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Stylized Facts and Discrete Stochastic Volatility Models

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Abstract

This dissertation paper highlights the ability of the discrete stochastic volatility models to predict some important properties of the data, i.e. leptokurtic distribution of the returns, slowly decaying autocorrelation function of squared returns, the Taylor effect and the asymmetric response of volatility to return shocks. Although, there are many methods proposed for stochastic volatility model estimation, in this paper Markov Chain Monte Carlo techniques were considered. It was found that the existent specifications in the stochastic volatility literature are consistent with the empirical properties of the data. Thus, from this point of view the discrete stochastic volatility models are reliable tools for volatility estimation.

1. Introduction

Stochastic volatility (SV) models find many financial applications, such as volatility estimation and forecasting, option pricing, asset allocation, and risk management. They are also considered in the literature as an alternative to ARCH-type models, introduced by Engle (1982). Although, the improvement of stochastic volatility models corresponds in time with the ARCH-type models, the former models are less popular in the empirical literature because of their complexity and difficulty of estimation. After the seminal work of Jacquier, Polson and Rossi (1994), who perform fully Bayesian inference through Markov Chain Monte Carlo scheme, a vast literature on the Bayesian analysis of SV models have appeared.

In this dissertation paper, it is investigated the ability of the stochastic volatility models to capture some important properties of the data, i.e. the so called stylized facts. If they do not predict this properties, their usefulness in volatility estimation and prediction is questionable. The stylized facts considered here are: leptokurtic distribution, slow decaying autocorrelation function, Taylor effect and the asymmetric response of volatility to the return shocks.

The dissertation paper is organized as follows. Section 2 reviews the main papers from the literature that treat the topic of the stylized facts predicted by the SV models. Section 3 presents the stochastic volatility models subject to estimation and stylized facts prediction testing. The SV models are estimated using Markov Chain Monte Carlo techniques that are described in Section 4. This section also discusses the Deviance Information Criterion that is a tool for model comparison. Section 5 describes the data used for model estimation and also emphasizes the stylized facts. In Section 6, it is analyzed the ability of the SV models to predict the stylized facts highlighted in the previous section and the last section concludes.

2. Literature review

The stochastic volatility (SV) models are considered in the literature as a successful alternative to the class of Autoregressive Conditionally Heteroscedastic (ARCH) models introduced by Engle (1982) and generalized by Bollerslev (1986) and others. As it is stated in the literature, the first paper that considers time changing volatility is due to Clark (1973). A very simple SV model was proposed by Taylor (1986), while Hamilton (1989) considers a simple discrete SV. Hull and White (1987) introduced the continuous-time diffusion model, which become widely used in the option-pricing literature. The basic SV model that appears in the empirical literature is a discrete approximation of the Hull and White (1987) continuous time stochastic volatility model.

Although the SV models were developed in parallel with the ARCH-type models, they are less popular because of their estimation complexity. The latent volatility enters the model nonlinearly, which leads to a likelihood function depending upon high-dimensional integrals. A variety of estimation procedures has been proposed to overcome this difficulty, including the Generalized Method of Moments (Melino and Turnbull, 1990), the Quasi Maximum Likelihood (Harvey et al. 1994 and Ruiz, 1994), the Efficient Method of Moments (Gallant et al., 1997), and Markov Chain Monte Carlo procedures (Jacquier et al.,1994 ; Kim et al., 1998).

Following the approach developed by Terasvirta (1996) for the GARCH models, Liesenfeld and Jung (1997) showed that the lognormal SV model does not adequately account for the leptokurtic distribution of the returns and slowly decaying autocorrelation function of the squared returns simultaneously. As a solution to this problem they propose that a heavier-tailed distribution to be used for the errors of the returns. Bai, Russell and Tiao (2003) using a different approach, conclude that lognormal SV model generates leptokurtosis but often not sufficiently large to explain the sample kurtosis.

Taylor (1986) observed that the autocorrelations of the absolute returns are larger then the autocorrelations of the squared returns. Granger and Ding (1995) denote this property of the financial returns as *Taylor effect*. Ding, Granger and Engle (1993) suggested that the autocorrelation function of absolute returns raised to power θ is actually maximized when θ is one.

Malmsten and Terasvirta (2004) and Galan, Perez and Ruiz (2004) analyze the *Taylor effect* in the context of the autoregressive stochastic volatility model. Veiga (2007) extended their approach for another two SV models, namely Long Memory Autoregressive Stochastic Volatility model and Two Factor Long Memory Stochastic Volatility model.

Harvey and Shephard (1996) provided one of the first econometric treatments of an asymmetric SV model using quasi maximum likelihood method. The asymmetric effect was achieved by considering a contemporaneous negative correlation between return shocks and volatility shocks. Another well known asymmetric model was proposed by Jacquier, Polson and Rossi (2004). In their model, the asymmetric response of the volatility is predicted by a negative correlation between return shocks and lagged volatility shocks. Yu (2004) showed that the specification with contemporaneous correlation is superior to that with the inter-temporal correlation.

Yu (2004) propose an asymmetric SV model that resemble the EGARCH model specification elaborated by Nelson (1991) and extended the news impact curve introduced in the literature by Engle and Ng (1993). An asymmetric SV model that resemble GJR model (Glosten, Jagannathan, Runkle, 1993) was also proposed by Asai and McAleer (2004).

There are many other specifications in the literature that are not discussed in this paper, for example, Jensen (2004) develops semiparametric inference for long memory SV models, So et al. (1998) and Carvalho and Lopez (2004) accommodates Markov jumps in the log-volatilities. In an excellent review, Asai et al. (2006) describes some important multivariate stochastic volatility models that exist in the literature.

3. Stochastic Volatility Models

In the theoretical finance literature, the SV model is often formulated in terms of stochastic differential equation of the form:

$$\begin{cases} ds(t) = \sigma(t) \cdot dB_1(t) \\ d\ln\sigma^2(t) = \alpha + \beta \cdot \ln\sigma^2(t) \cdot dt + \eta \cdot dB_2(t) \end{cases} \tag{1}$$

where s(t) represents the logarithm of the asset price, $\sigma^2(t)$ the volatility of the asset return, $B_1(t)$ and $B_2(t)$ are two Brownian motions. First we consider that the two Brownian motions are independent ($corr(dB_1, dB_2)=0$).

In the empirical literature the above continuous time model is discretized via Euler –Maruyama approximation. Using the notations $s(t+1) – s(t) = y(t)$, $B_1(t+1)- B_1(t) = u_t$, $B_2(t+1)- B_2(t) = v_t$, $1+\beta = \varphi$, $\ln\sigma^2(t) = h_t$ and $\mu = \alpha(1+\varphi)$ the SV model becomes:

$$\begin{cases} y_t = \sigma_t \cdot u_t = \exp(h_t / 2) \cdot u_t \\ h_{t+1} = \mu + \varphi \cdot (h_t - \mu) + \eta \cdot v_t \end{cases} \tag{2}$$

where y_t is the asset return at the moment t, h_t is the log-volatility of the return, φ the persistence parameter (see section 6.1), η is the standard deviation of the log-volatility process, u_t and v_t are the return shock and volatility shock respectively with the conditions that $u_t \sim i.i.d.$, $v_t \sim i.i.d.N(0,1)$ and $corr(u_t, v_t) = 0$. We will denote this as the ***basic SV model***. Although, the assumption that the log-volatility is Gaussian may seem rather ad hoc, Andersen, Bollerslev, Diebold and Ebens (2001) and Andersen, Bollerslev, Diebold and Labys (2003) show that the empirical distribution of the log-volatility of several exchange rates and index returns could be approximated by the Normal distribution.

The main properties of the basic SV model have been reviewd by Ghysels, Harvey and Renault (1996) and Shephard (1996). The stochastic process considered for the log-volatility is stationary if the autoregressive parameter, φ, is in absolute value less then one. Furthermore, as it will be shown in section 6.1, the basic SV model imply a leptokurtic distribution for the returns series, even if it is assumed a Gaussian distribution for the error in the mean equation.

If it is also assumed that $u_t \sim i.i.d.N(0,1)$ then the basic SV model is also known in the literature as ***lognormal SV model***. Changing the specification for the return error distribution and considering $u_t \sim t\,[df]$ yields a second model denoted here as ***t SV model.***

If we consider that the two Brownian motions from continuous model (1) are correlated ($corr(dB_1, dB_2) = \rho$), which implies that the error terms from the discrete model are also correlated (corr(u_t, v_t) = ρ), then the model (2) becomes an ***asymmetric SV model***. The models mentioned above and other specifications will be presented in more details in the following sections, grouping them with respect to their capability of capturing some stylized facts of financial time series.

4. Model estimation and comparison

The stochastic volatility models were estimated in this paper using a Bayesian approach. This method implies the integration over posterior distribution to make inference about model parameters or to make predictions. Because the models estimated here involve high-dimensional probability distributions, the integration can be done only numerically. The most popular techniques in this domain are Markov Chain Monte Carlo methods (MCMC), originated in the statistical physics literature. MCMC are a class of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its stationary distribution. The state of the chain after a large number of steps is then used as a sample from the desired distribution.

Of course, this is not the only method that someone can use to estimate the SV models. In the literature, there are some other proposals, such as:

- Generalized Method of Moments (Melino and Turnbull (1990), and Sorenson (2000));
- Quasi – Maximum Likelihood (Harvey (1994));
- Efficient Method of Moments (Gallant (1997));
- Simulated Maximum Likelihood (Danielsson (1994), and Sandmann and Koopman (1998)).

Markov Chain Monte Carlo procedures for the SV models have been first suggested by Jacquier, Polson and Rossi (1994). They proposed a single mover algorithm which proved to have some drawbacks such as slow convergence, a highly dependent consecutive states and inefficient mixing. To improve the simulation efficiency Shephard and Pitt (1997), Kim, Shephard and Chib (1998), Chib, Nardari and Shephard (1998), Lisenfeld and Richard (2006) and Gerlach and Tuyl (2006) proposed multi-mover algorithms that sample the latent volatility vector in a single block.

Many of the algorithms needed to estimate SV models with the methods mentioned above have been implemented in low level programming languages such as C++ or FORTRAN. For example, the SVPack (Ox software for volatility models) implementation of Kim, Shephard and Chib (1998) is based on C++ code while the MCMC algorithm of Jacquier, Polson and Rossi (1994) was implemented using FORTRAN code. While the implementation of these specially tailored packages is numerically efficient, the requiring effort for writing and debugging a program is usually major.

I had two reasons for choosing MCMC techniques. First, among all the above methods, MCMC ranks as one as the best estimation tools (see for example, Andersen, Chung and Sorensen (1999)), and second, Bayesian analysis of stochastic volatility models can be easily implemented using WinBUGS (Bayesian Analysis Using Gibbs Sampling for Windows). Meyer and Yu (2000, 2006) advocated using the all-purposes Bayesian software, BUGS, to implement MCMC estimation for univariate and multivariate SV models and showed that BUGS provide a flexible environment to estimate these models. However, due to the single move Gibbs sampler, convergence can be slow. Therefore, to achieve a satisfactory precision for parameter estimates, a large number of iterations are needed, increasing the computational cost.

Skaug and Yu (2007) developed several algorithms to perform classical and Bayesian likelihood-based analysis of SV models using automatic differentiation (AD), combined with the Laplace approximation. They also demonstrated the ease with which univariate and multivariate SV models can be estimated using the latent variable module ADMB-RE of the of the software package AD Model Builder.

The Bayesian approach involves the specification of the full probability model, that is the specification of the ***likelihood*** , $p(y|\theta)$, and the ***prior distribution*** for the parameters, $p(\theta)$. The likelihood represents the probability of the data, $y = (y_1, y_2, \ldots, y_n)$, given the parameters, $\theta = (\mu, \varphi, \eta, \rho, \nu, h)$ where $h = (h_1, h_2, \ldots, h_n)$, and the prior distribution represents the prior knowledge about the parameter distribution. Having the likelihood and the priors one can calculate the joint probability distribution, $p(y,\theta)$:

$$p(y,\theta) = p(\theta) \cdot p(y \mid \theta)$$

After observing the data, Bayes theorem is used to determine the joint ***posterior distribution*** of the parameters, p(θ | y):

$$p(\theta | y) = \frac{p(\theta) \cdot p(y | \theta)}{\int p(\theta) \cdot p(y | \theta) d\theta} \propto p(\theta) \cdot p(y | \theta) \qquad (3)$$

The high-dimensional integral can be interpreted as the normalizing constant that makes the area under the posterior distribution to be one. It is common in the Bayesian statistics to ignore the normalizing constant and to write the posterior as a proportional distribution to the product of the joint prior distribution and likelihood.

For example, the joint posterior distribution for the lognormal SV model has the form:

$$p(\mu, \varphi, \eta, h_1, ..., h_n | y_1, ..., y_n) \propto p(\mu) \cdot p(\varphi) \cdot p(\eta) \cdot p(h_0 | \mu, \varphi, \eta) \cdot \prod_{t=1}^{n} p(h_t | h_{t-1}, \mu, \varphi, \eta) \cdot \prod_{t=1}^{n} p(y_t | h_t)$$

where $p(h_t \mid h_{t-1}, \mu, \varphi, \eta)$ is the probability distribution function for h_t conditional on the model parameters, which is $N(\mu + \varphi h_{t-1}, \eta^2)$, and $p(y_t \mid h_t)$ is the likelihood of the return y_t, which is also Gaussian, i.e. $y_t \sim N(0, \exp(h_t))$.

For the t_SV model, the joint posterior distribution also contains the pr ior distribution for the parameter ν (degrees of freedom), and the likelihood is the Student-t distribution, i.e. $y_t \sim t(0, \exp(h_t), \nu)$.

For the asymmetric SV model proposed by Harvey and Shephard (1994), the likelihood and prior for log-volatility are Gaussian, i.e. $y_t \sim N((\rho/\eta)*\exp(h_t/2)*(h_{t+1} - \mu - \varphi*(h_t - \mu)), \exp(h_t)*(1-\rho^2))$ respectively $h_{t+1} \sim N(\mu + \varphi*(h_t - \mu), \eta^2)$.

