CreditMetrics™ – Technical Document

The benchmark for understanding credit risk

- A value-at-risk (VaR) framework applicable to *all institutions worldwide that carry credit risk in the course of their business.*
- A full portfolio view addressing credit event correlations which can identify the costs of over concentration and benefits of diversification in a mark-to-market framework.
- Results that drive: *investment decisions, risk-mitigating actions,* consistent risk-based *credit limits,* and rational *risk-based capital allocations.*

New York
April 2, 1997

J.P. Morgan

Co-sponsors:
Bank of America
Bank of Montreal
BZW
Deutsche Morgan Grenfell
KMV Corporation
Swiss Bank Corporation
Union Bank of Switzerland

This *Technical Document* describes CreditMetrics™, a framework for quantifying credit risk in portfolios of traditional credit products (loans, commitments to lend, financial letters of credit), fixed income instruments, and market-driven instruments subject to counterparty default (swaps, forwards, etc.). This is the first edition of what we intend will be an ongoing refinement of credit risk methodologies.

Just as we have done with RiskMetrics™, we are making our methodology and data available for three reasons:

1. We are interested in promoting greater transparency of credit risk. Transparency is the key to effective management.

2. Our aim is to establish a benchmark for credit risk measurement. The absence of a com mon point of reference for credit risk makes it difficult to compare different approaches to and measures of credit risk. Risks are comparable only when they are measured with the same yardstick.

3. We intend to provide our clients with sound advice, including advice on managing their credit risk. We describe the CreditMetrics™ methodology as an aid to clients in under standing and evaluating that advice.

Both J.P. Morgan and our co-sponsors are committed to further the development of CreditMetrics™ as a fully transparent set of risk measurement methods. This broad sponsorship should be interpreted as a signal of our joint commitment to an open and evolving standard for credit risk measurement. We invite the participation of all parties in this continuing enterprise and look forward to receiving feedback to enhance CreditMetrics™ as a benchmark for measuring credit risk.

CreditMetrics™ is based on, but differs significantly from, the risk measurement methodology developed by J.P. Morgan for the measurement, management, and control of credit risk in its trading, arbitrage, and investment account activities. **We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks.** CreditMetrics™ is nothing more than a high-quality tool for the professional risk manager in the financial markets and is not a guarantee of specific results.

CreditMetrics™—Technical Document

This book

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This is the reference document for CreditMetrics™. It is meant to serve as an introduction to the methodology and mathematics behind statistical credit risk estimation, as well as a detailed documentation of the analytics that generate the data set we provide.

This document reviews:

- the conceptual framework of our methodologies for estimating credit risk;
- the description of the obligors' credit quality characteristics, their statistical description and associated statistical models;
- the description of credit exposure types across "market-driven" instruments and the more traditional corporate finance credit products; and
- the data set that we update periodically and provide to the market for free.

In the interest of establishing a benchmark in a field with as little standardization and precise data as credit risk measurement, we have invited five leading banks, Bank of America, BZW, Deutsche Morgan Grenfell, Swiss Bank Corporation, and Union Bank of Switzerland, and a leading credit risk analytics firm, KMV Corporation, to be co-sponsors of CreditMetrics. All these firms have spent a significant amount of time working on their own credit risk management issues, and we are pleased to have received their input and support in the development of CreditMetrics. With their sponsorship we hope to send one clear and consistent message to the marketplace in an area with little clarity to date.

We have also had many fruitful dialogues with professionals from Central Banks, regulators, competitors, and academics. We are grateful for their insights, help, and encouragement. Of course, all remaining errors and omissions are solely our responsibility.

How is this related to RiskMetrics™?

We developed CreditMetrics to be as good a methodology for capturing counterparty default risk as the available data quality would allow. Although we never mandated during this development that CreditMetrics must resemble RiskMetrics, the outcome has yielded philosophically similar models. One major difference in the models was driven by the difference in the available data. In RiskMetrics, we have an abundance of daily liquid pricing data on which to construct a model of conditional volatility. In CreditMetrics, we have relatively sparse and infrequently priced data on which to construct a model of unconditional volatility.

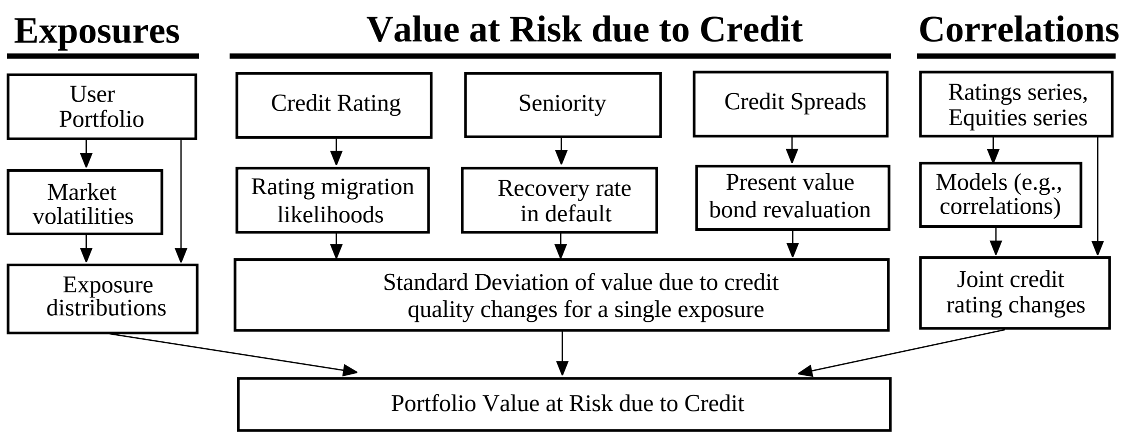
What is different about CreditMetrics?

Unlike market risks where daily liquid price observations allow a direct calculation of value-at-risk (VaR), CreditMetrics seeks to *construct* what it cannot directly *observe*: the volatility of value due to credit quality changes. This constructive approach makes CreditMetrics less an exercise in fitting distributions to observed price data, and more an exercise in proposing models which explain the changes in credit related instruments.

And as we will mention many times in this document, the models which best describe credit risk do not rely on the assumption that returns are normally distributed, marking a significant departure from the RiskMetrics framework.

In the end, we seek to balance the best of all sources of information in a model which looks across broad historical data rather than only recent market moves and across the full range of credit quality migration — upgrades and downgrades — rather than just default.

Our framework can be described in the diagram below. The many sources of information may give an impression of complexity. However, we give a step-by-step introduction in the first four chapters of this book which should be accessible to all readers.



One of our fundamental techniques is *migration analysis*, that is, the study of changes in the credit quality of names through time. Morgan developed transition matrices for this purpose as early as 1987. We have since built upon a broad literature of work which applies migration analysis to credit risk evaluation. The first publication of transition matrices was in 1991 by both Professor Edward Altman of New York University and separately by Lucas & Lonski of Moody's Investors Service. They have since been published regularly (see Moody's Carty & Lieberman [96a][1] and Standard & Poor's *Creditweek* [15-Apr-96]) and are also calculated by firms such as KMV.

Are RiskMetrics and CreditMetrics comparable?

Yes, in brief, RiskMetrics looks to a horizon and estimates the *value-at-risk* across a distribution of historically estimated realizations. Likewise, CreditMetrics looks to a horizon and constructs a distribution of historically estimated credit outcomes (rating migrations including potentially default). Each credit quality migration is weighted by its likelihood (transition matrix analysis). Each outcome has an estimate of change in value (given by either credit spreads or studies of recovery rates in default). We then aggregate volatilities across the portfolio, applying estimates of correlation. Thus, although the relevant time horizon is usually longer for credit risk, with CreditMetrics we compute credit risk on a comparable basis with market risk.

[1] Bracketed numbers refer to year of publication.

What CreditMetrics is not

We have sought to add value to the market's understanding of credit risk estimation, not by replicating what others have done before, but rather by filling in what we believe is lacking. Most prior work has been on the estimation of the relative likelihoods of default for individual firms; Moody's and S&P have long done this and many others have started to do so. We have designed CreditMetrics to accept as an input any assessment of default probability[2] which results in firms being classified into discrete groups (such as rating categories), each with a defined default probability. It is important to realize, however, that these assessments are only inputs to CreditMetrics, and not the final output.

We wish to estimate the *volatility of value* due to changes in credit quality, not just the *expected loss*. In our view, as important as default likelihood estimation is, it is only one link in the long chain of modeling and estimation that is necessary to fully assess credit risk (volatility) within a portfolio. Just as a chain is only as strong as its weakest link, it is also important to diligently address: (i) uncertainty of exposure such as is found in swaps and forwards, (ii) residual value estimates and their *uncertainties*, and (iii) credit quality *correlations* across the portfolio.

How is this document organized?

One need not read and fully understand the details of this entire document to understand CreditMetrics. This document is organized into three parts that address subjects of particular interest to our diverse readers.

Part I **Risk Measurement Framework**
This section is for the general practitioner. We provide a practicable framework of how to think about credit risk, how to apply that thinking in practice, and how to interpret the results. We begin with an example of a single bond and then add more variation and detail. By example, we apply our framework across different exposures and across a portfolio.

Part II **Model Parameters**
Although this section occasionally refers to advanced statistical analysis, there is content accessible to all readers. We first review the current academic context within which we developed our credit risk framework. We review the statistical assumptions needed to describe discrete credit events; their mean expectations, volatilities, and correlations. We then look at how these credit statistics can be estimated to describe what happened in the past and what can be projected in the future.

Part III **Applications**
We discuss two implementations of our portfolio framework for estimating the *volatility of value due to credit quality changes*. The first is an analytic calculation of the mean and standard deviation of value changes. The second is a simulation approach which estimates the full distribution of value changes. These both embody the same modeling framework and

[2] These assessments may be agency debt ratings, a user's internal ratings, the output of a statistical default prediction model, or any other approach.

produce comparable results. We also discuss how the results can be used in portfolio management, limit setting, and economic capital allocation.

Future plans

We expect to update this *Technical Document* regularly. We intend to further develop our methodology, data and software implementation as we receive client and academic comments.

CreditMetrics has been developed by the Risk Management Research Group at J.P. Morgan. Special mention must go to Greg M. Gupton who conceived of this project and has been working on developing the CreditMetrics approach at JPMorgan for the last four years. We welcome any suggestions to enhance the methodology and adapt it further to the changing needs of the market. We encourage academic studies and are prepared to supply data for well-structured projects.

Acknowledgments

We would like to thank our co-sponsors for their input and support in the writing and editing of this document. In particular, we thank the KMV Corporation, which has been a pioneer in developing portfolio approaches to credit risk, and whose work has influenced many of the methods presented here.

We thank numerous individuals at J.P. Morgan who participated in this project, as well as professionals at other banks and academic institutions who offered input at various levels. Also, this document could not have been produced without the contributions of our consulting editor, Margaret Dunkle. We apologize for any omissions.

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Part I

Risk Measurement Framework

Overview of Part I

This section describes the risk measurement framework used in CreditMetrics. We emphasize the basic ideas and illustrate them by means of simple examples. Later in Parts II and III, we give a more detailed treatment of CreditMetrics including methodology and data issues.

Part I is organized into the following four chapters:

- **Chapter 1: Introduction to CreditMetrics.** In this chapter, we discuss the merits and challenges of pursuing a quantitative portfolio approach to measuring credit risk. We give a summary of what we hope to achieve and the scope of our application. Using simple examples of one- and two-bond portfolios, we explain the ideas, methodology and data requirements of CreditMetrics.

- **Chapter 2: Stand-alone risk calculation.** In this chapter, we provide details of how CreditMetrics estimates credit risk for a single bond. We discuss how we directly calculate the standard deviation of value due to credit quality changes.

- **Chapter 3: Portfolio risk calculation.** In this chapter, we extend the credit risk calculation to a portfolio containing two bonds and introduce the notion of *correlations,* which will be central to our treatment of risk at the portfolio level. Again, we illustrate the credit risk calculation for this portfolio with the help of a simple example. This two-bond "portfolio" will serve to illustrate all the methodology we need to calculate credit risk across a portfolio of any size.

- **Chapter 4: Differing exposure types.** For simplicity, we have limited our discussion in the previous two chapters to bonds. In this chapter, we discuss how CreditMetrics addresses other instruments such as: receivables, loans, commitments to lend, financial letters of credit, swaps and forwards. We emphasize that our risk modeling framework is general, and we discuss the data necessary to extend it to other exposure types.

Chapter 1. Introduction to CreditMetrics

CreditMetrics is a tool for assessing portfolio risk due to changes in debt value caused by changes in obligor credit quality. We include changes in value caused not only by possible default events, but also by upgrades and downgrades in credit quality. Also, we assess the value-at-risk (VaR) – the volatility of value – not just the expected losses. Importantly, we assess risk within the full context of a portfolio. We address the correlation of credit quality moves across obligors. This allows us to directly calculate the diversification benefits or potential over-concentrations across the portfolio.

For example, suppose we invest in a bond. Credit risk arises because the bond's value in one year can vary depending on the credit quality of its issuer. In general, we know that the value of this bond will decline with a downgrade or default of its issuer – and appreciate if the credit quality of the obligor improves. Value changes will be relatively small with minor up(down)grades, but could be substantial – 50% to 90% are common – if there is a default. This is far from the more *normally distributed* market risks that VaR models typically address.

In this chapter, we step through our CreditMetrics methodology and data in a survey fashion to give the broad picture of what we hope to achieve. Specifically, we will:

- establish the link between the process of credit quality migration and the resulting changes in debt value;
- illustrate the resulting risk assessment with the simple example of a single bond;
- discuss the benefits and challenges to a portfolio approach and use a two-bond example to show how we address a full portfolio;
- extend our focus to specific credit instruments other than bonds; and
- summarize the data required for any credit instrument.

The result of our efforts will be measures of value-at-risk due to credit quality changes. These measures will assist in the evaluation, deployment and management of credit risk taking across both a portfolio and marginal transactions. These measures are consistent with the – perhaps more familiar – value-at-risk models which are used for market risks.

1.1 The portfolio context of credit

Credit risk has become perhaps the key risk management challenge of the late 1990s. Globally, institutions are taking on an increasing amount of credit risk. As credit exposures have multiplied, the need for more sophisticated risk management techniques for credit risk has also increased.

Of course, credit risk can be managed – as it has been – by more rigorous enforcement of traditional credit processes such as stringent underwriting standards, limit enforcement and counterparty monitoring. However, risk managers are increasingly seeking to quantify and integrate the overall credit risk assessment within a VaR statement which captures exposure to market, rating change, and default risks.

In the end, a better understanding of the credit portfolio will help portfolio managers to better identify pockets of concentration and opportunities for diversification. Over time, positions can be taken to best utilize risk-taking capacity – which is a scarce and costly resource. Managers can then make risk versus return trade-offs with knowledge of not only the expected credit losses, but also the uncertainty of loss.

1.1.1 The need for a portfolio approach

The primary reason to have a quantitative portfolio approach to credit risk management is so that we can more systematically address *concentration risk*. Concentration risk refers to additional portfolio risk resulting from increased exposure to one obligor or groups of correlated obligors (e.g., by industry, by location, etc.).

Traditionally, portfolio managers have relied on a qualitative feel for the concentration risk in their credit portfolios. Intuitive – but arbitrary – exposure-based credit limits have been the primary defense against unacceptable concentrations of credit risk. However, fixed exposure limits do not recognize the relationship between risk and return.

A more quantitative approach such as that presented here allows a portfolio manager to state credit lines and limits in units of marginal portfolio volatility. Furthermore, such a model creates a framework within which to consider concentrations along almost any dimension (industry, sector, country, instrument type, etc.).

Another important reason to take a portfolio view of credit risk is to more rationally and accountably address portfolio diversification. The decision to take on ever higher exposure to an obligor will meet ever higher marginal risk – risk that grows geometrically with the concentration on that name. Conversely, similar additional exposure to an equally rated obligor who has relatively little existing exposure will entail less risk. Indeed, such names may be individually risky, but offer a relatively small marginal contribution to overall portfolio risk due to diversification benefits.

Finally, by capturing portfolio effects (diversification benefits and concentration risks) and recognizing that risk accelerates with declining credit quality, a portfolio credit risk methodology can be the foundation for a rational risk-based capital allocation process.[1]

There are also more practical reasons for a more quantitative approach to credit risk:

- Financial products have become more complex. The growth of derivatives activity has created uncertain and dynamic counterparty exposures that are significantly more challenging to manage than the static exposures of more traditional instruments such as bonds or loans. End-users and providers of these instruments need to identify such exposures and understand their credit, as well as related market, risks.

[1] A capital measure reflecting these economic factors is a fundamental departure from the capital adequacy measures mandated for bank regulation by the Bank for International Settlements ("BIS"). The BIS risk-based capital guidelines do not distinguish high quality and well-diversified portfolios from low quality and concentrated portfolios. Some bank regulators, recognizing that the BIS regulatory capital regime can create uneconomic decision incentives and misleading presentation of the level of a bank's risk, are increasingly looking to internal economic models for a better understanding of a bank's credit risk.

- The proliferation of credit enhancement mechanisms: third-party guarantees, posted collateral, margin arrangements, and netting, makes it increasingly necessary to assess credit risk at the portfolio level as well as at the individual exposure level.

- Improved liquidity in secondary cash markets and the emergence of credit derivatives have made possible more active management of credit risk based on rational pricing. Proper due diligence standards require that institutions thoroughly review existing risks before hedging or trading them.

- Innovative new credit instruments explicitly derive value from correlation estimates, or credit events such as upgrades, downgrades or default. We can best understand these in the context of a portfolio model that also explicitly accounts for credit quality migrations.

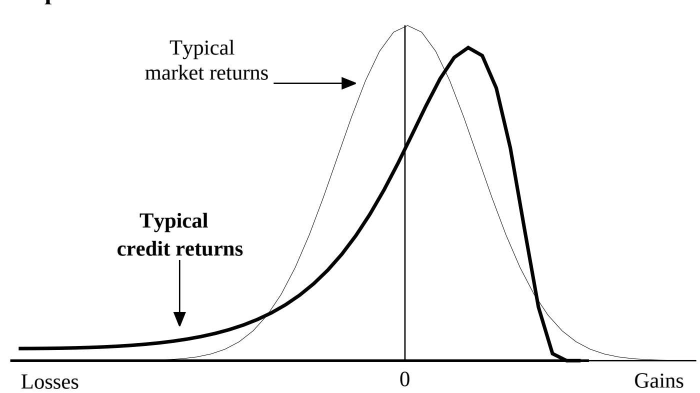
Above, we discussed why a *portfolio* approach to credit risk is necessary. In the following section, we discuss why estimating portfolio credit risk is a much harder problem than estimating portfolio market risk.

1.1.2 Challenges in estimating portfolio credit risk

Modeling portfolio risk in credit portfolios is neither analytically nor practically easy. For instance, modern portfolio theory has taken enormous strides in its application to equity price risks. However, fundamental differences between credit risks and equity price risks make equity portfolio theory problematic when applied to credit portfolios. There are two problems.

The first problem is that equity returns are relatively symmetric and are well approximated by normal or Gaussian distributions. Thus, the two statistical measures – mean (average) and standard deviation of portfolio value – are sufficient to help us understand market risk and quantify percentile levels for equity portfolios. In contrast, credit returns are highly skewed and fat-tailed (see *Chart 1.1)*. Thus, we need more than just the mean and standard deviation to fully understand a credit portfolio's distribution.

Chart 1.1
Comparison of distribution of credit returns and market returns



This long downside tail of the distribution of credit returns is caused by defaults. Credit returns are characterized by a fairly large likelihood of earning a (relatively) small profit through net interest earnings (NIE), coupled with a (relatively) small chance of losing a fairly large amount of investment. Across a large portfolio, there is likely to be a blend of these two forces creating the smooth but skewed distribution shape above.

The second problem is the difficulty of modeling correlations. For equities, the correlations can be directly estimated by observing high-frequency liquid market prices. For credit quality, the lack of data makes it difficult to estimate any type of credit correlation directly from history. Potential remedies include either: (i) assuming that credit correlations are uniform across the portfolio, or (ii) proposing a model to capture credit quality correlations that has more readily estimated parameters.

In summary, measuring risk across a credit portfolio is as necessary as it is difficult. With the CreditMetrics methodology, we intend to address much of this difficulty.

1.2 Types of risks modeled

A distinction is often drawn between “market” and “credit” risk. But increasingly, the distinction is not always clear (e.g., volatility of credit exposure due to FX moves). The first step, then, is to state exactly what risks we will be treating.

CreditMetrics estimates portfolio risk due to credit *events*. In other words, it measures the uncertainty in the forward value of the portfolio at the risk horizon caused by the possibility of obligor credit quality changes – both up(down)grades and default.

In addition, CreditMetrics allows us to capture certain market risk components in our risk estimates. These include the market-driven volatility of credit exposures like swaps, forwards, and to a lesser extent, bonds. For these instruments, volatility of value due to credit quality changes is increased by this further volatility of credit exposure.[2] Typically, market volatilities are estimated over a daily or monthly risk horizon. However, since credit is generally viewed over a larger horizon, market-driven exposure estimates should match the longer credit risk horizon.

1.3 Modeling the distribution of portfolio value

In this section, we begin to introduce some key modeling components: specification of which rating categories[3] to employ, probabilities of migrations between these categories, revaluation upon an up(down)grade, and valuation in default.

For this section, we will be satisfied to obtain the distribution of outcomes; we will leave until *Section 1.4* the calculation of standard deviations and percentile levels.

[2] As a matter of implementation, the estimation of market-driven exposure is performed in a J.P. Morgan software product called FourFifteen™ which uses RiskMetrics data sets of market volatility and correlation to analyze market risk. However, the software implementation of CreditMetrics, CreditManager™, can accept market-driven exposures from any source.

[3] By “rating categories”, we mean any grouping of firms of similar credit quality. This includes, but is in no way limited to, the categories used by rating agencies. Groups of firms which KMV has assigned similar expected default frequencies could just as easily be used as “rating categories.”

1.3.1 Obtaining a distribution of values for a single bond

To begin, let us use S&P's rating categories. Consider a single BBB rated bond which matures in five years. For the purposes of this example, we make two choices. The first is to utilize the Standard & Poor's rating categories and corresponding transition matrices.[4] The second is to compute risk over a one year horizon. Of course, other risk horizons may certainly be appropriate. Refer to *Section 2.5* for a discussion of how to choose a risk horizon.

Our risk horizon is one year; therefore we are interested in characterizing the range of values that the bond can take at the end of that period. Let us first list all possible credit outcomes that can occur at the end of the year due to credit events:

- the issuer stays at BBB at the end of the year;
- the issuer migrates up to AAA, AA, or A or down to BB, B, or CCC; or
- the issuer defaults.

Each outcome above has a different likelihood or probability of occurring. We derive these from historical rating data, which we will discuss at the end of the chapter. For now, we assume that the probabilities are known. That is, for a bond starting out as BBB, we know precisely the probabilities that this bond will end up in one of the seven rating categories (AAA through CCC) or defaults at the end of one year. These probabilities are shown in *Table 1.1*.

Table 1.1
Probability of credit rating migrations in one year for a BBB

Year-end rating	Probability (%)
AAA	0.02
AA	0.33
A	5.95
BBB	86.93
BB	5.30
B	1.17
CCC	0.12
Default	0.18

Note that there is a 86.93% likelihood that the bond stays at the original rating of BBB. There is a smaller likelihood of a rating change (e.g., 5.95% for a rating change to single-A), and a 0.18% likelihood of default.

So far we have specified: (i) each possible outcome for the bond's year-end rating, and (ii) the probabilities of each outcome. Now we must obtain the value of the bond under

[4] Throughout Part I, we will consistently follow one set of credit quality migration likelihoods to aid clarity of exposition. This set of migration likelihoods happens to be taken from Standard & Poor's. There are however a variety of data providers and we in no way wish to give the impression that we endorse one over any other.

each of the possible rating scenarios. What value will the bond have at year-end if it is upgraded to single-A? If it is downgraded to BB?

To answer these questions, we must find the new present value of the bond's remaining cash flows at its new rating. The discount rate that enters this present value calculation is read from the forward zero curve that extends from the end of the risk horizon to the maturity of the bond. This zero curve is different for each forward rating category.

To illustrate, consider our five-year BBB bond. Say the face value of this bond is $100 and the coupon rate is 6%. We want to find the value of the bond at year-end if it upgrades to single-A. Assuming annual coupons for our example, at the end of one year we receive a coupon payment of $6 from holding the bond. Four coupon payments ($6 each) remain, as well as the principal payment of $100 at maturity.

To obtain the value of the bond assuming an upgrade to single-A, we discount these five cash flows (four coupons and one principal) with interest rates derived from the forward zero single-A curve. We leave aside the details of this calculation until the next chapter. Here we just note that the calculations result in the following values at year-end across all possible rating categories.

In *Table 1.2*, in the non-default state, we show the coupon payment received, the forward bond value, and the total value of the bond (sometimes termed the *dirty price* of the bond). In the default state, the total value is due to a *recovery rate* (51.13% in this example), which we discuss in detail in the next chapter. Note that as expected, the value of the bond increases if there is a rating upgrade. Conversely, the value decreases upon rating downgrade or default. There is also a rise in value as the BBB remains BBB which is commonly seen when the credit spread curve is upward sloping.

Table 1.2
Calculation of year-end values after credit rating migration from BBB ($)

Rating	Coupon	Forward Value	Total Value
AAA	6.00	103.37	109.37
AA	6.00	103.10	109.19
A	6.00	102.66	108.66
BBB	6.00	101.55	107.55
BB	6.00	96.02	102.02
B	6.00	92.10	98.10
CCC	6.00	77.64	83.64
Default	–	51.13	51.13

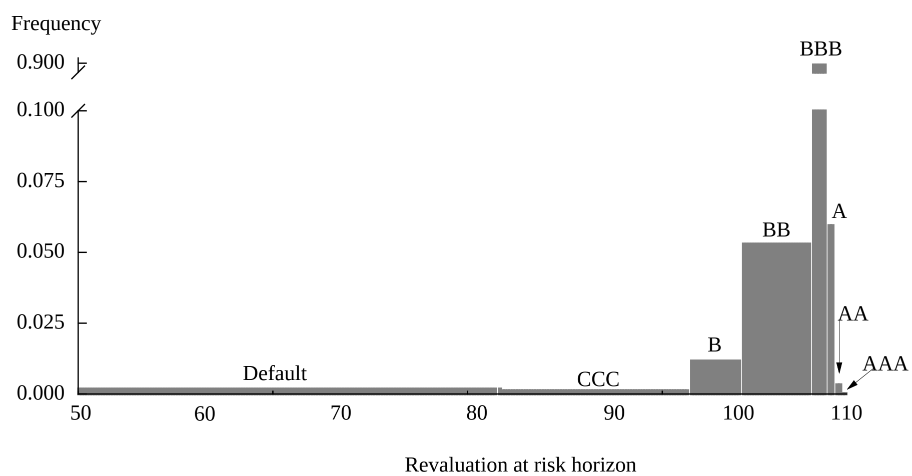
Let us summarize what we have achieved so far. First, we have obtained the probabilities or likelihoods for the original BBB bond to be in any given rating category in one year (*Table 1.1*). Further, we have also obtained the values of the bond in these rating categories (*Table 1.2*). The information in *Tables 1.1* and *1.2* is now used to specify the distribution of value of the bond in one year, as shown in *Table 1.3*.

Table 1.3
Distribution of value of a BBB par bond in one year

Year-end rating	Value ($)	Probability (%)
AAA	109.37	0.02
AA	109.19	0.33
A	108.66	5.95
BBB	107.55	86.93
BB	102.02	5.30
B	98.10	1.17
CCC	83.64	0.12
Default	51.13	0.18

The value distribution is also shown graphically in *Chart 1.2*. Note that in the chart the horizontal axis represents the value, and the vertical axis represents the probability. The distribution of value tells us the possible values the bond can take at year-end, and the probability or likelihood of achieving these numbers.

Chart 1.2
Distribution of value for a 5-year BBB bond in one year



In the next section, we define the credit risk estimate for this value distribution. First, however, we will discuss how we can generalize this probability distribution to a portfolio with more than just one instrument.

1.3.2 Obtaining a distribution of values for a portfolio of two bonds

So far we have illustrated the treatment of a stand-alone five-year BBB bond. Now we will add a single-A three year bond to this portfolio. This bond pays annual coupons at the rate of 5%. We want to obtain the distribution of values for this two-bond portfolio in one year. Just as in the one-bond case, to characterize the distribution of values, we need to specify the portfolio's possible year-end values and the probability of achieving these values. Now, we already know that the BBB bond can have one of the eight values at year-end, as shown in *Table 1.2*. Similarly, we can calculate the corresponding year-end values for the single-A rated bond. Again, we leave the details of the calculation to the next chapter, but simply state the results in *Table 1.4*.

Table 1.4
Year-end values after credit rating migration from single-A ($)

Year-end rating	Coupon	Forward Value	Total Value
AAA	5.00	101.59	106.59
AA	5.00	101.49	106.49
A	5.00	101.30	106.30
BBB	5.00	100.64	105.64
BB	5.00	98.15	103.15
B	5.00	96.39	101.39
CCC	5.00	73.71	88.71
Default	–	51.13	51.13

Next, we combine the possible values for the individual bonds (*Tables 1.2* and *1.4*) to obtain the year-end values for the portfolio as a whole. Since either of the bonds can have any of eight values in one year as a result of rating migration, the portfolio can take on 64 (that is, 8 • 8) different values. We obtain the portfolio's value at the risk horizon in each of the 64 states by simply adding together the values for the individual bonds.

Thus, as an example, consider the top left cell in *Table 1.5*, which reads 215.96. This cell corresponds to the outcome that both the BBB and single-A bonds upgrade to AAA at the end of the year. From *Table 1.2*, the year-end value of the original BBB bond is $109.37 if it upgrades to AAA. Further, from *Table 1.4*, the year-end value of the original single-A bond is $106.59 if it upgrades to AAA. Thus the portfolio as a whole has a value of $215.96 (= $109.37 + $106.59) in the first of 64 states. By similarly calculating the values of the portfolio in the other states, we obtain the results shown in *Table 1.5*.

Table 1.5
All possible 64 year-end values for a two-bond portfolio ($)

Obligor #1 (BBB)		Obligor #2 (single-A)							
		AAA	AA	A	BBB	BB	B	CCC	Default
		106.59	106.49	106.30	105.64	103.15	101.39	88.71	51.13
AAA	109.37	215.96	215.86	215.67	215.01	212.52	210.76	198.08	160.50
AA	109.19	215.78	215.68	215.49	214.83	212.34	210.58	197.90	160.32
A	108.66	215.25	215.15	214.96	214.30	211.81	210.05	197.37	159.79
BBB	107.55	214.14	214.04	213.85	213.19	210.70	208.94	196.26	158.68
BB	102.02	208.61	208.51	208.33	207.66	205.17	203.41	190.73	153.15
B	98.10	204.69	204.59	204.40	203.74	201.25	199.49	186.81	149.23
CCC	83.64	190.23	190.13	189.94	189.28	186.79	185.03	172.35	134.77
Default	51.13	157.72	157.62	157.43	156.77	154.28	152.52	139.84	102.26

So *Table 1.5* shows the portfolio taking on 64 possible values at the end of a year depending on the credit rating migration of the two bonds. These values range from $102.26 (when both bonds default) to $215.96 (when both bonds are upgraded to AAA).

We have illustrated the different possible values for the portfolio at the end of the year. To obtain the portfolio value distribution, the remaining piece in the puzzle is the likelihood or probability of observing these values. So we must estimate the likelihood of observing each of the 64 states of *Table 1.5* in one year.

Those 64 “joint” likelihoods must reconsile with each set of eight likelihoods which we have seen for the bonds on a stand-alone basis. In *Table 1.1* we showed the eight likelihoods for the BBB bond to be in each rating category in one year. Similarly, the corresponding likelihoods for the single-A rated bond are displayed in *Table 1.6*.

Table 1.6
Probability of credit rating migrations in one year for a single-A

Year-end rating	Probability (%)
AAA	0.09
AA	2.27
A	91.05
BBB	5.52
BB	0.74
B	0.60
CCC	0.01
Default	0.06

Again, we derive these likelihoods from historical rating data. We will briefly touch on the data issues at the end of the chapter, but leave aside the details for later (see *Chapter 6)*. Here we just note the numbers as they are given to us.

We must now estimate the 64 joint likelihoods[5] so that we can calculate the volatility of value in our two-bond example. These joint likelihoods must satisfy the constraint of summing to the stand-alone likelihoods in *Tables 1.1* and *1.6*. While doing this, we can also specify that they reflect some desired correlation (i.e., a correlation equal to 0.0).

This is simple if the rating outcomes on the two bonds are independent of each other. In this case the joint likelihood is simply a product of the individual likelihoods from *Tables 1.1* and *1.6*. Thus, for example, assuming independence, the likelihood that both bonds will maintain their original rating at year-end is simply equal to the product of 86.93% (the probability of BBB bond staying BBB from *Table 1.1*) and 91.05% (the probability of a single-A bond staying as single-A from *Table 1.6*) which is equal to 79.15%.

Unfortunately, this picture is simplistic. In reality, the rating outcomes on the two bonds are not independent of each other, because they are affected at least in part by the same macro-economic factors. Thus, it is extremely important to account for correlations between rating migrations in an estimation of the risk on a portfolio. We introduce our model for correlations in *Chapter 3* and describe the model in detail in *Chapter 8.*

[5] By joint likelihoods, we mean the chance that the two obligors undergo a given pair of rating migrations, for example, the first obligor downgrades to BB while the second obligor remains at A.

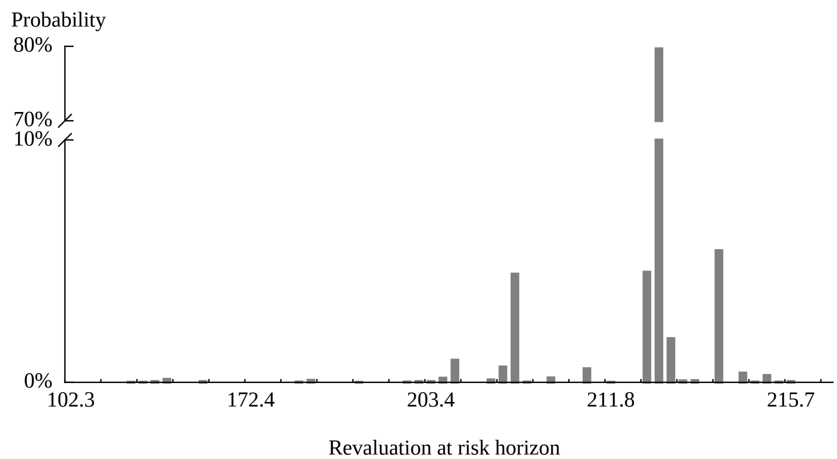
Here, we simply assume a correlation equal to 0.3 and take the resulting joint likelihoods as given. For example, for the case mentioned above where the two bonds maintain their original ratings at the end of the year, the actual joint likelihood value is 79.69%. For other portfolio states, the joint likelihood values are as shown in *Table 1.7* below.

Table 1.7
Year-end joint likelihoods (probabilities) across 64 different states (%)

Obligor #1 (BBB)		**Obligor #2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	Default
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.12	0.00	0.00	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

We now have all the data with which to specify the portfolio value distribution. Specifically, from *Table 1.5* we know all the different 64 values that the portfolio can have at the end of a year. From *Table 1.7* we know the likelihoods of achieving each of these 64 values. By plotting the likelihoods in *Table 1.7* and the values in *Table 1.5* on the same graph, we obtain the portfolio value distribution shown in *Chart 1.3*.

Chart 1.3
Distribution of value for a portfolio of two bonds



1.3.3 Obtaining a distribution of values for a portfolio of more than two bonds

In our examples of one and two bond portfolios, we have been able to specify the entire distribution of values for the portfolio. We remark that this becomes inconvenient, and finally impossible, to do this in practice as the size of the portfolio grows. Noting that for a three asset portfolio, there are 512 (that is, 8 times 8 times 8) possible joint rating states. For a five asset portfolio, this number jumps to 32,768, and in general, for a portfolio with N assets, there are 8^N possible joint rating states.

Because of this exponential growth in the complexity of the portfolio distribution, for larger portfolios, we utilize a simulation approach. A simulation is very much like the preceding example except that outcomes are sampled at random across all the possibile joint rating states. The result of such an approach is an estimate of the portfolio distribution, which for large portfolios looks more like a smooth curve and less like the collections of a few discrete points in *Charts 1.2* and *1.3*.

We remark that it is always possible to compute some portfolio risk measures analytically, regardless of the portfolio size, and discuss these in the following section.

1.4 Different credit risk measures

CreditMetrics can calculate two measures commonly used in risk literature to characterize the credit risk inherent in a portfolio: *standard deviation* and *percentile level*. Both measures reflect the portfolio value distribution and aid in quantifying credit risk. Neither is “best.” They both contribute to our understanding of the risk.

We emphasize that the credit risk model underlying both of these risk measures is the same. Therefore, the two risk measures reflect potential losses from the same portfolio distribution. However, they are different measures of credit risk.

The credit risk in a portfolio arises because there is variability in the value of the portfolio due to credit quality changes. Therefore, we expect any credit risk measure to reflect this variability. Loosely speaking, the greater the dispersion in the range of possible values, the greater the absolute amount at credit risk. With this background, we next provide two alternative measures of credit risk that we use in CreditMetrics.

1.4.1 Credit risk measure #1: standard deviation

The *standard deviation* is a symmetric measure of dispersion around the average portfolio value. The greater the dispersion around the average value, the larger the standard deviation, and the greater the risk. If the portfolio values are expressed in dollars, this standard deviation calculation also results in a dollar amount.

To illustrate the standard deviation calculation, we again refer to our two-bond portfolio. For this portfolio, the likelihoods of each state are shown in *Table 1.7*, and the values corresponding to these states are displayed in *Table 1.5*. To calculate the standard deviation, we first must obtain the *mean* value for the portfolio. This is obtained by multiplying the values with the corresponding probabilities and then adding the resulting values. Mathematically, the average value can be written as:

$$[1.1] \qquad Mean = p_1 \cdot V_1 + p_2 \cdot V_2 + \ldots + p_{64} \cdot V_{64}$$

where p_1 refers to the probability or likelihood of being in State 1 at the end of the risk horizon, and V_1 refers to the value in State 1.

Performing this simple calculation for our portfolio with the data from *Table 1.5* and *Table 1.7*, we find that the average value for the portfolio is $213.63. Now the standard deviation, which measures the dispersion between each potential migration value (V's) and this average value, is calculated as:

[1.2]

$$(\text{Standard Deviation})^2 = p_1 \cdot (V_1 - \text{Mean})^2 + p_2 \cdot (V_2 - \text{Mean})^2 + \ldots + p_{64} \cdot (V_{64} - \text{Mean})^2$$

Note that the above expression yields the squared standard deviation value, which is also known as the "variance." The square-root of this value is the standard deviation. The individual terms in the expression are of the form ($V_i - \textit{Average}$). This is consistent with our earlier comment that the standard deviation measures the dispersion of the individual values around the average value. Carrying out the above calculation for our example portfolio, we find that the portfolio standard deviation is $3.35.

The interpretation of standard deviation is difficult here because credit risk is not normally distributed. Thus, it is not possible to look up distribution probabilities in a normal table. The distribution of credit value is likely to have a long tail on the "loss" side and limited "gains" (see *Chart 1.1*). The length of this downside tail could be characterized by its length in standard deviations. For instance, the 99% tail is 1.70 standard deviation below the average (the 99.75% tail is 7.90 standard deviation below the average). By comparison, these distances for a normal distribution are 2.33 and 2.81 standard deviations respectively.

Because the standard deviation statistic is a symmetric measure of dispersion, it does not itself distinguish in our example between the gains side versus the losses side of the distribution. It cannot, for instance, distinguish in our example that the maximum upside value is only 0.70 standard deviations above the average while the maximum downside value is 33.25 standard deviations below the average.

To calculate the standard deviation we do not have to specify the entire distribution of portfolio values. Rather, we can operate pairwise across all pairs in the portfolio. We discuss this pairwise calculation in the remainder of this section. For now, simply note that since we do not have to rely on simulation to obtain the distribution of portfolio values, the standard deviation calculation is computationally simple and efficient.

1.4.2 Credit risk measure #2: percentile level

We define this second measure of risk as a specified *percentile level* of the portfolio value distribution. The interpretation of the percentile level is much simpler than the standard deviation: the lowest value that the portfolio will achieve 1% of the time is the 1st percentile.

Therefore, once we have calculated the 1st percentile level, the likelihood that the actual portfolio value is less than this number is only 1%. Thus the 1st percentile level number provides us with a probabilistic lower bound on the year-end portfolio value. Of course, there is no particular percentile level that is "best" (5%, 1%, 0.5%, etc.). The particular level used is the choice of the portfolio manager, and depends mostly on how the risk measure will be applied.

For normal distributions (or any other known distribution which is completely characterized by its mean and standard deviation), it is possible to calculate percentile levels from knowledge of the standard deviation. Unfortunately, normal distributions are mostly a characteristic of market risk.[6] In contrast, credit risk distributions are not typically symmetrical or bell-shaped. In particular, the distributions display a much fatter lower tail than a standard bell-shaped curve, as illustrated in *Chart 1.1*. Since we cannot assume that credit portfolio distributions are normal, nor can we characterize them according to any other standard distribution (such as the log-normal or Student-t), we must estimate percentile levels via another approach.

