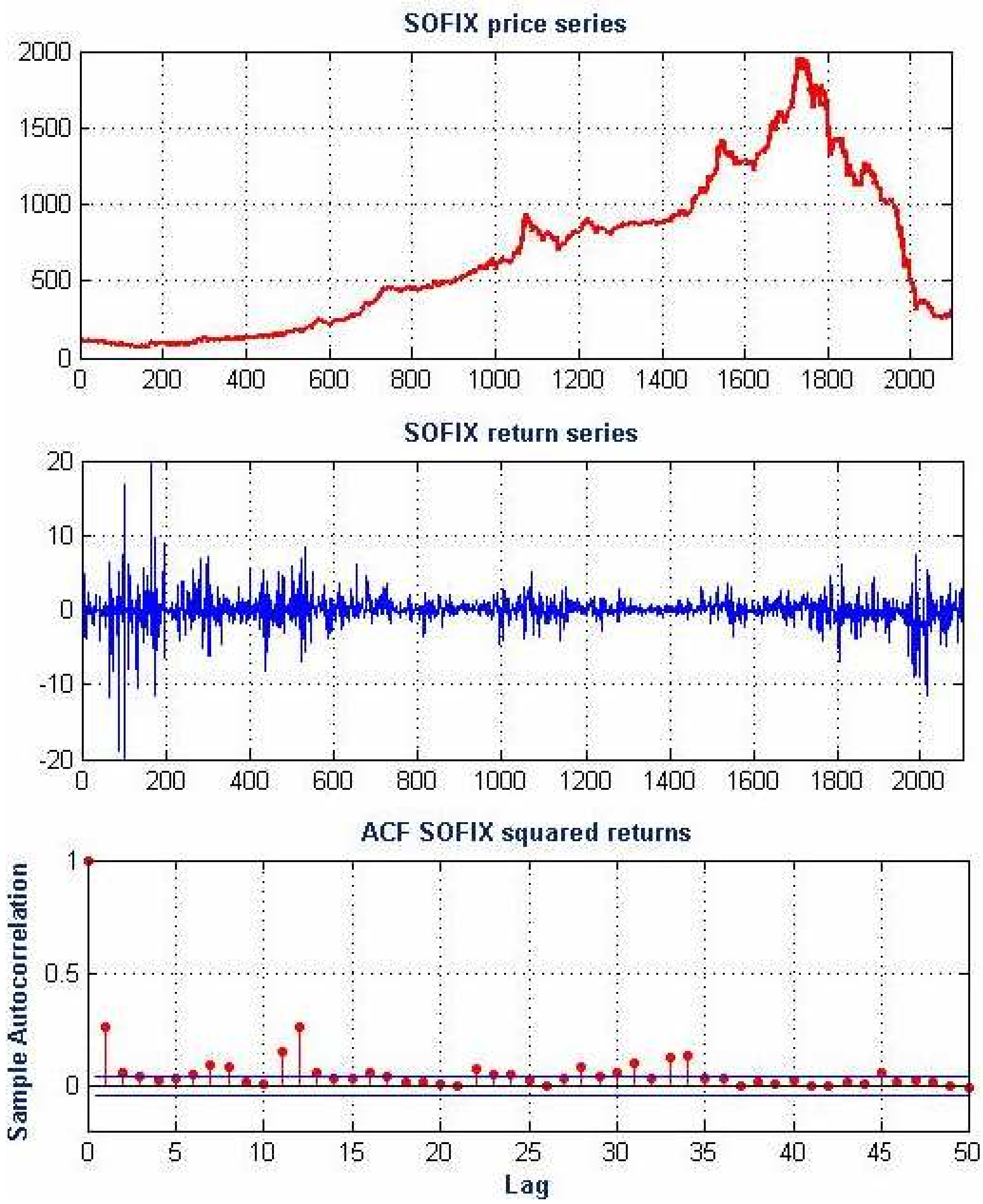

Fig.6 WIG Index