As priors I used very common distributions from the empirical literature (Kim, Shephard and Chib (1998), Meyer et al.(2000), Yu (2004)): $\mu \sim N(0, 25)$, $\varphi^* \sim$ *Beta* (20, 1.5), $\varphi = 2\varphi^*-1$, $\eta^2 \sim$ *InverseGamma* (2.5, 0.025), $\rho \sim$ *Uniform* (-1, 1) and $\psi \sim N(0, 25)$.

Because, it is not possible to derive an analytic expression for the posterior distribution of the SV models parameters, it is necessary to use MCMC techniques, such as ***Metropolis algorithm*** (Metropolis et al., 1953), ***Metropolis-Hastings algorithm*** (Hastings, 1970) and ***Gibbs sampling*** (Geman and Geman, 1984), ***the independence sampler*** (Tierney, 1994).

Like it was pointed above, MCMC is essentially Monte Carlo integration using Markov chains. MCMC draws samples from the required distribution (in our case the posterior distribution) by running a ingeniously constructed Markov chain for a long time. These techniques only require that the desired distribution to be known up to a constant of normalization. There are many ways of constructing these chains, but all of them, including the Gibbs sampler, are special cases of the general framework of Metropolis et al. (1953) and Hastings (1970).

For the Metropolis – Hastings algorithm, at each time t, the next state θ_{t+1} is chosen by first sampling a candidate point θ^* from a ***proposal distribution*** $q(.| \theta_t)$. The proposal distribution, theoretically, can be any parametric distribution, but the choice highly influences the convergence of the chains to the stationary distribution. The candidate point θ^* is then accepted with probability $k(\theta_t | \theta^*)$, where:

$$k(\theta_t | \theta^*) = \min\left(1, \frac{p(\theta^* | y) \cdot q(\theta_t | \theta^*)}{p(\theta_t | y) \cdot q(\theta^* | \theta_t)}\right)$$

If the candidate point is accepted, the next state becomes $\theta_{t+1} = \theta^*$, and if the candidate is rejected, the chain does not move.

The Metropolis algorithm considers only symmetric proposals, having the form $q(\theta^* | \theta_t) = q(\theta_t | \theta^*)$. Often it is convenient to choose a proposal which generates each component of θ^* conditionally independently, given θ_t. For the Metropolis algorithm the acceptance probability reduces to:

$$k(\theta_t | \theta^*) = \min\left(1, \frac{p(\theta^* | y)}{p(\theta_t | y)}\right)$$

A special case of the Metropolis algorithm is the ***random-walk Metropolis***, for which $q(\theta^* | \theta_t) = q(| \theta_t - \theta^* |)$.

The independence sampler is a Metropolis-Hasting algorithm whose proposal has the form $q(\theta^* | \theta_t) = q(\theta^*)$. For this the acceptance probability can e written as:

$$k(\theta_t | \theta^*) = \min\left(1, \frac{p(\theta^* | y) \cdot q(\theta_t)}{p(\theta_t | y) \cdot q(\theta^*)}\right)$$

In general, the independence sampler can work well or very badly. For the independence sampler to work well, q(.) should be a good approximation to p(.), and q(.) to be heavier-tailed than p(.).

Gibbs sampling, instead of updating the whole θ *en bloc*, it is often more convenient and computationally efficient to divide θ into components { $\theta_1, \theta_2, \ldots, \theta_m$} of possibly differing dimension, and then update these components one by one. If θ_{-i} comprise all elements of θ except θ_i, then the full conditional distribution $p(\theta_i \mid \theta_{-i})$ is the distribution of the i^th^ component of θ conditioning on all the remaining components:

$$p(\theta_i|\theta_{-i}) = \frac{p(\theta, y)}{\int p(\theta, y) d\theta_i}$$

For the Gibbs sampling, the proposal distribution for updating the i^th^ component of θ is:

$$q_i(\theta_i^*|\theta) = p(\theta_i^*|\theta_{-i})$$

The acceptance probability of the Gibbs sampler is 1; that is, Gibbs sampler candidates are always accepted. Thus Gibbs sampling consists purely in sampling from full conditional distributions. Sampling from full conditional distributions can be done with methods like:

- Inversion (Ripley, 1987);
- Rejection sampling (Ripley, 1987; Carlin and Gelfand, 1991);
- Ratio-of-uniforms method (Wakefield et al., 1991; Bennett, 1995);
- Adaptive rejection sampling (Gilks and Wild, 1992);
- Adaptive rejection Metropolis sampling (Gilks et al., 1995);
- Slice sampling (Neal, 1997), and other.

The inversion method, the adaptive rejection sampling and slice sampling are implemented in WinBUGS. This software also contains an expert system for choosing the appropriate sampling method for each full conditional distribution, thus the user is not required to specify the sampling method.

Sometimes the researcher has to choose between two or more models to measure a given phenomena, and he prefers the model that is less complex, that can be estimated easily, and with a satisfactory goodness of fit. Usually a model with more parameters, i.e. more complex, has also a better goodness of fit. Therefore, the researcher has to make a tradeoff between the complexity of the model and the goodness of fit.

In this paper for model selection it was used the Deviance Information Criterion (DIC) that was proposed by Spiegelhalter, Best, Carlin and Linde (2002). This is useful in Bayesian model selection problems where the posterior distributions of the models have been obtained by MCMC simulations. DIC is a generalization of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), also known as Schwartz Criterion, and can be computed in WinBUGS for many models, including stochastic volatility models. It combines a Bayesian measure of fit with a measure of complexity. Berg, Meyer and Yu (2004) demonstrate its usefulness in the model selection process for the family of stochastic volatility models

Many model checking criteria (Carlin and Louis, 1996; Gelman, Carlin, Stern and Rubin, 1996; Gilks, Richardson and Spiegelhalter, 1996; Key, Pericchi and Smith, 1999) have been proposed and discussed before the development of DIC. While Bayes factors (e.g. Kass and Raftery, 1995) have been viewed for many years as the only correct way to carry out Bayesian model comparison, they have come under increasing criticism (Lavine and Schervish, 1999). One serious drawback is that they are not well-defined when using improper priors which are typically the case in practice when employing noninformative priors. For stochastic volatility models, informative and thus proper prior distributions are usually employed and Bayes factors are well defined. Nonetheless, the number of unknown parameters in SV models exceeds the number of observations because of the latent volatilities. Computation of the Bayes factor requires the marginal likelihoods and thus a marginalization over the parameter vectors in each model. If the dimension of the parameter space is large, these extremely high-dimensional integration problems pose a formidable computational challenge.

In the context of SV models Kim, Shephard and Chib (1998) and Chib, Nardari and Shephard (2002) have shown how to compute Bayes factors using the marginal likelihood approach of Chib (1995) and evaluating the marginal likelihood at the posterior mean using ***particle filtering*** (Kitagawa, 1996; Pitt and Shephard, 1999). Bayes factors remain a computationally intensive task and are not particularly user-friendly tool for practitioners.

Other well known information criterions are BIC (Bayesian Information Criterion) and AIC (Akaike Information Criterion). Either criterion requires the specification of the number of free parameters in each model. In the SV models the parameters are augmented by the latent volatilities which are not independent, because they exhibit a Markovian dependence structure, therefore they can not be considered as free parameters. Thus, neither BIC nor AIC can be used for SV models comparison.

Following the original suggestion of Dempster (1974) for model choice in Bayesian framework DIC computation is based on posterior distribution of the log-likelihood or the ***deviance***, D(.), which is computed as:

$$D(\theta) = -2 \cdot \log\left(p\left(y|\theta\right)\right)$$

where θ represents the model parameters, y are the data, and p(y|θ) is the likelihood. The expectation $\overline{D} = E[D(\theta)]$, i.e. the ***posterior mean of the deviance***, is a measure of how well the model fits the data (in the WinBUGS output it is denoted as *Dbar*); the larger this is, the worse the fit.

Another component of the DIC is the ***effective number of parameters*** of the model which is computed as:

$$p_D = \overline{D} - D\left(\overline{\theta}\right)$$

where $\overline{\theta} = E(\theta)$ and $D\left(\overline{\theta}\right)$ is the ***deviance of the posterior means*** (in the WinBUGS output it is denoted as *Dhat*). The larger p_D, the easier it is for the model to fit the data. This term accounts for the model complexity. Putting together the elements defined above, the Deviance Information Criterion (DIC) is calculated as:

$$DIC = p_D + \overline{D} \qquad (4)$$

After computing the DIC, the model with a lower value for this criterion is preferred. It is clear that, as the number of parameter number of parameters also means a higher complexity of the model and p_D term compensates for this effect.

5. Data

To test whether the stochastic volatility models predict the leptokurtic distributions of the returns, the slowly decaying autocorrelation function of the squared returns and the Taylor effect, I used seven exchange rates (CHF/RON, EUR/RON, GBP/RON, JPY/RON, NOK/RON, SEK/RON, USD/RON) and five indices (BET-C, CAC-40, DAX, FTSE-100, MIB-30). The exchange rates time series are from National Bank of Romania data base and they are daily series from 4 January 2000 to 4 June 2007. The indices data were provided by REUTERS and they are also daily series from 4 January 2000 to 4 April 2007.

Summary statistics for this time series are reported in tabel 1. For all time series considered here, the null hypothesis for the Jarque-Bera test is rejected and also they reveal high levels for kurtosis, i.e. their distributions are leptokurtic. Among the clasical summary statistics, table 1 also reports the autocorrelation function at the first lag, ACF(1), that will be used in section 6.1 when discussing the theoretical and empirical combination between kurtosis and ACF(1).

To show that the time series used in this paper also have the property of slow decaying autocorrelation function and the Taylor effect, the autocorrelation functions for squared returns and for absolute returns are illustrated in figure 1. It can be observed that excepting JPY/RON, GBP/RON and NOK/RON, both stylized facts are predicted by the data. For JPY/RON, GBP/RON and NOK/RON series, the autocorrelation function for squared returns and for absolute returns are overlapping and it is not clear whether they share the Taylor effect property.

When analyzing the stochastic volatility models that are designed to predict the asymmetric response of volatility to return shocks, I extend the data with another eight stocks time series. I selected the most liquid stocks listed on the Bucharest Stock Exchange: Impact (IMP), Oltchim (OLT), Banca Transilvania (TLV), SIF-1, SIF-2, SIF-3, SIF-4, SIF-5. They are daily close prices and cover the period 5 January 2000 to 5 June 2007.

6. Stylized facts

6.1 Leptokurtic distribution and slow decaying autocorrelation function

In this section we will analyze the ability of the SV models to capture adequately two stylized facts: the leptokurtic distribution of returns and the slow decaying autocorrelation function of squared returns. These are analyzed together, because, as will be shown later, they are related. Specifically, the ACF of the squared returns depends on the kurtosis of the returns.

Liesenfeld and Jung (1997) demonstrate that the lognormal SV model is too restrictive to account adequately for both regularities mentioned above simultaneously, and the substitution of the normal distribution for the parameter u_t by a heavy-tailed distribution, such as Student-t, can solve the problem. Their approach is similar to Terasvirta (1996), who analyzed ACF – excess kurtosis relationship for GARCH model. Later, Malmsten and Terasvirta (2004) extended this study of the ACF - kurtosis relationship for SV model, GARCH model and EGARCH model.

Liesenfeld and Jung (1997) showed that the kurtosis of the returns is given by the following relation:

$$k = \frac{E\left(y_t^4\right)}{\left[E\left(y_t^2\right)\right]^2} = E\left(u_t^4\right)\cdot \exp\left(\sigma_h^2\right) \qquad where \quad \sigma_h^2 = \frac{\eta^2}{1-\varphi^2} \tag{5}$$

In relation (5) σ_h represents the unconditional variance of log-volatility (h_t) and is a function of the volatility of the return log-volatility (η^2) and the persistence parameter (φ). From this relation we an argue that the *theoretical* kurtosis of the return distribution can be attributed to the theoretical kurtosis of the return error distribution ($E(u^4)$) and to the kurtosis due to the variability of the return log-volatility.

In the case of the conditional normality assumption for the return, $E(u^4)$ is 3, which means that the theoretical kurtosis of the return distribution is greater then 3. This excess kurtosis implied by the ***lognormal SV model*** is consistent with the empirical observations mentioned earlier in section 5.

The theoretical autocorrelation function (ACF) of the squared return implied by the ***basic SV model*** can be shown to have the form:

$$ACF(t) = \frac{\exp(\sigma_h^2 \cdot \varphi^t) - 1}{E(u_t^4) \cdot \exp(\sigma_h^2) - 1} \qquad for\ t = 1, 2, 3... \tag{6}$$

From the relation (6) it can be concluded that the ACF decay by an exponentially rate that depends by the parameter φ. Figure 2 depicts the ACF as a function of φ and t fixing the other parameters. It is obvious from this figure that as the parameter φ increase, the ACF decrease with a slower rate. This is the reason why the parameter φ is referred as ***the persistence parameter***.

According to the relation (5) and (6) the theoretical autocorrelation function can be rewritten as a function of the theoretical kurtosis of the return distribution (k), persistence parameter (φ) and the kurtosis of the error distribution ($E(u^4)$):

$$ACF(t) = \frac{\left[\frac{k}{E(u_t^4)}\right]^{\varphi^t} - 1}{k - 1} \tag{7}$$

Equation (7) represents the theoretical ACF – kurtosis relationship implied by the ***basic SV model*** which can be particularized for the ***lognormal SV model*** by imposing the restriction that $E(u^4)$ is 3. The theoretical relation between the kurtosis of the return distribution and the autocorrelation of lag 1 for the squared return is illustrated in figure 3.a for different values for the persistence parameter (φ = {0.7, 0.9, 0.95, 0.99}). This figure also plots the empirical ACF – kurtosis relationship for the exchange rates (the blue points) and the indices (the red points) described in the data section.

The usual empirical values obtained in the literature for the persistence parameter are above 0.9. From figure 3.a it can be observed that the theoretical ACF(1)-kurtosis relationship implied by the ***lognormal SV model*** can be a good approximation for the indices time series, but not also for the exchange rates time series considered here. The ***lognormal SV model*** would have captured the empirical combination between ACF and kurtosis for the exchange rate series if the persistence parameter would have been around 0.7 (implying a faster decaying ACF), which is not empirically feasible. This means that for some time series the ***lognormal SV model*** can not capture simultaneously the slow decaying ACF of the squared returns and relatively high kurtosis of the return distribution. In other words, for a given ACF(1) the empirical kurtosis is higher then the theoretical kurtosis implied by the SV model with normal return errors. Although, the lognormal SV model can simulate returns with a kurtosis grater than 3, as it was shown in relation 3, this excess kurtosis implied by the model is not enough to capture the empirical kurtosis. This drawback of the ***basic SV model*** with normal return errors can be solved by using a distribution with fatter tails.