To calculate a percentile level, we must first specify the full distribution of portfolio values. For portfolios consisting of more than two exposures, this requires a simulation approach, which may be time-consuming. Our approach will be to generate possible portfolio scenarios at random according to a Monte Carlo framework. While the generation of scenarios may be time consuming, once we obtain these scenarios, the calculation of the 1^{st} percentile level is simple. To do this, we first sort the portfolio values in ascending order. Given these sorted values, the 1^{st} percentile level is the one below which there are exactly 1% of the total values. So if the simulation generates 10,000 portfolio values, the 1^{st} percentile level is the 100th largest among these.

Percentile levels may have more meaning for portfolios with many exposures, where the portfolio can take on many possible values. We may still consider our example portfolio with two bonds, however. For this portfolio, we estimate the 1^{st} percentile to be $204.40. Note that this amount is $9.23 (= $213.63 – $204.40) less than the mean portfolio value. Thus, using the 1^{st} percentile, we estimate the amount at credit risk to be $9.23, while using (one) standard deviation, we estimate this value at $3.35. Thus we see that the two measures give different values and so must be interpreted differently.

These different computational requirements introduce a trade-off between using the standard deviation and using the percentile level. The percentile level is intuitively appealing to use, because we know precisely what the likelihood is that the portfolio value will fall below this number. On the other hand, it is often much faster to compute the standard deviation. Users should evaluate this trade-off carefully and use the risk measure that best fits their purpose. Further discussion of this issue is presented in *Chapter 12*.

1.5 Exposure type differences

Up to this point our examples have used bonds, but the concepts that we have described in this chapter are equally applicable to other exposure types. The other exposure types we consider are receivables, loans, commitments to lend, financial letters of credit and market-driven instruments such as swaps and forwards.

Recall from *Section 1.3* that we derive both of our credit risk measures from the portfolio value distribution. Two components characterize this distribution. The first is the likelihood of being in any possible portfolio state. The second is the value of the portfolio in each of the possible states. Only the calculation of future values is different for different instrument categories. The likelihoods of being in each credit quality state are the same for all instrument categories since these are tagged to the obligor rather than to each of its

[6] See, for example, *RiskMetrics™—Technical Document,* 4th Edition*, 1996*.

obligations. In the remainder of this section, we briefly discuss the different exposure types in CreditMetrics. We provide a more detailed treatment in *Chapter 4*.

1.5.1 Receivables

Many commercial and industrial firms will have credit exposure to their customers through receivables, or trade credit. We suggest that the risk in such exposures be addressed within this same framework. It will commonly be the case that receivable will have a "maturity" which is shorter than the risk horizon (e.g., one year or less). This would simplify matters in that there would be no need to revalue the exposure upon up(down)grade. But even if revaluing is necessary, the credit risk is – in concept – no different than the risk in a comparable bond issued to the customer, and so it can be revalued accordingly.

1.5.2 Bonds and loans

For bonds, as we discussed in *Section 1.3*, the value at the end of the risk horizon is the present value of the remaining cash flows. These cash flows consist of the remaining coupon payments and the principal payment at maturity. To discount the cash flows, we use the discount rates derived from the forward zero curve for each specific rating category. This forward curve is calculated as of the end of the risk horizon.

We treat loans in the same manner as bonds, revaluing in each future rating state by discounting future cash flows. This revaluation accounts for the change in the value of a loan which results from the likelihood changing that the loan will be repaid fully.

1.5.3 Commitments

A loan commitment is a facility which gives the obligor the option to borrow at his own discretion. In practice, this essentially means both a loan (equal to the amount currently drawn on the line) and an option to increase the amount of the loan up to the face amount of the facility. The counterparty pays interest on the drawn amount, and a fee on the undrawn amount in return for the option to draw down further. For these exposures three factors influence the revaluation in future rating states:

- the amount currently drawn;
- expected changes in the amount drawn *that are due to credit rating changes*; and
- the spreads and fees needed to revalue both the drawn and undrawn portions.

All of these factors may be affected by covenants specific to a particular commitment. The details of commitment revaluation and typical covenants are discussed in *Section 4.3*.

1.5.4 Financial letters of credit

A financial or stand-by letter of credit is treated as an off balance sheet item until it is actually drawn. When it is drawn down its accounting treatment is just like a loan. However, the obligor can draw down at his discretion and the lending institution typically has no way to prevent a drawdown even during a period of obligor credit distress. Thus, for risk assessment purposes, we argue that the full nominal amount should be considered "exposed." This means that we suggest treating a financial letter of credit – whether or not any portion is actually drawn – exactly as a loan.

Note that there are other types of letters of credit which may be either securitised by a specific asset or project or triggered only by some infrequent event. The unique features of these types of letters of credit are not currently addressable within the current specification of CreditMetrics.

1.5.5 Market-driven instruments

For instruments whose credit exposure depends on the moves of underlying market rates, such as swaps and forwards, revaluation at future rating states is more difficult. The complexity for these instruments comes from the fact that if a swap, for example, is marked to market and is currently out-of-the-money to us, then a default by the counterparty does not influence the swap's value, since we will still make the payments we owe on the swap.[7] On the other hand, if the swap is in-the-money to us, then we expect payments, and do not receive the full amount in the case of a counterparty default. So in general, the credit exposure at any time to a market-driven instrument is the maximum of the transaction's net present value or zero.

The methodology we propose for market-driven instruments is applicable to single instruments, such as swaps or forwards, or to groups of swaps, forwards, bonds, or other instruments whose exposures can be netted. Thus, any set of cash flows which are settled together (typically, these will all be exposures to the same counterparty) can be considered as one market-driven instrument.

In cases of default, we estimate the future value of market-driven instruments using the expected exposure of the instrument at the risk horizon. This expected exposure depends both on the current market rates and their volatilities. In non-default states, the revaluation consists of two parts: the present value of future cashflows, and the amount we might lose if the counterparty defaults at some future time. The second part, the expected loss, depends on the average market-driven exposure over the remaining life of the instrument (which is estimated in a similar fashion to the expected exposure mentioned above), the probability that the counterparty will default over the same time (which is determined by the credit rating at the risk horizon), and the recovery rate in default.

Details of this methodology and a discussion of the various exposure calculations appear in *Section 4.5*.

[7] The exact settlement will depend on the covenants particular to the swap, but this is a reasonable assumption for explanatory purposes.

1.6 Data issues

Given a choice of which rating system (that is, what groupings of similar credits) will be used, CreditMetrics requires three types of data:

- likelihoods of credit quality migration, including default likelihoods;
- likelihoods of joint credit quality migration; and
- valuation estimates (e.g. bonds revalued at forward spreads) at the risk horizon given a credit quality migration.

Together, these data types result in the portfolio value distribution, which determines the absolute amount at risk due to credit quality changes.

1.6.1 Data required for credit migration likelihoods

We showed these individual likelihoods for BBB and single-A rating separately in *Tables 1.1* and *1.6* respectively, but this information is more compactly represented in matrix form as shown below in *Table 1.8*. We call this table a *transition matrix.* The ratings in the first column are the starting or current ratings. The ratings in the first row are the ratings at the risk horizon. For example, the likelihoods in *Table 1.8* corresponding to an initial rating of BBB are represented by the BBB row in the matrix. Further, note that each row of the matrix sums to 100%.

Table 1.8
One-year transition matrix (%)

Initial rating	**Rating at year-end (%)**							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (15 April 96)

Transition matrices can be calculated by observing the historical pattern of rating change and default. They have been published by S&P and Moody's rating agencies, and can be computed based on KMV's studies, but any provider's matrix is welcome and usable within CreditMetrics.[8] The transition matrix should, however, be estimated for the same time interval as the risk horizon over which we are interested in estimating risks. For instance, a semi-annual risk horizon would use a semi-annual rather than one-year transition matrix.

[8] As we discuss later in *Chapter 6*, adjustments due to limited historical data may sometimes be desirable.

1.6.2 Data required for joint likelihood calculations

Individual likelihoods are just one component of the portfolio joint likelihood. Earlier we stated that the joint likelihood is not simply the product of the individual likelihoods. This is because using the product as joint likelihood implicitly assumes that the pairwise rating outcomes are *independent* of each other, which is generally not true.

Therefore, to have joint likelihoods, we need to do one of two things. First, we may historically tabulate joint credit rating moves just as we historically tabulated single credit rating moves in the transition matrix. Second, we may propose a model for how the credit ratings of multiple names evolve together, and estimate the requisite correlation parameters for the model. We discuss several approaches to estimating joint likelihoods in *Chapter 8.*

1.6.3 Data required for portfolio value calculation

Each instrument type requires sufficient data to calculate the change in value for each possible credit quality migration. These have been detailed in *Section 1.5*. In general, there are three generic types:

1. Coupon rate and term of maturity are required for: receivables, loans, letters of credit, and bonds in order to revalue them.
2. In addition to *(1)* above, we require the drawn and undrawn portions for a loan commitment and the spread/fees for both portions.
3. Market-driven instruments, including swaps, forwards, and to a lesser extent bonds, require an examination of exposures which is detailed in *Section 4.5*.

It is interesting to note that an obligor's exposures across instruments can be estimated on a netted basis.

1.7 Advanced modeling features

CreditMetrics incorporates provisions to model additional parameters that make the credit risk estimate more precise. One such provision is for cases such as swaps and forwards, where the amount subject to credit risk is itself driven by market rates. Thus, not only is it uncertain in these cases whether a counterparty will default or experience a change in credit quality, but it is also uncertain what the loss will be in the event of a default. Estimates of the exposures in these cases rely on market rates and volatilities. Thus, as mentioned before, our software implementation of CreditMetrics, CreditManager™, takes market-driven exposures as an import from an external source, such as the current version of J.P. Morgan's FourFifteen™.

Separately, recoveries in the event of default are notoriously uncertain. Thus, we allow for the treatment of recoveries as random quantities. We present mean and standard deviation estimates for recoveries in *Chapter 7*. Standard deviation estimates of recovery value also are available from public research; these are provided in the CreditMetrics data set.

Chapter 2. Stand-alone risk calculation

This chapter illustrates the methodology used by CreditMetrics for calculating the credit risk for a single or stand-alone exposure. In *Chapter 1*, we summarized this methodology with the help of a BBB rated bond. Here, we discuss in detail each of the steps outlined in *Chapter 1*, using the same BBB example for illustration. Specifically:

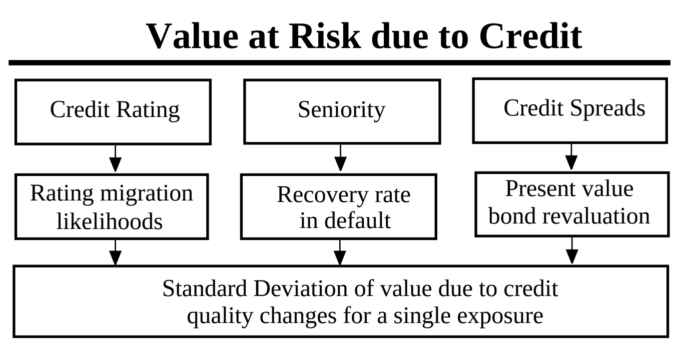
- we describe an individual obligor and how his credit rating implies both a default likelihood and the likelihoods for possible credit quality migrations;
- we describe a credit exposure and how its seniority standing implies a loss rate (that is, loss in the event of default);
- we describe credit spreads over the default free yield and their implication for the bond value upon up(down)grade in credit quality; and
- we assemble all of these pieces to estimate volatility of value due to credit quality changes.

2.1 Overview: Risk for a stand-alone exposure

There are three steps to calculating the credit risk for a “portfolio” of one bond, as illustrated in *Chart 2.1* below:

- *Step 1:* The senior unsecured credit rating of the bond’s issuer determines the chance of the bond either defaulting or migrating to any possible credit quality state at the risk horizon.
- *Step 2:* The seniority of the bond determines its recovery rate in the case of default. The forward zero curve for each credit rating category determines the value of the bond upon up(down)grade. Both of these aid revaluation of the bond.
- *Step 3:* The likelihoods from Step 1 and the values from Step 2 then combine in our calculation of volatility of value due to credit quality changes.

Chart 2.1
Our first “road map” of the analytics within CreditMetrics

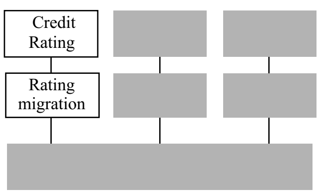


Readers who are familiar with RiskMetrics will see that the framework for credit risk shown above is different from the market risk framework. This is because the quality and availability of credit data are generally much different. Therefore, we *construct* what we cannot directly *observe*. In the process, we model the mechanisms of changes in value rather than try to observe value changes.

In the following sections, we detail each step used in CreditMetrics to quantify the risk of a stand-alone exposure. We illustrate these steps with our senior unsecured 5-year BBB rated bond. This bond pays an annual coupon at the rate of 6%. We also include this example in the CHAP01.XLS Excel spreadsheet, which is available on our web site location (*http://www.jpmorgan.com*).

The calculations performed in this chapter assume a risk horizon of one year. This choice is somewhat arbitrary. However, at the end of this chapter we discuss some of the issues surrounding the choice of this risk horizon.

2.2 Step #1: Credit rating migration



In our model, risk comes not only from default but also from changes in value due to up(down)grades. Thus, it is important for us to estimate not only the likelihood of default but also the chance of *migrating* to any possible credit quality state at the risk horizon. So we view default as just one of several “states of the world” that may exist for this credit one period from now.

The likelihood of any credit rating migration in the coming period is conditioned on the senior unsecured credit rating of the obligor.[1] *Chart 2.2* shows the credit quality migration likelihoods for obligors currently rated A, AAA, and BBB. For our BBB bond, the rightmost diagram is applicable.

Chart 2.2
Examples of credit quality migrations (one-year risk horizon)

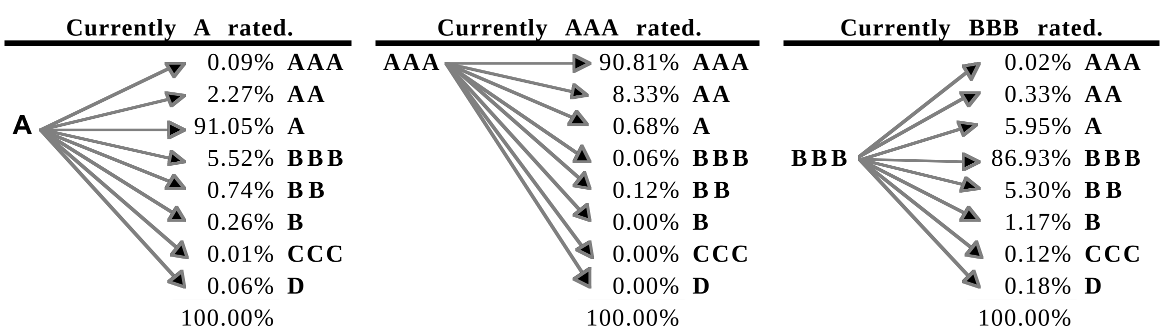


Chart 2.2 says, for example, that there is a 5.30% chance that a BBB rated credit will downgrade to a BB rating within one year. There are several common patterns among the three examples. Intuitively, we see that the most likely credit rating one year from now is the current credit rating. The next most likely ratings are one letter grade above or below. The only absolute rule about credit quality migrations is that the likelihoods

[1] There are some academic studies which condition the estimation of default likelihood upon not only the current credit rating but also whether the specific debt issue is new: see for instance Altman [89]. While this is sound and has its applications, we believe that many users will have dealings with established – not just new – obligors.

must sum to 100% since these are all the "states of the world" that are possible. Rather than showing each rating's credit quality migration likelihoods separately, it is often convenient to think of them in a square table, or transition matrix, as shown below in *Table 2.1*.

Table 2.1
One-year transition matrix (%)

Initial Rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (15 April 96)

To read this table, find today's credit rating on the left and follow along that row to the column which represents the rating at the risk horizon. For instance, the leftmost bottom figure of 0.22% says that there is a 0.22% likelihood that a CCC rated credit will migrate to AAA at the end of one year.

We derived the transition matrix in *Table 2.1* from rating migration data published by S&P. Thus the leftmost bottom figure of 0.22% means that 0.22% of the time (over the 15-year history from which this data was tabulated) a CCC-rated credit today migrated to AAA in one year. Of course, migrating from CCC to AAA within one year is highly unusual and likely represents only one instance in the historical data.[2] This presents a practical problem: results based on limited data are subject to estimation errors. Later, in *Chapter 6*, we discuss the anticipated long-term behavior of credit migrations, which would tend to mitigate this estimation noise.

As we have mentioned, it is possible to create transition matrices for any system of grouping similar credits. Again, we refer to these groupings loosely as rating categories. Regardless of how the rating categories are constructed and of how many categories there are, it is necessary to specify the default likelihood for each category, and the likelihoods that firms in one category migrate to any other. In addition, as we will see in the following section, for the purposes of revaluation, it is necessary to provide a credit spread to correspond to each category as well.

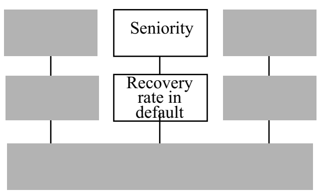
[2] The only adjustment we made to S&P's data was for the "no-longer-rated" migrations. The CCC row in this transition matrix, sourced from S&P, is based upon 561 firm/years worth of observation with 79 occurrences of a transition to "no longer rated." Across all rows in this transition matrix, there are more than 25,000 firm/years worth of observation, with most being in the BBB-to-AA rows.

2.3 Step #2: Valuation

In Step 1, we determined the likelihoods of migration to any possible credit quality states at the risk horizon. In Step 2, we determine the values at the risk horizon for these credit quality states. Value is calculated once for each migration state; thus there are (in this example) eight revaluations in our simple one-bond example.

These eight valuations fall into two categories. First, in the event of a default, we estimate the recovery rate based on the seniority classification of the bond. Second, in the event of up(down)grades, we estimate the change in credit spread that results from the rating migration. We then perform a present value calculation of the bond's remaining cash flows at the new yield to estimate its new value.

2.3.1 Valuation in the state of default



If the credit quality migration is into default, the likely residual value net of recoveries will depend on the seniority class of the debt. In CreditMetrics, we offer several historical studies of this dependence.[3] *Table 2.2* below summarizes the recovery rates in the state of default as reported by one of the available studies.

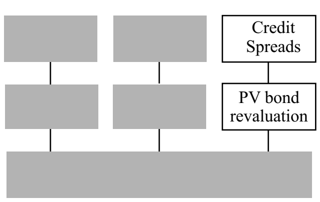
Table 2.2
Recovery rates by seniority class (% of face value, i.e., "par")

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Carty & Lieberman [96a] —Moody's Investors Service

In this table, we show the mean recovery rate (middle column) as well as the standard deviation of the recovery rate (last column). Our example BBB bond is senior unsecured. Therefore, we estimate its mean value in default to be 51.13% of its face value – which in this case we have assumed to be $100. Also from *Table 2.2*, the standard deviation of the recovery rate is 25.45%.

2.3.2 Valuation in the states of up(down)grade



If the credit quality migration is to another letter rating rather than to default, then we must revalue the exposure by other means.

To obtain the values at the risk horizon corresponding to rating up(down)grades, we perform a straightforward present value bond revaluation. This involves the following steps:

[3] There is also a recent study (see Altman & Kishore [96]) which conditions recovery rates on industry participations of the obligor in addition to seniority class.

1. Obtain the forward zero curves for each rating category. These forward curves are stated as of the risk horizon and go to the maturity of the bond.
2. Using these zero curves, revalue the bond's remaining cash flows at the risk horizon for each rating category.

Let us illustrate the above steps with the help of our BBB bond example. Recall that this bond has a five-year maturity, and pays annual coupons at the rate of 6%. Assume that the forward zero curves for each rating category has been given to us. We show an example in *Table 2.3* below.

Table 2.3
Example one-year forward zero curves by credit rating category (%)

Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

First, let us determine the cash flows which result from holding the bond position. Recall that our example bond pays an annual coupon at the rate of 6%. Therefore, assuming a face value of $100, the bond pays $6 each at the end of the next four years. At the end of the fifth year, the bond pays a cash flow of face value plus coupon, which equals $106 in this case.

Now, let us calculate the value *V* of the bond at the end of one year assuming that the bond upgrades to single-A. This calculation is described by the formula below:

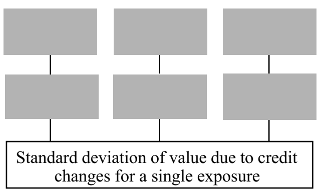
$$[2.1] \qquad V = 6 + \frac{6}{(1+3.72\%)} + \frac{6}{(1+4.32\%)^2} + \frac{6}{(1+4.93\%)^3} + \frac{6}{(1+5.32\%)^4} = 108.66$$

In the above formula, we use the forward zero rates for the single-A rating category from *Table 2.3*. To calculate the value of the bond in a rating category other than single-A, we would substitute the appropriate zero rates from the table. After completing these calculations for different rating categories, we obtain the values in *Table 2.4.*

Table 2.4
Possible one-year forward values for a BBB bond plus coupon

Year-end rating	Value ($)
AAA	109.37
AA	109.19
A	108.66
BBB	107.55
BB	102.02
B	98.10
CCC	83.64
Default	51.13

2.4 Step #3: Credit risk estimation



We now have all the information that we need to estimate the volatility of value due to credit quality changes for this one exposure on a stand-alone basis. That is, we know the likelihood of all possible outcomes – all up(down)grades plus default – and the distribution of value within each outcome. These likelihoods and values, which we obtain from Steps 1 and 2 respectively, are shown in *Table 2.5* below.

Table 2.5
Calculating volatility in value due to credit quality changes

Year-end rating	Probability of state (%)	New bond value plus coupon ($)	Probability weighted value ($)	Difference of value from mean ($)	Probability weighted difference squared
AAA	0.02	109.37	0.02	2.28	0.0010
AA	0.33	109.19	0.36	2.10	0.0146
A	5.95	108.66	6.47	1.57	0.1474
BBB	86.93	107.55	93.49	0.46	0.1853
BB	5.30	102.02	5.41	(5.06)	1.3592
B	1.17	98.10	1.15	(8.99)	0.9446
CCC	0.12	83.64	1.10	(23.45)	0.6598
Default	0.18	51.13	0.09	(55.96)	5.6358
		Mean =	$107.09	Variance =	8.9477
				Standard deviation =	$2.99

The figures in the first two columns – likelihoods of migration and value in each state – have been discussed in *Sections 2.2,* and *2.3,* respectively. Here we use these two columns to calculate the risk estimate.

2.4.1 Calculation of standard deviation as a measure of credit risk

Recall from Chapter 1 that there are two useful measures of credit risk that one can use: standard deviation and percentile level. First we consider the calculation of the standard deviation. For this, we have to first obtain the average value (the mean).

Note that the mean is just the probability-weighted average of the values across all rating categories including default. As shown in *Table 2.5*, this is estimated at $107.09, which includes the $6.00 coupon in all non-default states. The standard deviation then measures the dispersion between the individual values and this mean. After completing the calculations, we observe that the standard deviation of value changes due to credit is $2.99. This (or some scale of this) is one measure of the absolute amount that is at credit risk.

In the above calculation of standard deviation, we used a recovery value of $51.13 for the case of default. (This is the expected recovery rate for a senior unsecured bond from *Table 2.2*.) While discussing the results presented in this table, we pointed out that there is an uncertainty or standard deviation associated with this recovery rate. This uncertainty adds to the overall credit risk of holding the bond position. Before we describe how to account for this recovery rate uncertainty, let us present the standard deviation calculation in a manner that will help us later to incorporate recovery rate uncertainty.

Let p_i be the probability of being in any given state and μ_i be the value within each state (the first and second columns of *Table 2.5* respectively). Given this, we calculate the mean μ and the standard deviation σ using the formulae below:

[2.2]

$$\mu_{Total} = \sum_{i=1}^{s} p_i \mu_i = \begin{pmatrix} 0.02\% \cdot 109.37 + \\ 0.33\% \cdot 109.19 + \\ 5.95\% \cdot 108.66 + \\ 86.93\% \cdot 107.55 + \\ 5.30\% \cdot 102.02 + \\ 1.17\% \cdot 98.10 + \\ 0.12\% \cdot 83.64 + \\ 0.18\% \cdot 51.13 \end{pmatrix} = 107.09$$

$$\sigma_{Total} = \sqrt{\sum_{i=1}^{s} p_i \mu_i^2 - \mu_{Total}^2} = \sqrt{\begin{pmatrix} 0.02\% \cdot 109.37^2 + \\ 0.33\% \cdot 109.19^2 + \\ 5.95\% \cdot 108.66^2 + \\ 86.93\% \cdot 107.55^2 + \\ 5.30\% \cdot 102.02^2 + \\ 1.17\% \cdot 98.10^2 + \\ 0.12\% \cdot 83.64^2 + \\ 0.18\% \cdot 51.13^2 \end{pmatrix} - 107.09^2} = 2.99$$

The above formula is overly simple in that it allows the bond to only take on a mean value within each state. In general, the bond can take on a distribution of values within each state. In particular, there is well documented uncertainty surrounding the recovery rate in default. We incorporate this added uncertainty as follows:

[2.3]

$$\mu_{Total} = \sum_{i=1}^{s} p_i \mu_i \qquad \sigma_{Total} = \sqrt{\sum_{i=1}^{s} p_i(\mu_i^2 + \sigma_i^2) - \mu_{Total}^2}$$

$$= \begin{pmatrix} 0.02\% \cdot 109.37 + \\ 0.33\% \cdot 109.19 + \\ 5.95\% \cdot 108.66 + \\ 86.93\% \cdot 107.55 + \\ 5.30\% \cdot 102.02 + \\ 1.17\% \cdot 98.10 + \\ 0.12\% \cdot 83.64 + \\ 0.18\% \cdot 51.13 \end{pmatrix} \qquad = \sqrt{\begin{pmatrix} 0.02\% \cdot (109.37^2 + 0^2) + \\ 0.33\% \cdot (109.19^2 + 0^2) + \\ 5.95\% \cdot (108.66^2 + 0^2) + \\ 86.93\% \cdot (107.55^2 + 0^2) + \\ 5.30\% \cdot (102.02^2 + 0^2) + \\ 1.17\% \cdot (98.10^2 + 0^2) + \\ 0.12\% \cdot (83.64^2 + 0^2) + \\ 0.18\% \cdot (51.13^2 + 25.45^2) \end{pmatrix} - 107.09^2}$$

$$= 107.09 \qquad = 3.18$$

Note that the expected value or mean calculation remains the same as before. The only difference is in the standard deviation calculation, where we add a component σ_i representing the uncertainty in recovery value in the defaulted state $i = 8$.

For a derivation of this formula for the standard deviation, refer to *Appendix D*. This inclusion of the uncertainty in recovery rates increases the standard deviation from $2.99 to $3.18 (a 6.32% increase).

Finally, note the zero values in the standard deviation formula. These zeros represent the uncertainty of value in the up(down)grade states. Just as there is an uncertainty in value in the default state, we expect an uncertainty in value in the other rating up(down)grade states. This would be caused by the uncertainty of credit spreads within each credit rating category. For now, we have set this credit spread uncertainty to zero since it is unclear what portion of it is systematic versus diversifiable. If we ever have sufficient data to resolve this issue, we hope to allow credit spread volatility in future versions of CreditMetrics.

2.4.2 Calculation of percentile level as a measure of credit risk

Standard deviation is just one of two useful credit risk measures. The other risk measure is the percentile level.

Say we are interested in determining the 1st percentile level for our bond. This is the level below which our portfolio value will fall with probability 1%. Again, 1% is not the only percentile level we advocate for the reader. There are good reasons why different users should use different percentile levels. For the sake of illustration, however, we concentrate on the calculation of the 1st percentile level.

As we have mentioned before, percentile levels are more meaningful statistics for large portfolios, where the portfolio can take on many different portfolio values. It is also the case that for these large portfolios (in fact, for any portfolio with much more than two assets), it is necessary to perform simulations to compute percentile levels. Nonetheless, in order to provide an example, we compute percentile levels for our single bond.

Table 2.5 displays the likelihood that our bond will be in any given credit rating at the risk horizon and the value at each credit rating. We start from the bottom of the table, the state of default, and move upwards towards the AAA rating state. We keep a running total of the likelihoods as we move up. The value at which this running total first becomes equal to or greater than 1% is the 1st percentile level.

Let us go through the procedure: The likelihood of being in the defaulted state is 0.18%. This is less than 1%, so we move up to the CCC state. The combined likelihood of being in default or CCC state is 0.30% (sum of 0.18% and 0.12%). This is also less than 1%; so we move up again, this time to B rating state. The combined likelihood of being in default, CCC, or B is now equal to 2.17% (sum of 0.30% and 1.17%). This now exceeds 1%. We therefore stop here and read off the corresponding value from the B row. This value, which is equal to \$98.10, is the 1st percentile level value. This is \$8.99 below the mean value.

So far we have used an arbitrary risk horizon of one year. Below, we discuss issues surrounding the choice of risk horizon.

2.5 Choosing a time horizon

Much of the academic and credit agency data is stated on an annual basis. This is a convention rather than a requirement. It is important to note, that there is nothing about the CreditMetrics methodology that requires a one-year horizon. Indeed, it is difficult to support the argument than any one particular risk horizon is *best*. Illiquidity, credit relationships, and common lack of credit hedging instruments can all lead to prolonged risk-mitigating actions.

The choice of risk horizon raises two practical questions:

- Should a practitioner use only one risk horizon or many?
- Is there any firm basis for saying that any one particular horizon is best?

2.5.1 Should there be one horizon or many?

The choice of time horizon for risk measurement and risk management is not clear because there is no explicit theory to guide us. However, the one thing that *is* clear is that comparisons between alternatives must be made at the same risk horizon.

Many different security types bear credit risk. One of the common arguments in favor of multiple credit risk horizons is that they allow us to calculate risk at horizons tailored to each credit security type. For instance, it may be that interest rate swaps are more liquid than loans. The managers for each security-type (e.g., loans versus swaps) may wish to see their security type calculated at their own risk horizon. However, the risk estimates for these different subportfolios cannot be aggregated if there is a mismatch in time horizons.

2.5.2 Which horizon might be "best"?

Almost any risk measurement system is better at stating *relative* risk than it is at stating *absolute* risk. Since relative risk measurements will likely drive decisions, the choice of risk horizon is not likely to make an appreciable difference. The key element to any risk information system is the resulting *risk-mitigating actions*; any given risk horizon is likely to lead to the same qualitative decisions.

Although these actions may differ among institutions, the risk horizon is not likely to be significantly less than a quarter for a bank with loans, commitments, financial letters of credit, etc. On the other side, the natural turnover due to the ongoing maturity and reinvestment of positions provides appreciable room for risk-mitigating action even for highly illiquid instruments. Thus, using as a convention a one year risk horizon – not unlike the convention of annualized interest rates – is common.

Even if risk-mitigating actions are performed daily, recalculating risk at a longer horizon can still provide guidance to changes in relative risk. An analogy is driving a car. A car's instrument panel serves perfectly well when reporting speed at kilometers per hour even though driving decisions are made far more often, perhaps every second and every meter. So too, risk stated over the coming year can guide risk-mitigating actions.

2.5.3 Computing credit risk on different horizons

Two CreditMetrics modeling parameters must change to address different risk horizons:

- the credit instrument revaluation formulas change to perform the revaluation computation for the alternate time horizon; and

- the likelihoods of credit quality migration, as shown in the transition matrix, must be restated to the new risk horizon.

One way of doing the latter is simply to multiply the short-horizon transition matrices to obtain the transition matrix for a longer horizon. (For example, a two-year transition matrix could be obtained by multiplying the one-year transition matrix with itself.) Unfortunately, however, this methodology ignores the issue of *autocorrelation* in the credit quality changes over multiple time horizons. A non-zero autocorrelation would indicate that successive credit quality moves are not statistically independent between adjoining periods.

This issue of autocorrelation surfaces for market risk calculations also. For instance, some markets tend to exhibit *mean reversion* (that is, a tendency for prices to return to some long-term stable level), autocorrelation prevents us from translating daily volatilities to monthly or yearly volatilities in a simple way.

Regrettably, the issue of time period interdependencies can also arise for credit quality migrations. For instance, Altman & Kao [92b] find that there is positive autocorrelation in S&P downgrades, so a downgrade implies a higher likelihood of a downgrade in the following period. We confirm this, looking at the S&P rating data, and also find that an upgrade tends to lead to a "quiet" period. Note, however, that this finding applies in particular to the S&P rating system, and other credit assessment approaches are not necessarily subject to this problem. We discuss these and other issues surrounding transition matrices in *Chapter 6***.**

In this chapter we have discussed the essence of the CreditMetrics methodology. The next two chapters extend our framework across a portfolio of exposures and across different exposure types beyond a simple bond.

Chapter 3. Portfolio risk calculation

In *Chapter 2*, we explained the methodology used by CreditMetrics to obtain the credit risk for a stand-alone exposure. Here, we extend our methodology to a "portfolio" of two exposures. The chapter is organized as follows:

- we elaborate on the joint likelihoods in the credit quality co-movements;
- we extend our credit risk calculation for stand-alone exposure (discussed in *Chapter 2*) to the multiple exposure case; and
- we discuss the calculation of marginal risk estimation, which identifies over-concentrations within a portfolio and thus suggests potential risk-mitigating actions.

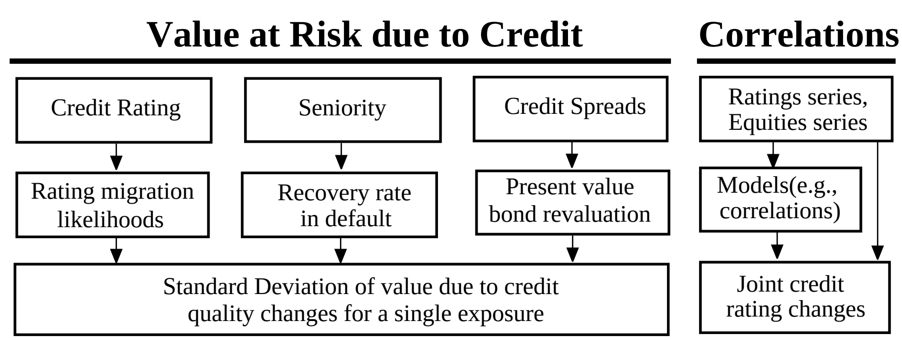
For clarity, we discuss the required steps to calculate credit risk across a portfolio with an example portfolio consisting of the following two specific bonds:

- *Bond #1:* BBB rated, senior unsecured, 6% annual coupon, five-year maturity
- *Bond #2:* A rated, senior unsecured, 5% annual coupon, three-year maturity

This example portfolio is the same as the one that we considered in *Chapter 1* when we were highlighting the steps in the calculation of portfolio credit risk. Here, we discuss the same steps as in *Chapter 1,* but in greater detail. Also, we point out that Bond #1 is the one for which we estimated the credit risk on a stand-alone basis in *Chapter 2*.

We now update the "road map" for CreditMetrics in *Chart 3.1* to show the additional work needed to address a portfolio. The reader can compare this chart with the prior chapter's corresponding *Chart 2.1* for a stand-alone exposure. Note that there is one significant addition. We must now estimate the contribution to risk brought by the effects of non-zero credit quality *correlations*. Thus, we must estimate *joint likelihoods* in the credit quality *co-movements*.

Chart 3.1
Our second "road map" of the analytics within CreditMetrics



Understanding joint likelihoods will allow us to properly account for the portfolio diversification effects. Correlation will, for example, determine how often losses occur in multiple exposures at the same time. Our volatility of value – our risk – will be lower if the correlation between credit events is lower. However, we do not elaborate here on the connection between correlation and joint likelihoods. Rather, we assume that the joint likelihoods are given to us. Later, in *Chapter 8*, we will discuss several different methods for determining joint likelihoods of credit quality migrations.

3.1 Joint probabilities

We have seen that, with the major ratings from AAA to CCC, there are eight possible outcomes for an obligor's credit quality in one year. Now we are interested in two obligors considered together. For us to estimate this joint risk, we need to consider all possible combinations of states between the two obligors. There are eight times eight or sixty-four possible states to which the two credits might migrate at the risk horizon.

The simplest way of obtaining the joint likelihoods is to just assume that these are the product of the individual likelihoods. Thus, as shown in *Table 3.1* below, the joint likelihood that the two obligors maintain their initial ratings is equal to 79.15%. This is the product of 86.93% (the likelihood that the BBB rated bond remains a BBB) and 91.05% (the likelihood that the single-A rated bond remains a single-A).

$$\underbrace{79.15\%}_{\substack{\textit{Chance both}\\ \textit{retain current rating}}} = \underbrace{86.93\%}_{\substack{\textit{Chance a BBB}\\ \textit{remains at BBB}}} \cdot \underbrace{91.05\%}_{\substack{\textit{Chance an A}\\ \textit{remains at A}}}$$

By repeating this type of calculation for all the 64 states we then fill the joint likelihood table shown below. However, calculation will be true only for the simplest case where the two obligors' credit rating changes are statistically independent.

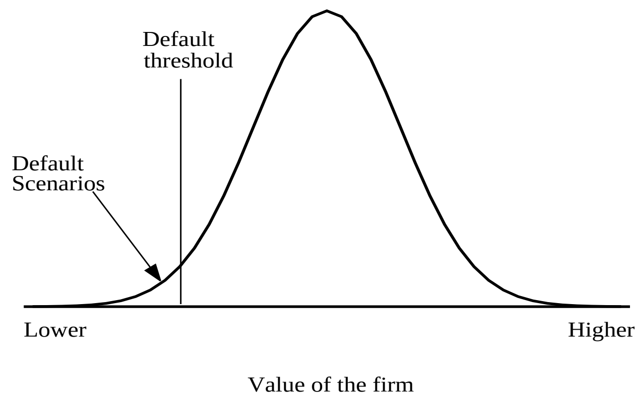
Table 3.1
Joint migration probabilities with zero correlation (%)

Obligor #1 (BBB)		**Obligor #2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	Default
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.01	0.30	0.02	0.00	0.00	0.00	0.00
A	5.95	0.01	0.14	5.42	0.33	0.04	0.02	0.00	0.00
BBB	86.93	0.08	1.98	79.15	4.80	0.64	0.23	0.01	0.05
BB	5.30	0.00	0.12	4.83	0.29	0.04	0.01	0.00	0.00
B	1.17	0.00	0.03	1.06	0.06	0.01	0.00	0.00	0.00
CCC	0.12	0.00	0.00	0.11	0.01	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.16	0.01	0.00	0.00	0.00	0.00

Assuming a zero correlation like this is too simplistic. It is unrealistic since these credit movements are affected in part by the same macro economic variables. In order to capture this effect, we will introduce in *Chapter 8* a model which links firm asset value to firm credit rating. We touch briefly on this model here.

In *Chart 3.2* we illustrate a framework for thinking about default as a function of the underlying (and volatile) value of the firm. This framework was first proposed by Robert Merton (see Merton [74]), and is often referred to as the *option theoretic* valuation of debt. It builds upon Black and Scholes option pricing model by stating that the credit risk component of a firm's debt can be valued like a put option on the value of the underlying assets of the firm. Under the Merton model, underlying firm value is random with some distribution. If the value of assets should happen to decline so much that the value is less than amount of liabilities outstanding, which we refer to as the default threshold, then it will be impossible for the firm to satisfy its obligations and it will thus default.

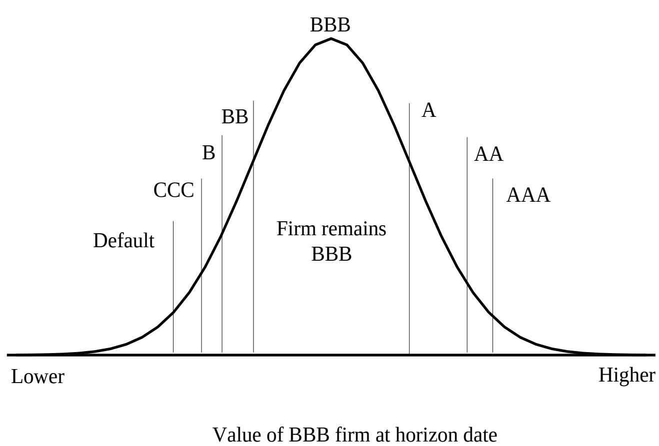
Chart 3.2
Model of firm value and its default threshold



We do not suggest here that default likelihoods must be estimated based on the volatility of underlying firm value. CreditMetrics assumes that each obligor will be labeled with a credit rating, which in turn will be associated with a default likelihood. It is unimportant to CreditMetrics *how* default likelihoods are estimated. We treat them as input parameters.

The Merton model can be easily extended to include rating changes. The generalization involves stating that in addition to the default threshold, there are credit rating up(down)grade thresholds as well. The firm's asset value relative to these thresholds determines its future rating, as illustrated in *Chart 3.3*.

Chart 3.3
Model of firm value and generalized credit quality thresholds



In the end, we have a link between the underlying firm value and the firm's credit rating, and can build the joint probabilities for two obligors from both this and a knowledge of the correlation between the two obligors' firm values. Again, this approach is developed in detail in *Section 8.4*. In *Table 3.2*, we present joint likelihoods which result from an application of this model.