Other distributions used in the literature that allow $E(u^4)$ to be greater then three are the Student-t and generalized error distribution (described by Box and Tiao (1973)). In this paper, it was considered the Student-t distribution to account for higher kurtosis. The density function of the Student-t distribution with mean zero and variance one is given by the following relation:

$$f(u_t) = \frac{1}{\sqrt{\pi \cdot (\nu - 2) \cdot \left(1 + \frac{u_t^2}{\nu - 2}\right)^{df+1}}} \cdot \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \quad , \nu > 2 \qquad (8)$$

where the parameter ν represents the degrees of freedom. For df approaching infinity the t-distribution approaches a normal distribution. If u_t is Student-t distributed with the density function (8) then its kurtosis is given by:

$$E(u_t^2) = 3 \cdot \frac{\nu - 2}{\nu - 4} \quad , \quad for \quad \nu > 4 \qquad (9)$$

Imposing the condition (9) to the relation (7) one can obtain the theoretical ACF – kurtosis relationship implied by the ***t_SV model.*** This connection is depicted in figure 3.b considering only the first lag for the autocorrelation function and setting the df parameter to the value 10 which imply a kurtosis of 4 for the return error (u_t). As the degrees of freedom parameter increase, the kurtosis of the return errors goes toward three and the curves from figure 3.b moves to the up and left part of the figure approaching the theoretical ACF-kurtosis relation implied by the ***lognormal SV model***. This means that the t_SV model is more flexible and the lognormal model can be viewed as a special case of the t_SV model when ν is very large (and $E(u^4)$ is very close to 3). In conclusion, the ***t_SV model*** can capture more adequately the excess kurtosis and the slower decaying ACF simultaneously.

As it was pointed above, to plot the ACF – kurtosis relation for the ***t_SV model*** it was used for the parameter ν a value of 10. To determine a more precise value for the kurtosis of the return error, I have estimated in WinBugs the ***t_SV model*** for the seven exchange rate series and for the five nieces series, where the degrees of freedom parameter (ν) was introduced as a hyper-parameter. For the comparison purpose, I have also estimated the ***lognormal SV model***. The estimation results are presented in table 2 and table 3.

The estimates for the log-volatility mean (μ), persistence parameter (φ) and for the standard deviation of the log-volatility process (η) - or the precision ($1/\eta$) for some series – are very similar between models. Except for the BET-C who has a persistence parameter close to 0.85, all the other indices present a very high persistence above 0.99. For the exchange rates the persistence varies between 0.936 for the NOK/RON (lognormal model) and 0.997 for the USD/RON exchange rate (both models). The log-volatility mean estimate is around -10.5 for exchange rates, which implies that the daily standard deviation varies around of a mean of approximately 0.525 %. For the indices the log-volatility mean is lower, it varies from -7.93 (DAX, t_SV model) to -9.33 (BET-C, t_SV model).

The estimates for the degrees of freedom parameter varies between 13.85 (SEK/RON) and 20.47 (USD/RON) for exchange rates, and between 16.48 (MIB-30) and 25.49 (CAC-40). Using relation (9) one can estimate the kurtosis for the error u_t (3.502 for CHF/RON, 3.495 for EUR/RON and GBP/RON, 3.552 for JPY/RON, 3.425 for NOK/RON, 3.609 for SEK/RON, 3.364 for USD/RON) and compare it with the kurtosis of the return error in the lognormal model (that is 3).

Another way of illustrating that the empirical ACF – kurtosis relation is fitted better with the t_SV model then with the lognormal model is to plot on the same axes the empirical ACF, the ACF implied by the ***lognormal SV model*** and the ACF implied by the ***t_SV model.***. The values of the ACF implied by the two models were determined by evaluating the theoretical ACF with the estimated parameters. These plots are depicted in figure 4 where the green bar-type plot represents the empirical ACF, the red line is the theoretical ACF implied by the lognormal model and the blue line is the theoretical ACF implied by the t_SV model.

6.2 The Taylor effect

This section analyzes the presence of the Taylor effect in the context of stochastic volatility models. After examining 40 series of returns, Taylor (1986) observes that the sample autocorrelation of absolute returns seem to be larger than the sample autocorrelation of squares. This effect will be referred here as the Taylor effect in wade sense, because in the literature there are some other definitions given to this empirical feature.

Granger and Ding (1995), although they were the first who denote this empirical property of financial series as the "Taylor effect" their definition is more restrictive: *if y_t is the series of returns and acf(θ, k) represents the sample autocorrelation of the order k of $|y_t|^{\theta}$ then the Taylor effect is defined as acf(1,k) > acf(θ,k) for any θ different from 1*. This definition was based on the results found by Ding, Granger and Engle (1993) that the autocorrelations of the absolute returns raised to the power θ are maximized when θ is around 1. It must be mentioned that their analysis used only one time series, in particular S&P500 index. Later, Ding and Granger (1996), after an analysis that include several series of daily exchange rates and stock prices, changed their conclusion by observing that the maximum autocorrelation is not always obtained when $\theta = 1$ but for smaller values of θ.

Malmsten and Terasvirta (2004), define the Taylor effect as *acf(1,k) > acf(2,k)* , which means that autocorrelations of absolute returns are larger then the autocorrelations of the squared returns. They have also summarized the ability of three conditional volatility models to predict the Taylor effect. The three models are Generalized Autoregressive Conditionally Heteroscedasticity (GARCH) model, Exponential GARCH (EGARCH) model, Stochastic Volatility model (the basic SV).

The autocorrelation function of $|y_t|^{\theta}$ for the GARCH model is unknown, except for $\theta = 2$. Therefore, to test whether GARCH models represent the Taylor effect Ding, Granger and Engle(1993), He and Terasvirta (1999) used Monte Carlo simulations. The general, conclusion was that the GARCH models do not always generate the Taylor property. Furthermore, Malmsten and Terasvirta (2004) showed that for the EGARCH model the Taylor property holds for high values of the kurtosis. However, looking at their results, it is possible to observe that for empirically relevant values of the kurtosis, the difference between autocorrelations of squares and absolute returns is very small.

In the context of stochastic volatility models, the presence of the Taylor effect can be better analyzed, because there can be derived analytical expressions for the autocorrelation function of $|y_t|^\theta$ for any value of θ (Harvey (1998)). Harvey and Streibel (1998) show that for some particular autoregressive SV models the larger the variance of the volatility, the smaller the value of θ that maximize the autocorrelations. Galan, Perez and Ruiz (2004) analyze the circumstances under which the basic SV model can present the Taylor effect. Veiga (2007) extended their approach for another two SV models, namely Long Memory Autoregressive Stochastic Volatility model and Two Factor Long Memory Stochastic Volatility model. Both papers conclude that the SV models have difficulties in generating the Taylor effect.

Following Harvey (1998), the autocorrelation function of $|y_t|^\theta$ can be written as:

$$acf(\theta,\varphi,\eta,\nu,k)=\frac{\exp(\frac{\theta^2}{4}\cdot\sigma_h^2\cdot\varphi^k)-1}{\beta(\theta,\nu)\cdot\exp(\frac{\theta^2}{4}\cdot\sigma_h^2)-1}\quad,\ k>1 \tag{10}$$

$$\beta(\theta,\nu)=\begin{cases}\dfrac{\Gamma\left(\theta+\frac{1}{2}\right)\cdot\Gamma\left(\frac{1}{2}\right)}{\left\{\Gamma\left(\frac{\theta+1}{2}\right)\right\}^2}\ ,\ u_t\sim N(0,1)\\[2ex]\dfrac{\Gamma\left(\theta+\frac{1}{2}\right)\cdot\Gamma\left(-\theta+\frac{1}{2}\right)\cdot\Gamma\left(\frac{1}{2}\right)\cdot\Gamma\left(\frac{\nu}{2}\right)}{\left\{\Gamma\left(\frac{\theta+1}{2}\right)\cdot\Gamma\left(\frac{\nu-\theta}{2}\right)\right\}^2}\ ,\ u_t\sim t(0,1,\nu)\end{cases} \tag{11}$$

where σ_h^2 is defined in relation (5), and Γ(.) is the Gamma function. Relations (10) and (11) shows that the autocorrelation function at the order k of the absolute value of the returns razed to the power θ depends on the persistence parameter (φ), on the standard deviation of the log-volatility(η) and on the degrees of freedom parameter (ν) if the distribution for the error u_t is considered to be Student. It is obvious that, if the degree of freedom parameter tends to infinity then the autocorrelation function for Student errors tends to the autocorrelation function for Normal errors. This the reason why it will be used in this paper the notation acf(θ, φ, η, ∞,k) to denote the autocorrelation function for Normal errors.

It is obvious from relation (10) that the autocorrelation function is a very complicated non-linear function, and we can not have an analytical expression for the value of θ that maximize it. Figure 5 plots the autocorrelation function of order 1 for a standard deviation of the log-volatility (η) set to 0.15 (which is empirically reasonable) and for different (and relevant) values of the persistence parameter (φ). As mentioned above, setting ν to infinity means that we consider a normal distribution for the error of the returns. It is clear from this graphical representation that the higher the persistence parameter, the lower the value that maximizes the autocorrelation function. Furthermore, we can observe that parameter θ is larger then 1 for low values of φ, and tends toward 1 as φ increase.

To have a better idea about the values of the power parameter (θ) that maximize the function acf(.) for a given φ and k, I maximized this function with respect to θ setting the persistence parameter to the value 0.85, 0.9, 0.95, 0.96, 0.97, 0.98, 0.99 and the lag parameter to the value 1, 5, 10, 25, 50, 75, 100. This was implemented in Maple, using the function *NLPSolve* ("Non-Linear Programming Solve") from the *Optimization* toolbox. The results are reported in table 4. Values on the grey font represent the values of the power parameter that maximize the curves from figure 5, precisely, for a persistence parameter of 0.99 the power parameter is 1.05 and it increase to 1.59 for φ of 0.95. Another observation from table 4 is that as the lag parameter increase, the optimal value for the parameter θ decrease but very slowly. This is consistent with the conclusion of Galan, Perez and Ruiz (2004) that the optimal θ is approximately the same for different lags. According to this conclusion, to analyze how the other parameters affect the optimal value of θ, we set the lag parameter to1.

To investigate the influence of the parameter η to the optimal θ, the autocorrelation function was again maximized in Maple for different values of η (0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6) and for different values of φ (the same as in table 4). The results are reported in table 5. The optimal values of the parameter θ close to 1 are highlighted with grey color. It can be concluded that the higher the standard deviation of log-volatility, the lower the value of power parameter that maximize the theoretical autocorrelation function (see also figure 6). For high values φ and high values of η, the power parameter is less then 1, and for low values of φ and low values of η, the parameter on interest is larger then 1. This means, that the Taylor effect in the wide sense is predicted by the lognormal SV model only for some particularly values of φ and η: if φ is 0.99 then η must be higher then approx. 0.15, if φ is 0.98 then η must be higher then approx. 0.2, if φ is 0.97 then η must be higher then approx. 0.25 and so on. One can be interested in these combinations between φ and η implied by the theoretical autocorrelation function, because these can be compared with the empirical combinations.

To do this in a more comprehensive manner, consider first figure 7.a. This illustrates the autocorrelation function for lag 1 for three different values of power parameter (1, 1.5, and 2). For each value for the parameter θ we have a different surface. For low values of φ and η the three surfaces are very close, and the distance between them increases as φ and η increase. More importantly, where the blue surface is above the green and the red ones, we have the combinations between φ and η for which acf(θ = 2) is grater them acf(θ = 1), that is the lognormal SV model does not predict the Taylor effect. If the red surface is above the other two, it means that acf(θ = 1) is higher then acf(θ = 1.5) and acf(θ = 2) respectively. Here we can find the desired combinations between φ and η that permits the lognormal SV model to predict the Taylor effect. To verify, whether these theoretical combinations are also consistent with the empirical ones, a three dimensional representation similar to that from figure 7.a was transformed into a bi-dimensional representation as in figure 7.b where there are also plotted the empirical combinations between φ and η. The new representation replace the surface for θ = 1.5 with other relevant values.

The blue region from figure 7.b represents the φ-η combinations for which acf(θ = 2) > acf(θ = 1), but the acf(.) function is maximized by a θ higher then 1 and not necessary 2. For this region the lognormal SV model does not predict the Taylor effect. This is the region where we find the empirical φ-η combination for JPY/RON, NOK/RON and GBP/RON (more precisely, they are on the frontier with the green region) This means that for the three exchange rates mentioned above, the difference between autocorrelation of squared returns and autocorrelation of absolute returns implied by the lognormal SV model is negligible. This seems to be a good prediction, because we recall from figure 1, that for this particularly three time series the empirical autocorrelations functions are overlapping. The green region represents the φ-η combinations for which acf(θ = 1) > acf(θ = 2), but the acf(.) function is maximized by a θ higher then 1. In this region we find CHF/RON, SEK/RON, FTSE-100, DAX. In the red region acf(θ = 1) > acf(θ = 2), but the acf(.) function is maximized by a θ lower then 1. The empirical points that appear in this particular region correspond to BET-C and USD/RON. The white band represents the η-φ combinations for which the acf (.) function is maximized for a θ of 1 This last region is consistent with the Taylor effect in the restrictive sense as defined by Granger and Ding (1995). It can be observed that there are some time series that support this definition, namely EUR/RON, CAC-40, MIB-30.

Using the same approach as for the ***lognormal SV model***, the analysis of the Taylor property in the ***t_SV model*** is now straightforward. The additional parameter that appears in this model is the degree of freedom parameter (ν). An important observation for this model is that, the acf(θ = 2) is never higher then acf(θ = 1) for ν < 16 and for empirically relevant values for η and φ (that is η < 0.8, 0.8 < φ <0.999). In other words, for time series with ν lower then 16, the t_SV model always predict the Taylor effect. To test whether our estimates from table 3 have this property, we compare the empirical φ-η combinations with the theoretical ones, as we have already done for the ***lognormal SV model***. The time series were grouped according to their estimated degrees of freedom parameter as follows: for ν = 15 (SEK/RON and JPY/RON); for ν = 16 (CHF/RON, EUR/RON, GBP/RON and MIB-30); for ν = 20 (USD/RON, NOK/RON and BET-C); for ν =25 (CAC-40, DAX and FTSE-100).

Figure 8 illustrates the new results, for every group defined above. The first observation is that all the series considered here predict the Taylor effect in the context of the ***t_SV model***, even JPY/RON, NOK/RON, GBP/RON which in the context of the lognormal model do not share this property. This means that the it is possible for the t_SV model to overestimate the difference between autocorrelation of squared returns and autocorrelation of absolute returns. In the green region where acf (.) is maximized by a θ higher then 1 (but lower then 2) we have: CHF/RON, GBP/RON, JPY/RON, NOK/RON and SEK/RON. None of the indices considered lies in this region. In the red region where acf (.) is maximized by a θ lower then 1 we have: EUR/RON, USD/RON, BET-C, DAX and MIB-30. Finally, on the white band where acf (.) is maximized by a θ very close to 1, we have: CAC-40 and FTSE-100.

6.3 The asymmetric response of volatility to the return shocks

This section examines the asymmetric response of volatility to return shocks and discuss the shape of the news impact curve (NIC) implied by the asymmetric stochastic volatility models. Models that share the asymmetric property not only improve the ability to describe return dynamics, but also provide more accurate option prices.

This asymmetric property that we are interested here implies that a negative return shock has a higher impact on future volatility than does a positive shock of the same size. It seams that this effect is empirically relevant for stock returns and not necessarily for exchange rate returns. This is maybe the reason why the explanations for asymmetric response of volatility are from the stock market perspective. I found in the literature two explanations for the asymmetric response of volatility: the leverage effect and the feedback effect.

The leverage effect explain the asymmetry suggesting that when a bad news arrives on the market (i.e. a negative return shock), it decrease the value of the firm's equity and hence increase its leverage. In consequence, the equity becomes more risky and its volatility increase. Similarly, when good news arrives on the market, the leverage of the firm decrease and also its volatility. According to this effect there must be a negative relationship between future volatility and returns.

The feedback effect (French, Schwert and Stambauch (1987), Campbell and Hentschel (1992) and Wu (2001)) predicts that under the assumption of volatility clustering a large piece of news (good or bad) will lead to high volatility which tends to be followed by another large volatility. Since the volatility is priced, an increased in volatility should increase the required rate of return and hence decrease the stock price. Consequently, when a large piece of bad news arrives the decrease of the stock price is amplified by the higher volatility which also decreases the stock price. When a large piece of good news arrives the overall effect is ambiguous, because good news means positive shocks, but also higher volatility to be priced which determine the stock price to fall.

Although both effects can explain volatility asymmetry, they differ in how volatility responds to the god news. In particular, while the leverage effect predicts a downward movement of future expected volatility, the volatility feedback effect does not predict any relationship between volatility and returns when good news arrives.

In the option pricing literature, asymmetric stochastic volatility models dates back to 1987 (Hull and White), four years earlier than the first asymmetric ARCH model (Nelson, 1991). Even though, in the empirical finance the ARCH-type models were more popular than the SV models, and one of the reasons may be the difficulty in estimating SV models. As mentioned above, one of earliest asymmetric ARCH models is Nelson's (1991) Exponential Generalized Autoregressive Conditionally Heteroscedasticity (EGARCH):

$$\begin{cases} y_t = \sigma_t \cdot u_t = \exp(\frac{h_t}{2}) \cdot u_t \quad , \quad u_t \sim iid\ N(0,1) \\ h_t = \mu + \varphi \cdot h_{t-1} + \psi \cdot |u_{t-1}| + \beta \cdot u_{t-1} \end{cases} \tag{12}$$

where y_t is the return, σ_t^2 is the conditional variance, u_t is the standard return shock, and h_t is the log-volatility of returns.

Another popular asymmetric ARCH model is GJR model (Glosten, Jagannathan and Runkle, 1993) defined as:

$$\begin{cases} y_t = \sigma_t \cdot u_t = \exp(\frac{h_t}{2}) \cdot u_t \quad , \quad u_t \sim iid\ N(0,1) \\ \sigma_t^2 = \mu + \varphi \cdot \sigma_{t-1}^2 + \beta \cdot y_{t-1}^2 + \gamma \cdot y_{t-1}^2 \cdot I(y_{t-1} < 0) \end{cases} \tag{13}$$

where $I(y_{t-1} < 0) = 1$ if $y_{t-1} < 0$, and 0 otherwise.

Engle and Ng (1993) introduced the news impact curve (NIC) to analyze the relationship between current return shocks and expected volatility. Particularly, these relations can be determined by conditioning the expression for σ_{t+1}^2 (or h_{t+1}) on the information available at time t and earlier, and then considering σ_{t+1}^2 (or h_{t+1}) as a function of u_t.

Following Yu (2004), news impact curves can be defined as:

$$f(u_t) \equiv \left(\sigma_{t+1}^2 \middle| u_t, \sigma_t^2 = \overline{\sigma}^2, \sigma_{t-1}^2 = \overline{\sigma}^2, \ldots \right) \tag{14}$$

or

$$g(u_t) \equiv \left(h_{t+1} \middle| u_t, h_t = \overline{h}, h_{t-1} = \overline{h}, \ldots \right) \tag{15}$$

where $\overline{\sigma}^2$ and $\overline{h}$ represent the long run mean of σ_t^2 and h_t respectively. Also note that positive u_t corresponds to good news and negative u_t corresponds to bad news.

From relations (12) and (15) it can be easily shown than the news impact curve (NIC) for EGARCH model is:

$$g(u_t) = \begin{cases} \mu + \varphi \cdot \overline{h} + (\beta + \psi) \cdot u_t \ , & if \ u_t \geq 0 \\ \mu + \varphi \cdot \overline{h} + (\beta - \psi) \cdot u_t \ , & if \ u_t < 0 \end{cases} \tag{16}$$

The asymmetric response of volatility is present by the model if β is different from 0. If $\beta < 0$ and $\beta + \psi > 0$ then NIC is asymmetrically V-shaped, and if $\beta < \psi < -\beta$ then NIC is monotonically decreasing.

From relations (13) and (14) results that the NIC for GJR model is:

$$f(u_t) = \begin{cases} \mu + \varphi \cdot \overline{\sigma}^2 + \beta \cdot \overline{\sigma}^2 \cdot u_t^2 \ , & if \ u_t \geq 0 \\ \mu + \varphi \cdot \overline{\sigma}^2 + (\beta + \gamma) \cdot \overline{\sigma}^2 \cdot u_t^2 \ , & if \ u_t < 0 \end{cases} \tag{17}$$

The NIC for this model is asymmetrically U-shaped for $\beta > 0$ and $\gamma > 0$ (which is empirically relevant).

Specifying an asymmetric SV model was until recently an open question. There was a puzzle whether one should use a contemporaneous or an inter-temporal correlation between return shocks and volatility shocks. In section 3 , using Euler – Maruyama approximation we obtained the discrete version if the continuous stochastic volatility model (1), very popular in the option pricing literature:

$$\begin{cases} y_t = \sigma_t \cdot u_t = \exp(h_t / 2) \cdot u_t \\ h_{t+1} = \mu + \varphi \cdot (h_t - \mu) + \eta \cdot v_t \end{cases} \tag{2}$$

In addition, if we impose the condition:

$$\text{Corr}(u_t, v_t) = \rho \qquad (18)$$

then we have the discrete SV model (referred here as ***asv.hs***) estimated by Harvey and Shephard (1996) using quasi maximum likelihood method, or, if we impose the condition:

$$\text{Corr}(u_t, v_{t-1}) = \rho \qquad (19)$$

then we are dealing with Jacquier, Polson and Rossi (2004) SV model (referred here as ***asv.jpr***).For estimation purpose, it is common to rewrite the model (2) with condition (18) (the ***asv.hs*** model) in a Gaussian nonlinear state space form with uncorrelated measurement and transition equation error as:

$$\begin{cases} y_t = \sigma_t \cdot u_t = \exp(h_t / 2) \cdot u_t \\ h_{t+1} = \mu + \varphi \cdot (h_t - \mu) + \rho \cdot \eta \cdot u_t + \eta \cdot \sqrt{1-\rho^2} \cdot w_t \end{cases} \qquad (20)$$

where $w_t = (v_t - \rho \cdot u_t)/\sqrt{1-\rho^2}$ and hence is iid N(0,1), and corr (u_t , w_t) = 0.

In the same way, using the transformation $w_t = (v_{t-1} - \rho \cdot u_t)/\sqrt{1-\rho^2}$, the log-volatility function for the ***asv.jpr model*** can be written as:

$$h_t = \mu + \varphi \cdot (h_{t-1} - \mu) + \rho \cdot \eta \cdot u_t + \eta \cdot \sqrt{1-\rho^2} \cdot w_t \qquad (21)$$

Yu (2004.a) provides both theoretical and empirical evidence that the correct timing should be the contemporaneous correlation specification. From the theoretical perspective, the ***asv.jpr model*** has two drawbacks: first, it is not consistent with the efficient market hypothesis, and second, the leverage effect is not warranted. To be precise, for the ***asv.hs model*** the expression $\frac{\partial h_{t+1}}{\partial y_t} = \rho \cdot \eta \cdot \sigma_t^{-1}$ is always negative if the leverage effect exists (ρ < 0), however for the ***asv.jpr.model***, the expression $\frac{\partial h_t}{\partial y_t} = \frac{\rho \cdot \eta \cdot \sigma_t^{-1}}{1 + \frac{1}{2} \cdot \eta \cdot \rho \cdot u_t}$ can be negative but also positive (although the numerator is always negative, the denominator can have both signs). From the empirically point of view, Yu (2004.a) estimated both models for S&P500 index and compared them through Bayes factors. His conclusion was that the ***asv.hs model*** fits the data better which makes it also empirically superior to ***asv.jpr model***.

There is no doubt that theoretically the ***asv.jpr model*** has some drawbacks, but empirically maybe the S&P500 was a bad choice for ***asv.jpr model*** and a good one for the ***asv.hs model***. To see whether for another time series the empirical results are different or not, I considered 15 time series: two exchange rates (EUR/RON and USD/RON), five indices (BET-C, CAC-40, DAX, FTSE-100 and MIB-30) and eight well known stocks listed on Bucharest Stock Exchange (IMP, OLT, SIF 1, SIF 2, SIF 3, SIF 4, SIF 5 and TLV). The model comparison was made for this time with a more recent proposed tool, precisely Deviance Information Criterion (DIC).

The model estimates are presented in table 6. First it ca be noticed that for the exchange rates data and for the foreign indices, both models predict a high persistence parameter (0.9833 (asv.hs) and 0.9767 (asv.jpr) for EUR/RON ,0.99 (both models) for USD/RON; 0.9871 (asv.hs) and 0.9904 (asv.jpr) for CAC-40; 0.9861 (asv.hs) and 09903 (asv.jpr) for DAX, 0.9848 (asv.hs) and 0.9874 (asv.jpr) for FTSE-100, 0.98 (both models) for the MIB-30). This is not also the case for the Romanian stock market data, where the persistence is relatively low (0.8385 (asv.hs) and 0.837 (asv.jpr) for BET-C; 0.8876 (asv.hs) and 0.8881 (asv.jpr) for IMP; 0.8498 (asv.hs) and 0.8554 (asv.jpr) for OLT; 0.8782 (asv.hs) and 0.8787 (asv.jpr) for SIF 1; 0.8782 (asv.hs) and 0.8766 (asv.jpr) for SIF 2; 0.869 (asv.hs) and 0.8674 (asv.jpr) for SIF3; 0.8841 (asv.hs) and 0.8851 (asv.jpr) for SIF 4; 0.8092 (asv.hs) and 0.8228 (asv.jpr) for SIF 5 and finally 0.8635 (asv.hs) and 0.8601(asv.jpr) for TLV). Moreover, the estimates for the persistence parameter are very close between the two models.

Regarding the correlation coefficient, I found no evidence for any asymmetric response of Romanian stocks volatility. The asymmetry parameter is statistically significant (and with the right sign) only for the four foreign indices (-0.82 (asv.hs) and -0.853 (asv.jpr) for CAC-40; -0.7731 (asv.hs) and -0.8003 (asv.jpr) for DAX; -0.8233 (asv.hs) and -0.8141 (asv.jpr) for FTSE -100; and finally -0.6911(asv.hs) and -0.7701 (asv.jpr) for MIB-30)).

Table 7 reports the values for the Deviance Information Criterion (DIC) for model comparison purpose. The results are, surprisingly, in favor of ***asv.jpr model***. For 7 series from 15 DIC for asv.jpr is lower than DIC for asv.hs suggesting that the former fits better the data. For 5 series from 15 the two models have approximately the same DIC, and only for 3 series asv.hs model is empirically superior.

Yu (2004.b) generalized the news impact curve developed for ARCH-type models by Engle and Ng (1993) as discussed above, to incorporate also the SV models. Therefore, he propose to fix information dated at time t or earlier at a constant, evaluate lagged h_{t+1} at the long run mean of h_t (noted as h), and then define the NIC to be the relation between $E(h_{t+1})$ and u_t. To maintain the notation from (15), the proposed NIC can be written as:

$$G(u_t) = E(h_{t+1} | u_t , h_t = h, h_{t-1} = h, \dots) \quad (22)$$

It can be observed from relation (22), that instead of examining the relationship between future volatility and return shocks, Yu considered the relationship between the expected future volatility and return shocks. The ARGH-type models having a deterministic conditional volatility function, the expectation of σ_{t+1}^2 is the same as σ_{t+1}^2 and NIC proposed by Engle and Ng (1993) are a special case of the more general NIC proposed by Yu (2004 b).

From (20) and (22) one can infer that the general NIC for Harvey and Shephard model (***asv.hs model***) has the form:

$$G(u_t) = \mu + \varphi \cdot (h - \mu) + \rho \cdot \eta \cdot u_t \quad (23)$$

This is a linear function in u_t with a slope of $\rho\eta$. The parameter η represents the standard deviation of the log-volatility so it is always positive, which imply that the monotonicity of the NIC is determined by the sign of ρ. The leverage effect is predicted by the ***asv.hs model*** if $\rho < 0$ (which is typically the case in practice).