Table 3.2
Joint migration probabilities with 0.30 asset correlation (%)

Obligor #1 (BBB)		**Obligor #2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	Default
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.12	0.00	0.00	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

There are at least four interesting features in the joint likelihood table above:

1. The probabilities across the table necessarily sum to 100%.
2. The most likely outcome is that both obligors simply remain at their current credit ratings. In fact, the likelihoods of joint migration become rapidly smaller as the migration distance grows.
3. The effect of correlation is generally to increase the joint probabilities along the diagonal drawn through their current joint standing (in this case, through BBB-A).
4. The sum of each column or each row must equal the chance of migration for that obligor standing alone. For instance, the sum of the last row must be 0.18%, which is the default likelihood for Obligor #1 (BBB) in isolation.

With this discussion of the joint likelihood, we turn our attention next to the credit risk calculation for our example two-bond portfolio. Specifically, we show how we extend our credit risk calculation from the stand-alone exposure case to the multiple exposure portfolio case.

3.2 Portfolio credit risk

As mentioned in *Chapter 1*, to calculate the volatility of value due to credit quality changes, we need two types of information for each of the 64 joint states between two obligors: joint likelihoods and revaluation estimates. In the previous section, we covered the joint likelihoods across co-movements in credit quality. Here, we discuss the revaluation of the two exposures – given the credit quality migration – for each of the 64 states. We will see that these data are then combined for two obligors in a fashion very similar to what we have already shown for one obligor.

We now must determine the 64 possible values of the portfolio at the risk horizon. This is easy since the value that the portfolio takes on in each pairwise credit rating class is simply the sum of the individual values. The portfolio values in each possible joint rating state are given in *Table 1.5*.

It is again noteworthy that the greatest potential for changes in value are on the downside. Indeed, the value is relatively flat across most of *Table 1.5*. It is only when either (or both) obligors suffer a downturn that the change in value becomes great. This same data was illustrated in *Chart 1.3* showing the frequency distribution of values.

We next focus on the calculation of the two risk measures for the portfolio, namely the standard deviation and the percentile level. As far as the portfolio standard deviation is concerned, we use the same formula in the stand-alone exposure case of *Chapter 2*. The only difference is that now we have 64 possible states rather than just eight in the stand-alone case. We illustrate this standard deviation calculation for the two-bond portfolio below:

[3.1]

$$\text{Mean: } \mu_{Total} = \sum_{i=1}^{S=64} p_i \mu_i = 213.63$$

$$\text{Variance: } \sigma^2_{Total} = \sum_{i=1}^{S=64} p_i \mu_i^2 - \mu^2_{Total} = 11.22 \qquad \{\text{std. dev. is } 3.35\}$$

The probabilities and the values that enter the standard deviation calculation are read off *Tables 3.2* and *1.5* respectively. Also, for simplicity, the above calculation ignores the additional contribution to the portfolio risk from the uncertainty (i.e., the standard deviation) in the recovery rate value. Note from the calculation that the mean and standard deviation for the portfolio are $213.63 and $3.35 respectively.

Recall from *Chapter 2* that the mean and standard deviation of our BBB bond were $107.09 and $2.99 respectively. For the single-A bond the comparable statistics are a mean of $106.55 and a standard deviation of $1.49. So we see that the means or expected values sum directly, but the risk – as measured by standard deviations – is much less than the summed individuals due to diversification.

At this point, although we have only discussed the calculation of standard deviation for a two-asset portfolio, we have presented all of the components necessary to calculate the standard deviation for any portfolio. For an arbitrary portfolio, we first identify all pairs of assets. We then consider each pair of assets as a subportfolio and compute its variance using the methods described in this section. Finally, we combine these variances with the variances for individual assets and arrive at a portfolio standard deviation. The details of this calculation are discussed in *Chapter 9* and *Appendix A*.

After having calculated the portfolio standard distribution, we next calculate a second measure of credit risk, that is, the percentile level. Assume that we are interested in calculating the 1st percentile level. Again, we point out that there is no fixed rule to prefer any given percentile level over another.

Recall from *Chapter 1* and *Chapter 2* that this calculation is quite simple. All we have to do is to find the portfolio value such that the likelihoods of all the values less than this sum to 1%. Since in this case, the portfolio has no more than two assets, we may simply examine the probabilities and values shown in *Table 3.2*, and obtain a 1^{st} percentile level number of \$204.40. This is \$9.23 below the mean value. Of course, for larger portfolios, it is not possible to calculate percentile levels analytically – we would have to perform a simulation.

This finishes our discussion of the calculation of the credit risk measures for the example portfolio of two bonds. In the next section, we introduce the concept of marginal risk. This concept enables us to understand where the risks are concentrated in the portfolio, and with which exposures we benefit due to diversification.

3.3 Marginal risk

We saw in *Chapter 2* how the credit risk can be calculated for an individual bond on a stand alone basis. However, the decision to hold a bond or not is likely to be made within the context of some existing portfolio. Thus, the more relevant calculation is the marginal increase to the portfolio risk that would be created by adding a new bond to it.

Let us first illustrate the calculation of marginal risk by using the standard deviation as a risk measure. Recall that the standard deviation of our one-bond (BBB-rated) portfolio in *Chapter 2* was \$2.99. The portfolio standard deviation increased to \$3.35 once we added the second, single-A rated bond. The marginal standard deviation of this second bond is therefore equal to \$0.36, which represents the difference between \$3.35 and \$2.99. Note that this marginal standard deviation is much smaller than the stand-alone standard deviation of the second bond, which is \$1.49. This is because of the diversification effect that is in turn caused by the fact that the year-end values of the individual bonds are not perfectly correlated.

We describe next how we extend our marginal risk calculation to percentile levels, again with the caveat that this approach is most appropriate for large portfolios. Recall that the BBB-rated bond had a mean value of \$107.09 and a 1^{st} percentile level value of \$98.10. This percentile level is therefore \$8.99 below the mean. Once the single-A rated bond is added, the two-bond portfolio has a mean of \$213.63 and a 1^{st} percentile level of 204.40. This percentile level is \$9.23 below the mean. We can now calculate the marginal risk of the single-A rated bond as the difference between \$9.23 and \$8.99, which is equal to \$0.24. On a stand-alone basis, the single-A rated bond has a 1^{st} percentile level value of \$103.15, which is \$3.39 below the mean value of \$106.55. This difference between the marginal risk (\$0.24) and the stand-alone risk (\$3.39) is again due to diversification.

We remark that marginal risk statistics are sometimes defined in a slightly different way. Where we define marginal risk to be the contribution of one asset to the total portfolio risk, others define it to be the marginal impact on portfolio risk of increasing an exposure by some small amount. While the two definitions do differ, they both serve the purpose of measuring an exposure's risk contribution to a portfolio, accounting for the effects of diversification, or lack thereof.

This concludes our discussion of marginal risk. In the next chapter, we show how CreditMetrics treats other asset types: receivables, loans, loan commitments, financial letters of credit and market-driven instruments such as swaps and forwards.

Chapter 4. Differing exposure types

So far we have demonstrated the methodology used in CreditMetrics with the help of portfolios consisting of only bonds. In *Chapter 2,* we discussed the case of a stand-alone bond exposure. In *Chapter 3*, we extended this stand-alone credit risk methodology to a portfolio of two bonds.

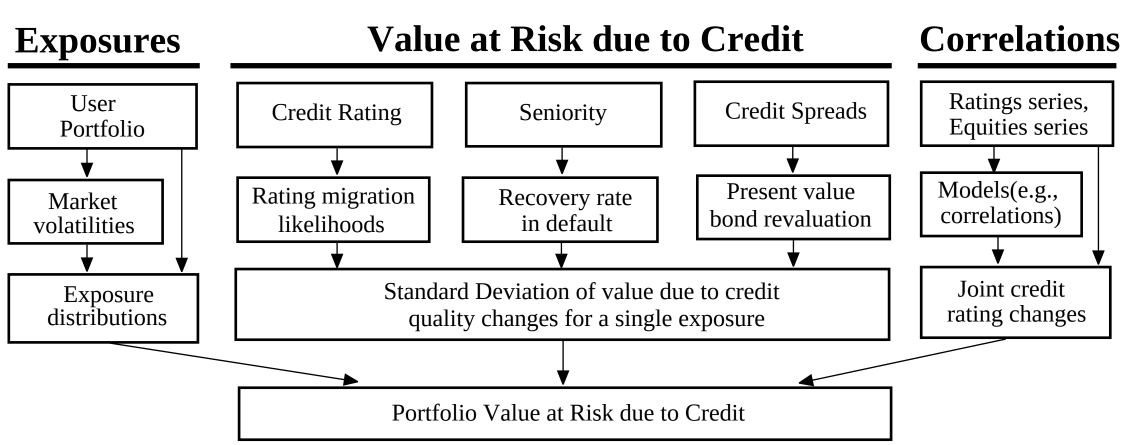
As discussed in *Chapter 1,* however, CreditMetrics is not limited to bonds. Rather, CreditMetrics is capable of estimating most any credit risk type limited only by the data available to revalue exposures upon up(down)grade and default. As a matter of implementation, we have included the following generic exposure types:

1. non-interest bearing receivables (trade credit);
2. bonds and loans;
3. commitments to lend;
4. financial letters of credit; and
5. market-driven instruments (swaps, forwards, etc.)

This chapter explains how CreditMetrics addresses different exposure types. Recall that for bonds, the analysis consisted of two steps: (i) specifying the likelihoods and joint likelihoods of obligors experiencing a credit quality change, and (ii) calculating the new values given each possible rating change at the risk horizon. For each of the additional exposure types covered in this chapter, the first step is identical as for bonds. Thus, here we only need to describe how to revalue each new exposure type in the event of a rating change or default. Thus, the goal of this chapter is to provide the reader with methods for constructing versions of *Table 1.2* for a variety of exposure types.

We show below in *Chart 4.1* the final "road map" for credit risk analytics within CreditMetrics. Note that this map now includes the provision for different exposure types.

Chart 4.1
Our final "road map" of the analytics within CreditMetrics



Before we describe how we treat different exposure types in CreditMetrics, let us summarize the CreditMetrics analytics shown in the chart above.

- The *credit exposure* is the amount subject to either changes in value upon credit quality up(down)grade or loss in the event of default. In practice, we must estimate this amount for: (i) commitments, where the exposure may change due to additional drawdowns, and (ii) market-driven instruments (swaps, forwards, etc.), where the exposure depends on the movement of market variables (e.g., FX or interest rates).

- The obligor's current long-term senior unsecured credit rating indicates its likelihood of credit quality migration that will be applied to all the obligor's obligations.

- The seniority standing and instrument type of each transaction indicates the recovery rate of that exposure in default.

- The transaction details and the forward zero rates for each credit rating category determine the changes in value of each transaction upon obligor up(down)grade.

- The joint likelihoods of credit quality movements for any pair of obligors is estimated through a treatment of the correlation between obligors.

- The portfolio standard deviation of changes in value due to credit quality changes are then calculated directly – in *closed form*.

In the following sections, we discuss how CreditMetrics treats different exposure types.

4.1 Receivables

Corporations which do not hold loans or bonds as exposures may still be subject to credit risk through payments due from their customers. Non interest bearing receivables (also called *trade credit*), are at risk to changes in credit quality of their customers. It is necessary, then, to consider such receivables on a comparable basis with any other risky credit instrument.

In concept, we treat receivables in the same way as we treated the bonds in the previous chapters. For receivables which become due beyond the risk horizon, we treat the cashflow as if it were a zero coupon bond paying on the receivable date, and revalue the cashflow based on the bond spreads in each rating category. If there are more applicable spreads available, specific to receivables, it would certainly be reasonable to use these in place of the bond spreads for the purposes of this revaluation.

Often, a receivable will be due before the risk horizon. In this situation, it is not even necessary to revalue in different rating categories. Either the payment is made, and we "revalue" at the receivable's face amount, or there is a default, and we revalue based on some recovery rate. Thus, in the case of a $1mm receivable due in nine months, where the risk horizon is one year and the recovery rate is, say 30%, we revalue the exposure at $1mm in each non-default state, and at $300,000 (= 30% times $1mm) in default.

We know of no systematic study of recovery rate experience for corporate receivables and so suggest that users take senior unsecured bond recovery experience as a guide.

4.2 Bonds and loans

In *Chapter 2*, we described how to revalue bonds in each future rating state using the forward interest rate curves for each credit rating. We generated a forward curve based on the credit spread curve for that rating, and discounted the future cash flows of the bond. In the case of default, the bond's value was taken to be a recovery fraction multiplied by the face value of the bond.

In concept, we treat a loan as a par bond, revaluing the loan using loan forward curves upon up(down)grade and applying a loan recovery rate to the principal amount in the case of default. This revaluation upon up(down)grade accounts for the decreasing likelihood that the full amount of the loan will be repaid as the obligor undergoes rating downgrades, and the increasing likelihood of repayment if the obligor is upgraded.

As an alternative, bonds may be treated in the same way as the market-driven instruments described in *Section 4.4*. In that section, we discuss the differences between the two approaches.

4.3 Loan commitments

A loan commitment is composed of a drawn and undrawn portion. The drawdown on the loan commitment is the amount currently borrowed. Interest is paid on the drawn portion, and a fee is paid on the undrawn portion. Typical fees on the undrawn portion are presented in *Table 4.1*. When we revalue a loan commitment given a credit rating change, we must therefore account for the changes in value to both portions. The drawn portion is revalued exactly like a loan. To this we add the change in value of the undrawn portion. As a practical matter, each lending institution is the best judge of its own pricing for loan commitments. Thus, it is quite appropriate for each institution to utilize its own pricing for this revaluation rather than using generic spreads in a downloadable data set.

Table 4.1
Fee on undrawn portion of commitment (b.p.)

Year-end rating	Fee: undrawn portion
AAA	3
AA	4
A	6
BBB	9
BB	18
B	40
CCC	120

Because loan commitments give the obligor the option of changing the size of a loan, loan commitments can dynamically change the portfolio composition. The amount drawn down at the risk horizon is closely related to the credit rating of the obligor (see *Table 4.2*). For example, if an obligor deteriorates, it is likely to draw down additional funds. On the other hand, if its prospects improve, it is unlikely to need the extra borrowings.

Note that it is not uncommon for loan commitments to have covenants that can reduce the credit risk. For example, if the loan rate not only floats with interest rate levels but also has credit spreads which change upon up(down)grade – a repricing grid – then the value of the facility will remain essentially unchanged across all up(down)grade categories. Thus, the risk will have been reduced because the only volatility of value remaining will be the potential loss in the event of default.

The worst possible case for a commitment is that the counterparty draws down the full amount and then defaults. It is intuitive, then, to treat a commitment as if it were a loan, with principal equal to the full commitment line. This is certainly the simplest approach to commitments, and from a risk perspective, the most conservative.

In practice, it has been seen that commitments are not always fully drawn in the case of default, and hence, that the risk on a commitment is less than the risk of a fully drawn loan. In order to model commitments more accurately, it is necessary to estimate not only the amount of the commitment which will be drawn down in the case of default, but also the amount which will be drawn down (or paid back) as the counterparty undergoes credit rating changes. For purposes of explanation in this section, we will rely on one study of commitment usage, but this is certainly an area where each firm should apply its own experience.

In the remainder of this section, we present a framework to calculate, given one source of data on commitment usage, the change in value of a commitment in each possible credit rating migration. We do this via an example.

Consider a three-year $100mm commitment to lend at a fixed rate (on the drawn portion) of 6% to a currently A rated obligor. For ease of illustration, we will assume the credit spreads of *Section 1.3.2*, so that the change in value of any drawn amount due only to changes in credit quality (and neglecting changes in commitment usage) will be the same as for bond example in that section. Our example will have $20mm currently drawn down with the remaining $80mm undrawn and charged at a fee of 6 b.p.

In each credit rating, the commitment's revaluation estimate at the risk horizon will depend on both:

- estimates of the change in amount drawn due to credit quality changes; and
- estimates of the change in value for both the drawn and undrawn portions.

We will address each of these in turn. Our first task is to estimate changes in drawdown given each possible credit rating change. Part of this estimation, the drawdown in default, can be directly taken from a published study. Asarnow & Marker (A&M) [95] have examined the average drawdown (of normally unused commitment) in the event of default (see *Table 4.2*). We may use this information to estimate how much of the undrawn amount of our commitment will be drawn in the case of a default. Thus, if our counterparty defaults, it will draw an additional 71% of the $80mm undrawn amount, or $56.8mm. Added to the current drawn amount of $20mm, this results in an estimated drawdown in default of $76.8mm.

Table 4.2
Average usage of commitments to lend

Credit rating	Average commitment usage	Usage of the normally unused commitment in the event of default
AAA	0.1%	69%
AA	1.6%	73%
A	4.6%	71%
BBB	20.0%	65%
BB	46.8%	52%
B	63.7%	48%
CCC	75.0%	44%

Source: Asarnow & Marker [95]

We also expect that there will be a credit rating related change in drawdown in all non-default states. One suggestion of this behavior is also evidenced in the A&M study. We see that the average commitment usage varies directly with credit rating. How to best apply this information is open to question. We offer the following method as a suggestion to initiate discussion rather than as a definitive result. We fully anticipate that, as a matter of implementation, each institution will substitute its own study based on its unique experience.

The following table estimates the changes in drawdown attributable to credit rating changes. As an example, consider the change to BBB from our initial rating of single-A. A&M found that the average draw increased from 4.6% to 20.0% between single-A and BBB. In other words, the undrawn portions moved from 95.4% to 80.0% or a reduction of 16.1% (= 100% - [80.0%/95.4%]). Thus, with our example initial drawdown of 20%, we estimate that an additional 12.9% (=16.1% * 80%) will be drawn down in the case of a migration to BBB. We present estimates of change in drawdown for all possible migrations in *Table 4.3*. Note that in cases of upgrades, we actually estimate a negative change in drawdown, corresponding to the counterpart paying back some amount.

Table 4.3
Example estimate of changes in drawdown

Year-end rating	Current Drawdown	Change in Drawdown	Estimate of New Drawdown
AAA	20.0	-19.6	0.4
AA	20.0	-13.0	7.0
A	20.0	0.0	20.0
BBB	20.0	12.9	32.9
BB	20.0	35.4	55.4
B	20.0	49.6	69.6
CCC	20.0	59.0	79.0
Default	20.0	56.8	76.8

It is against this estimate of the new drawdown amounts at the risk horizon that we now apply our revaluation estimates – to both the drawn and undrawn portions. Referring to *Table 4.4*, we see for instance that when the obligor downgrades to BB, the change in value is negative 2.7%, or -$1.5mm due to credit spread widening on the fixed rate dawn portion.

There is also a change in the value of the fees collected on the undrawn amount. Recall that at present, we are collecting 6 b.p. on the undrawn amount of $80mm. As the undrawn amount changes, so too do the fees we collect. Thus in the case of downgrade to BB, we collect fees on $35.4mm less. (This $35.4mm corresponds to the additional draw in this case, as seen in *Table 4.3*.) In default, we lose the all of the fees which we were receiving, the full 6 b.p. on the full $80mm. The change in value of the fees[1] is also given in *Table 4.4*, as well as the total change in value for the commitment, for all possible rating migrations.

Table 4.4
Revaluations for $20mm initially drawn commitment

Year-end rating	Drawdown at Year-end	Change in Value (%)	Change in Value ($mm)	Change in fees ($mm)	Total Value Change ($mm)
AAA	0.4	0.6	0.0	0.01	0.01
AA	7.0	0.5	0.0	0.01	0.01
A	20.0	0.3	0.1	0.00	0.10
BBB	32.9	-0.3	-0.1	-0.01	-0.11
BB	55.4	-2.7	-1.5	-0.02	-1.52
B	69.6	-4.3	-3.0	-0.03	-3.03
CCC	79.0	-16.3	-12.9	-0.04	-12.94
Default	76.8	-51.8	-39.8	-0.05	-39.85

Several observations are in order:

- the expected percentage drawn down in default is the most important factor;
- fees have a relatively small impact on the revaluations;
- it is possible to have negative revaluations greater than the current drawdown; and
- covenants that reset the drawdown spread upon an up(down)grade would reduce the volatility of value in all non-default states – keeping value close to par.

Thus we have obtained the revaluations estimates in each future credit rating state. The calculation of credit risk for the commitment is now simply a matter of applying the techniques of the previous two chapters.

4.4 Financial letters of credit (LCs)

There are times when an obligor may desire to have the option to borrow even if there is no immediate need to borrow. In such a case, an outright loan would be inefficient since its proceeds may sit in the obligor's hands under-employed. What is typically needed in this case is a *financial letter of credit*. With this type of off balance sheet access to

[1] To be precise, the change in fees should also account for a new discount function corresponding to the new rating, but we ignore this effect for this example.

funds, there is assurance that funds will be available even when other sources of funding may dry up due perhaps to credit quality deterioration.

We argue that such an exposure is comparable in risk to an outright term loan Indeed, the market typically prices these instruments comparable to loans. The obligor will almost assuredly drawdown the LC as it approaches credit distress. Thus, in all the cases where there can be a default, there will also be full exposure just like a loan. We suggest that LCs be treated identically to loans, including the use of the credit spread curves and recovery rates that have been estimated for loans.

Note that a *financial* letter of credit is distinguished from either *performance* or a *trade* letter of credit. Performance LCs are typically secured by the income generating ability of a particular project and trade LCs are triggered only infrequently by non credit related events. Both of these would have smaller risk than financial LCs.

4.5 Market-driven instruments

We have so far described how CreditMetrics calculates the credit risk for receivables, bonds, loans, and commitments. In this section, we explain how CreditMetrics treats derivative instruments that are subject to counterparty default (swaps, forwards, etc.). Our discussion focuses on the example of swaps.

Throughout this document, we refer to these types of exposures as *market-driven* instruments. In these transactions, credit risk and market risk components are intimately coupled because of an inherent optionality. This optionality stems from the fact that we face a loss on the transaction if the counterparty defaults *only* if we are in-the-money (i.e., the obligor owes us money on a net present value basis). This complicates CreditMetrics' handling of derivatives exposures.

We remark that to treat products like swaps in full detail, it would be necessary to propose an integrated model of credit and market risk. Such a model would describe both the correlations of swap exposures across a portfolio (capturing, for instance, that swaps based on the same interest rate would tend to go in- or out-of-the-money together), and the correlations between credit and market moves (for instance, that swap counterparties might be more likely to default in one interest regime than in another). Our goal here is not to provide a fully integrated model, but to capture the most crucial influences of market volatilities to the credit risks of market-driven instruments.

4.5.1 Credit risk calculation for swaps

Swaps are treated within CreditMetrics consistent with the way bonds and loans are treated. However, the revaluation of swaps in each credit quality state at the risk horizon is much more complicated than that of either bonds or loans. Credit loss occurs when both of the following two conditions are satisfied:

1. The counterparty undergoes a credit quality change.
2. The swap transaction is out-of-the-money for the counterparty, that is, the counterparty owes money on the swap transaction on a net present value basis.

The market and credit risk calculations for swaps are therefore intimately related. All things remaining equal, the greater the market volatility, the greater the amount exposed to loss during an unfavorable credit event.

To summarize, optionality (i.e., credit exposure to the swap counterparty only if the counterparty is out-of-the-money) is the feature that makes swaps distinct from bonds. Although the exposure in the case of bonds is also market-driven, there is no optionality involved, since the issuer of debt is always "out-of-the-money." In other words, the net present value for swaps can be either positive or negative for the counterparties. However, the net present value for the issuer is always negative.

We next describe how the swap is reevaluated in each possible rating state at the risk horizon. The purpose of this exercise is to fill the "value" table analogous to *Table 2.4* for bonds. Essentially, we represent the value of the swap as a difference of two components:

1. The first component is equal to the forward *risk-free*[2] value of the swap cash flows. This hypothetical value is obtained by finding the forward value of the swap cash flows by using the government rates rather than the swap rates; therefore the first component is the same for all forward credit rating states.
2. The second component represents the loss expected on the swap due to a default net of recoveries by the counterparty on the remaining cash flows of the swap. By "remaining" we mean all cash flows that occur after the risk horizon (assumed to be one year). Since the probability of this default varies by rating category, the second component varies from one rating category to another.

Finally, the revaluation of the swap in any rating category is obtained by subtracting the second (expected default loss) component from the first (risk-free value) component.

Note that this valuation scheme essentially values the swap as if it were risk-free, and then subtracts a penalty (the expected loss) to account for the risk due to the credit quality of the counterparty. The intuition behind this procedure for calculating the swap value is straightforward. First we calculate the value assuming that there is no risk whatsoever of the counterparty's default, using the government (i.e., credit-risk-free) rates for this calculation. We then subtract from this credit-risk-free value the amount that we can expect to lose due to a counterparty default. The probability of default is obviously an important factor driving this latter component. A second factor is *optionality,* which is implied in the fact that we have exposure to the counterparty only if the counterparty is in-the-money. The value of this optionality is determined from the amount by which the swap is expected to be in-the-money, and also from the volatility of interest rates.

An enhancement of this procedure might be to account for not only the expected loss due to credit, but also for the random nature of the swap exposure. This could be achieved by redefining the expected loss penalty to include a measure of how much the swap is likely to fluctuate in value. The result would be that two swaps with the same expected losses in each rating state would be distinguished by the amount of uncertainty in their losses.

[2] Here we mean risk-free from a credit risk perspective.

We next detail the calculation of the two components of the swaps value at the risk horizon in each possible credit rating state. First, however, let us restate mathematically the revaluation of the swap:

[4.1] $$\textit{Value of swap in 1 year}_{\textit{Rating R}} = \textit{Risk-free value in 1 year} - \textit{Expected loss in years 1 through maturity}_{\textit{Rating R}}$$

where "R" in the above expression can be any possible credit rating category including default.

As mentioned before, the calculation of the risk-free value of the swap in one year is straightforward. All we need do is discount the future swap cash flows occurring between Year 1 through maturity by the forward government zero curve. The calculation of the expected loss, on the other hand, is complicated. This is because of the optionality component in swap exposure explained earlier.

For each forward non-default credit rating, the expected loss can be written as:

[4.2] $$\textit{Expected loss}_{\textit{Rating R}} = \textit{Average exposure}_{\textit{Year 1 through maturity}} \bullet \textit{Probability of default in years 1 through maturity}_{\textit{Rating R}} \bullet (1 - \textit{Recovery Fraction})$$

The average exposure represents the average[3] of several expected exposure values calculated at different forward points over the life of the swap starting from the end of first year. We use average exposure in the expected loss expression above to account for the possibility of swap counterparty defaulting at any point in time between the end of the first year and maturity. Each of the expected exposure values that enter the average exposure calculation requires a modified Black-Scholes computation to account for the inherent optionality feature. As a result, the average exposure calculation for swaps is quite complicated and time-consuming. We refer the interested reader to other sources for a more thorough treatment of the expected and average exposure calculations.[4]

The second term that enters the expected loss calculation is the probability of default for each rating category between Year 1 and the maturity of the swap. For example, if the maturity of the swap is five years, then the four-year probability of default is required for each of the rating categories AAA through CCC. These probabilities can be obtained by multiplying the one-year transition matrix four times to generate the four-year transition matrix. The four-year default probabilities can then be simply read off from the last (i.e., default) column of this transition matrix.

Two assumptions are implicit in this method of generating the long-term default probabilities. First, we assume that the transition process is stationary in that the same transition matrix is valid from one year to another. Second, we assume that there is no autocorrelation in rating movements from one year to another.

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[3] The expected exposures are weighted by the appropriate discount factors for this average calculation.

[4] One such source is "On measuring credit exposure," *RiskMetrics™ Monitor,* J.P. Morgan, March 1997. Also, J.P. Morgan plans to provide a software tool in the near future that enables the user to calculate average and expected exposures. This tool will be based on the RiskMetrics market risk methodology, a software implementation of which is currently being marketed by J.P. Morgan under the name FourFifteen.

The method provided so far enables us to calculate the value of the swap in each of the non-default credit rating categories. To calculate the value in case of a default during the risk interval, we must modify this procedure somewhat. This is mainly due to the fact that the average exposure calculation over the life of the swap does not make any sense here, since we know for sure that the swap counterparty has defaulted during the risk interval. Therefore we write the expected loss in the defaulted state as:

[4.3] $$\textit{Expected loss}_{Default} = \textit{Expected exposure}_{Year\ 1} \cdot (1 - \textit{Recovery Fraction}).$$

The implicit assumption in the above expression is that the risk interval is relatively short, say one year, as compared to the maturity of the swap. If the risk interval is much longer than this, say several years, it will be more accurate to replace the expected exposure with the average exposure value calculated over the risk interval. This is because the swap counterparty can default at any point over the longer risk interval, and the expected exposures at these points can be very different from the expected exposure at the risk horizon. In this case, therefore, the average exposure is a more suitable measure.

This concludes our explanation of the value calculation for swaps at the risk horizon. Let us next consider an example.

Assume a three-year fixed for floating swap on \$10 mm notional beginning January 24, 1997. Let the risk horizon be one year and the recovery rate in case of default be 0.50.

On January 24, 1997, the average exposure at the end of one year is calculated to be equal to be \$61,627. This represents an average of the expected exposures between the end of one year and the end of three years (a two-year time period). Now, given a two-year default likelihood of 0.02% for the AA rating category, the value of the swap at the end of risk horizon in the AA rating category is equal to:

[4.4] $$\begin{aligned} &\textit{FV in 1 year} - p_{AA} \cdot AE \cdot (1 - R) = \textit{FV in 1 year} - 0.0002 \cdot 61{,}627 \cdot (1 - 0.5) \\ &= FV - \$6 \end{aligned}$$

where FV refers to the forward value, and AE refers to the average exposure in one year. Similarly, given a 33.24% default likelihood for the CCC rating category, the corresponding value of the swap in the CCC rating category is equal to:

[4.5] $$\textit{FV in 1 year} - p_{CCC} \cdot AE \cdot (1 - R) = \textit{FV in 1 year} - 0.3344 \cdot 61{,}627 \cdot (1 - 0.5) = FV - \$10{,}304.$$

Next, let us consider what happens in default. The expected exposure at the end of the year is calculated on January 24, 1997 to be equal to \$101,721. Given a recovery rate of 50%, the value in the defaulted state is equal to:

[4.6] $$\textit{FV in 1 year} - EE \cdot (1 - R) = \textit{FV in 1 year} - 101{,}721 \cdot (1 - 0.5) = FV - \$50{,}860$$

where EE refers to the expected exposure in one year.

In *Table 4.5* we summarize the value of the swap in each possible credit rating states at the risk horizon. We do not specify the risk-free component (FV), for two reasons:

1. This calculation is relatively straightforward and involves valuing the future cash flows with the risk-free yield.
2. More importantly, it is conceivable that, at least for the lower credit ratings and default, the expected loss value far exceeds the risk-free forward value of the swap itself. This is especially true when the swap value is near par, the risk interval is quite small, and the interest rates are not changing too rapidly. In this circumstance, it suffices to set the risk-free value term to zero and just use the expected loss term in the credit risk calculation.

Table 4.5
Value of swap at the risk horizon in each rating state
“FV” represents the risk-free forward value of the swap cash flows in one year.

Year-end rating	Two-year default likelihood (%)	Value ($)
AAA	0.00	FV – 1
AA	0.02	FV – 6
A	0.15	FV – 46
BBB	0.48	FV – 148
BB	2.59	FV – 797
B	10.41	FV – 3,209
CCC	33.24	FV – 10,304
Default	—	FV – 50,860

We next discuss a refinement to the calculation of the expected loss value in rating states AAA through CCC that produces more accurate expected loss numbers. For the sake of clarity we did not address it earlier; we now explain it below.

Recall that we calculate the expected loss value for rating states AAA through CCC by multiplying the average exposure by the probability of default for the desired rating category. Both the average exposure and the default probability are valid from Year 1 through the maturity of the swap. Also, the average exposure represents the average of the expected exposures calculated at several points between Year 1 and maturity.

Given this, we can more accurately calculate the expected loss component as follows:

1. Calculate the expected exposure in, say, one-year increments between Year 1 and maturity.
2. Weigh each of the expected exposures by a probability factor. This factor represents the probability that the counterparty defaults in the year in which the expected exposure is calculated, given that it does not default before then.
3. Add these weighted expected exposures after adjusting for the time value of money effect.

The result is a expected-loss calculation which reflects reality more accurately than if we were simply to multiply the average exposure by a single default probability. This is because by thus breaking the expected loss calculation into smaller pieces at different time horizons, we properly account for the timing of default.

4.5.2 Extension for forwards and multiple transactions

The methodology that we have presented above for swaps can be used in exactly the same manner for forwards. Furthermore, it can be easily extended to the case in which there are several transactions with the same counterparty and netting is enforceable. These transactions do not all have to be swaps, but can represent a variety of market-driven instruments, including forwards.

The methodology outlined for swaps can be extended to a portfolio of different instrument types with the same counterparty as follows. First, all the cash flows from the different transactions conducted by the same counterparty are netted to yield the resulting net cash flows. (Of course, this netting is done according to the particular netting arrangements that are in place with the counterparty.) Next, the swaps methodology is used to revalue these net cash flows in different rating categories at the risk horizon. Once again, this value comprises risk-free forward value and a expected loss value, both of which are calculated in exactly the same manner as for swaps.

Part II

Model Parameters

Overview of Part II

We have seen in the previous section the general overview, scope and type of results of CreditMetrics. Now we will give more detail to the main modeling parameters used in the CreditMetrics calculation: our sources of data, how we use the data to estimate parameters and why we have made some of the modeling choices we did. There is no single step in the methodology that is particularly difficult; there are simply a lot of steps. We devote a chapter to each major parameter and have tried to present each chapter as a topic which can be read on its own. Although we encourage the reader to study all chapters, reading only a particular chapter of interest is also possible.

Part II is organized into four chapters providing a detailed description of the major parameters within the CreditMetrics framework for quantifying credit risks. Our intent has been to make this description sufficiently detailed so that a practitioner can independently implement this model. This section is organized as follows:

- **Chapter 5: Overview of credit risk literature.** To better place our efforts within the context of prior research in the credit risk quantification field, we give a brief overview of some of the relevant literature.

- **Chapter 6: Default and credit quality migration.** We present an underlying *model of the firm* within which we integrate the process of firm default and, more generally, credit quality migrations. We argue that default is just a special case of a more general process of credit quality migration.

- **Chapter 7: Recovery rates.** Since changes in value are – naturally – greatest in the state of default, our overall measure of credit risk is sensitive to the estimation of recovery rates. We also model the uncertainty of recovery rates.

- **Chapter 8: Credit quality correlations.** The portfolio view of any risk requires an estimation of – most generally – joint movement. In practice, this often means estimating correlation parameters. CreditMetrics requires the joint likelihood of credit quality movements between obligors. Since the observation of credit events are often rare or of poor quality, it is difficult to further estimate their correlations of credit quality moves. We show that the results of several different data sources corroborate each other and might be used to estimate credit quality correlations.

Chapter 5. Overview of credit risk literature

One of our explicit goals is to stimulate broad discussion and further research towards a better understanding of quantitative credit risk estimation within a full portfolio context. We have sought to make CreditMetrics as competent as is possible within an objective and workable framework. However, we are certain that it will improve with comments from the broad community of researchers.

Extensive previous work has been done towards developing methodologies for estimating different aspects of credit risk. In this chapter, we give a brief survey of the academic literature so that our effort with CreditMetrics can be put in context and so that researchers can more easily compare our approach to others. We group the previous academic research on credit risk estimation within three broad categories:

- estimating particular individual parameters such as expected default frequencies or expected recovery rate in the event of default;
- estimating volatility of value (often termed *unexpected* losses) with the assumption of bond market level diversification; and
- estimating volatility of value within the context of a specific portfolio that is not perfectly diversified.

Also, there have been several papers on credit *pricing*, starting with Merton [74], which discuss debt value as a result of firm risk estimation in an option-theoretic framework. There is more recent work in this area which has focused on incorporating corporate bond yield spreads in valuation models, see Ginzburg, Maloney & Willner [93], Jarrow, Lando & Turnbull [96] and Das & Tufano [96]. For CreditMetrics, we have chosen to focus on the risk assessment side rather than focus on the pricing side.

5.1 Expected losses

Expected losses are driven by the expected probability of default and the expected recovery rate in default. We cover recovery rate expectations in much more detail in *Chapter 7* and so will devote this discussion to the expected default likelihood. The problem of estimating the chance of counterparty default has been so difficult that many systems devote all their efforts to this alone. Certainly, if the underlying estimates of default likelihood are poor, then a risk management system is unlikely to make up for this deficiency in its other parts. We will discuss three approaches that are used in practice:

- the accounting analytic approach which is the method used by most rating agencies;
- statistical methods which encompass quite a few varieties; and
- the option-theoretic approach which is a common academic paradigm for default.

We emphasize that CreditMetrics is not another rating service. We assume that exposures input into CreditMetrics will already have been labeled into discrete rating categories as to their credit quality by some outside provider.

As we discuss in *Chapter 6,* a transition matrix for use by CreditMetrics can be fit to any categorical rating system which has historical data. Indeed, we would argue that each credit scoring system should be fit with its own transition matrix. For some users with their own internal rating systems, this will be a necessary first step before applying CreditMetrics to their portfolios. If these systems have limited historical data sets available, then an estimation algorithm that expresses long-term behavior may be desirable (see *Section 6.4*).

5.1.1 Accounting analytic approach

Perhaps the most widely applied approach for estimating firm specific credit quality is fundamental analysis with the use of financial ratios. Such *accounting analytic* methods focus on leverage and coverage measures, coupled with an analysis of the quality and stability of the firm's earnings and cash flows. A good statement of this approach is in Standard and Poor's *Debt Rating Criteria*.[1] These raw quantitative measures are then tempered by the judgment and experience of an industry specialist. This broad description is generally the approach of the major debt rating agencies. This approach yields discrete ordinal groups (e.g., alphabetic ratings) which label firms by credit quality.

We are aware of at least 35 credit rating services worldwide. Also, it is common that financial institutions will maintain their own in-house credit rating expertise. However, letter (or numerical) rating categories by themselves only give an ordinal ranking of the default likelihoods. A quantitative credit risk model such as CreditMetrics cannot utilize ratings without additional information. Each credit rating label must have a statistical meaning such as a specific default probability (e.g., 0.45% over a one-year horizon).

Table 5.1
Moody's corporate bond average cumulative default rates (%)

Years	1	2	3	4	5
Aaa	0.00	0.00	0.00	0.07	0.23
Aa1	0.00	0.00	0.00	0.31	0.31
Aa2	0.00	0.00	0.09	0.29	0.65
Aa3	0.09	0.15	0.27	0.42	0.60
A1	0.00	0.04	0.49	0.79	1.01
A2	0.00	0.04	0.21	0.57	0.88
A3	0.00	0.20	0.37	0.52	0.61
Baa1	0.06	0.39	0.79	1.17	1.53
Baa2	0.06	0.26	0.35	1.07	1.70
Baa3	0.45	1.06	1.80	2.87	3.69
Ba1	0.85	2.68	4.46	7.03	9.52
Ba2	0.73	3.37	6.47	9.43	12.28
Ba3	3.12	8.09	13.49	18.55	23.15
B1	4.50	10.90	17.33	23.44	29.05
B2	8.75	15.18	22.10	27.95	31.86
B3	13.49	21.86	27.84	32.08	36.10

Source: Carty & Lieberman [96a]
— Moody's Investors Service

The two major U.S. agencies, S&P and Moody's, have published historical default likelihoods for their letter rating categories. An example from Moody's is shown in *Table 5.1*.

There have been many studies of the historical default frequency of corporate publicly rated bonds. These include Altman [92], [88], [87], Altman & Bencivenga [95], Altman & Haldeman [92], Altman & Nammacher [85], Asquith, Mullins & Wolff [89], Carty & Lieberman [96a] and S&P CreditWeek [96]. These studies are indispensable, and it is important to highlight some important points from them:

- the evolution and change in the original issue high yield bond market is unique in its history and future high yield bond issuance will be different;
- most of the default history is tagged to U.S. domestic issuers who are large enough to have at least an S&P or Moody's rating; and
- the definition of "default" has itself evolved (e.g., it now typically includes "distressed exchanges").

Thus, use of these data must be accompanied by a working knowledge of how they were generated and what they represent.

[1] See: http://www.ratings.standardpoor.com/criteria/index.htm

On a more macro-economic level, researchers have found that aggregate default likelihood is correlated with measures of the business and credit cycle. For example, Fons [91] correlates aggregate defaults to GDP, while Jónsson & Fridson [96] examine also corporate profits, manufacturing hours, money supply, etc.

5.1.2 Statistical prediction of default likelihood

There is a large body of more statistically focused work devoted to building credit quality estimation models, which seek to predict future default. One can identify three basic approaches to estimating default likelihood: qualitative dependent variable models, discriminant analysis, and neural networks. All of these approaches are strictly quantitative and will at least yield a ranking of anticipated default likelihoods and often can be tuned to yield an estimate of default likelihood.

Linear *discriminant analysis* applies a classification model to categorize which firms have defaulted versus which firms survived. In this approach, a historical sample is compiled of firms which defaulted with a matched sample of similar firms that did not default. Then, the statistical estimation approach is applied to identify which variables (and in which combination) can best classify firms into either group. The best example of this approach is Edward Altman's Z-scores; first developed in 1968 and now offered commercially as Zeta Services Inc. This approach yields a continuous numerical score based on a linear function of the relevant firm variables, which – with additional processing – can be mapped to default likelihoods.

The academic literature is full of alternative techniques ranging from principal components analysis, self-organizing feature maps, logistic regression, probit/logit analysis and hierarchical classification models. All of these methods can be shown to have some ability to distinguish high from low default likelihoods firms. Authors who compare the predictive strength of these diverse techniques include Alici [95], Altman, Marco & Varetto [93], and Episcopos, Pericli & Hu [95].