Yu (2004.b) proposed also a general asymmetric model that includes the ***asv.hs model*** as well an ARCH term:

$$\begin{cases} y_t = \sigma_t \cdot u_t = \exp(h_t / 2) \cdot u_t \\ h_{t+1} = \mu + \varphi \cdot (h_t - \mu) + \psi \cdot |y_t| + \rho \cdot \eta \cdot u_t + \eta \cdot \sqrt{1-\rho^2} \cdot w_t \end{cases} \quad (24)$$

In addition to model (23) the new general model (24) (denoted hereafter as ***g.asv model***) incorporates the absolute value of returns as in the EGARCH model. It is assumed in (24) that w_t is iid N(0,1) and corr (u_t , w_t) = 0. Precisely, it can be observed that this general model includes the following specifications:

- the ***asv.hs model***, when $\psi = 0$;
- an asymmetric ARCH model, when $\rho = +/- 1$;
- a symmetric ARCH model (but not the standard one), when $\eta = 0$;
- the ***basic SV model*** with an ARCH term, when $\rho = 0$;
- the ***basic SV model*** when $\rho = 0$ and $\psi = 0$.

It must be noted that none of the previous SV models discussed in this paper include any ARCH terms. The NIC for this model is:

$$G(u_t) = \begin{cases} \mu + \varphi \cdot (h - \mu) + (\rho \cdot \eta + \psi \cdot \sigma) \cdot u_t & if \ \ u_t \geq 0 \\ \mu + \varphi \cdot (h - \mu) + (\rho \cdot \eta - \psi \cdot \sigma) \cdot u_t & if \ \ u_t < 0 \end{cases} \tag{25}$$

The NIC is asymmetric for $\rho\eta$ different from zero. If $\rho < 0$ and $\psi\sigma > -\rho\eta$, the NIC is asymmetrically V-shaped and if $\rho < \psi\sigma < -\rho\eta$, the NIC is monotonically decreasing.

The ***g.asv model*** is highly related with the asymmetric SV model proposed by Asai and McAleer (2004) which is also an extension of the model estimated by Harvey and Shephard (1996). The conditional log-volatility is given by the following stochastic relation:

$$h_{t+1} = \mu + \varphi \cdot (h_t - \mu) + \gamma \cdot I(y_t < 0) + \rho \cdot \eta \cdot u_t + \eta \cdot \sqrt{1 - \rho^2} \cdot w_t \tag{26}$$

It is not difficult to observe from (26) that Asai and McAleer's model resemble the GJR specification, by introducing the indicator function ($I(y_t < 0)$).

Estimating the ***g.asv model***, one can determine which of the four incorporated models is confirmed by the data. The estimates for the ***g.asv model*** are reported in table 8. The time series used to estimate this model are: USD/RON, EUR/RON, BET-C, FTSE-100, MIB-30, SIF 1, SIF 2, SIF 5 and OLT. None of the nine series support the symmetric or the asymmetric ARCH specification, i.e. none of the series involved here have the parameter ρ very close to one or the parameter η equal to 0. This could be an evidence for the superiority of the SV specification with latent volatility over ARCH specification with deterministic volatility. BET-C, SIF 1 and OLT support the ***basic SV model***; SIF 2, SIF 5, and USD/RON support the ***basic SV model*** with an ARCH term, and FTSE-100 and EUR/RON support the ***asv.hs model***. For the Italian index (MIB-30) all parameters are statistically significant different from zero.

7. Conclusions

This dissertation paper, analyze the ability of the discrete stochastic volatility models to capture four stylized facts: leptokurtic distribution of the returns, slowly decaying autocorrelation function, Taylor effect and asymmetric response of volatility to return shocks.

The first two above mentioned stylized facts are discussed together because they are related, i.e. there can be derived a theoretical relation between autocorrelation and kurtosis. Comparing the theoretical autocorrelation – kurtosis relation with the empirical one, it was observed that the lognormal SV model for a given autocorrelation the theoretical kurtosis is not enough to capture the high excess kurtosis observed empirically. This problem was solved by imposing a fatter – tail distribution (here the Student-t distribution) for the return shocks. Therefore, the t_SV model (stochastic volatility model with Student distributed return shocks), captures more adequately slowly decaying autocorrelation function and excess kurtosis simultaneously, for empirically relevant values for the persistence parameter.

The ability of the SV models to predict Taylor effect is also analyzed for lognormal SV model and for t_SV model as well. Although, both models can predict Taylor effect, it seems that the t_SV model overestimates the difference between autocorrelation of the absolute returns and the autocorrelation of the squared returns.

In the literature there are some SV models that incorporate the asymmetric response of volatility to return shocks. In this dissertation, were discussed only two specifications, that is the Harvey and Shephard (1996) model and Jacquier, Polson and Rossi (2004) model. In the first model the asymmetric volatility is predicted by a negative correlation between volatility shocks and contemporaneous returns shocks, which differs from the second model where the it is assumed an inter-temporal correlation between volatility shocks an lagged return shocks. Although, from a theoretical perspective the model of Jacquier, Polson and Rossi (2004) has some drawbacks (i.e. it can not predict the asymmetry for all possible values of the parameters), from the empirical point of view, this model seems to fit the data better then the Harvey and Shephard (1996) model.

Estimating a general stochastic volatility model (proposed by Yu (2004)) which incorporates some ARCH terms, it was found that the stochastic specification for the volatility is more appropriate. Actually, it was found no evidence to support the ARCH models specification.

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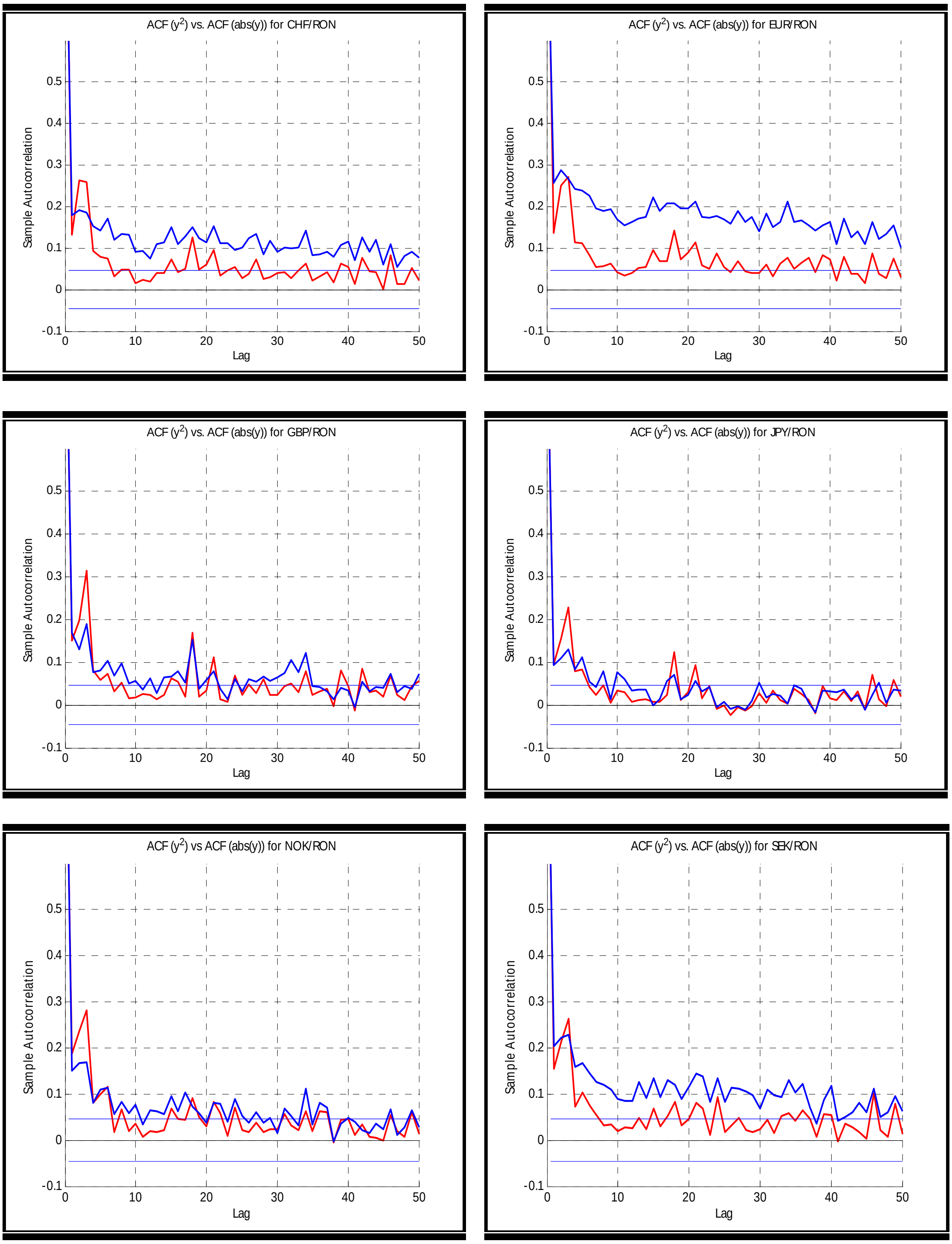
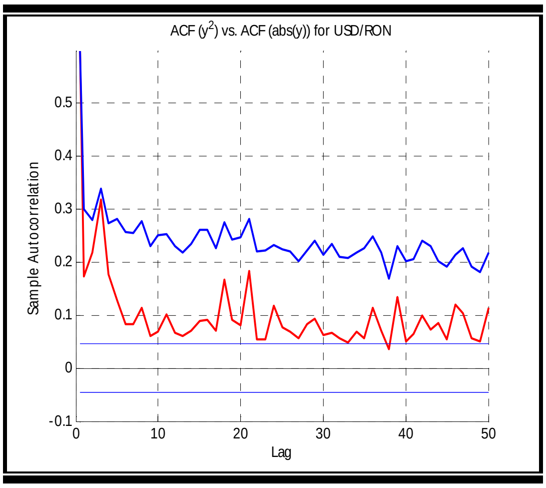
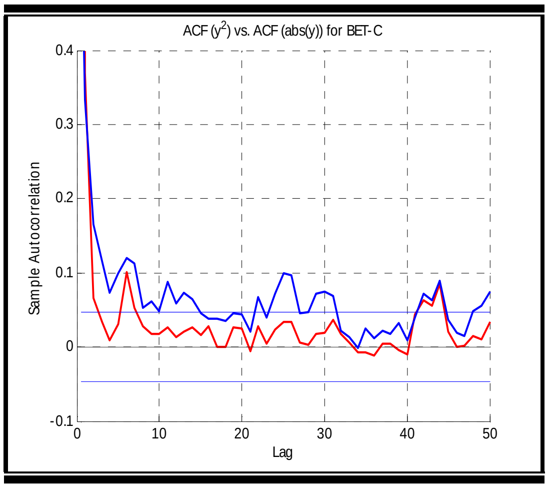
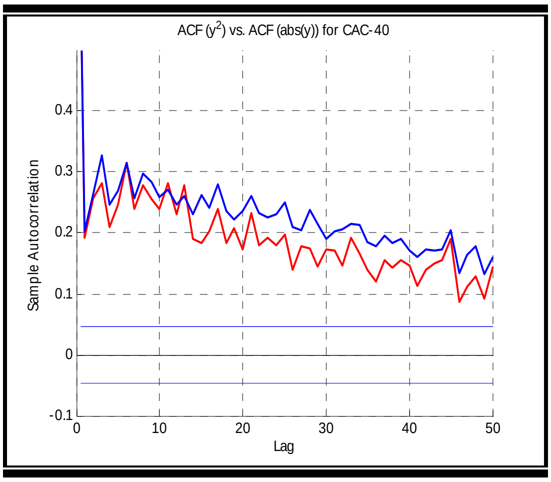
Appendix

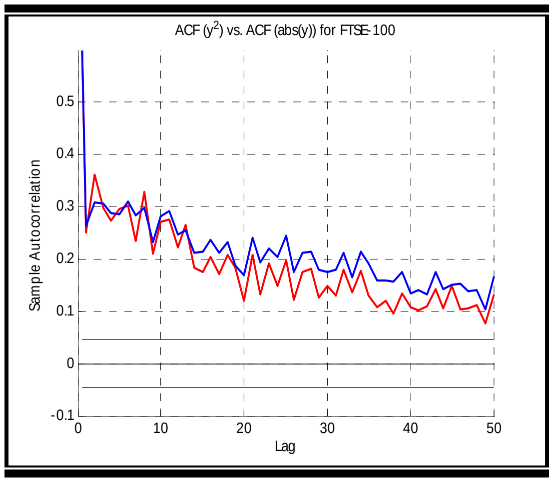
Figure 1. Autocorrelation function for squared returns (red line) and absolute returns (blue line)

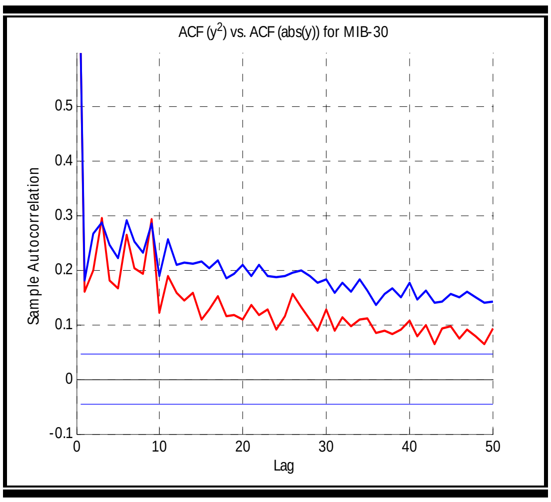



ACF (y^2) vs. ACF (abs(y)) for USD/RON
Sample Autocorrelation
Lag


ACF (y^2) vs. ACF (abs(y)) for BET-C
Sample Autocorrelation
Lag


ACF (y^2) vs. ACF (abs(y)) for CAC-40
Sample Autocorrelation
Lag


ACF (y^2) vs. ACF (abs(y)) for FTSE-100
Sample Autocorrelation
Lag


ACF (y^2) vs. ACF (abs(y)) for MIB-30
Sample Autocorrelation
Lag

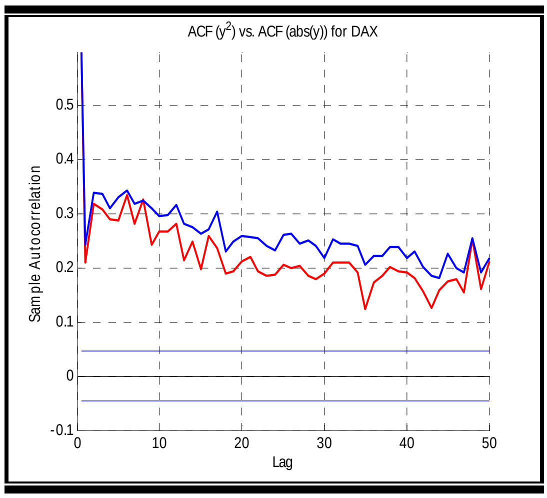

ACF (y^2) vs. ACF (abs(y)) for DAX
Sample Autocorrelation
Lag

Figure 2. Theoretical ACF(t,φ) for the basic SV model

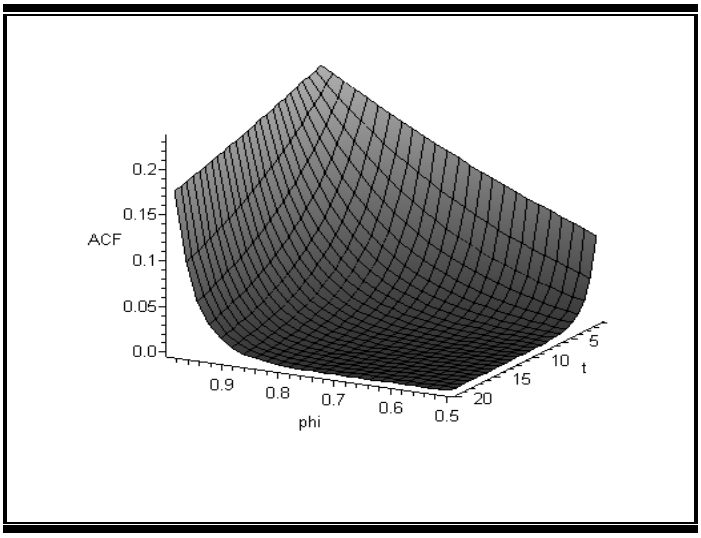
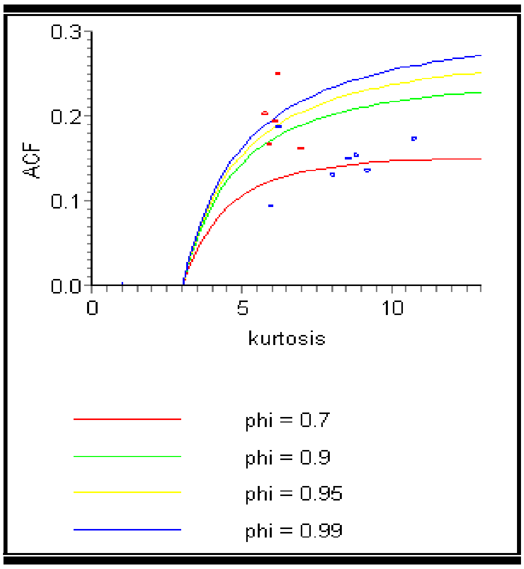


Figure 3. Theoretical ACF(1) – kurtosis relation

a). for the lognormal SV model



b). for t_SV model

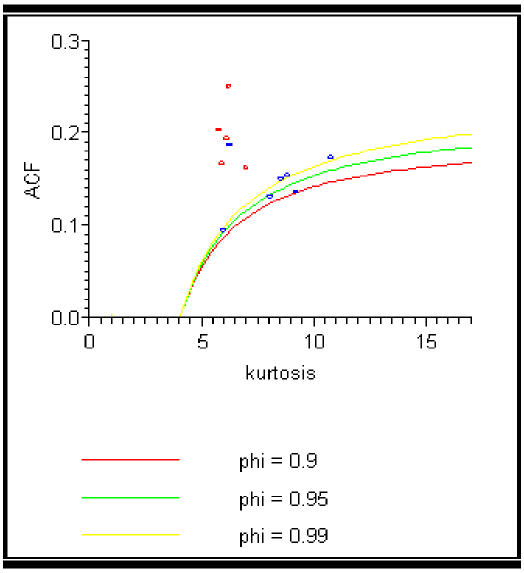


Figure 4. Empirical, theoretical lognormal and t_SV model ACFs

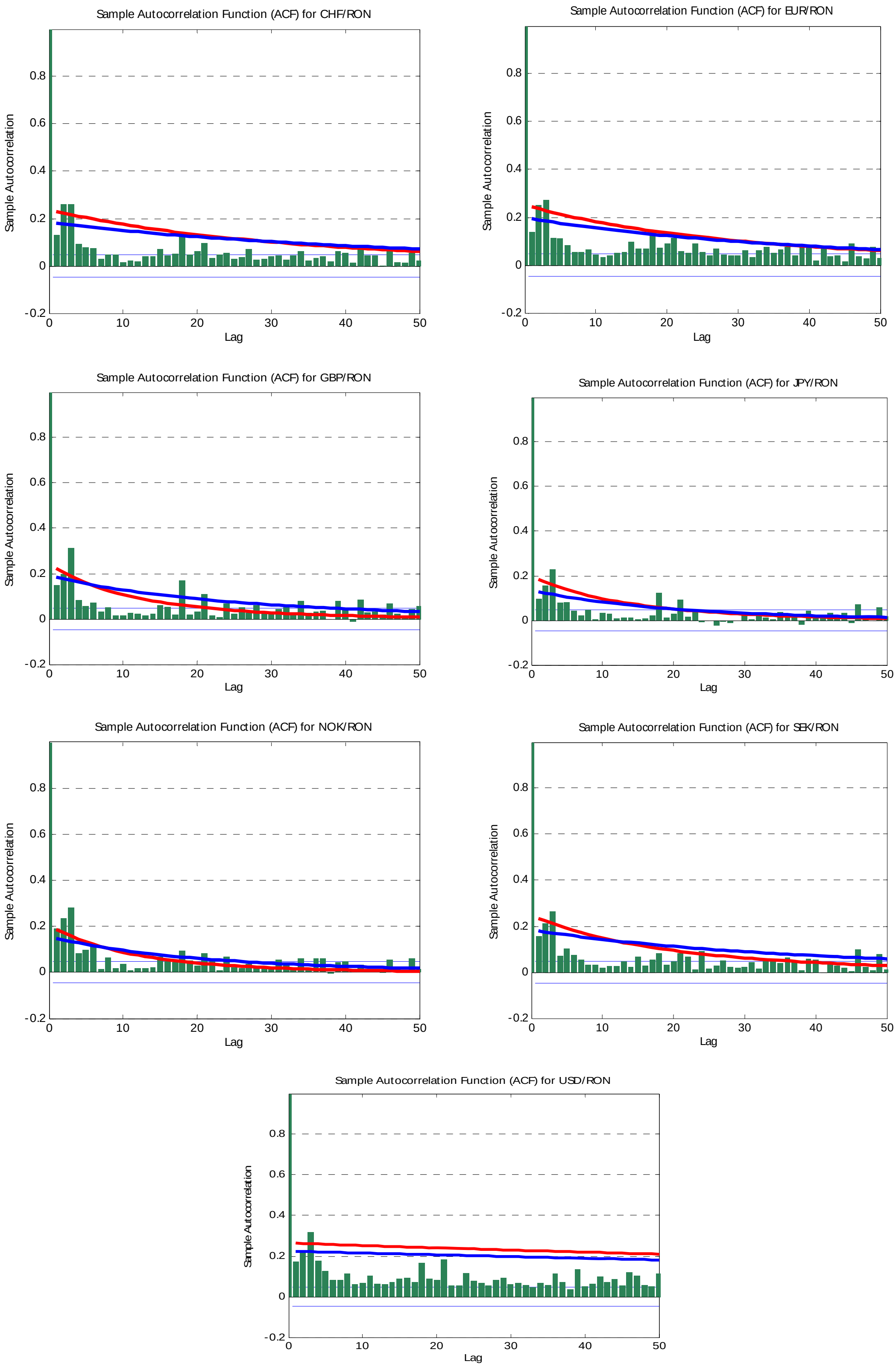
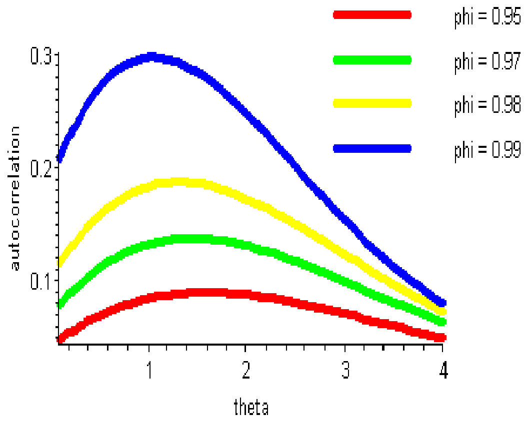


Figure 5. acf(θ, φ, 0.15, ∞, 1)

Figure 6. acf(θ, 0.98, η, ∞, 1)



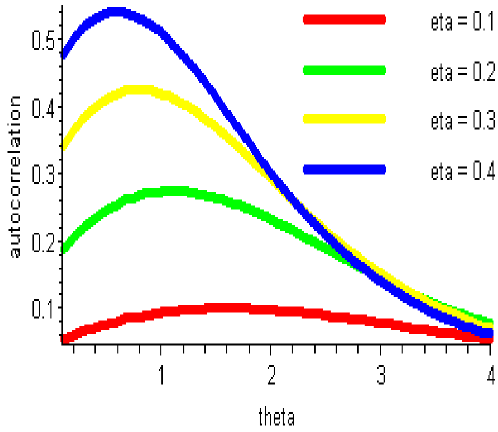
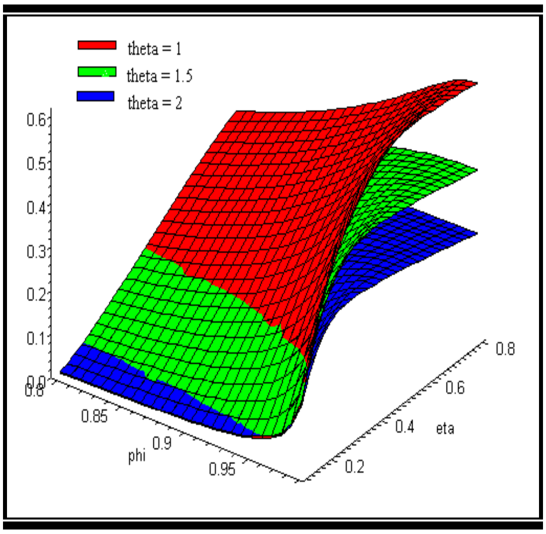


Figure 7. Combinations between φ and η consistent with Taylor effect for the lognormal SV model

a. Theoretical combinations between φ and η

b. Theoretical and empirical combinations between φ and η



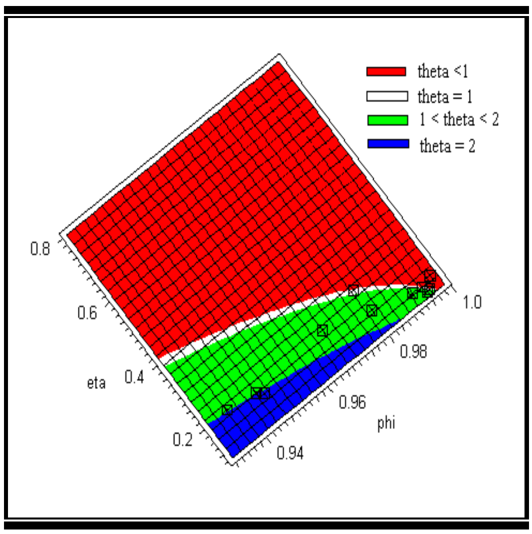
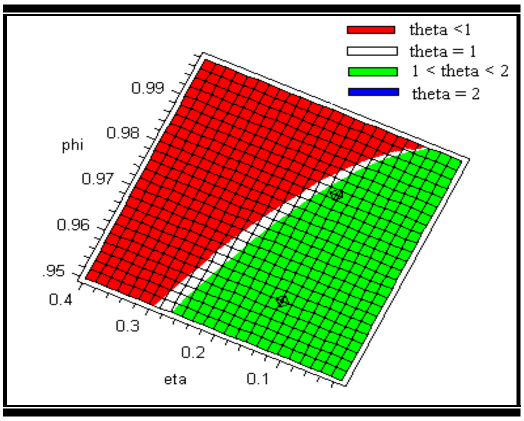
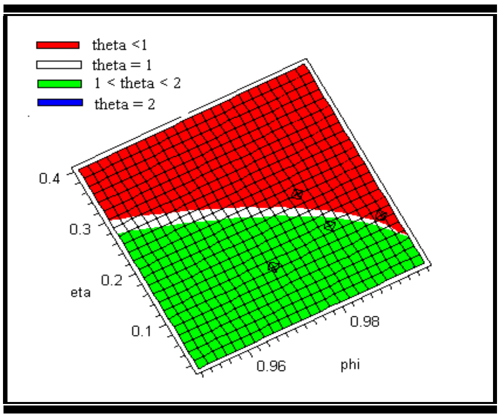


Figure 8. Theoretical and empirical combinations between φ and η consistent with Taylor effect for the t_ SV model

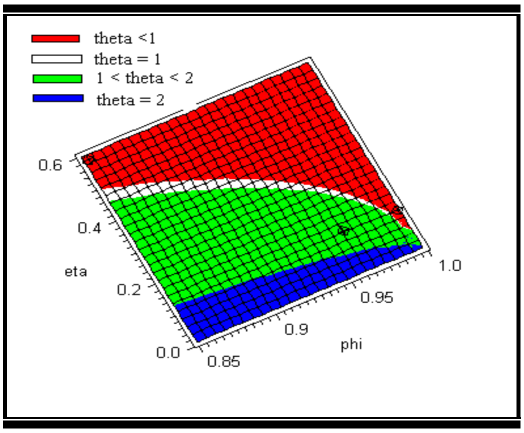
a. JPY, SEK (v =15)



b. CHF, EUR, GBP, MIB-30 (v =16)



c. USD, BET-C, NOK (v =20)



d. CAC-40, DAX, FTSE-100 (v =25)