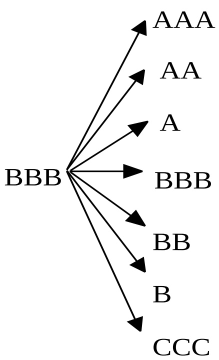
The application of *neural network* techniques to credit scoring include Dutta & Shekhar [88], Kerling [95], and Tyree & Long [94]. The popular press reports commercial applications of neural networks to large volume credit decisions such as credit card authorizations, but there do not appear to be commercial application yet of these neural network techniques for large corporate credits.

5.1.3 Option-theoretic approach

The *option-theoretic* approach was proposed by Fisher Black and Myron Scholes in the context of option pricing, and subsequently developed by Black, Cox, Ingersoll, and most notably, Robert Merton. In this view, a firm has a market value which evolves randomly through time as new information about future prospects of the firm become known. Default occurs when the value of the firm falls so low that the firm's assets are worth less than its obligations. This approach has served as an academic paradigm for default risk, but it is also used as a basis for default risk estimation. The leading commercial exemplar of this approach is KMV. In general, this method yields a continuous numeric value such as the number of standard deviations to the threshold of default, which – with additional processing – can be mapped to default likelihoods

5.1.4 Migration analysis

Chart 5.1
Credit migration



Understanding the potential range of outcomes that are possible is fundamental to risk assessment. As illustrated in *Chart 5.1*, knowing today's credit rating allows us to estimate from history the possible pattern of behaviors in the coming period. More specifically, if an obligor is BBB today, then chances are the obligor will be BBB in one year's time; but it may be up(down)graded. *Table 5.2* shows that, for instance, 86.93% of the time a BBB-rated obligor will remain a BBB, but there is a 5.30% chance that a BBB will downgrade to a BB in one year.

Table 5.2
Credit quality migration likelihoods for a BBB in one year

	AAA	AA	A	BBB	BB	B	CCC	Default
BBB	0.02%	0.33%	5.95%	86.93%	5.30%	1.17%	0.12%	0.18%

One of our fundamental techniques is *migration analysis*. Morgan developed transition matrices for our own use as early as 1987. We have since built upon a broad literature of work which applies migration analysis to credit risk evaluation. The first publication of transition matrices was in 1991 by both Professor Edward Altman of New York University and separately by Lucas & Lonski of Moody's Investors Service. They have since been published regularly (see Moody's Carty & Lieberman [96a] and Standard & Poor's *Creditweek* [15-Apr-96]) and can be calculated by firms such as KMV.

There have been studies of their predictive power and stationarity (Altman & Kao [91] and [92]). More recently, several practitioners (see Austin [92], Meyer [95], and Smith & Lawrence [95]) have used migration analysis to better estimate an accounting-based *allowance for loan and lease losses* (what we would term *expected default losses*). Also, these tools have been used to both estimate (Crabbe [95]) and even potentially improve Lucas [95b]) holding period returns. Finally, academics have constructed arbitrage free credit pricing models (see Ginzburg, Maloney and Willner [93], Jarrow, Lando & Turnbull [96] and Das & Tufano [96]). In CreditMetrics, we extend this literature by showing how to calculate the volatility of value due to credit quality changes (i.e., the potential magnitude of *unexpected* losses) rather than just expected losses.

5.2 Unexpected losses

The volatility of losses, commonly termed *unexpected* losses, has proven to be generally much more difficult to estimate than expected losses. Since it is so difficult to explicitly address correlations there have been a number of examples where practitioners take one of two approaches. First, they have applied methods which are statistically easy by addressing either the special case of correlations all equaling zero (perfectly uncorrelated) or correlations all equaling one (perfectly positively correlated). Neither of these is realistic.

Second, they have taken a middle road and assumed that their specific portfolio will have the same correlation effects as some index portfolio. The index portfolio can either be the total credit market ("full" diversification) or a sector index. Thus, the hope would be that statistics drawn from observing the index of debt might be applied through analogy to the specific portfolio. The institution's portfolio would be assumed to have the same correlations and profile of composition as the overall credit markets. These approaches can be grouped into two categories which we discuss in turn:

- historical default volatility; and
- volatility of holding period returns.

Although these may yield some estimate of general portfolio risk, they both suffer from an inability to do meaningful marginal analysis. These techniques would not allow the examination of marginal risk brought by adding some specific proposed transaction. There would also be no guide to know which specific names contribute disproportionate risk to the portfolio.

5.2.1 Historical default volatility

Historical default volatility is available from public studies: see for example *Table 5.3,* which is taken from Carty & Lieberman [96a]. There are several hypotheses to explain why default rates would be volatile:

- defaults are simply random events and the number of firms in the credit markets is not large enough to smooth random variation;
- the volume of high yield bond issuance across years is uneven; and
- the business cycle sees more firms default during downturns versus growth phases.

All three hypotheses are likely to have some truth for the corporate credit markets.

Table 5.3
Volatility of historical default rates by rating category

	Default rate standard deviations (%)	
Credit rating	**One-year**	**Ten-year**
Aaa	0.0	0.0
Aa	0.1	0.9
A	0.1	0.7
Baa	0.3	1.8
Ba	1.4	3.4
B	4.8	5.6

Source: Carty &Lieberman [96a] — Moody's Investors Service

The problem with trying to understand the volatility of individual exposures in this fashion is that it must be viewed within a portfolio.

5.2.2 Volatility of holding period returns

The volatility of default events is only one component of credit risk. Thus, it may also be useful to examine the volatilities of total holding period returns. A number of academic studies have performed this exercise. For corporate bonds, there are two studies by Bennett, Esser & Roth [93] and Wagner [96]. For commercial loans, there are studies by Asarnow [96] and Asarnow & Marker [95].

Once the historical return volatility is estimated – perhaps grouped by credit rating, maturity bucket, and industry/sector – some practitioners have applied them to analogous exposures in the credit portfolio. In this approach, portfolio diversification is addressed only to the extent that the portfolio under analysis is assuming to be analogous to the credit market universe. Again, there is the obvious problem of diversification differences. But there are also three practical concerns with this approach:

- historical returns are likely to poorly sample returns given credit quality migrations (including defaults) which are low-frequency but important[2];
- the data as it has been collected would require a standard deviation estimate over a sample size of less than 30 and so the standard error of the estimate is large; and
- the studies listed have commingled all sources of volatility – including interest rate fluctuations – rather than just volatility in value due to credit quality changes.

This general approach is sometimes termed the RAROC approach. Implementations vary, but the idea is to track a benchmark corporate bond (or index) which has liquidity and observable pricing. The resulting estimate of volatility of value is then used to proxy for the volatility of some exposure (or portfolio) under analysis.

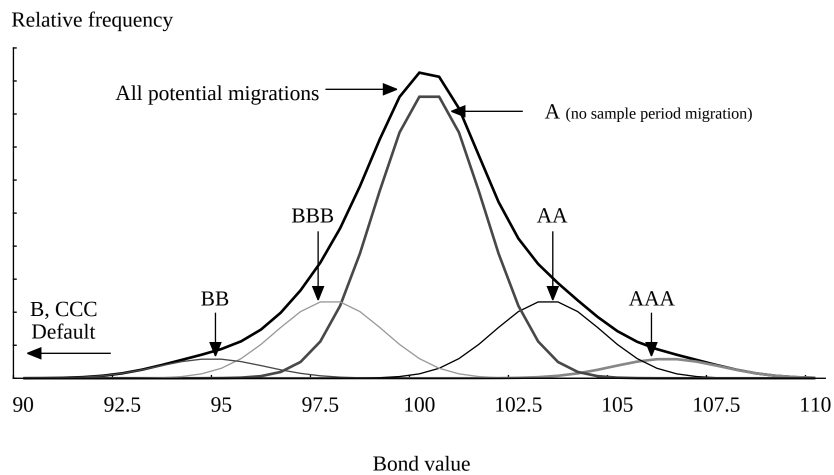
Potential problems with this approach arise because of its relative inefficiency in estimating infrequent events such as up(down)grades and defaults. Observing some benchmark bond in this fashion over, say, the last year, will yield one of two qualitative results. First, the benchmark bond will neither upgraded nor downgraded and the resulting observed volatility will be (relatively) small. Second, the benchmark bond will have *realized* some credit quality migration and the resulting observed volatility will be (relatively) large.

This process of observing volatility should be unbiased over many trials. However, the estimation error is potentially high due to the infrequent but meaningful impact of credit quality migrations on value. Our approach in CreditMetrics uses long term estimate of migration likelihood rather than observation within some recent sample period and so should avoid this problem.

Consider *Chart 5.2* below. Bonds within each credit rating category can be said also to have volatility of value due to day-to-day credit spread fluctuations. The RAROC approach seeks to measure these fluctuations, but will also sometimes *realize* a potentially large move due to a credit rating migration. Our approach is probabilistic. CreditMetrics assumes that all migrations might have been realized and each is weighted by the likelihoods of migration which we argue is best estimated using long term data.

[2] The credit quality migration and revaluation mechanism in CreditMetrics gives a weight to remote but possible credit quality migrations according to their long-term historical frequency without regard to how a short-term (perhaps one year) sampling of bond prices would – or would not – have observed theses.

Chart 5.2
Construction of volatility across credit quality categories



5.3 A portfolio view

Any analysis of a group of exposures could be called a portfolio analysis. We use the term here to mean a Markowitz-type analysis where the total risk of a portfolio is measured by explicit consideration of the relationships between individual risks and exposure amounts in a variance-covariance framework. This type of analysis was originated by Harry Markowitz, and has subsequently gone through considerable development, primarily in application to equity portfolios.

A growing number of major institutions estimate the portfolio effects of credit risk in a Markowitz-type framework. However, most institutions still rely on an intuitive assessment as to what level of over concentration to any one area may lead to problems. Thus, bank lenders, for instance, typically set exposure limits against several types of portfolio concentrations, such as industrial sector, geographical location, product type, etc. Lacking the guidance of a model, these groupings tend to be subjective rather than statistical.

For example, industrial sectors are generally defined by aggregating four-digit Standard Industrial Classification (SIC) codes into 60 or fewer groupings. This implies that the banker is assuming that credit quality correlations are higher within an industry or sector and lower between industries or sectors. It is not clear from the data that this is necessarily true. Although this is likely true for *commodity process* industries like oil refining and wood/paper manufacture, we believe it would be less true for *proprietary technology* industries like pharmaceuticals and computer software.

Modern portfolio theory is commonly applied to market risk. The volatilities and correlations necessary to calculate portfolio market volatility are generally readily measurable. In contrast, there has been relatively little academic literature on the problem of measuring diversification or over-concentration within a credit portfolio. To do this requires an understanding of credit quality correlations between obligors.

So, if we were interested in modeling the coincidence of *just* defaults, we might follow Stevenson & Fadil [95]. They constructed 33 industry indices of default experience as listed in Dun & Bradstreet's *Business Failure Record*. The correlation between these indices was their industry level estimate default correlation. While this approach is fine in concept, it suffers from the infrequency of defaults over which to correlate.

To get around this problem, another approach is to construct indices of, not just defaulted firms, but default *likelihoods* of all firms. We know of two services which publish quantitatively estimated default likelihood statistics across thousands of firms: KMV Corporation and Zeta Services. Gollinger & Morgan [93] used time series of default likelihoods (Zeta-Scores™ published by Zeta Services) to estimate default correlations across 42 industry indices. Neither of these studies has been realized in a practicable implementation.

In contrast to these academic suggestions, there is a practicable framework which is a commercial offering by KMV Corporation. In brief, they estimate the value of a firm's debt within the option theoretic framework first described in Merton [74]. Both expected default frequencies (EDFs) and correlations of default expectation are addressed within a consistent – and academically accepted – model-of-the-firm.

The approach practiced by KMV is to look to equity price series as a starting point to understanding the volatility of a firm's underlying (unlevered) asset value moves. Asset value moves can be taken to be approximately normally distributed. These asset values can in turn be mapped ordinally (one-to-one) to credit quality measure, as illustrated in *Chart 3.3*. An assumption of bivariate normality between firms' asset value moves then allows credit quality correlations to be estimated from equity prices series. This is the model on which we have constructed the equity-based correlation estimation in *Chapter 8*. J.P. Morgan has talked with KMV for at least four years on this approach to correlation and we are grateful for their input.

Chapter 6. Default and credit quality migration

A fundamental source of risk is that the *credit quality* of an obligor may change over the risk horizon. "Credit quality" is commonly used to refer to only the relative chance of default. As we show here, however, CreditMetrics makes use of an extended definition that includes also the volatility of up(down)grades. In this chapter we do the following:

- detail our model-of-the-firm which relates changes in underlying firm value to the event of credit distress;
- generalize this model to incorporate up(down)grades in credit quality;
- discuss the historical tabulation of transition matrices by different providers;
- discuss anticipated long-term behavior of transition matrices; and
- detail an approach to estimate transition probabilities which is sensitive to both the historical tabulation and anticipated long-term behavior.

6.1 Default

As discussed in the previous chapter, credit rating systems typically assign an alphabetic or numeric label to rating categories. By itself, this only gives an ordinal ranking of the default likelihoods across the categories. A quantitative framework, such as CreditMetrics, must give meaning to each rating category by linking it with a default probability.[1]

In the academic research, even the definition of the default event has evolved over time. Up to 1989, it was common to look for only missed interest or principal payments (see Altman [87]). Since then, starting with Asquith, Mullins & Wolff [89], researchers realized that distressed exchanges can play an important role in default statistics. Also default rates can be materially different depending upon the population under study. If rates are tabulated for the first few years of newly issued, then the default rate will be much lower than if the population broadly includes all extant debt.

6.1.1 Defining credit distress

For our purposes in CreditMetrics, we look to the following characteristics when we speak of the likelihood of credit distress:

- default rates which have been tabulated weighted by obligors rather than weighted by number of issues or dollars of issuance;
- default rates which have been tabulated broadly upon all obligors rather than just those with recent debt issuance; and

[1] Rating agencies commonly also include a judgment for differing recovery rates in their subordinated and structured debt rating. For instance, although senior and subordinated debt to a firm will encounter what we term "credit distress" at the exact same time, the anticipated recovery rate for subordinated is lower and thus it is given a lower rating. It is the senior rating that we look to as the most indicative of credit distress likelihood.

- default rates which are tabulated by senior rating categories (subordinated ratings include recovery rate differences, which are separate from the *likelihood* of default).

This last point is worth elaborating. We utilize credit ratings as an indication of the chance of default and credit rating migration likelihood. However, there are clearly differences in rating – to different debt of the same firm – between senior and subordinated classes. The rating agencies assign lower ratings to subordinated debt in recognition of differences in anticipated recovery rate in default. It is certainly true that senior debt obligations may be satisfied in full during bankruptcy procedures while subordinated debt is paid off only partially. In this circumstance we would say that the firm – and so *all* its debts – encountered *credit distress* even though only the subordinated class realized a *default*. Thus we take the senior credit rating as most indicative of the chance of a firm encountering *credit distress*.

6.1.2 Fitting probabilities of default with a transition matrix

Based on historical default studies from both Moody's and S&P credit rating systems, we have transition matrices which include historically estimated one-year default rates. These are included as part of the dataset for CreditMetrics. Of course, there are many rating agencies beyond S&P and Moody's. There are two ways of using alternative credit rating systems depending upon what historical information is available.

- If individual rating histories are available, then tabulating a transition matrix would give first direct estimate of the transition likelihoods including default.

- If all that is available are cumulative default histories by rating category,[2] then the transition matrix which "best replicates" this history can be estimated.

In the absence of historical information, perhaps a one-to-one correspondence could be made to established rating systems based on each credit category's rating criteria.

6.2 Credit quality migration

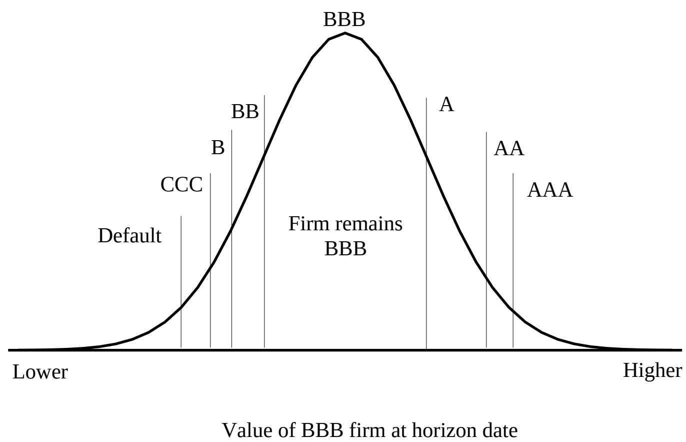
Credit rating migrations can be thought of as an extension of our model of firm defaults discussed in *Section 3* and illustrated again in *Chart 6.1*. We say that a firm has some underlying value – the value of its assets – and changes in this value suggest changes in credit quality. Certainly it is the case that equity prices drop precipitously as a firm moves towards bankruptcy. If we take the default likelihood as given by the credit rating of the firm, then we can work backwards to the "threshold" in asset value that delimits default. This is treated more formally in *Section 8.4*.

Likewise, just as our firm default model uses the default likelihood to place a threshold below which a firm is deemed to be in default, so also do the rating migration probabilities define thresholds beyond which the firm would be deemed to up(down)grade from its current credit rating. The data which drive this model are the default likelihood and credit rating migration likelihoods for each credit rating. We can compactly represent these rating migration probabilities using a transition matrix model (e.g., *Table 6.2*).

[2] Moody's terms these aggregated groupings "cohorts" and S&P terms them "static pools."

In essence, a transition matrix is nothing more than a square table of probabilities. These probabilities give the likelihood of migrating to any possible rating category (or perhaps default) one period from now given the obligor's credit rating today.

Chart 6.1
Model of firm value and migration



Many practical events (e.g., calls, enforced collateral provisions, spread resets) can be triggered by a rating change. These actions can directly affect the realized value within each credit rating category. For instance, a *pricing grid* – which predetermines a credit spread schedule given changes in credit rating – can reduce the volatility of value across up(down)grades.[3] Thus, we find it very convenient to explicitly incorporate awareness of rating migrations into our risk models.

6.3 Historical tabulation

We can tabulate historical credit rating migration probabilities by looking at time series of credit ratings over many firms. This technique is both powerful and limited. It is powerful in that we can freely model different volatilities of credit quality migration conditioned on the current credit standing. Said another way, each row in the transition matrix describes a volatility of credit rating changes that is unique to that row's initial credit rating. This is clearly an advantage since migration volatilities can vary widely between initial credit rating categories. There are, however, two assumption that we make about transition matrices. They are:

1. We assume that all firms tagged with the "correct" rating label. By this we mean that the rating agencies' are diligent in consistently applying credit rating standards across industries and countries (i.e., a "Baa" means the same for a U.S. electric utility as it does for a French bank). Of course, there is no reason that transition matrices could not be tabulated more specifically to reflect potential differences in the historical migration likelihoods of industries or countries. One caveat to this refinement might be the greater "noise" introduced by the smaller sample sizes.

[3] The securitised form of this structure is called a Credit-Sensitive Note (CSN) and is discussed in more detail in Das & Tufano [96].

2. We assume that all firms tagged with a given rating label will act alike. By this we mean that the full spectrum of credit migration likelihoods – not just the default likelihood – is similar for each firm assigned to a particular credit rating.

There are several sources of transition matrices, each specific to a particular credit rating service.[4] We advocate maintaining this correspondence even though it is common for practitioners to use shorthand assumptions, e.g., Moody's Baa is "just like" S&P's BBB, etc. Here we list three of these sources: Moody's, S&P, and KMV. Each is shown for the major credit rating categories – transition matrices which cover the minor (+/-) credit rating are also available, but are not shown here.

6.3.1 Moody's Investors Service transition matrix

Moody's utilizes a data set of 26 years' worth of credit rating migrations over the issuers that they cover. These issuers are predominantly U.S.-based firms, but are including more and more international firms. The transition matrix is tabulated upon issuers conditioned on those issuers continuing to be rated at the end of the year. Thus there is no concern with having to adjust for a *no-longer-rated* "rating."

Table 6.1
Moody's Investors Service: One-year transition matrix

Initial Rating	Rating at year-end (%)							
	Aaa	Aa	A	Baa	Ba	B	Caa	Default
Aaa	93.40	5.94	0.64	0	0.02	0	0	0
Aa	1.61	90.55	7.46	0.26	0.09	0.01	0	0.02
A	0.07	2.28	92.44	4.63	0.45	0.12	0.01	0
Baa	0.05	0.26	5.51	88.48	4.76	0.71	0.08	0.15
Ba	0.02	0.05	0.42	5.16	86.91	5.91	0.24	1.29
B	0	0.04	0.13	0.54	6.35	84.22	1.91	6.81
Caa	0	0	0	0.62	2.05	4.08	69.20	24.06

Source: Lea Carty of Moody's Investors Service

6.3.2 Standard & Poor's transition matrix

It happens that the transition matrix published by Standard & Poor's includes a *no-longer-rated* "rating," and so we pause to discuss this issue. The majority of these withdrawals of a rating occur when a firm's only outstanding issue is paid off or its debt issuance program matures. Yet our assumption is that CreditMetrics will be applied to obligations with a known maturity. So there should be no N.R. category in application.

Thus, it makes sense to eliminate the N.R. category and gross-up the remaining percentages in some appropriate fashion. We do this as follows. Since S&P describes that they track bankruptcies even after a rating is withdrawn, the default probabilities are already fully tabulated. We believe that there is no systematic reason correlated with credit rat-

[4] KMV is not a credit rating service. They quantitatively estimate Expected Default Frequencies (EDF) which are continuous values rather than categorical labels using an option theoretic approach.

ing stating which would explain rating removals. We thus adjust all remaining migration probabilities on a *pro rata* basis as shown in *Table 6.2* below:

Table 6.2
Standard & Poor's one-year transition matrix – adjusted for removal of N.R.

Initial Rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek April 15, 1996

Both of these tables are included in the CreditMetrics data set.

6.3.3 KMV Corporation transition matrix

Both of the above transition matrices were tabulated by credit rating agencies. In contrast, the sample transition matrix shown in Table 6.3 was constructed from KMV EDFs (expected default frequency) for non-financial companies in the US using data from January 1990 through September 1995. Each month, the rating group based on the EDF of each company for that month was compared against the rating group it was in 12 months hence, based on its EDF at that date. This gave a single migration. There are an average of 4,780 companies in the sample each month, resulting in a total of 329,803 migration observations. Firms that disappeared from the sample were allocated into the rating categories proportionately to the population. Rating group #8 signifies default, which is treated as a a terminal event for the firm.

The purpose of this sample is to show how an alternative approach such as EDFs can be utilized to generate a transition matrix. EDFs are default probabilities measured on a continuous scale of 0.02% to 20.0%, but grouped into discrete “rating” ranges for application in CreditMetrics.

Table 6.3
KMV one-year transition matrices as tabulated from expected default frequencies (EDFs)

Initial Rating	Rating at Year-end (%)							
	1 (AAA)	2 (AA)	3 (A)	4 (BBB)	5 (BB)	6 (B)	7 (CCC)	8 (Default)
1 (AAA)	66.26	22.22	7.37	2.45	0.86	0.67	0.14	0.02
2 (AA)	21.66	43.04	25.83	6.56	1.99	0.68	0.20	0.04
3 (A)	2.76	20.34	44.19	22.94	7.42	1.97	0.28	0.10
4 (BBB)	0.30	2.80	22.63	42.54	23.52	6.95	1.00	0.26
5 (BB)	0.08	0.24	3.69	22.93	44.41	24.53	3.41	0.71
6 (B)	0.01	0.05	0.39	3.48	20.47	53.00	20.58	2.01
7 (CCC)	0.00	0.01	0.09	0.26	1.79	17.77	69.94	10.13

Source: KMV Corporation

Table 6.3 is presented as an example and will not be included in the CreditMetrics data set. Subscribers to KMV's Expected Default Frequencies utilize a measure of default probability that is on a continues scale rather than discrete groupings offered by a credit rating agency.

Both KMV and we ourselves advocate that each credit rating (or expected default frequency) be addressed by a transition matrix tailored to that system. For this reason, the example KMV transition matrix shown here will not be part of the CreditMetrics data set. Only subscribers to KMV's expected default frequency (EDF) data would be users of such a transition matrix and so KMV will be offering it as part of that subscription.

Although it would be fine to have some issuers within a portfolio evaluated with one service (i.e., financials evaluated by IBCA) and other issuers evaluated by another service (i.e., corporates and industrials by Moody's, say), it would be inappropriate to mix systems (i.e., S&P ratings applied to Moody's transition matrix).

6.4 Long-term behavior

In estimating transition matrices, there are a number of desirable properties that one wants a transition to matrix to have, but which does not always follow from straightforward compilation of the historical data. In general, it is good practice to impose at least some of the desirable properties on the historical data in the form of estimation constraints.

The nature and extent of the problems encountered will be a function of the particular rating system, the number of grades considered, and the amount of historical data available. The following discussion uses S&P ratings as the basis for explaining these issues and how they can be addressed.

Historical tabulation is worthwhile in its own right. However, as with almost any type of sampling, it represents a limited amount of observation with sampling error. In addition to what we have historically observed, we also have strong expectations about credit rating migrations. For instance, over sufficient time we expect that any inconsistencies in rank order across credit ratings will disappear. By *rank order*, we mean a consistent progression in one direction such as default likelihoods always increasing – never then decreasing – as we move from high quality ratings to lower quality ratings. We list three potential short-term sampling error concerns here:

- Output cumulative default likelihoods should not violate proper rank order. For instance, *Table 6.4* below shows that AAAs have defaulted more often at the 10-year horizon than have AAs.

- Limited historical observation yields "granularity" in estimates. For instance, the AAA row in *Table 6.2* above is supported by 1,658 firm-years worth of observation. This is enough to yield a "resolution" of 0.06% (i.e., only probabilities in increments of 0.06% – or 1/1658 – are possible).

- This lack of resolution may erroneously suggest that some probabilities are identically zero. For instance, if there were truly a 0.01% chance of AAA default, then we would have to watch for another 80 years before there would be a 50% chance of tabulating a non-zero AAA default probability.

There are other potential problems with historical sampling such as the business cycle and regime shifts (e.g., the restructuring of the high-yield market in the 1980's). But these will not be addressed here.

Table 6.4
Average cumulative default rates (%)

Term	**1**	**2**	**3**	**4**	**5 ...**	**7 ...**	**10 ...**	**15**
AAA	0.00	0.00	0.07	0.15	0.24 ...	0.66 ...	1.40 ...	1.40
AA	0.00	0.02	0.12	0.25	0.43 ...	0.89 ...	1.29 ...	1.48
A	0.06	0.16	0.27	0.44	0.67 ...	1.12 ...	2.17 ...	3.00
BBB	0.18	0.44	0.72	1.27	1.78 ...	2.99 ...	4.34 ...	4.70
BB	1.06	3.48	6.12	8.68	10.97 ...	14.46 ...	17.73 ...	19.91
B	5.20	11.00	15.95	19.40	21.88 ...	25.14 ...	29.02 ...	30.65
CCC	19.79	26.92	31.63	35.97	40.15 ...	42.64 ...	45.10 ...	45.10

Source: S&P CreditWeek, Apr. 15, 1996

6.4.1 Replicate historical cumulative default rates

The major rating agencies have published tables of cumulative default likelihood over holding periods as long as 20 years – reported in annual increments. If we ignore for the moment the issue of autocorrelation, then it is generally true that *"there exists some annual transition matrix which best replicates (in a least squares sense) this default history."* Said another way, we can always work backwards from a cumulative default table to an implied transition matrix. *Table 6.4* illustrates part of a cumulative default probability table published by Moody's.

Cumulative default rate tables like this can be fit fairly closely by a single transition matrix.[5] Thus, it is apparently true that defaults over time are closely approximated by a transition matrix model.[6] This is an important result. It demonstrates that the statistical behavior of credit rating migrations can be captured through a transition matrix model. CreditMetrics uses a transition matrix to model credit rating migrations not only because it is intuitive but also because it is an extremely powerful statistical tool.

Below we show a transition matrix that has been created using *nothing but* a least squares fit to the cumulative default rates in *Table 6.4*. At this point, we are most interested in showing that: (i) such a matrix can be derived and (ii) that the process of defaults is closely replicated by a Markov process. (We make no claim that *Table 6.5* is a faithful replication of the historically tabulated *Table 6.2*.)

Table 6.5
Imputed transition matrix which best replicates default rates

Initial Rating	Rating at year end (%)							
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	43.78	53.42	1.65	0.71	0.29	0.11	0.02	0.01
AA	0.60	90.60	6.20	1.45	0.93	0.16	0.04	0.01
A	0.22	2.84	92.97	3.12	0.56	0.14	0.07	0.07
BBB	2.67	3.29	12.77	75.30	5.07	0.60	0.14	0.17
BB	0.19	3.58	8.28	9.97	55.20	17.17	4.53	1.08
B	0.12	0.50	20.69	1.05	0.25	55.40	17.05	4.95
CCC	0.04	0.11	6.28	0.30	0.12	41.53	32.46	19.15

For comparison to *Table 6.4*, we show below in *Table 6.6* the cumulative default rates which result from this transition matrix. Again, the most important point is that *Table 6.4* and *Table 6.6* are quite close; thus the Markov process is a reasonable modeling tool. The median difference between them is 0.16% with a maximum error of 2.13%.

This "best fit" Markov process has yielded the side benefit of resolving non-intuitive rank order violations in its resulting cumulative default rates. For instance, our problem of AAA's having a 10 year default rate that was *greater* than AA's is now gone. This behavior – of non-crossing default likelihoods – is a feature that we would expect given very long sampling histories.

[5] Empirically, a transition matrix fit is not as good for cumulative default rates of *newly issued* debt (as opposed to the total debt population) due to a "seasoning" effect where sub-investment grades have an unusually low default likelihood in the first few years. This "seasoning" problem has not been apparent for bank facilities.

[6] A transition matrix model is an example of a *Markov Process*. A Markov Process is a state-space model which allows the next progression to be determined only by the current state and not information of previous states.

Table 6.6
Resulting cumulative default rates from imputed transition matrix (%)

Term	1	2	3	4	5	...	7	...	10	...	15
AAA	0.01	0.04	0.09	0.18	0.31	...	0.66	...	1.37	...	2.81
AA	0.01	0.06	0.15	0.27	0.44	...	0.85	...	1.63	...	3.12
A	0.07	0.17	0.30	0.46	0.65	...	1.11	...	1.94	...	3.50
BBB	0.17	0.41	0.78	1.25	1.79	...	2.95	...	4.60	...	6.83
BB	1.08	3.41	6.14	8.76	11.05	...	14.53	...	17.71	...	20.39
B	4.95	10.97	15.75	19.33	21.98	...	25.46	...	28.19	...	30.35
CCC	19.15	27.43	32.63	36.32	39.01	...	42.49	...	45.14	...	47.05

6.4.2 Monotonicity (non-crossing) barrier likelihoods

Cumulative default rates are just a special case of what we term "barrier" likelihoods. In general, we can ask, "what is the cumulative rate of crossing any given level of credit quality?" For instance, if we managed a portfolio which was not allowed to invest in sub-investment grade bonds, then we might be interested in the likelihood of any credit quality migrations which were to or across the BB rating barrier. The cumulative probabilities for crossing the "BB barrier" using the transition matrix in *Table 6.5* are as shown in *Table 6.7*. Notice that monotinicity (rank order) is violated for single-As.

Table 6.7
"BB barrier" probabilities calculated from *Table 6.6* **matrix (%)**

Term	1	2	3	4	5	...	7	...	10	...	15
AAA	0.46	1.40	2.54	3.80	5.09	...	7.74	...	11.71	...	18.13
AA	1.25	2.54	3.85	5.17	6.51	...	9.17	...	13.12	...	19.47
A	0.91	2.00	3.20	4.49	5.82	...	8.57	...	12.69	...	19.29
BBB	6.57	11.66	15.69	18.93	21.60	...	25.78	...	30.40	...	36.25

Just as we would expect very long-term historical observation to resolve violations of non-intuitive cumulative default rank order, we should expect resolution of barrier rank ordering. This table above shows that our imputed transition matrix violates this anticipated long-term behavior.

We can now replay the least squares fit we performed when we produced *Table 6.4* with the added constraint that all possible barrier probabilities must also be in rank order. *Table 6.8* shows these same BB barrier probabilities with our new fit. (In fact, there are six non-default "barriers" for seven rating categories and our fitting algorithm addressed them all.)

Table 6.8
“BB barrier” probabilities calculated from *Table 6.6* **matrix (%)**

Term	1	2	3	4	5	...	7	...	10	...	15
AAA	0.39	1.09	1.98	3.01	4.12	...	6.52	...	10.37	...	16.97
AA	1.07	2.19	3.36	4.57	5.82	...	8.39	...	12.36	...	19.01
A	1.13	2.42	3.82	5.29	6.80	...	9.88	...	14.48	...	21.73
BBB	5.88	10.72	14.77	18.18	21.11	...	25.89	...	31.34	...	38.13

This refinement was achieved with minimal change in the transition matrix’s fit to the cumulative default rates. The differences in predicted cumulative default rates averages only 0.06% (median is 0.02%) between the two fitted transition matrices. For comparison with *Table 6.5*, we show this new fit of our imputed transition matrix.

Table 6.9
Imputed transition matrix with default rate rank order constraint

	---Rating at year end (%)---							
Initial Rating	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	58.57	39.02	1.42	0.63	0.18	0.14	0.03	0.01
AA	0.71	89.45	7.47	1.39	0.72	0.18	0.05	0.02
A	0.25	3.83	91.15	3.73	0.77	0.14	0.07	0.06
BBB	2.07	2.26	10.03	80.29	4.53	0.50	0.15	0.18
BB	0.15	3.57	7.84	10.38	55.91	16.18	4.91	1.06
B	0.14	0.62	19.21	2.44	0.55	54.87	17.24	4.94
CCC	0.04	0.14	5.85	0.77	0.33	41.10	32.65	19.14

Perhaps the difference between *Table 6.5* and *Table 6.9* is that the weight of probabilities are generally moved towards the upper-left to lower-right diagonal. Also, without directly trying, we are moving towards a better approximation of the historical transition matrix shown in *Table 6.2*.

6.4.3 Steady state profile matches debt market profile

Another desirable dimension of “fit” for a transition matrix is for it to exhibit a long-term steady state that approximates the observed profile of the overall credit markets. By this we mean that – among those firms which do not default – there will be some distribution of their credit quality across the available credit rating categories. To represent the rating profile across the bond market, we have taken the following data (*Table 6.10*) from Standard & Poor’s *CreditWeek* April 15, 1996.

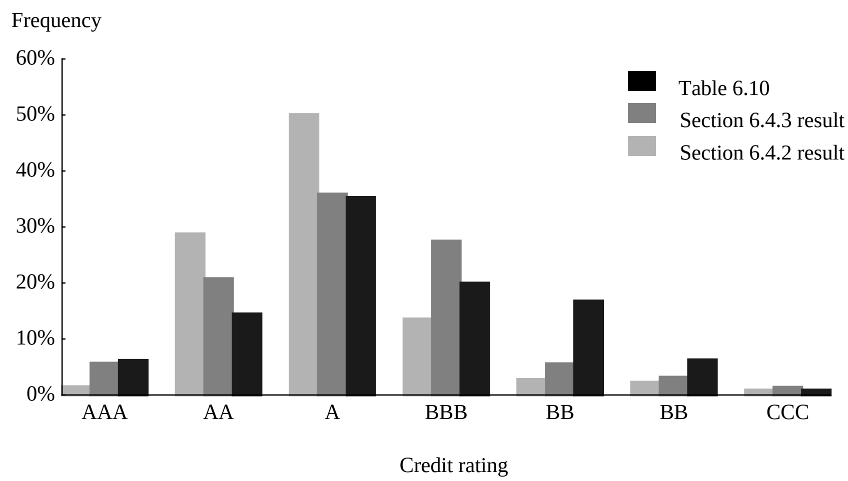
Table 6.10
Estimate of debt market profile across credit rating categories

S&P 1996	AAA	AA	A	BBB	BB	B	CCC
Count	85	200	487	275	231	87	13
Proportion	6.2%	14.5%	35.3%	20.0%	16.8%	6.3%	0.9%

Mathematically, our transition matrix Markov process will have two long-term properties (i.e., more than 100 periods). First, since default is an *absorbing* state, eventually all firms will default. Second, since the initial state has geometrically less influence on future states, the profile of non-defaulted firms will converge to some steady state regardless of the firm's initial rating.

As the chart below shows, our fitting algorithm can achieve a closer approximation of the anticipated long-term steady state. The transition matrix in *Table 6.9* shows too strong a tendency to migrate towards single-A. Once we add an incentive to fit the anticipated steady state, we see that a more balanced profile is achieved.

Chart 6.2
Achieving a closer fit to the long-term steady state profile



This additional soft constraint was accomplished with a negligible effect on the matrix's ability to replicate cumulative default likelihoods – and monotonicity in the barrier was still fully realized. Also, without directly trying, we are moving towards a better approximation of the historical transition matrix shown in *Table 6.2*.

6.4.4 Monotonicity (smoothly changing) transition likelihoods

Though it is certainly not a requirement of a transition matrix, our expectation is that there is a certain rank ordering the likelihood of migrations as follows:

1. Better ratings should never have a higher chance of default;
2. The chance of migration should become less as the migration distance (in rating notches) becomes greater; and

3. The chance of migrating to a given rating should be greater for more closely adjacent rating categories.

As an example, we will refer to *Table 6.10*. Since the default likelihoods ascend smoothly there is no violation of #1. However, since the chance that a single-B would migrate to a single-A is greater than either a migration to BBB or BB, there is a “violation” of #2. Also, since single-B has a greater chance of migrating to single-A than does an initial BB or BBB, there is a “violation” of rule #3. The reader can find other probabilities in this table which are not monotonic in our definition.

As before, we could add the soft constraint that our fitting algorithm should endeavor to mitigate these non-rank orderings of probabilities as it seeks to replicate the cumulative default likelihoods. However, as we discuss next, there is one last source of data that we should use in best estimating our transition matrix – an historically tabulated transition matrix. Any fitting algorithm that addresses smooth transition likelihoods would have to revisit these same probabilities when it includes knowledge of the historically tabulated transition matrix. So we address them both together below.

6.4.5 Match historically tabulated transition matrix

Standard & Poor’s historically tabulated transition matrix was shown above in *Table 6.2*. Up to now we have discussed some of the characteristics of transition matrices and methods of addressing these. Now we will bring all this together in *Table 6.11* to give an estimate of a one-year transition matrix which is rooted in the historical data and is also sensitive to our expectation of long-term behavior.

Table 6.11
Achieving a closer fit to the long-term steady state profile

Initial Rating	**Rating at year end (%)**							
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60

This transition matrix is meant to be close to the historically tabulated probabilities while being adjusted somewhat to better approximate the long-term behaviors we have discussed in this section. From a risk estimation standpoint we see that there are now small but non-zero probabilities of default imputed for AAAs and AAs.

Chapter 7. Recovery rates

Residual value estimation in the event of default is inherently difficult. At the time when a banker makes a loan or an investor buys a bond, it is in the belief not that the obligor will go bankrupt but that the instrument will outperform. So it can be especially difficult to imagine what the obligor's position will be in the unlikely event of default. Will it be a catastrophe which leaves no value to recover, or will it be a regrettable but well behaved wrapping up which affects only shareholders but leaves debt holders whole?

It is in the remote chance of an outright default that a credit instrument will realize its greatest potential loss. Across a typical portfolio, most of the credit risk will be attributable to default events. Investment grade credits will have relatively more of their volatility attributable to credit spread moves versus sub-investment grade credits, which will be primarily driven by potential default events. However, a typical portfolio will have a mixture of each, with most of the portfolio risk coming from the sub-investment grades. So the magnitude of any recovery rate in default is important to model diligently.

The academic literature in our bibliography focuses primarily on U.S. defaults post October 1, 1979 – the effective date of the 1978 Bankruptcy Reform Act. However, the general finding that recovery rates are highly uncertain with a distribution that can be modeled is applicable internationally.

In this chapter, we will discuss not only the estimation of mean expected recovery rate in default, but the important problem of addressing the wide uncertainty of recovery rate experience. This chapter is organized as follows:

- estimating recovery rate distributions, their mean and standard deviation, by seniority level and exposure type; and

- fitting a full distribution to recovery rate statistics while preserving the required 0% to 100% bounds.

We have seen much effort devoted to estimating recovery rates based on: (i) seniority ranking of debt, (ii) instrument type or use, (iii) credit rating *X*-years before default, and (iv) size and/or industry of the obligor. But the most striking feature of any historical recovery data is its wide uncertainty. Any worthwhile credit risk model must be able to incorporate recovery rate uncertainty in order to fully capture the volatility of value attributable to credit. However, once we contemplate volatility of recoveries, we must also address any potential correlation of recoveries across a portfolio. In this section, we estimate any potential correlation of recoveries across the book.

7.1 Estimating recovery rates

There are many practical problems in estimating recovery rates of debt in the event of default. Often there is no market from which to observe objective valuations, and if there are market prices available they will necessarily be within a highly illiquid market. Even if these issues are resolved there is the question of whether it is best to estimate values: (i) immediately upon announcement of default, (ii) after some reasonable period for information to become available – perhaps a month, or (iii) after a full settlement has been reached – which can take years.