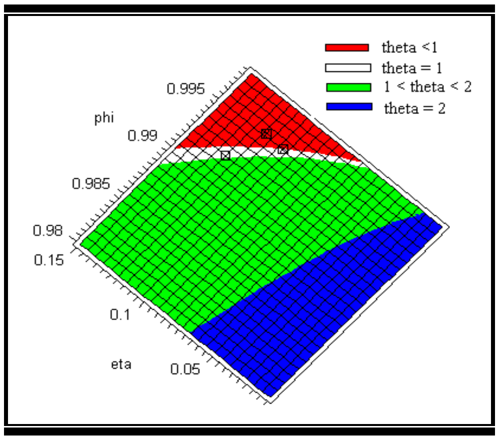


Table 1. Data summary statistics

a. exchange rates series

	CHF/RON	EUR/RON	GBP/RON	JPY/RON	NOK/RON	SEK/RON	USD/RON
Mean	0.000274	0.000290	0.000250	5.76E-05	0.000291	0.000247	0.000149
Median	0.000000	-9.35E-05	0.000240	-0.000228	0.000000	0.000000	0.000636
Maximum	0.035600	0.033856	0.034360	0.033269	0.037442	0.043314	0.032902
Minimum	-0.050280	-0.051064	-0.048380	-0.050789	-0.041556	-0.049305	-0.049684
Std. Dev.	0.006022	0.005741	0.005398	0.006596	0.006069	0.006130	0.005253
Skewness	0.206333	0.241668	-0.063077	0.073039	0.078485	0.259145	-0.317956
Kurtosis	8.007229	9.164532	8.523430	5.949496	6.203464	8.779745	10.71887
ACF(1)	0.132	0.137	0.151	0.095	0.188	0.155	0.174
Jarque-Bera	1987.858	3011.011	2403.780	686.7681	810.0873	2651.833	4723.850
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

b. indices series

	BET-C	CAC-40	DAX	FTSE-100	MIB-30
Mean	0.001332	0.000106	0.000164	3.75E-05	9.95E-05
Median	0.000872	0.000195	0.000552	0.000167	0.000323
Maximum	0.111222	0.072533	0.078452	0.060815	0.080837
Minimum	-0.121724	-0.073907	-0.064358	-0.054355	-0.077874
Std. Dev.	0.013377	0.014140	0.015869	0.011142	0.012293
Skewness	-0.390028	0.032458	0.077256	-0.071905	-0.130834
Kurtosis	13.02025	6.073581	5.741223	6.172357	6.930179
ACF(1)	0.37	0.195	0.251	0.163	0.168
Jarque-Bera	7841.211	733.6428	585.1510	782.8117	1204.334
Probability	0.000000	0.000000	0.000000	0.000000	0.000000

Tabel 2. Parameter estimates for the lognormal SV model

a. exchange rates series

time series	node	mean	sd	2.50%	median	97.50%
CHF/RON	mu	-10.41	0.2225	-10.81	-10.43	-9.963
	phi	0.9803	0.006725	0.9622	0.9809	0.9955
	eta	0.1661	0.0206	0.1322	0.1671	0.2063
EUR/RON	mu	-10.73	0.2794	-11.24	-10.76	-10.15
	phi	0.9797	0.006948	0.9588	0.9801	1
	eta	0.2332	0.0317	0.1813	0.232	0.2946
GBP/RON	mu	-10.64	0.09908	-10.84	-10.64	-10.46
	phi	0.9446	0.01958	0.9026	0.9471	0.9755
	eta	0.1891	0.03171	0.1421	0.1872	0.2342
JPY/RON	mu	-10.2	0.08811	-10.37	-10.2	-10.03
	phi	0.9461	0.01422	0.9145	0.9473	0.9716
	1/eta	32.81	10.12	18.8	30.36	55.22
NOK/RON	mu	-10.37	0.0856	-10.53	-10.37	-10.21
	phi	0.9362	0.019	0.8983	0.9404	0.9665
	eta	0.2021	0.03655	0.1555	0.1915	0.29
SEK/RON	mu	-10.48	0.1588	-10.79	-10.49	-10.17
	phi	0.9673	0.009773	0.9448	0.9689	0.9872
	eta	0.2063	0.03153	0.1597	0.2013	0.2709
USD/RON	mu	-12.83	0.4875	-13.77	-12.83	-11.89
	phi	0.9973	0.001652	0.9881	0.9976	1.007
	eta	0.1401	0.01364	0.1148	0.1392	0.1688

b. indices series

time series	node	mean	sd	2.50%	median	97.50%
BET-C	mu	-9.265	0.1031	-9.459	-9.267	-9.059
	phi	0.8434	0.02686	0.788	0.8446	0.8938
	eta	0.6121	0.06024	0.4995	0.6093	0.7251
CAC-40	mu	-8.255	0.3249	-8.83	-8.274	-7.579
	phi	0.9943	0.002916	0.9854	0.9948	1.003
	eta	0.1131	0.01105	0.0934	0.1127	0.1353
DAX	mu	-7.942	0.3818	-8.537	-7.961	-7.183
	phi	0.995	0.002913	0.9834	0.9954	1.006
	eta	0.114	0.01467	0.08455	0.1142	0.1438
FTSE-100	mu	-8.995	0.3567	-9.562	-9.043	-8.172
	phi	0.9911	0.004223	0.976	0.9919	1.003
	eta	0.1338	0.01556	0.1088	0.1317	0.1656
MIB-30	mu	-8.356	0.4051	-9.052	-8.39	-7.546
	phi	0.9939	0.003504	0.982	0.995	1.005
	eta	0.1302	0.01455	0.1042	0.1276	0.1598

Tabel 3. Parameter estimates for the t_SV model

a. exchange rates series

time series	node	mean	sd	2.50%	median	97.50%
CHF/RON	nu	15.94	4.004	9.834	15.45	25.01
	mu	-10.49	0.2601	-10.94	-10.52	-10.01
	phi	0.9859	0.005361	0.9708	0.9868	1.001
	1/eta	55.07	11.73	34.56	56.46	76.52
EUR/RON	nu	16.13	4.28	9.179	15.73	25.89
	mu	-10.78	0.3229	-11.35	-10.8	-10.11
	phi	0.9838	0.005598	0.9683	0.9844	0.9971
	eta	0.2062	0.02408	0.1649	0.2062	0.2501
GBP/RON	nu	16.13	4.462	9.529	15.62	25.88
	mu	-10.73	0.1315	-10.98	-10.73	-10.47
	phi	0.9725	0.008668	0.9532	0.9721	0.9889
	1/eta	79.64	17.99	49.64	80.19	119
JPY/RON	nu	14.86	4.384	8.782	14.14	25.48
	mu	-10.33	0.1049	-10.52	-10.33	-10.12
	phi	0.9593	0.01339	0.9282	0.9608	0.9814
	1/eta	52.45	17.28	26.29	53.52	92.84
NOK/RON	nu	18.1	4.775	11.83	16.97	30.37
	mu	-10.44	0.103	-10.64	-10.44	-10.24
	phi	0.9625	0.01437	0.9289	0.9657	0.9847
	eta	0.137	0.02686	0.09813	0.1349	0.1907
SEK/RON	nu	13.85	3.832	8.54	13.08	23.44
	mu	-10.59	0.2263	-10.98	-10.6	-10.12
	phi	0.9829	0.007135	0.9642	0.983	0.998
	eta	0.1431	0.02357	0.1062	0.1397	0.1865
USD/RON	nu	20.47	5.312	12.6	19.64	30.93
	mu	-12.87	0.6637	-14.27	-12.8	-11.85
	phi	0.9974	0.001677	0.9863	0.9973	1.008
	eta	0.1316	0.01636	0.1068	0.1343	0.1589

b. indices series

time series	node	mean	sd	2.50%	median	97.50%
BET-C	nu	19.87	5.135	12.26	19.06	31.11
	mu	-9.335	0.1075	-9.54	-9.336	-9.122
	phi	0.8526	0.03048	0.7949	0.8532	0.9081
	eta	0.5855	0.06789	0.4563	0.5874	0.7042
CAC-40	nu	25.49	5.53	16.46	25.2	37.75
	mu	-8.257	0.329	-8.844	-8.282	-7.647
	phi	0.9951	0.002652	0.9841	0.995	1.007

	eta	0.1061	0.01159	0.08383	0.1049	0.1324
DAX	nu	24.4	5.431	15.96	23.73	37.04
	mu	-7.932	0.4754	-8.722	-7.969	-7.136
	phi	0.9952	0.003014	0.9844	0.996	1.007
	eta	0.1192	0.01079	0.1006	0.118	0.1449
FTSE-100	nu	24.53	6.144	15.08	24.07	36.96
	mu	-9.01	0.5417	-9.672	-9.159	-8.034
	phi	0.992	0.004252	0.9786	0.9933	1.004
	eta	0.1267	0.01679	0.09959	0.1239	0.1603
MIB-40	nu	16.48	4.106	9.529	16.12	25.68
	mu	-8.459	0.3826	-9.138	-8.478	-7.723
	phi	0.9955	0.002601	0.984	0.9962	1.006
	eta	0.1136	0.01413	0.0921	0.113	0.1395

Table 4. Values of θ that maximize the function acf(θ, φ, 0.15, ∞, k)

eta=0.15	**the order of the autocorrelation function (k)**						
phi	***1***	***5***	***10***	***25***	***50***	***75***	***100***
0.85	1.8	1.76	1.71	1.69	1.69	1.69	1.69
0.9	1.74	1.68	1.64	1.6	1.59	1.59	1.59
0.95	**1.59**	1.54	1.49	1.41	1.37	1.36	1.36
0.96	1.53	1.48	1.43	1.35	1.3	1.28	1.28
0.97	**1.44**	1.4	1.35	1.27	1.21	1.18	1.17
0.98	**1.31**	1.27	1.23	1.15	1.08	1.04	1.02
0.99	**1.05**	1.02	0.99	0.94	0.87	0.82	0.79

Table 5. Values of θ that maximize the function acf(θ, φ, η, ∞, 1)

k=1	**Phi**						
eta	***0.85***	***0.9***	***0.95***	***0.96***	***0.97***	***0.98***	***0.99***
0.1	1.91	1.87	1.78	1.74	1.68	1.58	1.36
0.15	1.80	1.74	1.59	1.53	1.44	1.31	**1.05**
0.2	1.68	1.60	1.41	1.34	1.23	**1.09**	0.83
0.25	1.57	1.46	1.25	1.17	**1.06**	0.92	0.67
0.3	1.45	1.34	1.11	**1.03**	0.92	0.78	0.56
0.4	1.24	**1.11**	0.88	0.80	0.71	0.58	0.40
0.5	**1.06**	0.94	0.72	0.64	0.56	0.45	0.30
0.6	0.92	0.79	0.59	0.53	0.45	0.36	0.23

Table 6. Estimates for the asv.hs and asv.jpr models

a. exchange rate series

time series	model	node	mean	sd	2.50%	median	97.50%
EUR/RON	asv.hs	mu	-10.66	0.319	-11.24	-10.68	-9.974
	asv.jpr	mu	-10.79	0.2609	-11.28	-10.8	-10.25
	asv.hs	phi	0.9833	0.005936	0.968	0.984	0.9976
	asv.jpr	phi	0.9767	0.0071	0.9591	0.9772	0.9923
	asv.hs	rho	-0.1915	0.0755	-0.337	-0.1925	-0.04714
	asv.jpr	rho	0.1107	0.07831	-0.04612	0.1127	0.2604
	asv.hs	eta	0.2279	0.02608	0.1799	0.2265	0.2821
	asv.jpr	eta	0.2476	0.02759	0.1965	0.247	0.3016
USD/RON	asv.hs	mu	-12.66	0.8066	-14.02	-12.74	-11.03
	asv.jpr	mu	-12.56	0.7903	-13.93	-12.65	-11.07
	asv.hs	phi	0.9959	0.00362	0.9854	0.9971	1.002
	asv.jpr	phi	0.9958	0.003614	0.9833	0.9972	1.006
	asv.hs	rho	-0.0635	0.07451	-0.23	-0.05449	0.05759
	asv.jpr	rho	-0.06774	0.07075	-0.2214	-0.06128	0.05428
	asv.hs	eta	0.1417	0.01662	0.1131	0.1404	0.1779
	asv.jpr	eta	0.1411	0.01812	0.1118	0.1386	0.1779

b. stocks indices

time series	model	node	mean	sd	2.50%	median	97.50%
BET-C	asv.hs	mu	-9.263	0.1054	-9.466	-9.264	-9.053
	asv.jpr	mu	-9.308	0.1046	-9.511	-9.311	-9.1
	asv.hs	phi	0.8385	0.02836	0.7791	0.84	0.8902
	asv.jpr	phi	0.837	0.0298	0.7756	0.8386	0.8908
	asv.hs	rho	-0.03611	0.04989	-0.1335	-0.03637	0.06204
	asv.jpr	rho	0.06028	0.04497	-0.02819	0.06012	0.1478
	asv.hs	eta	0.6283	0.0619	0.5123	0.6262	0.7525
	asv.jpr	eta	0.6311	0.06355	0.5159	0.6292	0.755
CAC-40	asv.hs	mu	-8.768	0.1636	-9.056	-8.781	-8.416
	asv.jpr	mu	-8.166	0.1994	-8.523	-8.177	-7.739
	asv.hs	phi	0.9871	0.003046	0.9766	0.9872	0.9998
	asv.jpr	phi	0.9904	0.002115	0.9821	0.9905	0.9982
	asv.hs	rho	-0.8208	0.04479	-0.8984	-0.8247	-0.731
	asv.jpr	rho	-0.853	0.04163	-0.9211	-0.856	-0.7619
	asv.hs	eta	0.1407	0.01558	0.1092	0.14	0.1732
	asv.jpr	eta	0.1303	0.01101	0.1083	0.1303	0.1523
DAX	asv.hs	mu	-8.51	0.1709	-8.805	-8.522	-8.13
	asv.jpr	mu	-7.905	0.2322	-8.308	-7.921	-7.409
	asv.hs	phi	0.9861	0.003346	0.9738	0.9865	0.9978
	asv.jpr	phi	0.9903	0.002397	0.9799	0.9907	1.002