Since there have been academic studies, see Eberhart & Sweeney [92], which conclude that the bond market efficiently prices future realized liquidation values, we take comfort in those studies which poll/estimate market valuations about one month after the announcement of default. This certainly is the value which an active investor would face whether or not he chose to hold his position after the default event.[1]

We look to the following independent studies for use in CreditMetrics. These studies refine their estimates of recovery rate according to seniority type among bonds. Among bank facilities (e.g., loans, commitments, letter of credit) the studies have viewed these as a separate "seniority" class. It is clear from the data that the historical loan recovery rates have been higher than recovery rates for senior bonds. It is not clear whether this is attributable to differences in relationship, use of borrowing or security.

7.1.1 Recovery rates of bonds

For corporate bonds, we have two primary studies of recovery rate which arrive at similar estimates (see Carty & Lieberman [96a] and Altman & Kishore [96]). For bond recoveries we can look primarily to Moody's 1996 study of recovery rates by seniority class. This study has the largest sample of defaulted bond that we know of. *Table 7.1* is a partial representation of Table 5 from Moody's Investors Service, which shows statistics for defaulted bond prices (1/1/70 through 12/31/1995).

Table 7.1
Recovery statistics by seniority class
Par (face value) is $100.00.

	Carty & Lieberman [96a]			Altman & Kishore [96]		
Seniority Class	**Number**	**Average**	**Std. Dev.**	**Number**	**Average**	**Std. Dev.**
Senior Secured	115	$53.80	$26.86	85	$57.89	$22.99
Senior Unsecured	278	$51.13	$25.45	221	$47.65	$26.71
Senior Subordinated	196	$38.52	$23.81	177	$34.38	$25.08
Subordinated	226	$32.74	$20.18	214	$31.34	$22.42
Junior Subordinated	9	$17.09	$10.90	—	—	—

As this table shows, the subordinated classes are appreciably different from one another in their recovery realizations. In contrast, the difference between secured versus unsecured senior debt is not statistically significant. It is likely that there is a self-selection effect here. There is a greater chance for security to be requested in the cases where an underlying firm has questionable hard assets from which to salvage value in the event of default.

There is no public study we are aware of that seeks to isolate the effects of different levels of security controlling for the asset quality of the obligor firm. It becomes then a

[1] There are two studies, see Swank & Root [95] and Ward & Griepentrog [93], that report high average holding period returns for debt held between the default announcement and the ultimate bankruptcy resolution. These studies also note the high average uncertainty of returns and thus the market's risk pricing efficiency.

practical problem for the risk manager to judge on a bond-by-bond basis what adjustment is best made to recovery rate estimates for different levels of security.[2]

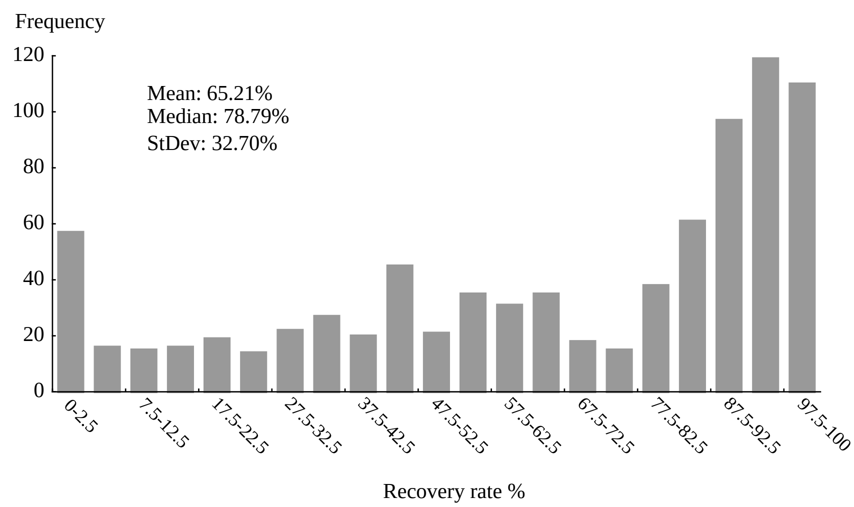
7.1.2 Recovery rate of bank facilities

For bank facilities, we again have two primary studies of recovery rate which arrive at similar estimates see (Asarnow & Edwards [95] and Carty & Lieberman [96b]). A&E track 831 commercial and industrial loan defaults plus 89 structured loans while C&L track 58 defaults of loans with Moody's credit ratings. Both studies treat bank facilities as essentially a seniority class of their own – with this being senior to all public bond seniority classes.

Moody's reports a 71% mean and 77% median recovery rate which is within sampling error of Asarnow & Edwards 65.21% mean and 78.79% median recovery rates. So these two studies are different by no more than 5%.

Chart 7.1 below is reproduced from A&E, and we have used it to estimate the standard deviation of recovery rates, of 32.7%, which is beyond the information reported by A&E.

Chart 7.1
Distribution of bank facility recoveries



Source: Asarnow & Edwards [95]

A legitimate concern is that all of the studies referenced here are either exclusively based on, or primarily driven by, U.S. bankruptcy experience. Since bankruptcy law and practice differs from jurisdiction to jurisdiction (and even across time within a jurisdiction), it is not clear that these historical estimates of recovery rate will be directly applicable internationally.

[2] For this reason, our software implementation of CreditMetrics, CreditManager, will allow recovery rate estimates to be overwritten on the individual exposure level

7.2 Distribution of recovery rate

Recovery rates are best characterized, not by the predictability of their mean, but by their consistently wide uncertainty. Loss rates are bounded between 0% and 100% of the amount exposed. If we did not know anything about recovery rate, that is, if we thought that all possible recovery rates were equally likely, then we would model them as a flat (i.e., uniform) distribution between the interval 0 to 1. Uniform distributions have a mean of 0.5 and a standard deviation of 0.29 ($\sigma = \sqrt{1/12}$). The standard deviations of 25.45% for senior unsecured bonds and 32.7% for bank facilities are on either side of this and so represent relatively high uncertainties.

We can capture this wide uncertainty and the general shape of the recovery rate distribution – while staying within the bounds of 0% to 100% – by utilizing a *beta distribution*. Beta distributions are flexible as to their shape and can be fully specified by stating the desired mean and standard deviation. *Chart 7.2* illustrates beta distributions for different seniority classes using some of statistics reported in *Table 7.1*.

Chart 7.2
Example beta distributions for seniority classes

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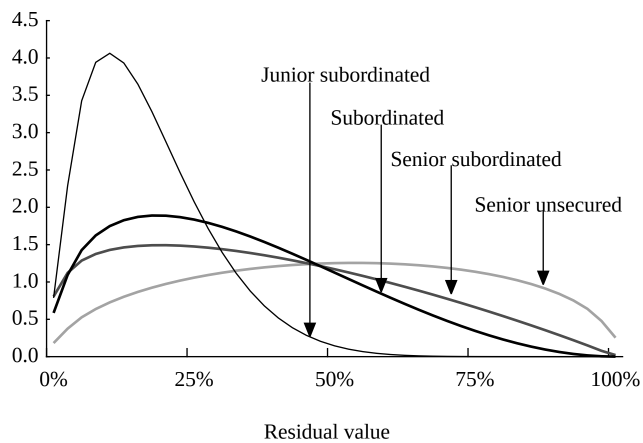


This full representation of the distribution is unnecessary for the analytic engine of CreditMetrics. It is used later in our simulation framework, where the shape of the full distribution is required.

Chapter 8. Credit quality correlations

Central to our view of credit risk estimation is a diligent treatment of the portfolio effect of credit. Whereas market risks can be diversified with a relatively small portfolio or hedged using liquid instruments, credit risks are more problematic. For credit portfolios, simply having many obligors' names represented within a portfolio does not assure good diversification (i.e., they may all be large banks within one country). When diversification is possible, it typically achieved by much larger numbers of exposures than for market portfolios.

The problem of constructing a Markowitz-type portfolio aggregation of credit risk has only recently been widely examined. We know of two academic papers which address the problems of estimating correlations within a credit portfolio: Gollinger & Morgan [93] use time series of default likelihoods (Zeta-Scores™ published by Zeta Services, Inc.) to correlate across 42 constructed indices of industry default likelihoods, and Stevenson & Fadil [95] correlate the default experience, as listed in Dun & Bradstreet's *Business Failure Record*, across 33 industry groups. Both of these studies note the practical difficulties of estimating default correlations.

Our portfolio treatment of credit risk was greatly influenced by various engagements with KMV, which has studied models of credit correlations for a number of years.

The structure of this chapter is as follows:

- First, we discuss evidence from default histories which supports our assertion that credit correlations actually exist.
- Second, we investigate the possibility of modeling joint rating changes directly using historical rating change data.
- Third, we discuss the estimation of credit correlations through the observation of bond spread histories.
- Fourth, we present a model which connects rating changes and defaults to movements in an obligor's asset value. This allows us to model joint rating changes across multiple obligors without relying on historical rating change or bond spread data.
- Last, we discuss methods to estimate the parameters of the asset value model, and present a dataset for this purpose.

8.1 Finding evidence of default correlation

In this section, before moving on to modeling correlations in credit rating changes, we examine several histories of rating changes and defaults in order to establish that such correlations in fact exist. One might claim that each firm is in many ways unique and its changes in credit quality often are driven by events and circumstances specific to that firm; this would argue for little correlation between different firms' rating changes and defaults. Thus, it would be desirable for us to first find evidence of defaults across a large body of companies.

We can do this by examining the default statistics reported by the major rating agencies over many years. Since the studies we consider are based on a very large number of observations, if defaults were uncorrelated, then we would expect to observe default rates which are very stable from year to year. On the other hand, if defaults were perfectly correlated, then we would observe some years where every firm in the study defaults and others where no firms default. That our observations lie somewhere between these two extremes (that is, we observe default rates which fluctuate, but not to the extent that they would under perfect correlation) is evidence that some correlation exists. We make this observation more precise below.

We will use the formula below to compute average default correlation ρ from the data; for a full derivation, see *Appendix F*.

$$[8.1] \qquad \rho = \frac{N\left(\frac{\sigma^2}{\mu - \mu^2}\right) - 1}{N - 1} \approx \frac{\sigma^2}{\mu - \mu^2}$$

where the approximation is for large values of N, the number of names covered by the data. In the formula, μ denotes the average default rate over the years in the study and σ is the standard deviation of the default rates observed from year to year.

Both Moody's and S&P publish default rate statistics which could be used to make this type of statistical inference of average default correlations. In *Table 8.1*, we use data from Tables 3 and 6 from Moody's most recent default study (see Carty & Lieberman [96a]).

We can infer from these figures that the number of firm-years supporting the default rate, μ, is in the thousands for all credit rating categories. Thus, our approximation formula for ρ is appropriate. However, there are only 25 yearly observations supporting the calculation of σ (and it is reported with significant rounding), so the confidence levels around the resulting inferred correlation will be high.

Table 8.1
Inferred default correlations with confidence levels

Credit rating category	Default rate μ	Standard deviation defaults σ	Implied default correlation ρ	Lower confidence $Pr\{\rho < X\} = 2.5\%$	Upper confidence $Pr\{\rho > X\} = 2.5\%$
Aa	0.03%	0.1%	0.33%	0.05%	1.45%
A	0.01%	0.1%	1.00%	0.15%	4.35%
baa	0.13%	0.3%	0.69%	0.29%	1.83%
ba	1.42%	1.4%	1.40%	0.79%	2.91%
B	7.62%	4.8%	3.27%	1.95%	6.47%

Source: Moody's 1970-1995 1-year default rates and volatilities (Carty & Lieberman [96a])

There are at least four caveats to this approach:

- the standard deviations of default rates, σ, are calculated over a very limited number of observations which lead to wide confidence levels;
- the underlying periodic default rates for investment grade categories are not normally distributed; thus the confidence levels for the investment grades will be wider than those calculated;
- the average default rate, μ, is assumed to be constant across all firms within the credit rating category and constant across time; and
- the approach is sensitive to the proportion of recession versus growth years which – in the 25-year sample – may not be representative of the future.

The inferred default correlations are all positive and – using the confidence interval technique discussed above – are all statistically greater than zero to at least the 97.5% level. This is a fairly objective indication that default events have statistically significant correlations which cannot be ignored in a risk assessment model such as CreditMetrics.

In fact, our needs go beyond estimations of default correlations; we must estimate the joint likelihood of any possible combination of credit quality outcomes. Thus, if the credit rating system recognizes eight states (i.e., *AAA, AA, …, CCC* plus *Default*), then between two obligors there are 8•8 or 64 possible joint states whose likelihoods must to be estimated.

8.2 Direct estimation of joint credit moves

Perhaps the most direct way to estimate joint rating change likelihoods is to examine credit ratings time series across many firms which are synchronized in time with each other. We have done this with a sample of 1,234 firms who have senior unsecured S&P credit ratings reported quarterly for as much as the last 40 quarters. We note that this data set does not include much of the default experience that S&P reports in their more comprehensive studies and stress that we have assembled this data set only to illustrate the principle that joint credit quality migration likelihoods can be estimated directly. With this method, it is possible to avoid having to specify a correlation estimate and an accompanying descriptive model.

Since we are interested in tabulating all possible pairwise combinations between firms, there are over 1.13 million pairwise combinations within our particular sample. In general, if a rating series data set offers N observations in a tabulated transition matrix then it will offer on the order of N^2 observations of joint migration. For a rating system with seven non-default categories, there will be 28 unique joint likelihood tales. In *Table 8.2* we show one of these 28 tabulated results for the case where one firm starts the period as a BBB and another firm starts the period with a single-A rating.

Table 8.2
Historically tabulated joint credit quality co-movements

Firm starting in BBB	Firm starting in A							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	0	0	0	0	0	0	0	0
AA	0	15	1,105	54	4	0	0	0
A	0	978	44,523	2,812	414	224	0	0
BBB	0	12,436	621,477	40,584	5,075	2,507	0	0
BB	0	839	41,760	2,921	321	193	0	0
B	0	175	7,081	532	76	48	0	0
CCC	0	55	2,230	127	18	15	0	0
Default	0	29	981	67	7	0	0	0

This yields our non-parametric estimate of joint credit quality probabilities to be as in *Table 8.3*:

Table 8.3
Historically tabulated joint credit quality co-movement (%)

Firm starting in BBB	Firm starting in A							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	-	-	-	-	-	-	-	-
AA	-	0.00	0.14	0.01	0.00	-	-	-
A	-	0.12	5.64	0.36	0.05	0.03	-	-
BBB	-	1.57	78.70	5.14	0.64	0.32	-	-
BB	-	0.11	5.29	0.37	0.04	0.02	-	-
B	-	0.02	0.90	0.07	0.01	0.01	-	-
CCC	-	0.01	0.28	0.02	0.00	0.00	-	-
Default	-	0.00	0.12	0.01	0.00	-	-	-

We emphasize again that this illustration is only to demonstrate a technique for estimating joint credit quality migration likelihoods directly. Unfortunately, our own access to the rating agency's data sets is inadequate to fully estimate a production quality study.

This method of estimation has the advantage that it does not make assumptions as to the underlying process, the joint distribution shape, or rely on distilling the data down to a single parameter – the correlation. However, it carries the limitation of treating all firms with a given credit rating as identical. So two banks would be deemed to have the same relationship as a bank and an oil refiner. In the following sections, we discuss a method of estimating credit quality correlations which are sensitive to the characteristics of individual firms.

8.3 Estimating credit quality correlations through bond spreads

A second way to estimate credit quality correlations using historical data would be to examine price histories of corporate bonds. Because it is intuitive that movements in bond prices reflect changes in credit quality, it is reasonable to believe that correlations of bond price moves might allow for estimations of correlations of credit quality moves. Such an approach has two requirements: adequate data on bond price histories and a model relating bond prices to credit events.

Where bond price histories are available, it is possible to estimate some type of credit correlation by first extracting credit spreads from the bond prices, and then estimating the correlation in the movements of these spreads. It is important to note that such a correlation only describes how spreads tend to move together. To arrive at the parameters we require for CreditMetrics (that is, likelihoods of joint credit quality movements), it is necessary to adopt a model which links spread movements to credit events.

Models of risky bonds typically have three state variables: the first is the risk free interest rate, the second is the credit spread, and the third indicates whether the bond has defaulted. A typical approach (see for example Duffee [95] or Nielsen and Ronn [94]) is to assume that the risk free rate and credit spread evolve independently[1] and that defaults are linked to the credit spread through some pricing model. This pricing model allows us to infer the probability of the issuer defaulting from the observed bond spread[2]. An extension of this type of model to two or more bonds would allow for the inference of default correlations from the correlation in bond spread moves.

While an approach of this type is attractive because it is elegant and consistent with other models of risky assets, its biggest drawback is practical. Bond spread data is notoriously scarce, particularly for low credit quality issues, making the estimation of bond spread correlations impossible in practice.

8.4 Asset value model

In this section, we present the approach which we introduced in *Chapter 3* and which we will use in practice to model joint probabilities of upgrades, downgrades, and defaults (all of which will be referred to generically as credit rating changes). We are motivated to pursue such an approach by the fact that practical matters (such as the lack of data on joint defaults) make it difficult to estimate such probabilities directly. Our approach here then will be indirect. It involves two steps:

1. Propose an underlying process which drives credit rating changes. This will establish a connection between the events which we ultimately want to describe (rating changes), but which are not readily observable, and a process which we understand and can observe.
2. Estimate the parameters for the process above. If we have been successful in the first part, this should be easier than estimating the joint rating change probabilities directly.

In this section, we propose that a firm's asset value be the process which drives its credit rating changes and defaults. This model is essentially the option theoretic model of Merton [74], which is discussed further in Kealhofer [95]. We describe the model which links changes in asset values to credit rating changes and explain how we parameterize

[1] The evolution of these quantities is generally modeled by diffusion processes with some drift and volatility.

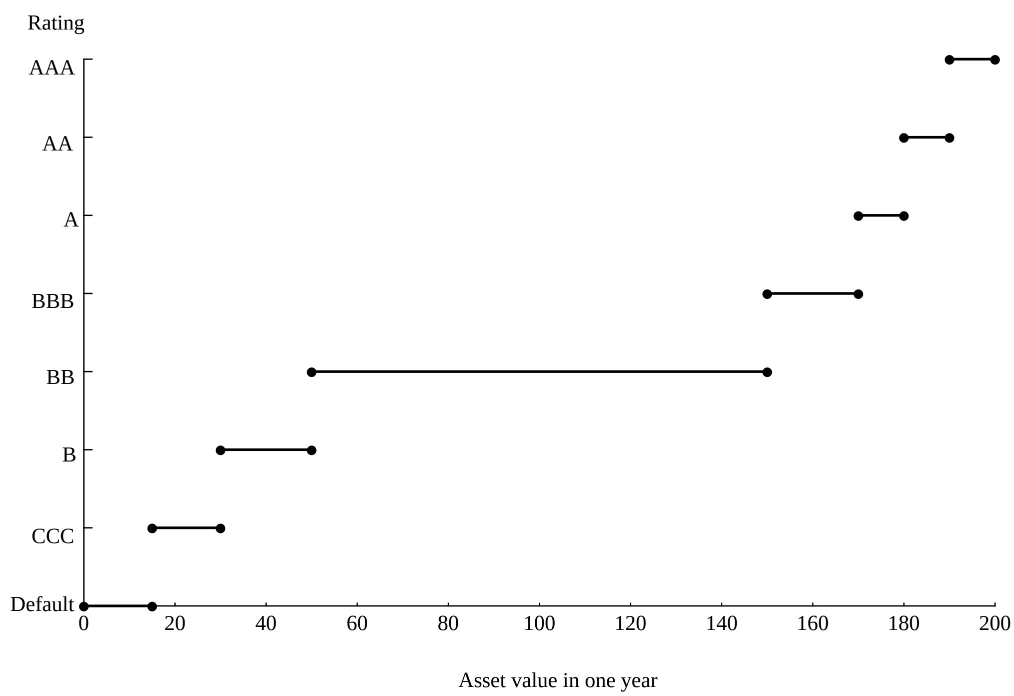
[2] This is similar to the inference of implied volatilities from observed option premiums.

the asset value model. We postpone the discussion of parameter estimation to *Section 8.5*.

It is evident that the value of a company's assets determines its ability to pay its debt holders. We may suppose then that there is a specific level such that if the company's assets fall below this level in the next year, it will be unable to meet its payment obligations and will default. Were we only treating value changes due to default, this would be a sufficient model. However, since we wish to treat portfolio value changes resulting from changes in credit rating as well, we need a slightly more complex framework.

Extending the intuition above, we assume there is a series of levels for asset value that will determine a company's credit rating at the end of the period in question. For example, consider a hypothetical company that is BB rated and whose assets are currently worth $100 million. Then the assumption is that there are asset levels such that we can construct a mapping from asset value in one year's time to rating in one year's time, as in *Chart 8.1*. Essentially, the assumption is that the asset value in one year determines the credit rating (or default) of the company at that time. The asset values in the chart which correspond to changes in rating will be referred to as asset value thresholds. We reiterate that we are not yet claiming to know what these thresholds are, only that this relationship exists.

Chart 8.1
Credit rating migration driven by underlying BB firm asset value



Assuming we know the asset thresholds for a company, we only need to model the company's change in asset value in order to describe its credit rating evolution. To do this, we assert that the percent changes in asset value (that is, asset "returns," which we will denote by R) are normally distributed, and parameterized by a mean μ and standard deviation (or volatility) σ. Note that this volatility is not the volatility of value of a credit instrument (which is an output of CreditMetrics) but simply the volatility of asset returns for a given name. For ease of exposition, we will assume $\mu=0$[3].

Given this parameterization of the asset value process, we may now establish a connection between the asset thresholds in the chart above and the transition probabilities for our company. Continuing with our example of the BB rated obligor, we read from the transition matrix that the obligor's one-year transition probabilities are as in the second column of *Table 8.4*.

On the other hand, from the discussion of asset thresholds above, we know that there exist asset return thresholds Z_{Def}, Z_{CCC}, Z_{BBB}, etc., such that if $R< Z_{Def}$, then the obligor goes into default; if $Z_{Def}<R<Z_{CCC}$, then the obligor is downgraded to CCC; and so on. So for example, if Z_{Def} were equal to -70%, this would mean that a 70% (or greater) decrease in the asset value of the obligor would lead to the obligor's default.

Since we have assumed that R is normally distributed, we can compute the probability that each of these events occur:

[8.2]
$$Pr\{Default\} = Pr\{R < Z_{Def}\} = \Phi(Z_{Def}/\sigma) ,$$
$$Pr\{CCC\} = Pr\{Z_{De(f<\ )}R < Z_{CCC}\} = \Phi(Z_{CCC}/\sigma) - \Phi(Z_{Def}/\sigma) ,$$

and so on. (Φ denotes the cumulative distribution for the standard normal distribution.) These probabilities are listed in the third column of *Table 8.4*.

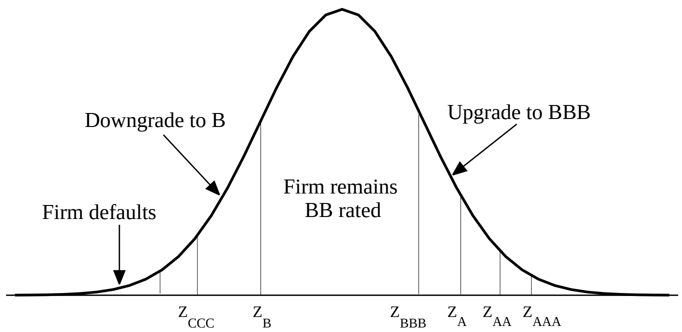
Table 8.4.
One year transition probabilities for a BB rated obligor

Rating	Probability from the transition matrix (%)(	Probability according to the asset value model
AAA	0.03	$1-\Phi(Z_{AA}/\sigma)$
AA	0.14	$\Phi(Z_{AA}/\sigma)-\Phi(Z_{A}/\sigma)$
A	0.67	$\Phi(Z_{A}/\sigma)-\Phi(Z_{BBB}/\sigma)$
BBB	7.73	$\Phi(Z_{BBB}/\sigma)-\Phi(Z_{BB}/\sigma)$
BB	80.53	$\Phi(Z_{BB}/\sigma)-\Phi(Z_{B}/\sigma)$
B	8.84	$\Phi(Z_{B}/\sigma)-\Phi(Z_{XXX}/\sigma)$
CCC	1.00	$\Phi(Z_{XXX}/\sigma)-\Phi(Z_{Def}/\sigma)$
Default	1.06	$\Phi(Z_{Def}/\sigma)$

The connection between asset returns and credit rating may be represented schematically as in *Chart 8.2*, where we present the return thresholds superimposed on the distribution of asset returns. The integral between adjacent thresholds corresponds to the probability that the obligor assumes the credit rating corresponding to this region.

[3] This likely will not be the case in practice, but for our purposes here, the value of μ will not influence the result. It is in fact true that σ does not influence the final result either – and the reader may choose to ignore σ in the expressions to follow – but we retain it for illustrative purposes.

Chart 8.2
Distribution of asset returns with rating change thresholds



Now in order to complete the connection, we simply observe that the probabilities in the two columns of the Table 1 must be equal. So considering the default probability, we see that $\Phi(Z_{Def}/\sigma)$ must equal 1.06%, which lets us solve for Z_{Def}:

[8.3] $$Z_{Def} = \Phi^{-1}(1.06\%) \cdot \sigma = -2.30\sigma,$$

where $\Phi^{-1}(p)$ gives the level below which a standard normal distributed random variable falls with probability p. Using this value, we may consider the *CCC* probability to solve for Z_{CCC}, then the *B* probability to solve for Z_B, and so on, obtaining the values in *Table 8.5*. Note there is no threshold Z_{AAA}, since any return over 3.43σ implies an upgrade to AAA.[4]

Table 8.5
Threshold values for asset return for a BBB rated obligor

Threshold	Value
Z_{AA}	3.43σ
Z_A	2.93σ
Z_{BBB}	2.39σ
Z_{BB}	1.37σ
Z_B	–1.23σ
Z_{CCC}	–2.04σ
Z_{Def}	–2.30σ

Now consider a second obligor, A rated. Denote this obligor's asset return by R′, the standard deviation of asset returns for this obligor by σ′, and its asset return thresholds

[4] We comment that to this point, we have not added anything to our model. For one obligor, we only need the transition probabilities to describe the evolution of credit rating changes, and the asset value process is not necessary. The benefit of the asset value process is only in the consideration of multiple obligors.

by Z'_{Def}, Z'_{CCC}, and so on. The transition probabilities and asset return thresholds are listed in *Table 8.6*.

Table 8.6
Transition probabilities and asset return thresholds for A rating

Rating	Probability	Threshold	Value
AAA	0.09%		
AA	2.27%	Z'_{AA}	$3.12\sigma'$
A	91.05%	Z'_{A}	$1.98\sigma'$
BBB	5.52%	Z'_{BBB}	$-1.51\sigma'$
BB	0.74%	Z'_{BB}	$-2.30\sigma'$
B	0.26%	Z'_{B}	$-2.72\sigma'$
CCC	0.01%	Z'_{CCC}	$-3.19\sigma'$
Default	0.06%	Z'_{Def}	$-3.24\sigma'$

At this point, we have described the motion of each obligor individually according to its asset value processes. To describe the evolution of the two credit ratings jointly, we assume that the two asset returns are correlated and normally distributed,[5] and it only remains to specify the correlation ρ between the two asset returns. We then have the covariance matrix for the bivariate normal distribution:

[8.4] $$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma\sigma' \\ \rho\sigma\sigma' & \sigma'^2 \end{pmatrix}$$

This done, we know how the asset values of the two obligors move together, and can then use the thresholds to see how the two credit ratings move together.

To be specific, say we wish to compute the probability that both obligors remain in their current credit rating. This is the probability that the asset return for the BB rated obligor falls between Z_B and Z_{BB} while at the same time the asset return for the A rated obligor falls between Z'_{BBB} and Z'_A. If the two asset returns are independent (i.e., $\rho=0$), then this joint probability is just the product of 80.53% (the probability that the BB rated obligor remains BB rated) and 91.05% (the probability that the A rated obligor remains A rated). If ρ is not zero, then we compute:

[8.5] $$Pr\{Z_B < R < Z_{BB}, Z'_{BBB} < R' < Z'_A\} = \int_{Z_B}^{Z_{BB}} \int_{Z'_{BBB}}^{Z'_A} f(r, r'; \Sigma)(dr')dr$$

where $f(r,r';\Sigma)$ is the density function for the bivariate normal distribution with covariance matrix Σ[6]. We may use the same procedure to calculate the probabilities of each of

[5] Technically, we assume that the two asset returns are bivariate normally distributed. We remark, however, that it is not necessary to use the normal distribution. Any multivariate distribution (including those incorporating fat tails or skewness effects) where the joint movements of asset values can be characterized fully by one correlation parameter would be applicable.

[6] The variables r and r' in Eq. [8.5] represent the values that the two asset returns may take on within the specified intervals.

the 64 possible joint rating moves for the two obligors. As an example, suppose that ρ=20%. We would then obtain the probabilities in *Table 8.7*.

Table 8.7
Joint rating change probabilities for BB and A rated obligors (%)

Rating of first company	**Rating of second company**								
	AAA	AA	A	BBB	BB	B	CCC	Def	Total
AAA	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.03
AA	0.00	0.01	0.13	0.00	0.00	0.00	0.00	0.00	0.14
A	0.00	0.04	0.61	0.01	0.00	0.00	0.00	0.00	0.67
BBB	0.02	0.35	7.10	0.20	0.02	0.01	0.00	0.00	7.69
BB	0.07	1.79	73.65	4.24	0.56	0.18	0.01	0.04	80.53
B	0.00	0.08	7.80	0.79	0.13	0.05	0.00	0.01	8.87
CCC	0.00	0.01	0.85	0.11	0.02	0.01	0.00	0.00	1.00
Def	0.00	0.01	0.90	0.13	0.02	0.01	0.00	0.00	1.07
Total	0.09	2.29	91.06	5.48	0.75	0.26	0.01	0.06	100

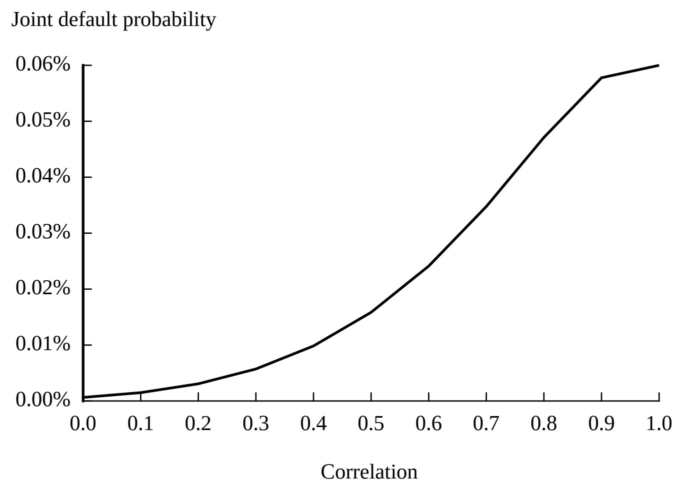
This table is sufficient to compute the standard deviation of value change for a portfolio containing only issues of these two obligors. Note that the totals for each obligor are just that obligor's transition probabilities. To compute the standard deviation for a larger portfolio, it is only necessary to repeat this analysis for each pair of obligors in the portfolio.[7]

The effect of the correlation merits further comment. Consider the worst case event for a portfolio containing these two obligors – that both obligors default. If the asset returns are independent, then the joint default probability is the product of the individual default probabilities, or 0.0006%. On the other hand, if the asset returns are perfectly correlated (ρ=1), then any time the A rated obligor defaults, so too does the BB rated obligor. Thus, the probability that they both default is just the probability that the A rated obligor defaults, or 0.06%, 100 times greater than in the uncorrelated case.

In *Chart 8.3*, we illustrate the effect of asset return correlation on the joint default probability for our two obligors.

[7] Note that if all pairs of obligors have the same correlation, then the maximum number of matrices like *Table 8.7* which would be needed is 28, regardless of the size of the portfolio. Notice that *Table 8.7* depends only on the ratings of the two obligors and on the correlation between them, and not on the particular obligors themselves. Thus, since there are only seven possible ratings for each obligor, there are only 28 possibilities for the ratings of each pair of obligors, and 28 possible matrices.

Chart 8.3
Probability of joint defaults as a function of asset return correlation



We have pointed out before that for pairs of obligors, it is only necessary to specify joint probabilities of rating changes and defaults, and that actual default correlations are not used in any calculations. However, many people are accustomed to thinking in terms of default correlations, and so we touch briefly on them here. For an asset correlation ρ_A, we have shown that it is possible to compute p_{12}, the probability that obligors 1 and 2 both default. The default correlation between these two obligors can then be written as

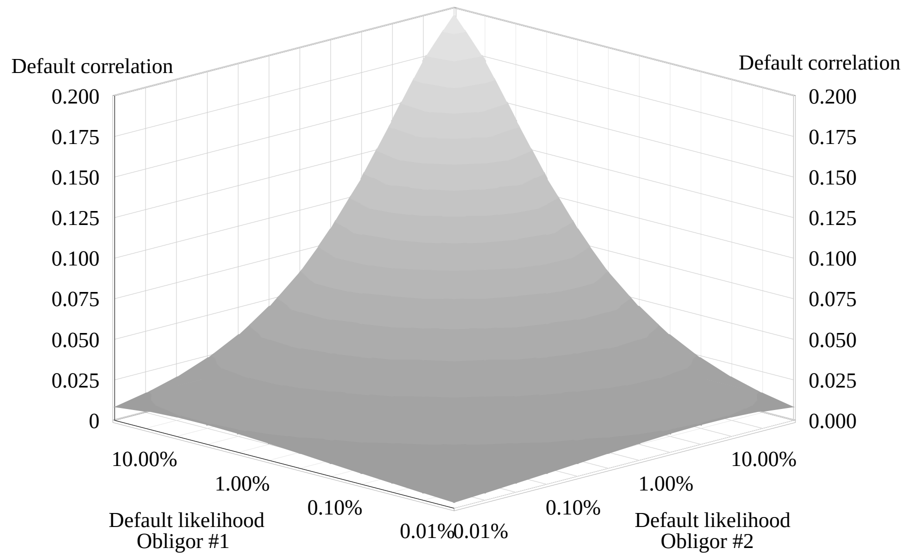
$$[8.6] \qquad \rho_D = \frac{p_{12} - p_1 p_2}{\sqrt{p_1(1-p_1)p_2(1-p_2)}},$$

where p_1 and p_2 are the probabilities that obligor 1 and obligor 2 default, respectively.

The translation from asset to default correlation lowers the correlation significantly. Asset correlations in the range of 40% to 60% will typically translate into default correlations of 2% to 4%. We see then that even the very small default correlation estimates in *Section 8.1* require that asset value moves exhibit relatively high correlations.

Chart 8.4 shows how the default correlation is a function of the two obligor's default probabilities. An asset correlation of 30% was assumed and default probabilities range from 1bp to above 10%. The high “mound” towards the back indicates that junk bond defaults will be far more correlated with each other than will investment grade defaults.

Chart 8.4
Translation of equity correlation to default correlation



Before moving on to estimation of parameters, we make one important observation: Equation [8.5] above does not depend on either of the volatilities σ or σ'. This may seem counterintuitive, that in a risk model we are ignoring asset volatility, but essentially all of the volatility we need to model is captured by the transition probabilities for each obligor. As an example, consider two obligors which have the same rating (and therefore the same transition probabilities), but where the asset volatility for one obligor is ten times greater than the other. We know that the credit risk is the same to either obligor. One obligor does have a more volatile asset process, but this just means that its asset return thresholds are greater than those of the other firm. In the end, the only parameters which affect the risk of the portfolio are the transition probabilities for each obligor and the correlations between asset returns.

The consequence of this is that we may consider *standardized* asset returns, that is, asset returns adjusted to have mean zero and standard deviation one. The only parameter to estimate then is the correlation between asset returns, which is the focus of the next section.

One last comment is that it is a simple matter to adjust for different time horizons. For example, to perform this analysis for a six-month time horizon, the only change is that we use the six-month transition probabilities to calibrate the asset return thresholds.

8.5 Estimating asset correlations

The user can pursue different alternatives to estimate firm asset correlations. The simplest is just to use some fixed value across all obligor pairs in the portfolio. This precludes the user having to estimate a large number (4,950 for a 100-obligor portfolio) of individual correlations, while still providing reasonable portfolio risk measures. However, the ability to detail risk due to overconcentration in a particular industry, for example, is lost. A typical average asset correlation across a portfolio may be in the range of 20% to 35%.[8]

For more specific correlations, there are independent data providers that can provide models which are independent of – but can be consistently used in – CreditMetrics. Below, we present our own interpretation of this type of underlying firm asset correlation estimation.

One fundamental – and typically very observable – source of firm-specific correlation information are equity returns. Here, we use the correlation between equity returns as a proxy for the correlation of asset returns. While this method has the drawback of overlooking the differences between equity and asset correlations, it is more accurate than using a fixed correlation, and is based on much more readily available data than credit spreads or actual joint rating changes.

In the best of all possible worlds, we could produce correlations for any pair of obligors which a user might request. However, the scarcity of data for many obligors, as well as the impossibility of storing a correlation matrix of the size that would be necessary, make this approach untenable. Therefore, we resort to a methodology which relies on correlations within a set of indices and a mapping scheme to build the obligor-by-obligor correlations from the index correlations.

Thus, to produce individual obligor correlations, there are two steps:

- First, we utilize industry indices in particular countries to construct a matrix of correlations between these industries. The result is that we obtain the correlation, for example, of the German chemical industry with the United States insurance industry. For reasons which will become clear below, we also report the volatility for each of these indices.[9]

- Next, we map individual obligors by industry participation. For example, a company might be mapped as 80% Germany and 20% United States, and 70% chemicals and 30% finance, resulting in 56% participation in the German chemicals industry, 24% in German finance, 14% in American chemicals, and 6% in American finance. Using these weights and the country-industry correlations from above, we obtain the correlations between obligors.

In *Section 8.5.1*, we discuss the data we provide and the methodology which goes into its construction. In the following subsection, we present an example to describe the methods by which the user specifies the weightings for individual obligors and arrives at individual obligor correlations. The last subsection is a generalization of this example.

[8] Based on conversations with Patrick H. McAllister in 1994 when he was an Economist at the Board of Governors of the Federal Reserve System. Part of his research inferred average asset correlations of corporate & industrial loan portfolios within mid-sized US banks to be in the range 20%-to-25%. Our own research suggests that it is easier to construct higher correlation portfolios versus lower correlation portfolios, hence a 20%-to-35% range.

[9] Recall from *Section 8.4* that volatilities do not figure into the model for joint rating changes. We will see that the volatilities of the indices are necessary, however, for mapping individual obligors to the indices.

8.5.1 Data

As mentioned above, we provide the user a matrix of correlations between industries in various countries. In this section, we discuss the data and the methods by which we construct this matrix.

In *Table 8.8*, we list the countries for which we provide data, along with the family of industry specific indices we use for each country. For each country, the broad country index used is the MSCI index. For countries where no index family appears, insufficient industry index data was available and we utilize only the data for the broad country index.

Table 8.8
Countries and respective index families

Country	Index family	Country	Index family
Australia	ASX	Mexico	Mexican SE
Austria		New Zealand	
Belgium		Norway	Oslo SE
Canada	Toronto SE	Philippines	Philippine SE
Finland	Helsinki SE	Poland	
France	SBF	Portugal	
Germany	CDAX	Singapore	All-Singapore
Greece	Athens SE	South Africa	
Hong Kong	Hang Seng	Spain	
Indonesia		Sweden	Stockholm SE
Italy	Milan SE	Switzerland	SPI
Japan	Topix	Thailand	SET
Korea	Korea SE	United Kingdom	FT-SE-A
Malaysia	KLSE	United States	S&P

In *Table 8.9*, we list the industries for which we provide indices in one or more of the countries. We choose these industry groups by beginning with the major groups used by Standard & Poor for the United States, and then eliminating groups which appear redundant. For instance, we find that the correlation between the Health Care and Pharmaceuticals indices is over 98%, and so consolidate these two groups into one, reasoning that the two indices essentially explain the same movements in the market.

Table 8.9
Industry groupings with codes

Grouping	Code	Grouping	Code
General country index	GNRL	Insurance	INSU
Automobiles	AUTO	Machinery	MACH
Banking & finance	BFIN	Manufacturing	MANU
Broadcasting & media	BMED	Metals Mining	MMIN
Chemicals	CHEM	Oil & gas – refining & marketing	OGAS
Construction & building materials	CSTR	Paper & forest products	PAPR
Electronics	ELCS	Publishing	PUBL
Energy	ENRG	Technology	TECH
Entertainment	ENMT	Telecommunications	TCOM
Food	FOOD	Textiles	TXTL
Health care & pharmaceuticals	HCAR	Transportation	TRAN
Hotels	HOTE	Utilities	UTIL

Because the industry coverage in each country is not uniform, we also provide data on MSCI worldwide industry indices. In a case such French chemicals, where there is no country-industry index, the user may then choose to proxy the French chemical index with a combination of the MSCI France index and the MSCI worldwide chemicals index. Finally, realizing that it may at times be more feasible to describe a company by a regional index rather than a set of country indices, we provide data on six MSCI regional indices. In the end, we select the indices for which at least three years of data are available, leaving us with 152 country-industry indices, 28 country indices, 19 worldwide industry indices, and 6 regional indices. The available country-industry pairs are presented in *Table 8.10*. For the specific index titles used in each case, refer to *Appendix I*.

Table 8.10
Country-industry index availability

Country	GNRL	AUTO	BFIN	BMED	CHEM	CSTR	ELCS	ENRG	ENMT	FOOD	HCAR	HOTE	INSU	MACH	MANU	MMIN	OGAS	PAPR	PUBL	TECH	TCOM	TXTL	TRAN	UTIL	Total
Australia	X		X	X	X	X		X		X			X					X					X		10
Austria	X																								1
Belgium	X																								1
Canada	X	X	X	X	X	X	X	X		X	X	X	X			X			X				X		15
Finland	X		X										X			X		X							5
France	X	X	X			X		X		X															6
Germany	X	X	X		X	X							X	X				X				X	X	X	11
Greece	X		X										X												3
Hong Kong	X		X																					X	3
Indonesia	X																								1
Italy	X		X		X					X						X		X							6
Japan	X		X	X	X	X	X	X		X	X		X	X		X	X	X				X	X		16
Korea	X		X		X	X				X			X	X		X		X				X	X		11
Malaysia	X		X													X									3
Mexico	X					X										X							X		4
New Zealand	X																								1
Norway	X		X										X												3
Philippines	X															X	X								3
Poland	X																								1
Portugal	X																								1
Singapore	X		X									X													3
South Africa	X		X													X									3
Spain	X																								1
Sweden	X		X		X	X												X							6
Switzerland	X		X		X	X	X																		5
Thailand	X		X		X	X	X	X		X	X	X	X	X		X		X	X	X		X	X		17
United Kingdom	X		X	X	X	X	X	X		X	X	X	X			X	X	X			X	X	X		17
United States	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	24
MSCI worldwide		X	X	X	X	X	X	X	X	X	X	X	X	X		X		X			X	X	X	X	19
Total	28	5	20	6	12	13	7	8	2	10	6	6	12	6	1	13	4	11	3	2	3	7	10	4	199

For each of the indices, we consider the last 190 weekly returns, and compute the mean and standard deviation of each return series. Thus, if we denote the t^{th} week's return on the k^{th} index by $R_t^{(k)}$, we compute the average weekly return on this index by

$$[8.7] \qquad \bar{R}^{(k)} = \frac{1}{T}\sum_{t=1}^{T} R_t^{(k)},$$

where T is 190 in our case, and the weekly standard deviation of return by

$$[8.8] \qquad \sigma_k = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(R_t^{(k)} - \bar{R}^{(k)}\right)^2}.$$

As mentioned above, we provide the user with the standard deviations (volatilities), and discuss their use in the next section. In addition, for all pairs of indices, we compute the covariance of weekly returns by

$$[8.9] \qquad \mathrm{COV}(k, l) = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t^{(k)} - \bar{R}^{(k)}\right)\left(R_t^{(l)} - \bar{R}^{(l)}\right),$$

and the correlation of weekly returns by

$$[8.10] \qquad \rho_{k,l} = \frac{\mathrm{COV}(k, l)}{\sigma_k \sigma_l}.$$

We provide these correlations to the user.

Note that our computations of volatilities and correlations differ from the standard volatility computations in RiskMetrics in that we weight all of the returns in each time series equally. The motivation for this is that we are interested in computing correlations which are valid over the longer horizons for which CreditMetrics will be used. The statistics here tend to be more stable over time, and reflect longer term trends, whereas the statistics in RiskMetrics vary more from day to day, and capture shorter term behavior.

Note also that the correlations we compute are based on historical weekly returns. It is therefore an assumption of the model that the weekly correlations which we provide are accurate reflections of the quarterly or yearly asset moves which drive the CreditMetrics model.

8.5.2 Obligor correlations – example

Now that we have described how to calculate correlations between country-industry pairs, it only remains to illustrate how to apply these to obtain correlations between individual obligors. The steps of this computation are as follows:

1. Assign weights to each obligor according to its participation in countries and industries, and specify how much of the obligor's equity movements are not explained by the relevant indices.
2. Express the standardized returns for each obligor as a weighted sum of the returns on the indices and a company-specific component.
3. Use the weights along with the index correlations to compute the correlations between obligors.

By specifying the amount of an obligor's equity price movements are not explained by the relevant indices, we are describing this obligor's firm-specific, or idiosyncratic, risk. Generally, prices for companies with large market capitalization will track the indices closely, and the idiosyncratic portion of the risk to these companies is small; on the other hand, prices for companies with less market capitalization will move more independently of the indices, and the idiosyncratic risk will be greater.

We will explain each of the steps above through an example.

Suppose we wish to compute the correlation between two obligors, ABC and XYZ. Assume that we decide that ABC participates only in the United States chemicals industry, and that its equity returns are explained 90% by returns on the United States chemicals index and 10% by company-specific movements. We assume that these company-specific movements are independent of the movements of the indices, and also independent of the company-specific movements for all other companies. Assume that XYZ participates 75% in German insurance and 25% in German banking and finance and that 20% of the movements in XYZ's equity are company-specific.

To apply these weights and describe the standardized returns for the individual obligors, we need the volatilities and correlations of the relevant indices. We present these in the *Table 8.11*. The volatilities listed are for weekly returns.

Table 8.11
Volatilities and correlations for country-industry pairs

Index	Volatility	Correlations		
		U.S. Chemicals	Germany Insurance	Germany Banking
U.S.: Chemicals	2.03%	1.00	0.16	0.08
Germany: Insurance	2.09%	0.16	1.00	0.34
Germany: Banking	1.25%	0.08	0.34	1.00

For the firm ABC, the volatility explained by the U.S. chemicals index is 90% of the firm's total volatility. The remainder is explained by ABC's firm specific movements. Thus, we consider two independent standard normal random variables, $r^{(USCm)}$ and $\hat{r}^{(ABC)}$, which represent the standardized returns of the U.S. chemical index and ABC's firm specific standardized returns, respectively. We then write ABC's standardized returns as

$$[8.11] \qquad r^{(ABC)} = w_1 r^{(USCm)} + w_2 \hat{r}^{(ABC)} .$$

We know that 90% of ABC's volatility is explained by the index, and thus we know that $w_1 = 0.9$. We also know that the total volatility must be one (since the returns are standardized), and thus $w_2 = \sqrt{1 - w_1^2} = 0.44$.

For XYZ, we proceed in a similar vein. We first figure the volatility of the index movements for XYZ, that is, the volatility of an index formed by 75% German insurance and 25% German banking, by

[8.12]

$$\hat{\sigma} = \sqrt{0.75^2 \cdot \sigma_{DeIn}^2 + 0.25^2 \cdot \sigma_{DeBa}^2 + 2 \cdot 0.75 \cdot 0.25 \cdot \rho(DeIn, DeBa) \cdot \sigma_{DeIn} \cdot \sigma_{DeBa}} = 0.017.$$

We then scale the weights so that the total volatility of the index portion of XYZ's standardized returns is 80%. Thus, the weight on the German insurance index is

$$[8.13] \qquad 0.8 \cdot \frac{0.75 \cdot \sigma_{DeIn}}{\hat{\sigma}} = 0.74,$$

and the weight on the German banking index is

$$[8.14] \qquad 0.8 \cdot \frac{0.25 \cdot \sigma_{DeBa}}{\hat{\sigma}} = 0.15.$$

Finally, in order that the total standardized return of XYZ have variance one, we know that the weight on the idiosyncratic return must be $\sqrt{1 - 0.8^2} = 0.6$.

At this point, we have what we will refer to as each firm's *standard weights*, that is, the weightings on standardized index returns which allow us to describe standardized firm returns. Recall that for our example we describe the returns for ABC and XYZ by:

$$[8.15] \qquad r^{(ABC)} = 0.90 r^{(USCm)} + 0.44 \hat{r}^{(ABC)},$$

and

$$[8.16] \qquad r^{(XYZ)} = 0.74 r^{(DeIn)} + 0.15 r^{(DeBa)} + 0.6 \hat{r}^{(XYZ)},$$

where $\hat{r}^{(ABC)}$ and $\hat{r}^{(XYZ)}$ are the idiosyncratic returns for the two firms. Since the idiosyncratic returns are independent of all the other returns, we may compute the correlation between ABC and XYZ by:

$$[8.17] \qquad \rho(ABC, XYZ) = 0.90 \cdot 0.74 \cdot \rho(USCm, DeIn) + 0.90 \cdot 0.15 \cdot \rho(USCm, DeBa) = 0.11$$

The above illustrates the method for computing correlations between pairs of obligors, and suggests a more general framework. In the next subsection, we present the same methods, but generalized to handle obligors with participations in more industries and countries.

Note that the index volatilities do not actually enter into the correlation calculations, but do play a role when we convert industry participations to standard weights. This allows us to account for cases like our example, where industry participation is split 75% and 25%, or 3 to 1, but since the industry with 75% participation (insurance) is more volatile than the other industry (banking), the standard weight on insurance is actually more than three times greater than the standard weight on banking.

8.5.3 Obligor correlations – generalization

To complete our treatment of obligor correlations, we provide generalizations of the methods above for computing standard weights and for calculating correlations from these weights.

First, to compute standard weights, consider a firm with industry participations of $\hat{w}_1$, w_2, and w_3, where the indices account for α of the movements of the firm's equity. We compute the firm's standard weights in the following steps:

Compute the volatility of the weighted index for the firm, that is,

$$[8.18]\quad \hat{\sigma} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2 + 2w_2w_3\rho_{2,3}\sigma_2\sigma_3 + 2w_1w_3\rho_{1,3}\sigma_1\sigma_3}$$

Scale the weights on each index such that the indices represent only α of the volatility of the firm's standardized returns. The scaling is as below:

$$[8.19]\quad w_1 = \alpha \cdot \frac{w_1\sigma_1}{\hat{\sigma}},\ w_2 = \alpha \cdot \frac{w_2\sigma_2}{\hat{\sigma}},\ \text{and}\ w_3 = \alpha \cdot \frac{w_3\sigma_3}{\hat{\sigma}}.$$

Compute the weight on the idiosyncratic returns by taking $\sqrt{1-\alpha^2}$.

The generalization to the case of four or more indices should be clear.

Now suppose we have *n* different firms with standard weightings on *m* indices, and we wish to compute the equity correlations between these firms. Let the correlation matrix for the indices be denoted by *C*. Since the weightings are on both the indices and the idiosyncratic components, we need to create a correlation matrix, $\bar{C}$, which covers both of these. This matrix will be *m*+*n* by *m*+*n*, and constructed as below:

$$\begin{array}{cc}
 & \begin{array}{cc} \overbrace{\hspace{6em}}^{m\text{ columns}} & \overbrace{\hspace{6em}}^{n\text{ columns}} \end{array} \\
\begin{array}{r} m\text{ rows}\left\{\vphantom{\begin{matrix}0\\ \vdots\\ \vdots\\ \vdots\\ 0\end{matrix}}\right. \\[2ex] n\text{ rows}\left\{\vphantom{\begin{matrix}0\\ \vdots\\ \vdots\\ 0\end{matrix}}\right. \end{array} &
\left(\begin{array}{c|c}
C & \begin{matrix} 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{matrix} \\
\hline
\begin{matrix} 0 & \cdots\cdots & 0 \\ \vdots & \cdots\cdots & \vdots \\ \vdots & \cdots\cdots & \vdots \\ 0 & \cdots\cdots & 0 \end{matrix} & \begin{matrix} 1 & 0 & \cdots & 1 \\ 0 & \ddots & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 \end{matrix}
\end{array}\right)
\end{array}$$

Thus, the upper left of $\bar{C}$ is the m by m matrix C, representing the correlations between indices; the lower right is the n by n identity matrix, reflecting that each firm's idiosyncratic component has correlation one with itself and is independent of the other firms' idiosyncratic components; and the remainder consists of only zeros, reflecting that there is no correlation between the idiosyncratic components and the indices. For the example in the previous subsection (where $m = 3$ and $n = 2$), we would have

$$[8.20] \qquad \bar{C} = \begin{bmatrix} 1 & 0.16 & 0.08 & 0 & 0 \\ 0.16 & 1 & 0.34 & 0 & 0 \\ 0.08 & 0.34 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

We then create a $m+n$ by n weight matrix W, where each column represents a different firm, and each row represents weights on indices and idiosyncratic components. Thus, in the k^{th} column of W, the first m entries will give the first firm's weights on the indices, the $m+n+k$ entry will give the firm's idiosyncratic weight, and the remaining entries will be zero. For our example, the matrix W would be given by

$$[8.21] \qquad W = \begin{bmatrix} 0.90 & 0 \\ 0 & 0.74 \\ 0 & 0.15 \\ 0.44 & 0 \\ 0 & 0.60 \end{bmatrix}.$$

The n by n matrix giving the correlations between all of the firms is then given by $W' \cdot \bar{C} \cdot W$.

Part III

Applications

Overview of Part III

To this point, we have detailed an analytic approach to compute the mean and standard deviation of portfolio value change, presented calculations for one- and two-asset portfolios, and discussed the inputs to these calculations. In this section we discuss approaches to computing risk measures other than standard deviation and apply the CreditMetrics methodology to a larger portfolio.

Both issues – alternative measures of risk and computations for a larger portfolio – point us to a central theme of this section: simulation. By this we mean the generation of future portfolio scenarios according to the models already discussed.

Implementation of a simulation approach involves a tradeoff. On the one hand, we are able to describe in much more detail the distribution of portfolio value changes; on the other, we introduce noise into what has been an exact solution for the risk estimates. We will continue to discuss this tradeoff as we go.

Part III is composed of four chapters which describe the methods and discuss the outputs of the CreditMetrics methodology for larger portfolios. The chapters dealing with simulation focus on computing advanced (beyond the mean and standard deviation) risk estimates. This section is organized as follows:

- **Chapter 9: Analytic portfolio calculation.** We extend the methods discussed in *Chapter 3* for computing the standard deviation and marginal standard deviation to a large (more than two instruments) portfolio.

- **Chapter 10: Simulation.** We address the assumptions necessary to specify the portfolio distribution completely, describe the Monte Carlo approach to this distribution, and discuss how to produce percentile levels as well as marginal statistics. We focus on computing advanced (beyond the mean and standard deviation) risk estimates for larger portfolios.

- **Chapter 11: Portfolio example.** We choose a portfolio of 20 instruments of varying maturities and rating and specify the asset correlations between their issuers. We then utilize the simulation approach of the previous section to estimate certain risk statistics and interpret these results in the context of the portfolio.

- **Chapter 12: Application of model outputs.** We consider how the analysis in *Chapter 11* might lead to risk management actions such as prioritizing risk reduction, setting credit risk limits, and assessing economic capital.

Chapter 9. Analytic portfolio calculation

In *Chapter 3*, we discussed the computation of the standard deviation of value change for a portfolio of two instruments. We refrained from extending this computation to larger portfolios, stating that the standard deviation of value for larger portfolios involves no different calculations than the standard deviation for two-asset portfolios. In this chapter, we illustrate this point for a three-asset portfolio, and discuss as well the calculation of marginal standard deviations for this portfolio. The generalization of these calculations to portfolios of arbitrary size is straightforward, and is detailed in *Appendix A*.

9.1 Three-asset portfolio

Our example is a portfolio consisting of three assets, all annual coupon bonds. We take the first two of these bonds to be issued by the BBB and A rated firms of *Chapter 3* and the third to be a two-year bond paying a 10% coupon and issued by a CCC rated firm. We will refer to the firms respectively as Firms 1, 2, and 3. Suppose that the Firm 1 issue has a notional amount of 4mm, the Firm 2 issue an amount of 2mm, and the Firm 3 issue an amount of 1mm. Denote by V_1, V_2, and V_3, the values at the end of the risk horizon of the three respective issues.

We present transition probabilities for the three firms in *Table 9.1* below, and revaluations in *Table 9.2*.

Table 9.1
Transition probabilities (%)

	Transition probability (%)		
Rating	**Firm 1**	**Firm 2**	**Firm 3**
AAA	0.02	0.09	0.22
AA	0.33	2.27	0.00
A	5.95	91.05	0.22
BBB	86.93	5.52	1.30
BB	5.30	0.74	2.38
B	1.17	0.26	11.24
CCC	0.12	0.01	64.86
Default	0.18	0.06	19.79

Table 9.2
Instrument values in future ratings ($mm)

	Value of issue ($mm)		
Future rating	**Firm 1**	**Firm 2**	**Firm 3**
AAA	4.375	2.132	1.162
AA	4.368	2.130	1.161
A	4.346	2.126	1.161
BBB	4.302	2.113	1.157
BB	4.081	2.063	1.142
B	3.924	2.028	1.137
CCC	3.346	1.774	1.056
Default	2.125	1.023	0551

Utilizing the methods of *Chapter 2* and the information in the tables above, we may compute the mean value for each issue:

[9.1] $$\mu_1 = \$4.28\text{mm},\ \mu_2 = \$2.12\text{mm},\text{ and }\mu_3 = \$0.97\text{mm},$$

giving a portfolio mean of $\mu_p = \$7.38\text{mm}$. We may also compute the variance of value for each of the three assets, obtaining

[9.2] $$\sigma^2(V_1) = 0.014,\ \sigma^2(V_2) = 0.001,\text{ and }\sigma^2(V_3) = 0.044.$$

Note that since the standard deviations are in units of ($mm), the units for $\sigma^2(V_1)$, $\sigma^2(V_2)$, and $\sigma^2(V_3)$ are ($mm)^2.

Now to compute σ_p, the standard deviation of value for the portfolio, we could use the standard formula

[9.3] $$\sigma_p^2 = \sigma^2(V_1) + \sigma^2(V_2) + \sigma^2(V_3) + 2 \cdot COV(V_1, V_2) + 2 \cdot COV(V_1, V_3) + 2 \cdot COV(V_2, V_3).$$

This would require the calculation of the various covariance terms. Alternatively, noting that

[9.4] $$\sigma^2(V_1 + V_2) = \sigma^2(V_1) + 2 \cdot COV(V_1, V_2) + \sigma^2(V_2),$$

we may express σ_p by

[9.5] $$\sigma_p^2 = \sigma^2(V_1 + V_2) + \sigma^2(V_1 + V_3) + \sigma^2(V_2 + V_3) - \sigma^2(V_1) - \sigma^2(V_2) - \sigma^2(V_3).$$

The above formula has the attractive feature of expressing the portfolio standard deviation in terms of the standard deviations of single assets (e.g. $\sigma(V_1)$) and the standard deviations of two-asset subportfolios (e.g. $\sigma(V_1 + V_2)$). Thus, to complete our computation of σ_p, it only remains to identify each two-asset subportfolio, compute the standard deviations of each, and apply Eq. [9.5].

The standard deviation for two-asset portfolios was covered in *Chapter 3*, and so in principle, we have described all of the portfolio calculations. We present the two-asset case again as a review. Consider the first pair of assets, the BBB and A rated bonds. In order to compute the variance for the portfolio containing only these assets, we utilize the joint transition probabilities in *Table 3.2*, which are an output of the asset value model of the previous chapter, with an assumed asset correlation of 30%. Along with these probabilities we need the values of this two-asset portfolio in each of the 64 joint rating states; we present these values in *Table 9.3*. Note that the values in *Table 9.3* differ from those in *Table 3.2* since the notional amounts of the issues in these two cases are different.

Table 9.3
Values of a two-asset portfolio in future ratings ($mm)

New rating for Firm 1 (currently BBB)	New rating for Firm 2 (currently A)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	6.51	6.51	6.50	6.49	6.44	6.40	6.15	5.40
AA	6.50	6.50	6.49	6.48	6.43	6.40	6.14	5.39
A	6.48	6.48	6.47	6.46	6.41	6.37	6.12	5.37
BBB	6.43	6.43	6.43	6.42	6.37	6.33	6.08	5.33
BB	6.21	6.21	6.21	6.19	6.14	6.11	5.86	5.10
B	6.06	6.05	6.05	6.04	5.99	5.95	5.70	4.95
CCC	5.48	5.48	5.47	5.46	5.41	5.37	5.12	4.37
Default	4.26	4.26	4.25	4.24	4.19	4.15	3.90	3.15

Applying Eq. [3.1] to the probabilities in *Table 3.2* and the values in *Table 9.3*, we then compute $\sigma^2(V_1 + V_2) = 0.018$. In a similar fashion, we specify that the asset correlations between the first and third and between the second and third obligors are also 30%, and then create analogs to *Table 3.2* and *Table 9.3*. This allows us to compute $\sigma^2(V_1 + V_3) = 0.083$ and $\sigma^2(V_2 + V_3) = 0.051$. Finally, we apply Eq. [9.6] to obtain $\sigma_p^2 = 0.093$, and thus $\sigma_p = \$0.305\text{mm}$.

The calculation of portfolio variance in terms of the variance of two-asset subportfolios may seem unusual to those accustomed to the standard covariance approach. We remark that we have all of the information necessary to compute the covariances and correlations between our three assets. Thus, since

$$[9.6] \qquad COV(V_1, V_2) = \frac{\sigma^2(V_1 + V_2) - \sigma^2(V_1) - \sigma^2(V_2)}{2},$$

we have $COV(V_1, V_2) = 0.0015$. Similarly, we obtain $COV(V_1, V_3) = 0.0125$ and $COV(V_2, V_3) = 0.0030$. This allows us to then compute correlations between the asset values using

$$[9.7] \qquad CORR(V_1, V_2) = \frac{COV(V_1, V_2)}{\sqrt{\sigma^2(V_1) \times \sigma^2(V_2)}}.$$

We then have $CORR(V_1, V_2) = 40.1\%$, $CORR(V_1, V_3) = 50.4\%$, and $CORR(V_2, V_3) = 45.2\%$. It is a simple matter then to check that the standard formula Eq. [9.1] yields the same value for σ_p as we computed above.

We refer to σ_p as the *absolute* measure of the portfolio standard deviation. Alternatively, we may express this risk in percentage terms; we thus refer to σ_p/μ_p (which is equal to 4.1% in our example) as the *percent* portfolio standard deviation. These notions of absolute and percent measures will be used for other portfolio statistics, with the percent statistic always representing the absolute statistic as a fraction of the mean portfolio value.

To extend this calculation to larger portfolios is straightforward. We present the details of this in *Appendix A*.

9.2 Marginal standard deviation

As defined in *Section 3.3*, the marginal standard deviation for a given instrument in a portfolio is the difference between the standard deviation for the entire portfolio and the standard deviation for the portfolio not including the instrument in question. Thus, since we now are able to compute the standard deviation for a portfolio of arbitrary size, the calculation of marginal standard deviations is clear.

Consider the Firm 1 issue in our portfolio above. We have seen that the standard deviation for the entire portfolio is $0.46mm. If we remove the Firm 1 issue, then the new portfolio variance is given by $\hat{\sigma}_p^2 = \sigma^2(V_2 + V_3) = 0.051$, making the new portfolio standard deviation $\hat{\sigma}_p = \$0.225\text{mm}$. The *marginal standard deviation* of the Firm 1 issue is then the difference between the absolute portfolio standard deviation and this figure, or $\sigma_p - \hat{\sigma}_p = \$0.080\text{mm}$. Thus, we see that we can reduce the total portfolio standard deviation by $0.080mm if we liquidate the Firm 1 issue While this is a measure of the absolute risk contributed by the Firm 1 issue, we might also wish to characterize the riskiness of this instrument independently of its size. To this end, we may express the marginal standard deviation as a percentage of μ_1, the mean value of the Firm 1 issue. We refer to this figure, 1.9% in this case, as the *percent marginal standard deviation* of this issue.

The difference between marginal and stand-alone statistics gives us an idea of the effect of diversification on the portfolio. Note that if we consider the Firm 1 issue alone, its standard deviation of value is $0.117mm. If this asset were perfectly correlated with the other assets in the portfolio, its marginal impact on the portfolio standard deviation would be exactly this amount. However, we have seen that the marginal impact of the Firm 1 issue is only $0.080mm, and thus that we benefit from the fact that this issue is not in fact perfectly correlated with the others.

The risk measures produced in this section may strike the reader as a bit small, particularly in light of the riskiness of the CCC rated issue in our example. This might be explained by the fact that the size of this issue is quite small in comparison with the other assets in the portfolio. However, since we have only considered the standard deviation to this point, it may be that to adequately describe the riskiness of the portfolio, we need more detailed information about the portfolio distribution. In order to obtain this higher order information, it will be necessary to perform a simulation based analysis, which is the subject of the following two chapters.

Chapter 10. Simulation

Our methodology up to this point has focused on analytic estimates of risk, that is, estimates which are computed directly from formulas implied by the models we assume. This analytical approach has two advantages:

1. **Speed**. Particularly for smaller portfolios, the direct calculations require fewer operations, and thus can be computed more quickly.
2. **Precision.** No random noise is introduced in the calculations and, therefore, no error in the risk estimates.

However, it has also two principal disadvantages. One is that for large portfolios, number 1 above is no longer true. The other is that by restricting ourselves to analytical approaches, we limit the available of statistics that can be estimated.

Throughout this document, we have discussed methods to compute the standard deviation of portfolio value; yet we have also stressed that this may not be a meaningful measure of the credit risk of the portfolio. To provide a methodology that better describes the distribution of portfolio values, we present in this chapter a simulation approach known as “Monte Carlo.”

The three sections of this chapter treat the three steps to a Monte Carlo simulation:

1. **Generate scenarios.** Each scenario corresponds to a possible “state of the world” at the end of our risk horizon. For our purposes, the “state of the world” is just the credit rating of each of the obligors in our portfolio.
2. **Value portfolio.** For each scenario, we revalue the portfolio to reflect the new credit ratings. This step gives us a large number of possible future portfolio values.
3. **Summarize results.** Given the value scenarios generated in the previous steps, we have an estimate for the distribution of portfolio values. We may then choose to report any number of descriptive statistics for this distribution.

We will continue to consider the example portfolio of the previous chapter: three two-year par bonds issued by BBB, A, and CCC rated firms. The notional values of these bonds are $4mm, $2mm, and $1mm.

10.1 Scenario generation

In this section, we will discuss how to generate scenarios of future credit ratings for the obligors in our portfolio. We will rely heavily on the asset value model discussed in *Section 8.4*. The steps to scenario generation are as follows:

1. Establish asset return thresholds for the obligors in the portfolio.
2. Generate scenarios of asset returns according to the normal distribution.
3. Map the asset return scenarios to credit rating scenarios.

In *Table 10.1* below, we restate the transition probabilities for the three issues.

Table 10.1
Transition probabilities (%)

	Transition Probability (%)		
Rating	**Firm 1**	**Firm 2**	**Firm 3**
AAA	0.02	0.09	0.22
AA	0.33	2.27	0.00
A	5.95	91.05	0.22
BBB	86.93	5.52	1.30
BB	5.30	0.74	2.38
B	1.17	0.26	11.24
CCC	0.12	0.01	64.86
Default	0.18	0.06	19.79

We then present in *Table 10.2*[1] the asset return thresholds for the three firms, which are obtained using the methods of *Section 8.4*.

Table 10.2
Asset return thresholds

Threshold	**Firm 1**	**Firm 2**	**Firm 3**
Z_{AA}	3.54	3.12	2.86
Z_{A}	2.78	1.98	2.86
Z_{BBB}	1.53	-1.51	2.63
Z_{BB}	-1.49	-2.30	2.11
Z_{B}	-2.18	-2.72	1.74
Z_{CCC}	-2.75	-3.19	1.02
Z_{Def}	-2.91	-3.24	-0.85

Recall that the thresholds are labeled such that a return falling just below a given thresholds corresponds to the rating in the threshold's subscript. That is, a return less than Z_{BB} (but greater than Z_B) corresponds to a rating of BB.

In order to describe how the asset values of the three firms move jointly, we state that the asset returns in for each firm are normally distributed, and specify the correlations for each pair of firms[2]. For our example, we assume the correlations in *Table 10.3*.

1 Recall the comment at the end of *Chapter 8* that asset return volatility does not affect the joint probabilities of rating changes. For this reason, we may consider standardized asset returns, and report the thresholds for these.

2 Technically, the assumption is that the joint distribution of the asset returns of any collection of firms is multivariate normal.

Table 10.3
Correlation matrix for example portfolio

	Firm 1	**Firm 2**	**Firm 3**
Firm 1	1.0	0.3	0.1
Firm 2	0.3	1.0	0.2
Firm 3	0.1	0.2	1.0

Generating scenarios for the asset returns of our three obligors is a simple matter of generating correlated, normally distributed variates. There are a number of methods for doing this – Cholesky factorization, singular value decomposition, etc. – for discussions of which see, for example, Strang [88]. In *Table 10.4*, we list ten scenarios which might be produced by such a procedure. In each scenario, the three numbers represent the standardized asset return for each of the three firms.

Table 10.4
Scenarios for standardized asset returns

Scenario	**Firm 1**	**Firm 2**	**Firm 3**
1	-0.7769	-0.8750	-0.6874
2	-2.1060	-2.0646	0.2996
3	-0.9276	0.0606	2.7068
4	0.6454	-0.1532	-1.1510
5	0.4690	-0.5639	0.2832
6	-0.1252	-0.5570	-1.9479
7	0.6994	1.5191	-1.6503
8	1.1778	-0.6342	-1.7759
9	1.8480	2.1202	1.1631
10	0.0249	-0.4642	0.3533

To fully specify our scenarios, it is only necessary to assign ratings to the asset return scenarios. For example, consider scenario 2 of *Table 10.4*. The standardized return for Firm 1 is –2.1060, which falls between Z_B (–2.18 from *Table 10.2*) and Z_{BB} (–1.49 from *Table 10.2*) for this name. This corresponds to a new rating of BB. For Firm 2, the return is –2.0646, which falls between Z_{BB} and Z_{BBB} for this name, corresponding to a new rating of BBB. Continuing this process, we may fill in *Table 10.5*, which completes the process of scenario generation

Table 10.5
Mapping return scenarios to rating scenarios

	Asset Return			New Rating		
Scenario	**Firm 1**	**Firm 2**	**Firm 3**	**Firm 1**	**Firm 2**	**Firm 3**
1	-0.7769	-0.8750	-0.6874	BBB	A	CCC
2	-2.1060	-2.0646	0.2996	BB	BBB	CCC
3	-0.9276	0.0606	2.7068	BBB	A	A
4	0.6454	-0.1532	-1.1510	BBB	A	Default
5	0.4690	-0.5639	0.2832	BBB	A	CCC
6	-0.1252	-0.5570	-1.9479	BBB	A	Default
7	0.6994	1.5191	-1.6503	BBB	A	Default
8	1.1778	-0.6342	-1.7759	BBB	A	Default
9	1.8480	2.1202	1.1631	A	AA	B
10	0.0249	-0.4642	0.3533	BBB	A	CCC

Notice that for this small number of trials, the scenarios do not correspond precisely to the transition probabilities in *Table 10.1*. (For example, in four of the ten scenarios, Firm 3 defaults, while the probability that this occurs is just 20%.) These random fluctuations are the source of the lack of precision in Monte Carlo estimation. As we generate more scenarios, these fluctuations become less prominent, but it is important to quantify how large we can expect the fluctuations to be. This is the topic of *Appendix B*.

10.2 Portfolio valuation

For non-default scenarios, this step is no different here than in the previous chapters. For each scenario and each issue, the new rating maps directly to a new value. To recall the specifics of valuation, refer back to *Chapter 4*.

For default scenarios, the situation is slightly different. We discussed in *Chapter 7* that recovery rates are not deterministic quantities but rather display a large amount of variation. This variation of value in the case of default is a significant contributor to risk. To model this variation, we obtain the mean and standard deviation of recovery rate for each issue in our portfolio according to the issue's seniority. For example, in our BBB rated senior unsecured issue, the recovery mean is 53% and the recovery standard deviation is 33%. For each default scenario, we generate a random recovery rate according to a beta distribution[3] with these parameters[4]. These recovery rates then allow us to obtain the value in each default scenario.

In the end, we obtain a portfolio value for each scenario. The results for the first ten scenarios for our example are presented in *Table 10.6*.

[3] Recall that the beta distribution only produces numbers between zero and one, so that we are assured of obtaining meaningful recovery rates.

[4] Note that we assume here that the recovery rate for a given obligor is independent of the value of all other instruments in the portfolio.

Table 10.6
Valuation of portfolio scenarios ($mm)

	Rating			Value			
Scenario	**Firm 1**	**Firm 2**	**Firm 3**	**Firm 1**	**Firm 2**	**Firm 3**	**Portfolio**
1	BBB	A	CCC	4.302	2.126	1.056	7.484
2	BB	BBB	CCC	4.081	2.063	1.056	7.200
3	BBB	A	A	4.302	2.126	1.161	7.589
4	BBB	A	Default	4.302	2.126	0.657	7.085
5	BBB	A	CCC	4.302	2.126	1.056	7.484
6	BBB	A	Default	4.302	2.126	0.754	7.182
7	BBB	A	Default	4.302	2.126	0.269	6.697
8	BBB	A	Default	4.302	2.126	0.151	6.579
9	A	AA	B	4.346	2.130	1.137	7.613
10	BBB	A	CCC	4.302	2.126	1.056	7.484

Note that for a given issue, the value is the same in scenarios with the same (non-default) credit rating. For defaults, this is not the case – the values of the Firm 3 issue in the default scenarios are different – since recovery rates are themselves uncertain. Thus, each default scenario requires an independently generated recovery rate.

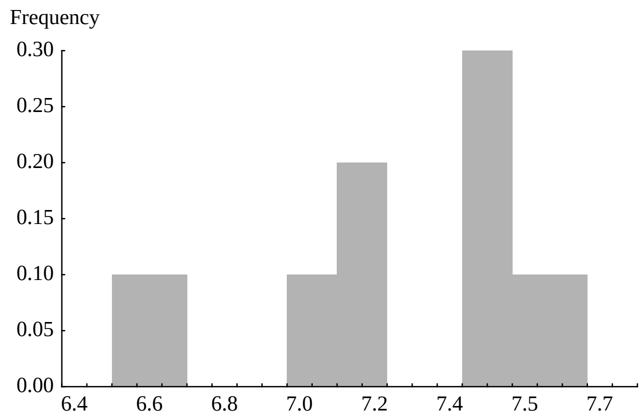
10.3 Summarizing the results

At this point, we have created a number of possible future portfolio values. The final task is then to synthesize this information into meaningful risk estimates.

In this section, we will examine a number of descriptive statistics for the scenarios we have created. In the section to follow, we will examine the same statistics, but for an example in which we consider a larger portfolio and a larger number of scenarios, so as to obtain more significant results.

In order to gain some intuition about the distribution of values, we first examine a plot of the ten scenarios for our example. This plot is presented in *Chart 10.1*. For a larger number of scenarios, we would expect this plot to become more smooth, and approach something like the histogram we will see in *Chart 11.1*.

Chart 10.1
Frequency plot of portfolio scenarios



Even for small number of scenarios, we begin to see the heavy downside tail typical of credit portfolio distributions.

The first statistics we examine are those which we are able to compute analytically: the mean and standard deviation of future portfolio value. Let $V^{(1)}, V^{(2)}, V^{(3)}, \ldots$ indicate the portfolio value in the respective scenarios. Then we may compute the sample mean (μ) and standard deviation (σ) of the scenarios as follows:

[10.1] $$\mu_p = \frac{1}{N}\sum_{i=1}^{N} V^{(i)} = \$7.24\text{mm and } \sigma_p = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(V^{(i)}-\mu\right)^2} = \$0.37\text{mm}$$

where N is the number of scenarios (in our case, N=10).

As we have mentioned before, the mean and standard deviation may not be the best measures of risk in that, since the distribution of values is not normal, we cannot infer percentile levels from the standard deviation. We are thus motivated to perform simulations in order to capture more information about the distribution of values. Estimates of percentile levels are straightforward. For example, to compute the tenth percentile given our scenarios, we choose a level (x) at which one of the ten scenarios is less than x and the other nine scenarios are greater than x. For our scenarios, this level is between \$6.58mm and \$6.70mm. This imprecision is due to simulation noise, but we will see in the next chapter that as we consider more scenarios, our estimates of percentiles become more precise.

To this point, we have considered only statistics which describe the portfolio distribution. We would also like to consider individual assets and to ascertain how much risk each asset contributes to the portfolio. To this end, we will describe marginal statistics.

We have discussed marginal standard deviations previously. This concept may be generalized, and we may compute a marginal analog of any of the statistics (standard deviation, percentile) discussed above. In general, the marginal statistic for a particular asset is the difference between that statistic for the entire portfolio and that statistic for the portfolio not including the asset in question. Thus, if we wish to compute the marginal tenth percentile of the third asset in our portfolio (the CCC rated bond), we take

$$[10.2] \quad \theta_{10}(V_1 + V_2 + V_3) - \theta_{10}(V_1 + V_2)$$

where V_1, V_2, and V_3 represent the future values of the first, second, and third assets, respectively, and θ_{10} represents the tenth percentile of the values in question. For the scenarios above, the tenth percentile for the entire portfolio is \$6.64mm, while that for just the first two assets is \$6.29mm; and thus the marginal standard deviation for the third asset is \$0.35mm. This marginal figure may be interpreted as the amount by which we could decrease the risk on our portfolio by removing the CCC rated bond.

As we have mentioned a number of times, the statistics obtained through Monte Carlo simulation are subject to fluctuations; any set of scenarios may not produce a sample mean or sample 5^{th} percentile which is equal to the true mean or 5^{th} percentile for the portfolio. Thus, it is important to quantify, given the number of scenarios which are generated, how close we expect our estimates of various portfolio statistics to be to their true value. In fact, a reasonable way to choose the number of scenarios to be generated is to specify some desired level of precision for a particular statistic, and generate enough scenarios to achieve this. Quantifying the precision of simulation based statistics is the subject of *Appendix B*.

Chapter 11. Portfolio example

In this chapter, we examine a more realistic example portfolio and discuss the results of a simulation-based analysis of this portfolio. The risk estimates are no different than those in the previous chapter, but should take on more meaning here in the context of a larger portfolio.

11.1 The example portfolio

In this chapter, we consider a portfolio of 20 corporate bonds (each with a different issuer) of varying rating and maturity. The bonds are listed in *Table 11.1*. The total market value of the portfolio is $68mm.

Table 11.1.
Example portfolio

Asset	Credit rating	Principal amount	Maturity (years)	Market value
1	AAA	7,000,000	3	7,821,049
2	AA	1,000,000	4	1,177,268
3	A	1,000,000	3	1,120,831
4	BBB	1,000,000	4	1,189,432
5	BB	1,000,000	3	1,154,641
6	B	1,000,000	4	1,263,523
7	CCC	1,000,000	2	1,127,628
8	A	10,000,000	8	14,229,071
9	BB	5,000,000	2	5,386,603
10	A	3,000,000	2	3,181,246
11	A	1,000,000	4	1,181,246
12	A	2,000,000	5	2,483,322
13	B	600,000	3	705,409
14	B	1,000,000	2	1,087,841
15	B	3,000,000	2	3,263,523
16	B	2,000,000	4	2,527,046
17	BBB	1,000,000	6	1,315,720
18	BBB	8,000,000	5	10,020,611
19	BBB	1,000,000	3	1,118,178
20	AA	5,000,000	5	6,181,784

Recall that for each asset, the credit rating determines the distribution of future credit rating, and thus also the distribution of future value. For the portfolio, however, we must also specify the asset correlations in order to describe the distribution of future ratings and values. For this example, we assume the correlations in *Table 11.2*.

Table 11.2
Asset correlations for example portfolio

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	1	0.45	0.45	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.45	1	0.45	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
3	0.45	0.45	1	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
4	0.45	0.45	0.45	1	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
5	0.15	0.15	0.15	0.15	1	0.35	0.35	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
6	0.15	0.15	0.15	0.15	0.35	1	0.35	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
7	0.15	0.15	0.15	0.15	0.35	0.35	1	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
8	0.15	0.15	0.15	0.15	0.35	0.35	0.35	1	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
9	0.15	0.15	0.15	0.15	0.35	0.35	0.35	0.35	1	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
10	0.15	0.15	0.15	0.15	0.35	0.35	0.35	0.35	0.35	1	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
11	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	1	0.45	0.45	0.45	0.45	0.2	0.2	0.2	0.1	0.1
12	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	1	0.45	0.45	0.45	0.2	0.2	0.2	0.1	0.1
13	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	1	0.45	0.45	0.2	0.2	0.2	0.1	0.1
14	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	0.45	1	0.45	0.2	0.2	0.2	0.1	0.1
15	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	0.45	0.45	1	0.2	0.2	0.2	0.1	0.1
16	0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	1	0.55	0.55	0.25	0.25
17	0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	0.55	1	0.55	0.25	0.25
18	0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	0.55	0.55	1	0.25	0.25
19	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.25	0.25	0.25	1	0.65
20	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.25	0.25	0.25	0.65	1

Observe that there are five groups of issuers (those for assets 1-4, 6-10, 11-15, 16-18, and 19-20, in the shaded areas of the table) within which the asset correlations are relatively high, while the correlations between these groups are lower. This might be the case for a portfolio containing issues from firms in five different industries; the correlations between firms in a given industry are high, while correlations across industries are lower.

11.2 Simulation results

Using the methodology of the previous chapter, we generate 20,000 portfolio scenarios, that is, 20,000 possible future occurrences in one year's time of the credit ratings for each of our issues. For each scenario, we then obtain a portfolio value for one year into the future. In Charts *11.1* through *11.3*, we present histograms of the portfolio value scenarios. Note the axes on each chart carefully. The first chart illustrates the distribution of the most common scenarios, the second moves a bit further into the left tail of the distribution, and the third shows the distribution of the most extreme 5% of all cases. The vertical axis, which represents relative frequency, is ten times smaller in the second chart than in the first, and twenty times smaller in the third chart than in the second.