	asv.hs	rho	-0.7731	0.04635	-0.8515	-0.7762	-0.6761
	asv.jpr	rho	-0.8003	0.04547	-0.8794	-0.8014	-0.7074
	asv.hs	eta	0.146	0.01479	0.1193	0.1456	0.1764
	asv.jpr	eta	0.1339	0.01448	0.108	0.1327	0.1633
FTSE-100	asv.hs	mu	-9.411	0.1419	-9.671	-9.418	-9.124
	asv.jpr	mu	-8.876	0.1871	-9.216	-8.885	-8.483
	asv.hs	phi	0.9848	0.003042	0.9731	0.9853	0.9956
	asv.jpr	phi	0.9874	0.002703	0.976	0.9874	0.9983
	asv.hs	rho	-0.8233	0.05061	-0.887	-0.8329	-0.7174
	asv.jpr	rho	-0.8141	0.04549	-0.8853	-0.8208	-0.7308
	asv.hs	eta	0.1555	0.01319	0.1274	0.1549	0.1834
	asv.jpr	eta	0.1503	0.01285	0.1255	0.15	0.1788
MIB-30	asv.hs	mu	-8.975	0.2023	-9.306	-8.996	-8.529
	asv.jpr	mu	-8.437	0.2649	-8.87	-8.468	-7.823
	asv.hs	phi	0.9839	0.004263	0.9705	0.9845	0.9961
	asv.jpr	phi	0.9879	0.003249	0.9765	0.9871	0.9987
	asv.hs	rho	-0.6911	0.06231	-0.7964	-0.6928	-0.5615
	asv.jpr	rho	-0.7701	0.04636	-0.8463	-0.775	-0.6672
	asv.hs	eta	0.156	0.01669	0.1244	0.1551	0.1907
	asv.jpr	eta	0.1493	0.01457	0.1226	0.1484	0.1783

c. stocks series

time series	model	node	mean	sd	2.50%	median	97.50%
IMP	asv.hs	mu	-7.318	0.1602	-7.621	-7.322	-6.991
	asv.jpr	mu	-7.338	0.1611	-7.639	-7.343	-7.006
	asv.hs	phi	0.8876	0.02249	0.8412	0.8888	0.9294
	asv.jpr	phi	0.8881	0.02362	0.8406	0.8881	0.9324
	asv.hs	rho	-0.01786	0.05276	-0.1228	-0.01755	0.08508
	asv.jpr	rho	0.05147	0.04821	-0.04313	0.05163	0.1466
	asv.hs	eta	0.6702	0.07084	0.5354	0.6686	0.809
	asv.jpr	eta	0.6722	0.07406	0.5304	0.6756	0.8105
OLT	asv.hs	mu	-7.112	0.1089	-7.321	-7.113	-6.897
	asv.jpr	mu	-7.102	0.1101	-7.318	-7.102	-6.888
	asv.hs	phi	0.8498	0.02704	0.7939	0.8509	0.899
	asv.jpr	phi	0.8554	0.0246	0.8018	0.8567	0.9
	asv.hs	rho	0.05232	0.05696	-0.0613	0.05318	0.1619
	asv.jpr	rho	0.09397	0.0489	-0.002101	0.09375	0.1915
	asv.hs	eta	0.6243	0.06384	0.512	0.6259	0.7401
	asv.jpr	eta	0.6094	0.0576	0.5049	0.606	0.7242
SIF 1	asv.hs	mu	-7.714	0.1258	-7.956	-7.716	-7.46
	asv.jpr	mu	-7.736	0.1235	-7.975	-7.737	-7.491
	asv.hs	phi	0.8782	0.02312	0.8301	0.8786	0.9204
	asv.jpr	phi	0.8787	0.0227	0.8312	0.8802	0.9195
	asv.hs	rho	0.033	0.05424	-0.07355	0.03272	0.138
	asv.jpr	rho	0.08516	0.04826	-0.01003	0.08559	0.1775
	asv.hs	eta	0.577	0.05679	0.4722	0.5779	0.6804
	asv.jpr	eta	0.5785	0.05656	0.4809	0.5755	0.6953

SIF 2	asv.hs	mu	-7.529	0.1172	-7.757	-7.53	-7.297
	asv.jpr	mu	-7.545	0.1157	-7.774	-7.545	-7.325
	asv.hs	phi	0.8782	0.02231	0.8284	0.8796	0.918
	asv.jpr	phi	0.8766	0.0213	0.8308	0.8784	0.9151
	asv.hs	rho	0.06804	0.05472	-0.03977	0.06819	0.1724
	asv.jpr	rho	0.124	0.0489	0.02982	0.124	0.2216
	asv.hs	eta	0.5394	0.05068	0.4476	0.5363	0.6404
	asv.jpr	eta	0.5516	0.04859	0.4688	0.5476	0.6529
SIF 3	asv.hs	mu	-7.688	0.1214	-7.922	-7.689	-7.448
	asv.jpr	mu	-7.717	0.1193	-7.947	-7.717	-7.479
	asv.hs	phi	0.869	0.02319	0.8243	0.8702	0.9103
	asv.jpr	phi	0.8674	0.02292	0.8182	0.8683	0.9088
	asv.hs	rho	0.02065	0.0547	-0.08688	0.02036	0.1267
	asv.jpr	rho	0.08192	0.04845	-0.01385	0.08252	0.1773
	asv.hs	eta	0.5934	0.05569	0.4913	0.5888	0.6875
	asv.jpr	eta	0.6013	0.05533	0.5001	0.6001	0.7114
SIF 4	asv.hs	mu	-7.604	0.1277	-7.849	-7.606	-7.348
	asv.jpr	mu	-7.623	0.125	-7.867	-7.624	-7.372
	asv.hs	phi	0.8841	0.02168	0.8374	0.8849	0.9244
	asv.jpr	phi	0.8851	0.02204	0.8393	0.8869	0.9237
	asv.hs	rho	0.01443	0.0538	-0.09136	0.01559	0.1167
	asv.jpr	rho	0.08004	0.04742	-0.01535	0.08091	0.1729
	asv.hs	eta	0.5646	0.05639	0.4607	0.5638	0.6781
	asv.jpr	eta	0.5607	0.05897	0.4613	0.5568	0.6719
SIF 5	asv.hs	mu	-7.587	0.09733	-7.776	-7.588	-7.395
	asv.jpr	mu	-7.578	0.1002	-7.773	-7.578	-7.376
	asv.hs	phi	0.8092	0.03078	0.7444	0.811	0.8658
	asv.jpr	phi	0.8228	0.02993	0.7604	0.8245	0.8776
	asv.hs	rho	0.0659	0.05152	-0.03206	0.06617	0.1658
	asv.jpr	rho	0.08262	0.04603	-0.007327	0.08206	0.1736
	asv.hs	eta	0.6785	0.06095	0.5699	0.676	0.8011
	asv.jpr	eta	0.6508	0.05943	0.5435	0.6493	0.767
TLV	asv.hs	mu	-8.277	0.1092	-8.49	-8.278	-8.059
	asv.jpr	mu	-8.315	0.1062	-8.524	-8.314	-8.103
	asv.hs	phi	0.8635	0.02499	0.8088	0.8646	0.9084
	asv.jpr	phi	0.8601	0.02741	0.7998	0.8623	0.9084
	asv.hs	rho	0.02461	0.05584	-0.08341	0.02386	0.1363
	asv.jpr	rho	0.1102	0.04946	0.01404	0.11	0.2085
	asv.hs	eta	0.5508	0.05601	0.4546	0.5472	0.6666
	asv.jpr	eta	0.559	0.0606	0.453	0.5538	0.6866

Table 7. DIC for asv.hs and asv.jpr models

a. exchange rates series

time series	model	Dbar	Dhat	pD	DIC
EUR/RON	asv.hs	-14481.7	-14663.5	181.792	-14299.9
	asv.jpr	-14451	-14603.9	152.928	-14298
USD/RON	asv.hs	-15329.8	-15415.4	85.596	-15244.2
	asv.jpr	-15327.1	-15409.9	82.85	-15244.2

b. stocks indices series

time series	model	Dbar	Dhat	pD	DIC
BET-C	asv.hs	-11940.3	-12265.1	324.756	-11615.6
	asv.jpr	-11948.2	-12280.1	331.899	-11616.3
CAC-40	asv.hs	-13455.2	-14688.4	1233.2	-12222
	asv.jpr	-13779.1	-15075.8	1296.71	-12482.4
DAX	asv.hs	-12725.1	-13828.4	1103.34	-11621.7
	asv.jpr	-12923.6	-14099.2	1175.56	-11748.1
FTSE-100	asv.hs	-14452.2	-15662.3	1210.09	-13242.1
	asv.jpr	-14340.4	-15535	1194.63	-13145.7
MIB-30	asv.hs	-13145.3	-14041.7	896.478	-12248.8
	asv.jpr	-13587.1	-14684.8	1097.63	-12489.5

c. stocks series

time series	model	Dbar	Dhat	pD	DIC
IMP	asv.hs	-8213.76	-8545.67	331.902	-7881.86
	asv.jpr	-8220.22	-8557.50	337.28	-7882.94
OLT	asv.hs	-7687.45	-8001.77	314.315	-7373.14
	asv.jpr	-7686.95	-8003.86	316.91	-7370.04
SIF 1	asv.hs	-8799.30	-9092.44	293.139	-8506.16
	asv.jpr	-8816.34	-9122.5	306.164	-8510.18
SIF 2	asv.hs	-8446.36	-8725.97	279.608	-8166.75
	asv.jpr	-8482.6	-8786.71	304.104	-8178.50
SIF 3	asv.hs	-8795.04	-9093.15	298.107	-8496.93
	asv.jpr	-8814.91	-9129.41	314.5	-8500.41
SIF 4	asv.hs	-8630.70	-8917.59	286.888	-8343.81
	asv.jpr	-8641.23	-8937.65	296.424	-8344.81
SIF 5	asv.hs	-8564.89	-8900.30	335.407	-8229.48
	asv.jpr	-8556.73	-8889.19	332.459	-8224.27
TLV	asv.hs	-9797.32	-10076.5	279.139	-9518.18
	asv.jpr	-9829.57	-10132.7	303.103	-9526.47

Table 8. Estimates for the g.asv model

a. indices series

time series	node	mean	sd	2.50%	median	97.50%	DIC
BET-C	mu	-9.042	0.4224	-9.57	-9.134	-7.93	
	phi	0.8572	0.04193	0.7741	0.8579	0.9394	
	psi	-2.004	3.906	-9.568	-1.915	5.96	
	rho	-0.03994	0.05087	-0.1387	-0.04058	0.05845	
	eta	0.6091	0.06728	0.477	0.6123	0.7362	-11620.5
FTSE-100	mu	-10.04	0.4511	-10.59	-10.17	-8.973	
	phi	0.9707	0.01129	0.9478	0.9705	0.9902	
	psi	2.762	2.134	-0.6063	2.7	6.965	
	rho	-0.8573	0.0514	-0.9308	-0.8626	-0.7584	
	eta	0.1585	0.01506	0.1315	0.1574	0.1862	-13511.2
MIB-30	mu	-7.832	0.5749	-8.9	-7.823	-6.749	
	phi	0.9892	0.004134	0.9775	0.9898	0.9995	
	psi	-1.39	0.625	-2.53	-1.402	-0.2187	
	rho	-0.6862	0.06054	-0.7816	-0.6909	-0.5594	
	eta	0.1557	0.01875	0.1252	0.1537	0.1965	-12237

b. stocks series

time series	node	mean	sd	2.50%	median	97.50%	DIC
SIF 1	mu	-6.188	1.03	-7.733	-6.324	-3.923	
	phi	0.9376	0.02844	0.8703	0.9441	0.975	
	psi	-3.678	1.582	-6.289	-3.88	0.05954	
	rho	0.02811	0.05632	-0.08099	0.0283	0.1376	
	eta	0.5191	0.05467	0.4229	0.5154	0.6346	-8533.27
SIF 2	mu	-4.823	1.292	-7.194	-4.806	-2.31	
	phi	0.9586	0.02128	0.8986	0.965	0.9817	
	psi	-4.337	1.188	-6.386	-4.442	-1.549	
	rho	0.07375	0.05685	-0.0362	0.07292	0.1862	
	eta	0.4629	0.04603	0.3856	0.4597	0.5604	-8210.13
SIF 5	mu	-3.232	1.448	-6.523	-3.135	-0.6454	
	phi	0.9663	0.02085	0.9083	0.9717	0.9832	
	psi	-6.004	1.133	-8.027	-6.048	-3.833	
	rho	0.05089	0.05681	-0.05907	0.05041	0.1631	
	eta	0.4831	0.05043	0.3995	0.4786	0.5989	-8267.79
OLT	mu	-7.32	0.2382	-7.692	-7.35	-6.773	
	phi	0.8151	0.043	0.7253	0.8171	0.8934	
	psi	1.719	1.666	-1.444	1.669	5.138	
	rho	0.05346	0.057	-0.05856	0.05374	0.1637	
	eta	0.6401	0.061	0.5266	0.6382	0.761	-7357.86

c. exchange rates series

time series	node	mean	sd	2.50%	median	97.50%	DIC
EUR/RON	mu	-10.07	0.583	-11.11	-10.11	-8.844	
	phi	0.9853	0.005621	0.971	0.9858	0.9966	
	psi	-2.409	1.954	-6.13	-2.424	1.607	
	rho	-0.183	0.0793	-0.3334	-0.1835	-0.02638	
	eta	0.2322	0.02498	0.1876	0.2311	0.2823	-14309.8
USD/RON	mu	-12.3	0.4892	-13.35	-12.25	-11.48	
	phi	0.9859	0.005951	0.9728	0.9855	0.9985	
	psi	4.457	2.267	0.3141	4.372	8.825	
	rho	-0.1729	0.08952	-0.3394	-0.1727	-0.00407	
	eta	0.1421	0.01759	0.1102	0.1414	0.1769	-15244.4