Chart 11.1
Histogram of future portfolio values – upper 85% of scenarios

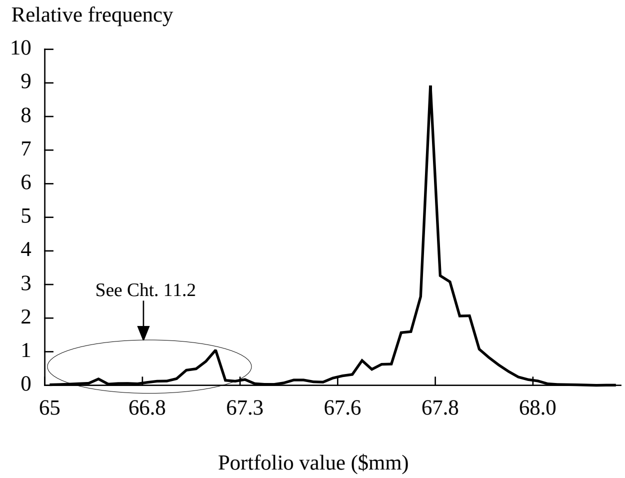


Chart 11.2
Histogram of future portfolio values – scenarios between 95th and 65th percentiles

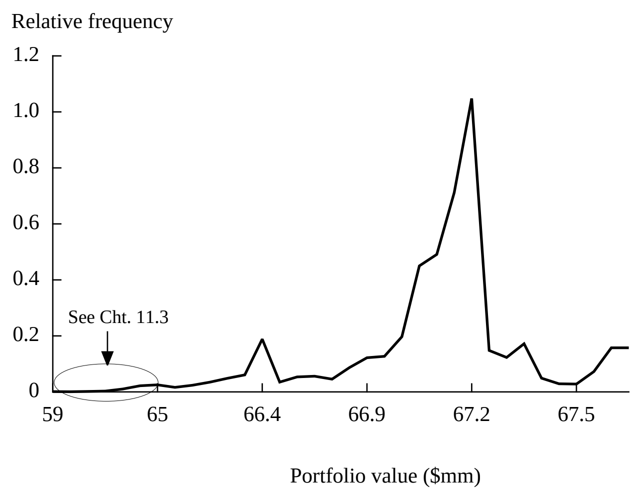
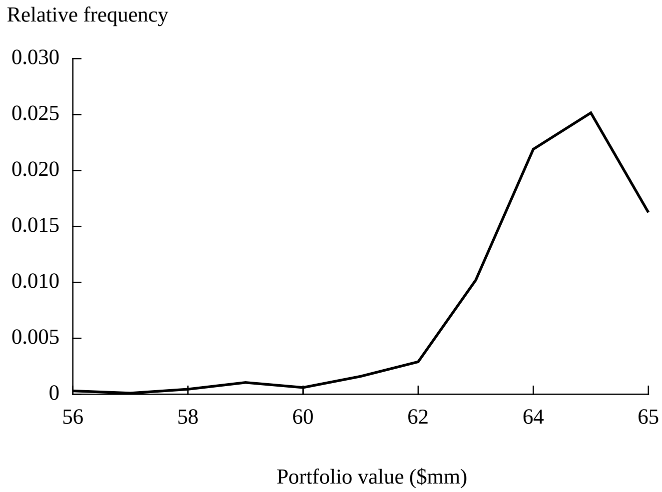


Chart 11.3
Histogram of future portfolio values – lower 5% of scenarios



We may make several interesting observations of these charts. First, by far the most common occurrence (almost 9% of all scenarios, exhibited by the spike near $67.8mm in *Table 11.1*) is that none of the issuers undergoes a rating change. Further, in well over half of the scenarios, there are no significant credit events, and the portfolio appreciates.

The second observation is the odd bimodal structure of the distribution. This is due to the fact that default events produce much more significant value changes than any other rating migrations. Thus, the distribution of portfolio value is driven primarily by the number of issues which default. The second hump in the distribution (the one between $67mm and $67.2mm) represents scenarios in which one issue defaults.

The two other humps further to the left in the distribution represent scenarios with two and three defaults, respectively. For larger portfolios, these humps become even more smoothed out, while for smaller ones, the humps are generally more prominent.

Regardless of the particulars of the shape of the value distribution, one feature persists: the heavy downward skew. Our example distribution is no different, displaying a large probability of a marginal increase in value along with a small probability of a more significant drop in value.

As in the previous chapter, the first two statistics we present are the mean and standard deviation of the portfolio value. For our case, we have:

- Mean portfolio value (μ) = $67,284,888.
- Standard deviation of portfolio value (σ) = $1,136,077.

As we have mentioned before, the mean and standard deviation may not be the best measures of risk in that, since the loss distribution is not normal, we cannot infer confidence levels from these parameters. We can however estimate percentiles directly from our scenarios.

For example, if we wish to compute the 5th percentile (the level below which we estimate that 5% of portfolio values fall), we sort our 20,000 scenarios in ascending order and take the 1000th of these sorted scenarios (that is, $64.98mm) as our estimate. (Our assumption is then that since 5% of the simulated changes in value were less than -$5.69mm, there is a 5% chance that the actual portfolio value change will be less than this level.) Here we see the advantage of the simulation approach, in that we can estimate arbitrary percentile levels, where in the analytic approach, because the portfolio distribution is not normal, we are only able to compute two statistics.

In *Table 11.3* below, we present various percentiles of our scenarios of future portfolio values. For comparison and in order to illustrate the non-normality of the portfolio distribution, we also give the percentiles which we would have estimated had we utilized the sample mean and standard deviation, and assumed that the distribution was normal.

Table 11.3
Percentiles of future portfolio values ($mm)

	Actual scenarios	**Normal distribution**	
Percentile	**Portfolio value ($mm)**	**Formula**	**Portfolio value ($mm)**
95%	67.93	$\mu+1.65\sigma$	69.15
50%	67.80	μ	67.28
5%	64.98	$\mu-1.65\sigma$	65.42
2.5%	63.97	$\mu-1.96\sigma$	65.06
1%	62.85	$\mu-2.33\sigma$	64.64
0.5%	61.84	$\mu-2.58\sigma$	64.36
0.1%	57.97	$\mu-3.09\sigma$	63.77

Using the scenarios, we estimate that 2.5% of the time (or one year in forty), our portfolio in one year will drop in value to $63.97mm or less. If we had used a normal assumption, we would have estimated that this percentile would correspond to only a drop to $65.06mm, a much more optimistic risk estimate.

On the other hand, if we examine the median value change (the 50% level), the normal assumption leads to a more pessimistic forecast: there is a 50% chance that the portfolio is less valuable than the mean value of $67.28mm. By contrast, the scenarios point to a higher mean, and thus to a greater than 50% chance that the portfolio value will exceed its mean.

Another interesting observation is that the 5th and 1st percentiles of the scenarios are 2 and 2.9 standard deviations, respectively, below the mean. This is further evidence that it is best not to use the standard deviation to infer percentile levels for a credit portfolio.

11.3 Assessing precision

In this section, we utilize the methods of *Appendix B* to give confidence bands around our estimated statistics, and examine how these confidence bands evolve as we increase the number of scenarios which we consider.

For the 20,000 scenarios in our example, we have the results shown in *Table 11.4*.

Table 11.4
Portfolio value statistics with 90% confidence levels ($mm)

Statistic	Lower bound	Estimate	Upper bound
Mean portfolio value	67.27	67.28	67.30
Standard deviation	1.10	1.14	1.17
5th percentile	64.94	64.98	65.02
1st percentile	62.66	62.85	62.97
0.5 percentile[1]	61.26	61.84	62.08
0.1 percentile[2]	56.11	57.97	58.73

[1] 1 in 200 chance of shortfall
[2] 1 in 1,000 chance of shortfall

For both the mean and standard deviation, and for the 5th and 1st percentiles, the confidence bands are reasonably tight, and we feel assured of making decisions based on our estimates of these quantities. For the more extreme percentiles, we see that the true loss level could well be at least 10% greater than our estimate. If we desire estimates for these levels, we would be best off generating more scenarios.

With regard to the question of how many scenarios we need to obtain precise estimates, we may examine the evolution of our confidence bands for each estimate as we consider more and more scenarios. We present this information for the six statistics above in the following charts.

Chart 11.4
Evolution of confidence bands for portfolio mean ($mm)

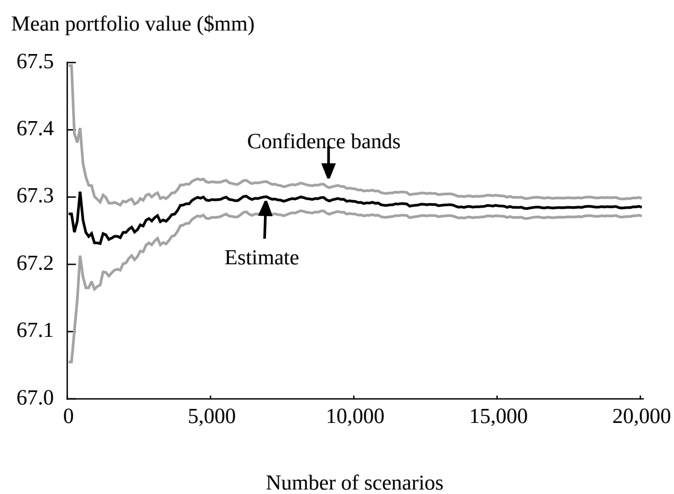


Chart 11.5
Evolution of confidence bands for standard deviation ($mm)

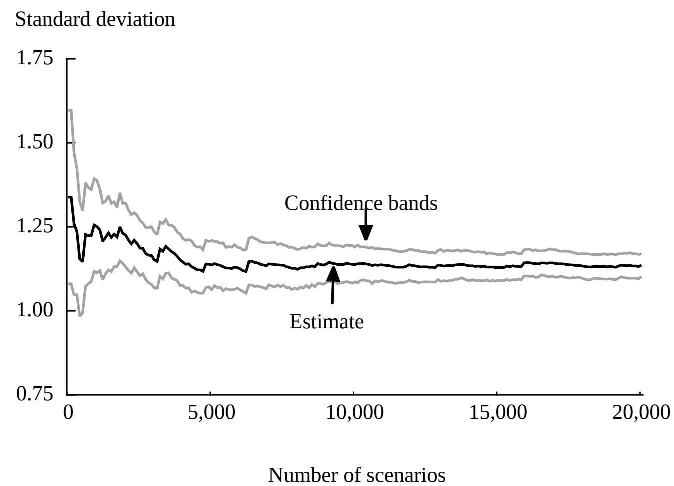


Chart 11.6
Evolution of confidence bands for 5th percentile ($mm)

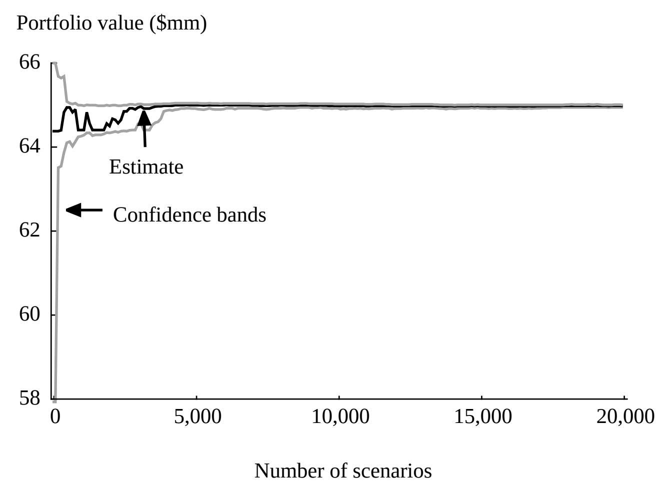


Chart 11.7
Evolution of confidence bands for 1st percentile ($mm)

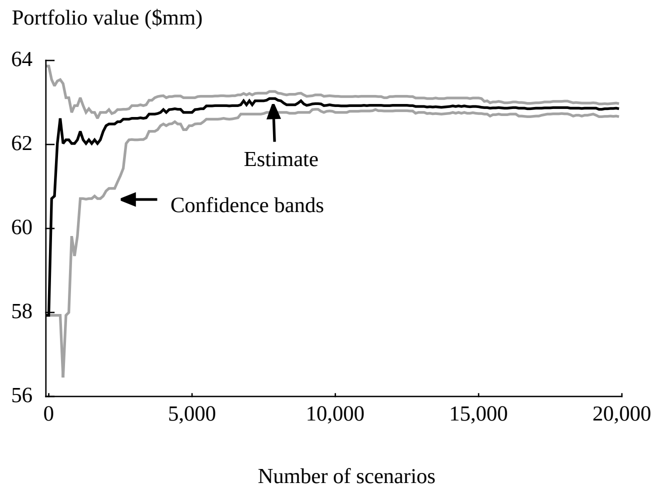


Chart 11.8
Evolution of confidence bands for 0.5 percentile ($mm)

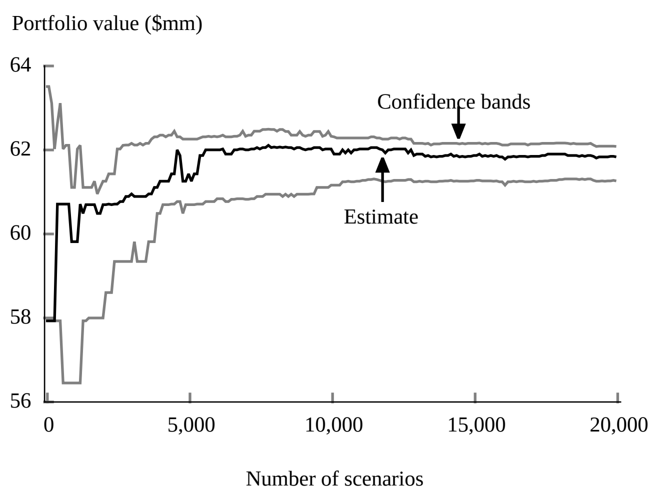
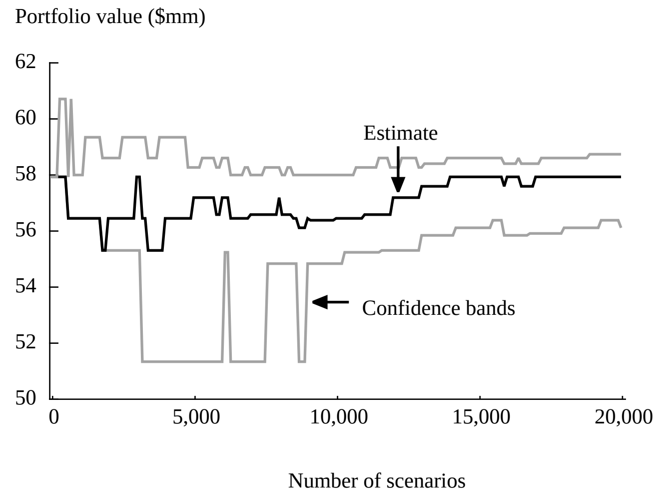


Chart 11.9
Evolution of confidence bands for 0.1 percentile ($mm)



It is interesting to note here that few of the plots change beyond about 10,000 scenarios; we could have obtained similar estimates and similar confidence bands with only half the effort. In fact, if we had been most concerned with the 5^{th} percentile, we might have been satisfied with the precision of our estimate after only 5000 trials, and could have stopped our calculations then. For the most extreme percentile level, note that the estimates and confidence bands do not change frequently. This is due to the fact that on average only one in one thousand scenarios produces a value which truly influences our estimate. This suggests that to meaningfully improve our estimate will require a large number of additional scenarios.

11.4 Marginal risk measures

To examine the contribution of each individual asset to the risk of the portfolio, we compute marginal statistics. Recall that for any risk measure, the marginal risk of a given asset is the difference between the risk for the entire portfolio and the risk of the portfolio without the given asset.

As an example, let us consider the standard deviation. For each asset in the portfolio, we will compute four numbers. First, we compute each asset's *stand-alone standard deviation* of value, that is the standard deviation of value for the asset computed without regard for the other instruments in the portfolio. Second, we compute the *stand-alone percent standard deviation*, which is just the stand-alone standard deviation expressed as a percentage of the mean value for the given asset. Third, we compute each asset's *marginal standard deviation*, the impact of the given asset on the total portfolio standard deviation. Last, we express this figure in percent terms, giving the *percent marginal standard deviation*. These four statistics are presented for each of the 20 assets in *Table 11.5*.

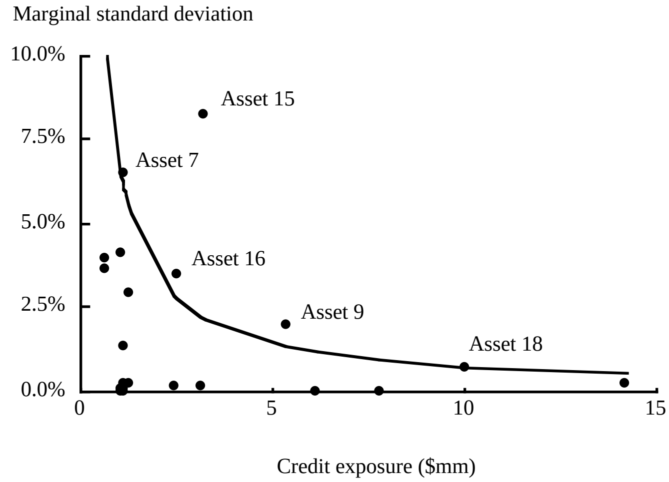
Table 11.5
Standard deviation of value change

		Stand-alone		Marginal	
Asset	**Credit rating**	**Absolute ($)**	**Percent**	**Absolute ($)**	**Percent**
1	AAA	4,905	0.06	239	0.00
2	AA	2,007	0.17	114	0.01
3	A	17,523	1.56	693	0.06
4	BBB	40,043	3.37	2,934	0.25
5	BB	99,607	8.63	16,046	1.39
6	B	162,251	12.84	37,664	2.98
7	CCC	255,680	22.67	73,079	6.48
8	A	197,152	1.39	35,104	0.25
9	BB	380,141	7.06	105,949	1.97
10	A	63,207	1.99	5,068	0.16
11	A	15,360	1.30	1,232	0.10
12	A	43,085	1.73	4,531	0.18
13	B	107,314	15.21	25,684	3.64
14	B	167,511	15.40	44,827	4.12
15	B	610,900	18.72	270,000	8.27
16	B	322,720	12.77	89,190	3.53
17	BBB	28,051	2.13	2,775	0.21
18	BBB	306,892	3.06	69,624	0.69
19	BBB	1,837	0.16	120	0.01
20	AA	9,916	0.16	389	0.01

The difference between the stand-alone and marginal risk for a given asset is an indication of the effect of diversification. We see in general that for the higher rated assets, there is a greater reduction from the stand-alone to marginal risk than for the lower rated assets. This is in line with our intuition that a much larger portfolio is required to diversify the effects of riskier credit instruments.

An interesting way to visualize these outputs is to plot the percent marginal standard deviations against the market value of each asset, as in *Chart 11.10*. Points in the upper left of the chart represent assets which are risky in percent terms, but whose exposure sizes are small, while points in the lower right represent large exposures which have relatively small chances of undergoing credit losses. Note that the product of the two coordinates (that is, the percent risk multiplied by the market value) gives the absolute marginal risk. The curve in *Chart 11.10* represents points with the same absolute risk; points which fall above the curve have greater absolute risk, while points which fall below have less.

Chart 11.10
Marginal risk versus current value for example portfolio



Based on the discussion above, we may identify with the aid of the curve the five greatest contributors to portfolio risk. Some of these "culprits" are obvious: Asset 7 is the CCC rated issue, and has a much larger likelihood of default, whereas Asset 18 is BBB rated, but is a rather large exposure.

On the other hand, the other "culprits" seem to owe their riskiness as much to their correlation with other instruments as to their individual characteristics. For instance, Asset 9 has a reasonably secure BB rating, but has a correlation of 35% with Asset 7, the CCC rated issue, while Asset 16 is rated B, but has a 55% correlation with Asset 18. Finally, the appearance of Asset 15 as the riskiest in absolute terms seems to be due as much to its 45% correlation with two other B issues as to its own B rating.

With this, we conclude the chapter. The reader should now an understanding of the various descriptors of the future portfolio distribution which can be used to assess risk. In the following chapter, we step away from the technical, and discuss what policy implications the assessment of credit risk might have, as well as how the use of a risk measure should influence the decision on precisely which measure to use.

Chapter 12. Application of model outputs

The measures of credit risk outlined in the preceding sections can have a variety of applications; we will highlight just a few:

- to set priorities for actions to reduce the portfolio risk;
- to measure and compare credit risks so that an institution can best apportion scarce risk-taking resources by limiting over-concentrations; and
- to estimate *economic capital* required to support risk-taking.

The objective of all of the above is to utilize risk-taking capacity more efficiently. Whether this is achieved by setting limits and insisting on being adequately compensated for risk, or by allocating capital to functions which have proven to take risk most effectively, is a policy issue. The bottom line is that in order to optimize the return we receive for the risk we take, it is necessary to measure the risk we take; and this is the contribution of CreditMetrics.

Note that we do not address the issue of credit pricing. Although credit risk can be an important input into a credit pricing decision, we believe that there are significant other determinants for pricing which are beyond the scope of CreditMetrics. These additional factors are non-trivial and so we have chosen to focus this current version on the already challenging task of risk estimation.[1]

12.1 Prioritizing risk reduction actions

The primary purpose of any risk management system is to direct *actions*. But there are many actions that may be taken towards addressing risk – so they must be prioritized. For this discussion, we will make reference to *Chart 12.1*, which is exactly like *Chart 11.10*, but for a hypothetical portfolio with a very large number of exposures.

There are at least two features of risk which are worth reducing, but the trade-off between them is judgmental: (i) absolute exposure size, and (ii) statistical risk level. Thus approaches include:

- reevaluate obligors having the largest *absolute size* (the lower right corner of the chart) arguing that a single default among these would have the greatest impact.
- reevaluate obligors having the highest *percentage level of risk* (the upper left corner of the chart) arguing that these are the most likely to contribute to portfolio losses.

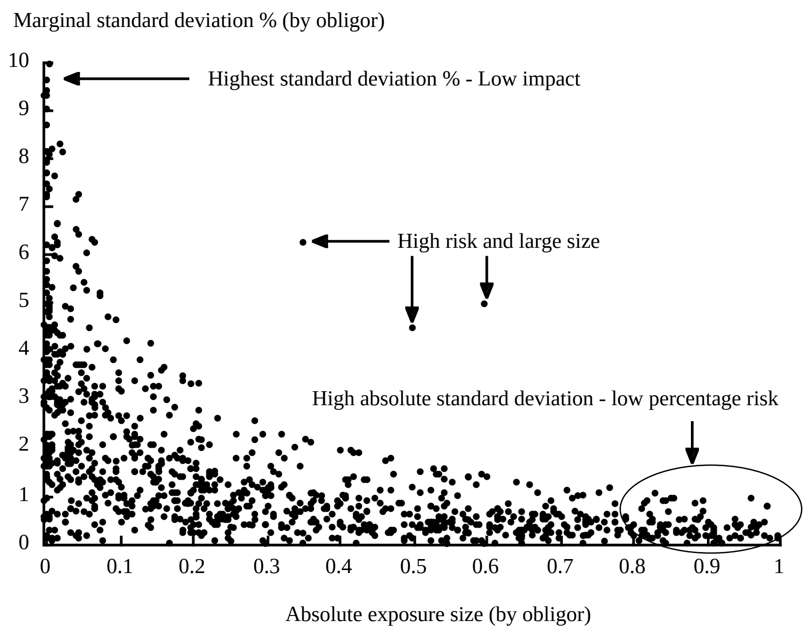
---

[1] Researchers interested in valuation and pricing models may refer to the following: Das & Tufano [96], Foss [95], Jarrow & Turnbull [95], Merton [74], Shimko, Tejima & Van Deventer [93], Skinner [94], and Sorensen & Bollier [94]. Other research on historical credit price levels and relationships includes: Altman & Haldeman [92], Eberhart, Moore & Roenfeldt [90], Fridson & Gao [96], Hurley & Johnson [96], Madan & Unal [96], Neilsen & Ronn [96], and Sarig & Warga [89].

- reevaluate obligors contributing the largest *absolute amount of risk* (points towards the upper right corner of the chart) arguing that these are the single largest contributors to portfolio risk.

Although all three approaches are perfectly valid, we advocate the last one, setting as the highest priority to address those obligors which are both relatively high percentage risk and relatively large exposure. These are the parties which contribute the greatest volatility to the portfolio. In practice, these are often "fallen angels," whose large exposures were created when their credit ratings were better, but who now have much higher percentage risk due to recent downgrades.

Chart 12.1
Risk versus size of exposures within a typical credit portfolio



Like *Chart 11.10*, this chart illustrates a risk versus size profile for a credit portfolio. Obligors with high percentage risk – and presumably high anticipated return – can be tolerated if they are small in size. Large exposures are typically allowed only if they have relatively small percentage risk levels. Unfortunately, the quality of a credit can change over time and a large exposure may have its credit rating downgraded (i.e., its point will move straight up in this chart). The portfolio will then have a large exposure with also a relatively large absolute level of risk. It is this type of obligor which we advocate addressing first.

Chart 12.1 does not completely describe the portfolio in question, however, as it does not address the issue of returns. Thus, there is another issue to consider when considering which exposures should be addressed: whether the returns on the exposures in question adequately compensate their risk. This is where the power of a portfolio analysis becomes evident. In general, it can be assumed that assets will be priced according to their risk on a stand-alone basis, or otherwise, in a CAPM (capital asset pricing model) framework, according to their correlation with a broad universe of assets. What this means is that a given asset may contribute differently to the risk of distinct portfolios, and yet yield the same returns in either case.

Consequently, we can imagine the following situation. Two managers identify a risky asset in their portfolios. It turns out that the two assets are of the same maturity, credit rating, and price, and are expected to yield equivalent returns. However, because of the structure of the two portfolios, if the managers swap these assets, the risk of both portfolios will be reduced without the expected return on either being affected. This might be the case if two managers are heavily concentrated in two different industries. By swapping similar risky assets, the managers reduce their concentration, and thus their risk, without reducing their expected profits.

We see then not only the importance of evaluating the contribution of each asset to the risk of the portfolio, but also the identification of how each asset makes its contribution. When the risk of an asset is due largely to concentrations particular to the portfolio, as in the example above, an opportunity could well exist to restructure the portfolio in such a way as to reduce its risk without altering its profitability.

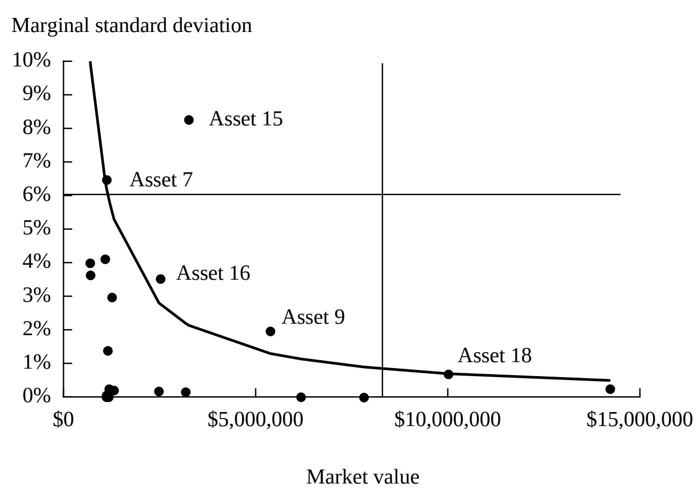
12.2 Credit risk limits

In terms of policy rigor, the next step beyond using risk statistics for prioritization is to use them for limit setting. Of course, what type of risk measure to use for limits, as well as what type of policy to take with regard to the limits, are management decisions. In this section, we discuss three aspects a user might consider with regard to using CreditMetrics for limit purposes: what type of limit to set, which risk measure to use for the limits, and what policy to employ with regard to the limits.

12.2.1 Types of credit risk limits

This section's discussion will make reference to *Chart 12.2*, which the reader might recognize as exactly the same as *Chart 11.10*, but with two additional barriers included.

Chart 12.2
Possible risk limits for an example portfolio



We might consider each of the three possibilities mentioned in the previous section as candidates for credit risk limits. We treat each in turn:

- *Set limits based on percentage risk.* This would correspond to a limit like the horizontal line in *Chart 12.2.* If we measured risk in absolute terms, this would correspond exactly to a limit on credit quality, that is, a limit restricting the portfolio to contain only exposures rated, say, B or higher. Since we measure risk in marginal terms, this limit would be slightly different in that it would also restrict exposures that are more correlated to the portfolio, since these contribute more to portfolio risk.

- *Set limits based on exposure size.* This would correspond to a limit like the vertical bar in *Chart 12.2.* Such a limit would restrict the portfolio to have no exposures, regardless of credit quality, above a given size.

- *Set limits based on absolute risk.* This would correspond to a limit like the curve in *Chart 12.2.* Such a limit would prevent the addition to the portfolio of any exposure which increased the portfolio risk by more than a given amount. In effect, this would cap the total risk of the portfolio at a certain amount above the current risk.

In the previous section, we argued that it is best to address exposures with the highest level of absolute risk first, since these have the greatest impact on the total portfolio risk. By the same token, it is most sensible to set limits in terms of absolute (rather than percent) risk. Moreover, limiting absolute risk is consistent with the natural tendencies of portfolio managers; in other words, it is perfectly intuitive to require that exposures which pose a greater chance of decreases in value due to credit be smaller, while allowing those with less chance of depreciating to be greater.

We see the natural tendency to structure portfolios in this way in both *Charts 12.1* and *12.2;* in both cases, the risk profiles tend to align themselves with the curve rather than with either the vertical or horizontal line. Thus, setting limits based on absolute risk would take the qualitative intuition that currently drives decisions and make it quantitative.

It is worth mentioning here that the risk limits we have discussed are not meant to replace existing limits to individual names. Limits based on the notion that there is a maximum amount of exposure we desire to a given counterparty, regardless of this counterparty's credit standing, are certainly appropriate. Such limits may be thought of as conditional, in that they reflect the amount we are willing to lose conditioned on a given counterparty's defaulting, and do not depend on the probability that the counterparty actually defaults. The limits proposed in this section are meant to supplement, but not replace, these conditional limits.

12.2.2 Choice of risk measure

Given a choice of what type of limit to implement, it is necessary next to choose the specific risk measure to be used. Essentially, there are two choices to make: first, whether to use a marginal or stand-alone statistic, and second, whether to use standard deviation, percentile level, or another statistic.

The arguments for using marginal statistics have been made before. These statistics allow the user to examine an exposure with regard to its effect on the actual portfolio, taking into accounts the effects of correlation and diversification. Thus, marginal statistics provide a better picture of the true concentration risk with respect to a given counterparty.

On the other hand, certain circumstances suggest the use of absolute risk measures for limits. For instance, suppose a portfolio contains a large percentage of a bond issue of a given name. Even if the name has a very low correlation with the remainder of the portfolio (meaning that the bond has low marginal risk), the position should be considered risky because of the liquidity implications of holding a large portion of the issue. Thus, it is important in this case to know the stand-alone riskiness of the position.

As to what statistic to use, we describe four statistics below, and discuss the applicability of each to credit risk limits.

As always, the easiest statistic to compute is the *standard deviation*. However, as a measure of credit risk, it has a number of deficiencies. First, the standard deviation is a "two-sided" measure, measuring the portfolio value's likely fluctuations to the upside or downside of the mean. Since we are essentially concerned with only the downside, this makes the standard deviation somewhat misleading. In addition, since distributions of credit portfolios are mostly non-normal, there is no way to infer concrete information about the distribution from just the standard deviation.

We have also discussed the use of *percentile levels* at some length. The advantages of this statistic are that it is easy to define and has a very concrete meaning. When we state the first percentile level of a portfolio, we know that this is precisely the level below which we can expect losses only one percent of the time. There is a price for this precision, however, as we cannot derive such a measure analytically, and must resort to simulations. Thus, our measure is subject to the random errors inherent in Monte Carlo approaches.

Another statistic which is often mentioned for characterizing risk is *average shortfall*. This statistic is defined as the expected loss given that a loss occurs, or as the expected loss given that losses exceed a given level. While this does give some intuition about a portfolio's riskiness, it does not have quite as concrete an interpretation as a percentile level. For instance, if we say that given a loss of over \$3mm occurs, we expect that loss to be \$6mm, we still do not have any notion of how likely a \$6mm loss is. Along the same lines, we might consider using the *expected excession of a percentile level*. For the 1st percentile level, this statistic is defined as the expected loss given that the loss is more extreme than the 1st percentile level. If this statistic were \$12mm, then the interpretation would be that in the worst 1 percent of all possible cases, we would expect our losses to be \$12mm. This is a very reasonable characterization of risk, but like percentile level and average shortfall, requires a simulation approach.

When choosing a risk statistic, it is important to keep in mind its application. For limits, and particularly for prioritization, it is not absolutely necessary that we be able to infer great amounts of information about the portfolio distribution from the risk statistics that we use. What is most important is that the risk estimates give us an idea of the *relative* riskiness of the various exposures in our portfolio. It is reasonable to claim that the standard deviation does this. Thus, for the purpose of prioritization or limit setting, it would be sensible to sacrifice the intuition we obtain from percentile levels or expected excessions if using the standard deviation provides us with significant improvements in computational speed.

12.2.3 Policy issues

The fundamental point of a limit is that it triggers action. There can be many levels of limits which we classify according to the severity of action taking in the case the limit is exceeded.

For informational limits, an excession of the limit might require more in-depth reporting, additional authorization to increase exposure size, or even supplemental covenant protection or collateral. The common thread is that exposures which exceed the limits are permitted, but trigger other actions which are not normally necessary.

Alternatively, one might set hard limits, which would preclude any further exposure to an individual name, industry, geographical region, or instrument type. In practice, one might implement both types of limits – an informational limit at some low level of risk or exposure and a hard limit at a higher level. And these limits might even be based on two different risk measures – a marginal measure at one level and an absolute measure at the other.

The assumption for both types of limits above is that the limits are in place before the exposures, and each exposure we add to the portfolio satisfies the limits. However, for the aforementioned fallen angels, this will not be the case. These exposures satisfy the risk limits when they are added to the portfolio, but subsequently exceed the limits due to a change in market rates or to a credit rating downgrade. Excessions of this type are essentially uncontrollable, although a portfolio manager might seek to reduce the risk in these cases by curtailing additional exposure, reducing existing exposure, or hedging with a credit derivative.

It is not uncommon to set limits at different levels of aggregation since different levels of oversight may occur at higher and higher levels. For instance, there might be limits on individual names, plus industry limits, plus sector limits, plus even an overall credit portfolio limit.

It should always be the case that a limit will be less than or equal to the sum of limits one level lower in the hierarchy. Thus, the financial sector limit should not be greater than the sum of limits to industries underneath it such as banks, insurers, brokers, etc. This will be true whether limits are set according to exposures (which can be aggregated by simply summing them) or according to risk (which can be aggregated only after accounting for diversification).

12.3 Economic capital assessment

For the purposes of prioritization and limit setting, the subjects of the first two sections, we examined risk measures in order to evaluate and manage individual exposures. The total risk of the portfolio might guide the limit-setting process, but it was the relative riskiness of individual exposures which most concerned us.

In this section, we examine a different application of credit risk measures, that of assessing the capital which a firm puts at risk by holding a credit portfolio. We are no longer trying to compare different exposures and decide which contribute most to the riskiness of the portfolio, but rather are seeking to understand the risk of the entire portfolio with regard to what this risk implies about the stability of our organization.

To consider risk in this way, we look at risk in terms of capital; but rather than considering the standard regulator or accounting view of capital, we examine capital from a risk management informational view. The general idea is that if a firm's liabilities are constant, then it is taking risk by holding assets that are volatile, to the extent that the asset volatility could result in such a drop in asset value that the firm is unable to meet its liability obligations.

This risk-taking capability is not unlimited, as there is a level beyond which no manager would feel comfortable. For example, if a manager found that given his asset portfolio, there was a ten percent chance for such a depreciation to occur in the next year as to cause organization-wide insolvency, then he would likely seek to decrease the risk of the asset portfolio. For a portfolio with a more reasonable level of risk, the manager cannot add new exposures indiscriminately, since eventually the portfolio risk will surpass the "comfort level." Thus, each additional exposure utilizes some of a scarce resource, which might be thought of as risk-taking capability, or alternately, as economic capital.

To measure or assess the economic capital utilized by an asset portfolio, we may utilize the distribution of future portfolio values which we describe elsewhere in this document. This involves a choice, then, of what statistic to use to describe this distribution. The choice is in some ways similar to the choice of risk statistic for limits which we discussed in the previous section; however, the distinct use of risk measures here make the decision different. For limits, we were concerned with individual exposures and relative measures; for economic capital, we are interested in a portfolio measure and have more need for a more concrete meaning for our risk estimate. These issues should become clear as we consider the risk statistics below.

For limits we could argue that the standard deviation was an adequate statistic in that it could capture the relative risks of various instruments. In this case, however, it is difficult to argue that a standard deviation represents a good measure of capital since we are unable to attach a concrete interpretation to this statistic. Yet this statistic is practical to compute and for this reason alone may be the logical choice.

As an indicator of economic capital, a percentile level seems quite appropriate. Using for example the 1st percentile level, we could define economic capital as the level of losses on our portfolio which we are 99% certain (or in the words of Jacob Bernoulli, "morally certain"[2]) that we will not experience in the next year. This fits nicely with our discussion of capital above. If it is our desire to be 99% certain of meeting our financial obligations in the next year, then we may think of the 1st percentile level as the risk we are taking, or as the economic capital which we are allocating to our asset portfolio. If this level ever reaches the point at which such a loss will prevent us from meeting obligations, then we will have surpassed the maximum amount of economic capital we are willing to utilize.

As with limits, we may consider average shortfall as a potential statistic. Yet just as in the case of limits, it is difficult to consider an expected shortfall of $6mm as a usage of capital since we do not know how likely such a loss actually is. On the other hand, the expected excession of a percentile level does seem worth consideration. Recall that if this statistic were $12mm at the first percentile level, then the interpretation would be

[2] As quoted in Bernstein [96].

that in the worst 1 percent of all possible cases, we would expect our losses to be $12mm. So like the percentile level above, this seems to coincide with our notion of economic capital, and thus seems a very appropriate measure.

All of the above measures of economic capital differ fundamentally from the capital measures mandated for bank regulation by the Bank for International Settlements (BIS). For a portfolio of positions not considered to be trading positions, the BIS risk-based capital accord of 1988 requires capital that is a simple summation of the capital required on each of the portfolio's individual transactions, where each transaction's capital requirement depends on a broad categorization (rather than the credit quality) of the obligor; on the transaction's exposure type (e.g., drawn loans versus undrawn commitments); and, for off-balance-sheet exposures, on whether the transaction's maturity is under one-year or over one year. The weaknesses of this risk-based structure – such as its one-size-fits-all risk weight for all corporate loans and its inability to distinguish diversified and undiversified portfolios – are increasingly apparent to regulators and market participants, with particular concern paid to the uneconomic incentives created by the regulatory regime and the inability of regulatory capital adequacy ratios to accurately portray actual bank risk levels. In response to these concerns, bank regulators are increasingly looking for insights in internal credit risk models that generate expected losses and a probability distribution of unexpected losses.[3]

12.4 Summary

In summary, the CreditMetrics methodology gives the user a variety of options to use for measuring economic capital which may in turn lead to further uses of CreditMetrics. We briefly touch on three applications of an economic capital measure: *exposure reduction*, *limit setting*, and *performance evaluation*.

An assessment of economic capital may guide the user to actions which will alter the characteristics of his portfolio. For example, if the use of economic capital is too high, it will be necessary to take actions on one or more exposures, possibly by prohibiting additional exposure, or else by reducing existing exposures by unwinding a position or hedging with a credit derivative. How to choose which exposures to treat could then be guided by the discussions in *Section 12.1*.

On the other hand, one might wish to use the measure of economic capital in order to aid the limit-setting process, assuring that if individual or industry level exposures are within the limits, then the level of capital utilization will be at an acceptable level.

A third use is performance evaluation. The traditional practice has been to evaluate portfolio managers based on return, leading to an incentive structure which encourages these managers to take on lower rated exposures in order to boost performance. Adding a measure of economic capital utilization allows for a more comprehensive measure of performance; when managers' returns are paired with such a risk measure, it can be seen which managers make the most efficient use of the firm's economic capital. Examining performance in this way retains the incentive to seek high returns, but penalizes for taking undue risks to obtain these returns.

3 See Remarks by Alan Greenspan, Board of Governors of the Federal Reserve System, before the 32nd Annual Conference on Bank Structure and Competition, FRB of Chicago, May 2, 1996.

By examining rates of return on economic capital and setting targets for these returns, a manager or firm goes a step beyond the traditional practice of requiring one rate of return on its most creditworthy assets and a higher rate on more speculative ones; the new approach is to consider a hurdle rate of return on risk, which is more clear and more uniform than the traditional practice. Identifying portfolios or businesses that achieve higher returns on economic capital essentially tells a manager which areas are providing the most value to the firm. And just as it is possible to allocate any other type of capital, areas where the return on risk is higher may be allocated more economic capital, or more risk-taking ability. By focusing capital on the most efficient parts of a firm or portfolio, profits are maximized, but within transparent, responsible risk guidelines.

Appendices

Appendices

In CreditMetrics we use certain general statistical formulas, data, and indices in several different capacities. We have chosen to address each of them in detail here in an appendix so that we may give them the depth they deserve without cluttering the main body of this *Technical Document*. These appendices include:

Appendix A: Analytic standard deviation calculation.

A generalization of the methods presented in *Chapter 9* to compute the standard deviation for a portfolio of arbitrary size.

Appendix B: Precision of simulation-based estimates.

Techniques to assess the precision of portfolio statistics obtained through simulation.

Appendix C: Derivation of the product of N random variables.

Used to: (i) combine the uncertainty of spread and exposure risk and (ii) for the derivation of risk across mutually exclusive outcomes.

Appendix D: Derivation of risk across mutually exclusive outcomes.

Used for both: the value variance of a position across *N*-states and the covariance between positions across *N*-states.

Appendix E: Derivation of the correlation of two binomials.

Used to link correlation between firms' value to their default correlations.

Appendix F: Inferring default correlations from default volatilities.

Used as alternative method to estimate default correlations which corroborates our equity correlation approach.

Appendix G: International bankruptcy code summary.

Contains this information in tabular format.

Appendix H: Model inputs.

Describes the CreditMetrics data files and required inputs.

Appendix I: Indices used for asset correlations.

Contains this information in tabular format.

Appendix A. Analytic standard deviation calculation

In *Chapter 9*, we presented the calculation of the standard deviation for an example three asset portfolio, and stated that the generalization of this calculation to a portfolio of arbitrary size was straightforward. In this appendix, we present this generalization in detail.

Consider a portfolio of n assets. Denote the value of these assets at the end of the horizon by $V_1, V_2, ..., V_n$; let these values' means be $\mu_1,\mu_2,...,\mu_n$ and their variances be $\sigma^2(V_1),\sigma^2(V_2),...,\sigma^2(V_n)$. The calculation of these individual means and variances is detailed in *Chapter 2*.

The value of the portfolio at the end of the forecast horizon is just $V_1+V_2+...+V_n$, and the mean value is $\mu_p=\mu_1+\mu_2+...+\mu_n$. To compute the portfolio standard deviation (σ_p), we may use the standard formula:

[A.1]
$$\sigma_p^2 = \sum_{i=1}^{n}\sigma^2(V_i) + 2\cdot\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} COV(V_i, V_j).$$

Alternatively, we may relate the covariance terms to the variances of pairs of assets,

[A.2]
$$\sigma^2(V_i+V_j) = \sigma^2(V_i) + 2\cdot COV(V_i, V_j) + \sigma^2(V_j),$$

and using this fact, express the portfolio standard deviation in terms of the standard deviations of subportfolios containing two assets:

[A.3]
$$\sigma_p^2 = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sigma^2(V_i+V_j) - (n-2)\cdot\sum_{i=1}^{n}\sigma^2(V_i).$$

As in *Chapter 9*, we see that the portfolio standard deviation depends only on the variances for pairs of assets and the variances of individual assets. This makes the computation of the portfolio standard deviation straightforward. We begin by computing the variances of each individual asset; we then identify all pairs of assets among the n assets in the portfolio[1] and compute the variances for each of these pairs using the methods in *Chapter 3*; finally, we apply Eq. [A.3].

[1] There will be $n\cdot(n-1)/2$ pairs.

Appendix B. Precision of simulation-based estimates

In *Chapter 10*, we presented a methodology to compute portfolio statistics using Monte Carlo simulation and mentioned that statistics which are estimated in this way are subject to random errors. In this appendix, we discuss how we may quantify the sizes of these errors, and thus discover how confident we may be of the risk estimates we compute. We devote one subsection each to the treatment of the sample mean, sample standard deviation, and sample percentile levels.

Throughout this section, we will use $V^{(1)}, V^{(2)}, V^{(3)}, \ldots, V^{(N)}$ to indicate the portfolio values across scenarios and $V^{[1]}, V^{[2]}, V^{[3]}, \ldots, V^{[N]}$ to indicate the same values sorted into ascending order (so that, for example $V^{[2]}$ is the second smallest value). Further, let μ_n denote the sample mean and σ_n the sample standard deviation of the first n scenarios.

B.1 Sample mean

Quantifying the error about our estimate of the mean portfolio value is straightforward. For large n, μ_n will be approximately normally distributed with standard deviation $\sigma_n / \sqrt{n}$. Thus, after generating n scenarios, we may say that we are 68%[2] confident that the true mean portfolio value lies between $\mu_n - \sigma_n / \sqrt{n}$ and $\mu_n + \sigma_n / \sqrt{n}$ and 90% confident the true mean lies between $\mu_n - (1.65 \cdot \sigma_n / \sqrt{n})$ and $\mu_n + 1.65 \cdot \sigma_n / \sqrt{n}$. Note that these bands will tighten as n increases.

B.2 Sample standard deviation

Our confidence in the estimate σ_n is more difficult to quantify since the distribution of the estimate is less well approximated by a normal distribution, and the standard deviation of the estimate is much harder to estimate.

The simplest approach here is to break the full set of scenarios into several subsets, compute the sample standard deviation for each subset, and examine how much fluctuation there is in these estimates. For example, if we have generated 20,000 portfolio scenarios, then we might divide these scenarios into fifty separate groups of 400. We could then compute the sample standard deviation within each group, obtaining fifty different estimates $\sigma^{(1)}, \sigma^{(2)}, \ldots, \sigma^{(50)}$. The sample standard deviation of these estimates, which we denote by s, is then an estimate for the standard error of σ_{400}. In order to extrapolate to an estimate for the standard error of σ_{20000}, we assume that the same scaling holds as with the sample mean, and take $s / \sqrt{50}$. Then we can say that we are approximately 90% confident that the true value of our portfolio standard deviation lies between $\mu_{2000} - (1.65 \cdot s / \sqrt{50})$ and $\mu_{2000} + (1.65 \cdot s / \sqrt{50})$ [3]. This procedure is commonly referred to as “jackknifing.”

For the sample mean and standard deviation, our approach to assessing precision was the same. Motivated by the fact that the estimates we compute are sums over a large number

[2] Since the probability that a normally distributed random variable falls within one standard deviation of its mean is 68%.

[3] This methodology is somewhat sensitive to the choice of how many separate groups to divide the sample into. We choose 50 here, but in practice suggest that the user experiment with various numbers in order to get a feel for the sensitivity of the confidence estimates to this choice.

of independent trials, we approximated the distributions of the estimates as normal. The rest of the analysis then focused on computing the standard errors for the estimates. Moreover, in some sense, the assessment of precision for estimates of these two statistics is somewhat redundant, as it is possible to obtain exact values in both cases.

In the next section, we treat estimates of percentile levels, for which neither of these points applies. Estimates are not just sums over the scenarios, and thus we cannot expect the distributions of the estimates to be normal; further, we have no way of computing percentile levels directly, and thus are much more concerned with the precision of our estimates.

B.3 Sample percentile levels

As an example, say we are trying to estimate the 5th percentile level, and let θ_5 be the true value of this level. Each scenario which we generate then (by definition) has a 5% chance of producing a portfolio value less than θ_5. Now consider 1000 independent scenarios, and let N_5 be the number of these scenarios which fall below θ_5. Note that N_5 follows the binomial distribution. Clearly, the expected value of N_5 is $1000 \cdot 5\% = 50$, while the standard deviation is $\sqrt{1000 \cdot 5\% \cdot (100\% - 5\%)} = 6.9$. For this many trials, it is reasonable to approximate the distribution of N_5 by the normal. Thus, we estimate that there is a 68% chance that N_5 will be between 50-6.9=43.1 and 50+6.9=56.9, and a slightly higher chance that N_5 will be between 43 and 57. Further, there is a 90% chance that N_5 falls between $50 - 1.65 \cdot 6.9 = 38.6$ and $50 + 1.65 \cdot 6.9 = 61.2$.

At this point we have characterized N_5. This may not seem particularly useful, however, since N_5 is not actually observable. In other words, since we do not actually know the level θ_5 (this is what we are trying to estimate), we have no way of knowing how many of our scenarios fell below θ_5. We assert that it is not necessary to know N_5 exactly, since we can gain a large amount of information from its distribution.

Observe that if N_5 is greater than or equal to 43, then at least 43 of our scenarios are less than θ_5. This implies that θ_5 is at least as large as the 43rd smallest of our portfolio values. (Recall that in our notation, this scenario is denoted by $V^{[43]}$.) On the other hand, if N_5 is less than or equal to 57, then it must be true that θ_5 is no larger than the 57th smallest of the portfolio values (that is, $V^{[57]}$). Thus, we have argued that the event

[B.1] $$43 \le N_5 \le 57$$

is exactly the same as the event

[B.2] $$V^{[43]} < \theta_5 < V^{[57]}.$$

Now since these two events are the same, they must have the same probability, and thus

[B.3] $$\Pr\{V^{[45]} < \theta_5 < V^{[57]}\} = \Pr\{43 \le N_5 \le 57\} = 68\%$$

and so we have a confidence bound for our estimate of θ_5. To recap, using 1000 scenarios, we estimate the 5th percentile portfolio value by the 50th smallest scenario, and state that we are 68% confident that the true percentile lies somewhere between the 43rd and 57th smallest scenarios.

In general, if we wish to estimate the p^{th} percentile using N scenarios, we first consider the number of scenarios that fall below the true value of this percentile. We characterize this number via the following:

[B.4]
$$\begin{aligned}
\text{lower bound:}\quad & l = N \cdot p - \alpha \cdot \sqrt{N \cdot p \cdot (1-p)} \\
\text{mean:}\quad & m = N \cdot p,\ s = \sqrt{N \cdot p \cdot (1-p)} \\
& \text{and} \\
\text{upper bound:}\quad & u = N \cdot p + \alpha \cdot \sqrt{N \cdot p \cdot (1-p)}
\end{aligned}$$

where α depends on the level of confidence which we desire. (That is, if we desire 68% confidence, then α=1, if we desire 90%, then α=1.65, etc.) If either l or m are not whole numbers, we round them downwards, while if u is not a whole number, we round upwards. We then estimate our percentile by $V^{[m]}$ and state with our desired level of confidence that the true percentile lies between $V^{[l]}$ and $V^{[u]}$.

For further discussion of these methods, see DeGroot [86], p. 563. Note that the only assumption we make in this analysis is that the binomial distribution is well approximated by the normal. In general, this will be the case as long as the expected number of scenarios falling below the desired percentile (that is, $N \cdot p$) is at least 20 or so. In cases where this approximation is not accurate, we may take the same approach as in this section, but characterize the distribution precisely rather than using the approximation. The result will be similar, in that we will obtain confidence bands on the number of scenarios falling below the threshold, and then proceed to infer confidence intervals on the estimated percentile.

Appendix C. Derivation of the product of *N* random variables

First we examine in detail the volatility of the product of two random variables. Let *X* and *Y* be any independent and uncorrelated distributions defined as follows:

[C.1] $X \sim \mu_x + \sigma_x \cdot Z_x \quad Y \sim \mu_y + \sigma_y \cdot Z_y$ ($where\ \sim denotes$ distributed as)

where all distributions, *Z*, are independent and standardized but can otherwise have any desired shape: normal, highly skewed, binomial, etc.

[C.2] $\sigma^2_{X \cdot Y} = E(X^2 \cdot Y^2) - E(X \cdot Y)^2$ (Textbook formula)

First, we will multiply out x · y.

[C.3] $X \cdot Y = \mu_x\mu_y + \mu_x\sigma_y Z_Y + \mu_y\sigma_x Z_X + \sigma_x Z_X \sigma_y Z_Y$

Since the expected value of Z is zero, the *E( )*'s simplify greatly.

[C.4]
$$E(X \cdot Y) = \mu_x\mu_y$$
$$E(X \cdot Y)^2 = \mu_x^2\mu_y^2 \quad \text{(Since: E(Z) = 0)}$$
$$E(X^2 \cdot Y^2) = \mu_x^2\mu_y^2 + \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2 \quad \text{(Since: } E(Z)^2 = 1\text{)}$$

Now $\sigma_{X \cdot Y}$ is only a matter of algebra.

[C.5]
$$\sigma^2_{X \cdot Y} = (\mu_x^2\mu_y^2 + \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2) - (\mu_x^2\mu_y^2)$$
$$= \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2$$
$$\boxed{\sigma^2_{X \cdot Y} = \sqrt{\mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2}}$$

By induction, we can we can extend the volatility estimation for the product of arbitrarily many independent events. First, the expectation of this product is simply the product of its expectations:

[C.6]
$$E\left(\prod_i^N \Phi_i\right) = \prod_i^N \mu_i \text{ where all } \Phi_i \sim \mu_i + \sigma_i \cdot Z_i$$
and all Z_i are standardized (0,1)

The variance of the product of *N* distributions will in general have, $2^N - 1$, terms. For the case of the product of three distributions, the result is:

[C.7]
$$VAR(\Phi_X \cdot \Phi_Y \cdot \Phi_Z) = \left(\begin{array}{l} + \mu_x^2\mu_y^2\sigma_z^2 + \mu_x^2\sigma_y^2\sigma_z^2 \\ + \mu_x^2\sigma_y^2\mu_z^2 + \sigma_x^2\mu_y^2\sigma_z^2 + \sigma_x^2\sigma_y^2\sigma_z^2 \\ + \sigma_x^2\mu_y^2\mu_z^2 + \sigma_x^2\sigma_y^2\mu_z^2 \end{array} \right)$$

In general, the pattern continues and can be denoted as follows for *N* distributions. In this notation, *j* and *m* denote sets whose elements comprise the product sums:

[C.8]
$$VAR\left(\prod_i^N \Phi_i\right) = \sum_{k=1}^{N}\left[\prod_{J \in s(N,k)}\left(\sigma_j^2 \cdot \prod_{m = S(N) - j} \mu_m^2\right)\right] - \prod_i^N \mu_i^2$$

where the sets $S(N) = \{1, 2, 3, \ldots, N\}$

and $s(N, k) = \{ j_i, \ldots j_k | 1 \le j_1 < \dot{\ldots} < j_k \le N, k \le N \}$.

Appendix D. Derivation of risk across mutually exclusive outcomes

Imagine that there were two alternative outcomes (subscripts *1* and *2*) that might occur in the event of default with probabilities of p_1 and p_2 which sum to the total probability of default. For completeness, subscript $\omega$is the case of no default. Each of these three cases has some distribution of losses denoted, $\Phi_i(x)$, with statistics, μ_i and σ_i.

Definitions: $1 = p_1 + p_2 + p_\omega$ and $\Phi_T(x) = p_1\Phi_1(x) + p_2\Phi_2(x) + p_\omega\Phi_\omega(x)$.

[D.1] Expected Total Loss

$$\begin{aligned} \mu_T &= \int x\Phi_T(x)dx \\ &= \int x(p_1\Phi_1(x) + p_2\Phi_2(x) + p_\omega\Phi_\omega(x))dx \\ &= p_1\mu_1 + p_2\mu_2 + p_\omega\mu_\omega \end{aligned}$$

[D.2] Variance of Total Loss

$$\begin{aligned} \sigma_T^2 &= \int (x-\mu_T)^2\Phi_T(x)dx \\ &= \int (x^2 - 2x\mu_T + \mu_T^2)(p_1\Phi_1(x) + p_2\Phi_2(x) + p_\omega\Phi_\omega(x))dx \\ &= \left(\begin{array}{c} \underbrace{p_1\int x^2\Phi_1(x) \quad + \quad p_2\int x^2\Phi_2(x) \quad + \quad p_\omega\int x^2\Phi_\omega(x)}_{\text{These simplify}} \\ -2\mu_T \underbrace{(p_1\mu_1 + p_2\mu_2 + p_\omega\mu_\varpi)}_{\text{Note that this equals } \mu_T \text{ see above}} \\ + \underbrace{\mu_T^2(p_1 + p_2 + p_\omega)}_{\text{Note that this sums to 1}} \end{array} \right) \\ &= \left(\begin{array}{c} p_1(\mu_1^2 + \sigma_1^2) + p_2(\mu_2^2 + \sigma_2^2) + p_\omega(\mu_\omega^2 + \sigma_\omega^2) \\ -2\mu_T^2 \\ +\mu_T^2 \end{array} \right) \\ &= p_1(\mu_1^2 + \sigma_1^2) + p_2(\mu_2^2 + \sigma_2^2) + p_\omega(\mu_\omega^2 + \sigma_\omega^2) - \mu_T^2 \end{aligned}$$

The above derivation requires a substitution for an integral that merits further discussion. The problem of multiplying a random variable by itself was addressed in the prior appendix note (see *Appendix C)*. If the two are the same distribution, then the correlation is simply 1.0.

[D.3] Mean of Product of Two Random Variables

$$\begin{aligned} \mu_{(i \cdot j)} &= \mu_i\mu_j + \rho\sigma_i\sigma_j && \text{See prior appendix note.} \\ &= \mu_i^2 + \sigma_i^2 && \text{Since } i = j \text{ and } \rho = 1.0 \\ &= \int x_2\Phi_i(x)dx && \text{Substitution made above.} \end{aligned}$$

For completeness, we have included terms describing the losses in the case of no default: μ_ω and σ_ω. But these are both zero since there will be no losses in the case of no default. Thus the overall total mean and standard deviation of losses in this process simplifies as follows:

[D.4] $$\sigma_T = \sqrt{\sum_{i=1}^{S} p_i(\mu_i^2 + \sigma_i^2) - \mu_T^2} \qquad \text{where } \mu_T = \sum_{i=1}^{S} p_i \mu_i$$

Appendix E. Derivation of the correlation of two binomials

The traditional textbook formula for covariance is shown below.

$$cov_{x,y} = \sum_{i=1}^{n} W_i(x_i - \mu_x)(y_i - \mu_y)$$

The expected probabilities, *p*'s, of the two binomials, *x* and *y*, are termed μ_x and μ_y respectively. Normally all the *n* observations would be equally weighted ($^1/_n$), but here the probability weights W_i will equal the likelihood of each possible outcome. For the joint occurrence of two binomials, there will be exactly four possible outcomes. We can simply list them explicitly. The probability weights W_i are easily calculated for the case of independence, but we will leave them as variables to allow for any degree of possible correlation. As shown below, defaults will have value 1 and non-defaults will have value 0.

[E.1]

Obligor Y		**Obligor X**	
Default	**No Default**	**Default**	**No Default**
1: X& Y default	3: Only X defaults	1: X& Y default	2: Only Y defaults
2: Only Y defaults	4: Neither defaults	3: Only X defaults	4: Neither defaults

$$cov_{x,y} = \begin{array}{l} W_1(1-\mu_x)(1-\mu_y) \\ +\, W_2(0-\mu_x)(1-\mu_y) \\ +\, W_3(1-\mu_x)(0-\mu_y) \\ +\, W_4(0-\mu_x)(0-\mu_y) \end{array}$$

The difficult problem in defining the probability weights *W*'s is knowing the correlated joint probability of default (cell #1 above). We will label this joint probability as α. Multiplying and simplifying the resulting formula, see below, yields an intuitive result for our covariance. If the joint default probability, α, is greater than the independent probability, (that is μ_x times μ_y), then the covariance is positive; otherwise it is negative.

[E.2]
$$cov_{x,y} = \begin{pmatrix} W_1(1-\mu_x)(1-\mu_y) \\ +W_2(0-\mu_x)(1-\mu_y) \\ +W_3(1-\mu_x)(0-\mu_y) \\ +W_4(0-\mu_x)(0-\mu_y) \end{pmatrix}$$

$$= \begin{pmatrix} [\quad\alpha\quad](1-\mu_x)(1-\mu_y) \\ +[\mu_y-\alpha](0-\mu_x)(1-\mu_y) \\ +[\mu_x-\alpha](1-\mu_x)(0-\mu_y) \\ +[1-\mu_x-\mu_y+\alpha](0-\mu_x)(0-\mu_y) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha-\alpha\mu_y-\alpha\mu_x+\alpha\mu_x\mu_y \\ -\mu_x\mu_y+\mu_x\mu_y^2+\alpha\mu_x-\alpha\mu_x\mu_y \\ -\mu_x\mu_y+\mu_x^2\mu_y+\alpha\mu_y-\alpha\mu_x\mu_y \\ +\mu_x\mu_y-\mu_x^2\mu_y-\mu_x\mu_y^2+\alpha\mu_x\mu_y \end{pmatrix}$$

$$= \alpha-\mu_x\mu_y$$

Now that we have derived the covariance as a function of the joint default probability, α, we can redefine α in terms of the correlation of our two binomials. Again, we can start with a textbook formula for the covariance:

[E.3]
$$\begin{array}{ccc} cov_{x,y} = \rho_{x,y}\sigma_x\sigma_y & & \alpha = \mu_x\mu_u + \rho_{x,y}\sigma_x\sigma_y \\ so & \text{thus} & and \\ \alpha-\mu_x\mu_y = \rho_{x,y}\sigma_x\sigma_y & & \rho_{x,y} = (\alpha-\mu_x\mu_y)/\sigma_x\sigma_y \end{array}$$

Interestingly, the above definition of α and ρ is identical the formula for the mean of the product of two correlated random variables as shown above (see *Appendix A)*. Importantly, this correlation ρ_{xy} is the resulting correlation of the joint binomials[4]. It does not represent some underlying firm-asset correlation that (via a bivariate normal assumption) might lead to correlated binomials. The σ's here are the usual binomial standard deviations, $\sqrt{\mu(1-\mu)}$. This formula for ρ_{xy} implies that there are bounds on ρ_{xy} since α is at least max(*0,* $\mu_x+\mu_y$-1) and at most min (μ_x, μ_y). Thus:

[E.4]
$$\frac{(max(0, \mu_x+\mu_y-1)-\mu_x\mu_y)}{\sigma_x\sigma_y} \le \rho_{x,y} \le \frac{(min(\mu_x, \mu_y)-\mu_x\mu_y)}{\sigma_x\sigma_y}$$

4 Other researchers have used this same binomial correlation, see Lucas [95a].

Appendix F. Inferring default correlations from default volatilities

For N firms in a grouping with identical default rate (i.e., within a single credit rating category), let X_1 be a random variable which is either 1 or 0 according to each firm's default event realization with mean default rate, $\mu(X_1)$, and binomial default standard deviation, $\sigma(X_1)$, defined as follows:

[F.1]
$$\left(X_i = \begin{cases} 1 \text{ if company } i \text{ defaults} \\ 0 \text{ otherwise} \end{cases} \right)$$

$$\mu_{CrRt} = \mu(X_i) = \frac{1}{N}\sum_i^N X_i$$

$$\sigma(X_i) = \sqrt{\mu_{CrRt}(1 - \mu_{CrRt})}$$

Let D represent the number of defaults, $D = \sum_i^N X_i$. So the variance of D is as follows:

[F.2]
$$\begin{aligned} VAR(D) &= \sum_i^N \sum_j^N \rho_{ij}\sigma(X_i)\sigma(X_j) \\ &= \sum_i^N \sum_j^N \rho_{ij}\sigma(X_i)^2 \qquad \text{Since all } i \text{ and } j \text{ have the same default rate.} \\ &= \sum_i^N \sum_j^N \rho_{ij}(\mu_{CrRt} - \mu_{CrRt}^2) \\ &= (\mu_{CrRt} - \mu_{CrRt}^2)\left[N + \sum_i^N \sum_{j \le i}^N \rho_{ij}\right] \end{aligned}$$

Rather than each ρ_{ij}, we are interested in the average correlation, $\bar{\rho}_{CrRt}$, and define this as follows

[F.3]
$$\bar{\rho}_{CrRt} = \left[\sum_i^N \sum_{j \le i}^N \rho_{ij}\right] / (N^2 - N)$$

and so we can now define

[F.4]
$$VAR(D) = (\mu_{CrRt} - \mu_{CrRt}^2)[N + (N^2 - N)\bar{\rho}_{CrRt}].$$

Across many firms we can observe the volatility of defaults, $\sigma_{CrRt}^2 = VAR(D/N)$, thus:

[F.5]
$$\sigma^2_{CrRt} = VAR\left(\frac{D}{N}\right) = \frac{VAR(D)}{N^2}$$
$$= (\mu_{CrRt} - \mu^2_{CrRt}) \cdot \frac{1 + (N-1)\bar{\rho}_{CrRt}}{N} \quad \therefore \quad \boxed{\bar{\rho}_{CrRt} = \frac{N\left(\dfrac{\sigma^2_{CrRt}}{\mu_{CrRt} - \mu^2_{CrRt}}\right) - 1}{N-1}}$$

This can be applied with good result in a simplified form if N is "large":

[F.6]
$$\bar{\rho}_{CrRt} = \frac{N\left(\dfrac{\sigma^2_{CrRt}}{\mu_{CrRt} - \mu^2_{CrRt}}\right) - 1}{N-1} = \frac{8,500\left(\dfrac{1.4\%^2_{Ba}}{1.42\%_{Ba} - 1.42\%^2_{Ba}}\right) - 1}{8,500 - 1} = 1.3886\%$$
$$= \frac{\sigma^2_{CrRt}}{\mu_{CrRt} - \mu^2_{CrRt}} = \frac{1.4\%^2_{Ba}}{1.42\%_{Ba} - 1.42\%^2_{Ba}} = 1.4002\%$$

The estimate of 8,5000 firm-years above stems from Moody's reporting of 120 firms being rated Ba one calendar year prior to default (8,500 ≅ 120/1.42%), see Carty & Lieberman [96a].

Appendix G. International bankruptcy code summary

The practical result of the seniority standing of debt will vary across countries according to local bankruptcy law. Of course, this will affect the likely recovery rate distributions. Major differences will apply to secured versus unsecured debt. The following summary table is reproduced from Rajan & Zingales [95] – who in turn reference Keiser [94], Lo Pucki & Triantis [94], and White [93].

Table G.1
Summary of international bankruptcy codes

Country	Forms of Liquidation	Forms of Reorganization	Management Control in Bankruptcy	Automatic Stay	Rights of Secured Creditors
United States	Chapter 7: Can be voluntary (management files) or involuntary (creditors file).	Chapter 11: Can be voluntary (management files) or involuntary (creditors file).	Trustee appointed in Chapter 7. Management stays in control in Chapter 11.	Automatic stay on any attempts to collect debt once filing takes place.	Secured creditors get highest priority in any attempts to collect payment are also stayed unless court or trustee approves
Japan	Court Supervised Liquidation (*Hasan*) and Special Liquidation (*Tokubetsu Seisan*). The latter is less costly and a broader set of firms are eligible to file.	Composition (*Wagi-ho*), Corporate Arrangement (*Kaisha Seiri*) and Reorganization (*Kaisha Kosei-ho*). The list in order of increasing eligibility. Only debtors file.	Third party is appointed except in composition and corporate arrangement.	All creditors are stayed except in court supervised liquidation and composition where only unsecured creditors are stayed.	Secured Creditors have highest priority and greater voting rights in renegotiation. However, can be subject on the petition that is filed.
Germany	Liquidation (*Konkursordnung*) can be requested by creditors or debtor. Management required to file as soon as it learns it is insolvent.	Composition (*Vergleich* or *Zwangvergleich*) can be filed for only by debtor.	Receiver appointed to manage firm.	Only unsecured creditors are stayed.	Secured creditors can recover their claims even after a bankruptcy filing. No stay for secured creditors.
France	Liquidation (*Liquidation Judicaire*)	Negotiated Settlement (*Reglement Amiable*) where a court appointed conciliator attempts a settlement with creditors and Judicial Arrangement (*Redressement Judiciare*).	Debtor loses control in liquidation. Debtor remains in control otherwise but submits to court appointed administrator's decisions in a judicial arrangement.	Stay on all creditors in judicial arrangement.	Secured creditors may lose status if court determines the security is necessary for continuation of the business, or if the securing asset is sold as part of settlement.
Italy	Bankruptcy (*Fallimento*)	Preventive Composition (*Concordato Preventino*)	Debtor is removed from control over the firm.	Stay on all creditors.	Secured creditors stayed in bankruptcy, through composition allowed only if enough value exists to pay secured creditors in full and 40% of unsecured creditor claims. Secured creditors follow administrative claims in priority.
United Kingdom	Members' voluntary winding up, Creditors' voluntary winding up, Compulsory winding up.	Administration, Administrative Receivership (usually ends in sale of business), and Voluntary Arrangement.	Debtor is removed from control except in members' voluntary winding up.	Stay on all creditors in administration, on unsecured only in liquidation, and no stay in a voluntary arrangement until a proposal is approved.	Secured creditor may prevent administration order by appointing his own receiver. A creditor with a fixed or floating charge can appoint an administrative receiver to realize the security and pay the creditor.
Canada	Liquidation proceedings much like Chapter 7 in the United States	Firms can file for automatic stay under the Companies Creditors Arrangement Act or the Bankruptcy and Insolvency Act.	Firm is in control in reorganizations while trustee is appointed for liquidation. Trustee may be appointed to oversee management in some reorganizations at the discretion of the court.	Stay on all creditors in reorganization.	Secured creditors have to give 10 days notice to debtor of intent to repossess collateral. Repossession even close to bankruptcy filing is permitted, but stayed after filing.

Appendix H. Model inputs

Available for free download from the Internet http://jpmorgan.com/ is a data set of all the elements described in this technical document and necessary to implement the CreditMetrics methodology. Here, we briefly list what is provided and the format in which it is available.

CreditMetrics data files include:

- country/industry index volatilities and correlations,
- yield curves,
- spread curves, and
- transition matrices.

H.1 Common CreditMetrics data format characteristics

In general, CreditMetrics data files are text (ASCII) files which use tab characters (ASCII code 9) as column delimiters, and carriage returns/line feeds as row delimiters.

Every CreditMetrics data file begins with a header, for example:

CDFVersion	v1.0
Date	02/15/1997
DataType	CountryIndustryVolCorrs

The header is followed by a row of column headers, followed by the data.

Cells in the data rows must contain data. If the value is unavailable or not applicable, the cell should contain the keyword NULL.

H.2 Country/industry index volatilities and correlations

This file is named **indxvcor.cdf**. The data represent the weekly volatilities and correlations discussed in *Chapter 8*.

CDFVersion	v1.0					
Date	02/15/1997					
DataType	CountryIndustryVolCorrs					
IndexName	Volatility					
MSCI Australia Index (.CIAU)	0.0171	1.0000	0.6840	0.6911	0.7343	0.6377
ASX Banks & Finance Index (.ABII)	0.0219	0.6840	1.0000	0.4360	0.4580	0.4436
ASX Media Index (.AMEI)	0.0257	0.6911	0.4360	1.0000	0.5528	0.3525

H.3 Yield curves

This file is named **yldcrv.cdf**. A yield curve is defined by currency . Allowable currencies are the standard three-letter ISO currency codes (e.g., CHF, DEM, GBP, JPY, USD).

CDFVersion	v1.0		
Date	02/15/1997		
DataType	YieldCurves		
Currency	CompoundingFrequency	Maturity	YieldToMaturity
CHF	1	1.0	0.055
CHF	1	2.0	0.05707

H.4 Spread curves

Bridge will be the initial data provider for credit spreads. Their contact number is (1-800) 828 - 8010.

Bridge credit spread data is derived through a compilation of information provided by major dealers including Citibank, CS First Boston, Goldman Sachs, Liberty Brokerage, Lehman Brothers, Morgan Stanley, Salomon Brothers and J.P. Morgan. A team of evaluators reviews the contributed information on a daily bvasis to ensure accuracy and consistency.

This file is named **sprdcrv.cdf**. A spread curve is defined by a combination of rating system, rating, and a yield curve (a yield curve being defined as a combination of currency and asset type). Allowable currencies are the standard 3-letter ISO currency codes (e.g. CHF, DEM, GBP, JPY, USD). Allowable asset types are BOND, LOAN, COMMITMENT, RECEIVABLE, and MDI.

Initial data is available only for USD and BOND

CDFVersion	v1.0					
Date	02/15/1997					
DataType	SpreadCurves					
RatingSystem	Rating	Currency	AssetType	CompoundingFrequency	Maturity	Spread
Moody8	Aaa	CHF	BOND	1	5.0	0.01118
Moody8	Aaa	CHF	BOND	1	3.0	0.00866
Moody8	Aaa	CHF	BOND	1	10.0	0.015811
Moody8	Aaa	CHF	BOND	1	15.0	0.019365
Moody8	Aaa	CHF	BOND	1	2.0	0.007071
Moody8	Aaa	CHF	BOND	1	20.0	0.022361

H.5 Transition matrices

This file is named **trnsprb.cdf**. This contains transition probabilities for both Moody's major and modified ratings, S&P major rating transition matrix, and J.P. Morgan derived matrices estimating long-term ratings behavior. Initially, they will have data for a one year risk horizon. However, the format supports other horizons.

The FromRating and ToRating columns of descriptive rating labels are included for readability. CreditMetrics only utilizes the numerical FromRating and ToRating columns.

CDFVersion	v1.0					
Date	02/15/97					
DataType	TransitionProbabilities					
RatingSystem	FromRank	ToRank	FromRating	ToRating	HorizonInMonths	Probability
Moody18	0	0	Aaa	Aaa	12	0.880784
Moody18	0	1	Aaa	Aa1	12	0.050303
Moody18	0	2	Aaa	Aa2	12	0.029015

H.6 Data Input Requirements to the Software Implementation of CreditMetrics

Table H.1
Required inputs for each issuer

Data Type	Description
Issuer name	Must be unique.
Credit Rating/Agency	Long term rating that applies to the issuer's senior unsecured debt regardless of the particular seniority class(es) listed as its exposure. Each rating has an agency (Moody's, S&P, etcetera)
Market Capitalization	Stock price times number of shares outstanding
Country & Industry	Proportion of sales assigned to specified countries and industries.
Issuer-specific risk	Volatility of issuer asset returns not explained by industry/country group(s).

Table H.2
Required inputs for each exposure type

Property	Bond	Loan	Commitment	MDI	Receivable
Issuer Name	x	x	x	x	x
Portfolio	x	x	x	x	x
Currency	x	x	x	x	x
Asset type	x	x	x	x	x
Par value	x	x			x
Maturity	x	x	x		x
Seniority class	x				
Recovery rate	x	x	x	x	x
Recovery rate std	x	x	x	x	x
Fixed or floating	x	x	x		
Coupon or spread	x	x	x		
Coupon frequency	x	x	x		
Current line			x	x	
Current drawdown			x		
Expected drawdown			x		
Duration				x	
Expected exposure				x	
Average exposure				x	
Forward value				x	

Appendix I. Indices used for asset correlations

	Asset Category	Index
Australia	General	MSCI Australia Index
	Banking and finance	ASX Banks & Finance Index
	Broadcasting and media	ASX Media Index
	Construction and building materials	ASX Building Materials Index
	Chemicals	ASX Chemicals Index
	Energy	ASX Energy Index
	Food	ASX Food & Household Goods Index
	Insurance	ASX Insurance Index
	Paper and forest products	ASX Paper & Packaging Index
	Transportation	ASX Transport Index
Austria	General	MSCI Austria Index
Belgium	General	MSCI Belgium Index
Canada	General	MSCI Canada Index
	Automobiles	Toronto SE Automobiles & Parts Index
	Banking and finance	Toronto SE Financial Services Index
	Broadcasting and media	Toronto SE Broadcasting Index
	Construction and building materials	Toronto SE Cement & Concrete Index
	Chemicals	Toronto SE Chemicals Index
	Hotels	Toronto SE Lodging, Food & Health Index
	Insurance	Toronto SE Insurance Index
	Food	Toronto SE Food Stores Index
	Electronics	Toronto SE Electrical & Electronics Index
	Metals mining	Toronto SE Metals Mines Index
	Energy	Toronto SE Integrated Oils Index
	Health care and pharmaceuticals	Toronto SE Biotechnology & Pharmaceuticals Index
	Publishing	Toronto SE Publishing & Printing Index
	Transportation	Toronto SE Transportation Index
Germany	General	MSCI Germany Index
	Automobiles	CDAX Automobiles Index
	Banking and finance	CDAX Investment Company Index
	Chemicals	CDAX Chemicals Index
	Construction and building materials	CDAX Construction Index
	Insurance	CDAX Insurance Index
	Machinery	CDAX Machinery Index
	Paper and forest products	CDAX Paper Index
	Textiles	CDAX Textiles Index
	Transportation	CDAX Transport Index
	Utilities	CDAX Utilities Index
Greece	General	MSCI Greece Index
	Banking and finance	Athens SE Banks Index
	Insurance	Athens SE Insurance Index
Finland	General	MSCI Finland Index
	Banking and finance	Helsinki SE Banks & Finance Index
	Metals mining	Helsinki SE Metal Index
	Paper and forest products	Helsinki SE Forest & Wood Index
	Insurance	Helsinki SE Insurance & Investment Index

	Asset Category	Index
France	General	MSCI France Index
	Automobiles	SBF Automotive Index
	Banking and finance	SBF Finance Index
	Construction and building materials	SBF Construction Index
	Energy	SBF Energy Index
	Food	SBF Food Index
Hong Kong	General	MSCI Hong Kong Index
	Banking and finance	Hang Seng Finance Index
	Utilities	Hang Seng Utilities Index
Indonesia	General	MSCI Indonesia Index
Italy	General	MSCI Italy Index
	Chemicals	Milan SE Chemical Current Index
	Banking and finance	Milan SE Financial Current Index
	Food	Milan SE Food & Groceries Current Index
	Paper and forest products	Milan SE Paper & Print Current Index
	Metals mining	Milan SE Mine & Metal Current Index
Japan	General	MSCI Japan Index
	Banking and finance	Topix Banking Index
	Broadcasting and media	Topix Communications Index
	Construction and building materials	Topix Construction Index
	Chemicals	Topix Chemical Index
	Electronics	Topix Electrical Appliances Index
	Food	Topix Foods Index
	Insurance	Topix Insurances Index
	Machinery	Topix Machinery Index
	Metals mining	Topix Mining Index
	Health care and pharmaceuticals	Topix Pharmaceuticals Index
	Paper and forest products	Topix Pulp and Paper Index
	Energy	Topix Electric Power and Gas Index
	Oil and gas -- refining and marketing	Topix Oil and Coal Products Index
	Textiles	Topix Textile Products Index
	Transportation	Topix Transportation Equipment Index
Korea	General	MSCI Korea Index
	Banking and finance	Korea SE Finance Major Index
	Construction and building materials	Korea SE Construction Major Index
	Chemicals	Korea SE Chemical Company Major Index
	Food	Korea SE Food & Beverage Major Index
	Insurance	Korea SE Insurance Major Index
	Machinery	Korea SE Fabricated Metal & Machinery Major Index
	Metals mining	Korea SE Mining Major Index
	Paper and forest products	Korea SE Paper Product Major Index
	Textiles	Korea SE Textile & Wear Major Index
	Transportation	Korea SE Transport & Storage Major Index
Malaysia	General	MSCI Malaysia Index
	Banking and finance	KLSE Financial Index
	Metals mining	KLSE Mining Index

	Asset Category	Index
Mexico	General	MSCI Mexico Index
	Transportation	Mexican SE Commercial & Transport Index
	Metals mining	Mexican SE Mining Index
	Construction and building materials	Mexican SE Construction Index
New Zealand	General	MSCI New Zealand Index
Norway	General	MSCI Norway Index
	Banking and finance	Oslo SE Bank Index
	Insurance	Oslo SE Insurance Index
Philippines	General	MSCI Philippines Index
	Metals mining	Philippine SE Mining Index
	Oil and gas -- refining and marketing	Philippine SE Oil Index
Poland	General	MSCI Poland Index
Portugal	General	MSCI Portugal Index
Singapore	General	MSCI Singapore Index
	Hotels	All-Singapore Hotel Index
	Banking and finance	All-Singapore Finance Index
Spain	General	MSCI Spain Index
Sweden	General	MSCI Sweden Index
	Banking and finance	Stockholm SE Banking Sector Index
	Construction and building materials	Stockholm SE Real Estate & Construction Index
	Chemicals	Stockholm SE Pharmaceutical & Chemical Index
	Paper and forest products	Stockholm SE Forest Industry Sector Index
Switzerland	General	MSCI Switzerland Index
	Banking and finance	SPI Banks Cum Dividend Index
	Construction and building materials	SPI Building Cum Dividend Index
	Chemicals	SPI Chemical Cum Dividend Index
	Electronics	SPI Electronic Cum Dividend Index
Thailand	General	MSCI Thailand Index
	Banking and finance	SET Finance Index
	Chemicals	SET Chemicals & Plastics Index
	Electronics	SET Electrical Components Index
	Technology	SET Electrical Products &Computers Index
	Construction and building materials	SET Building & Furnishing Materials Index
	Energy	SET Energy Index
	Food	SET Food & Beverages Index
	Health care and pharmaceuticals	SET Health Care Services Index
	Insurance	SET Insurance Index
	Hotels	SET Hotel & Travel Index
	Machinery	SET Machinery & Equipment Index
	Metals mining	SET Mining Index
	Paper and forest products	SET Pulp & Paper Index
	Publishing	SET Printing & Publishing Index
	Textiles	SET Textile Index
	Transportation	SET Transportation Index

	Asset Category	Index
United Kingdom	General	MSCI United Kingdom Index
	Banking and finance	FT-SE-A 350 Banks Retail Index
	Broadcasting and media	FT-SE-A 350 Media Index
	Construction and building materials	FT-SE-A 350 Building Materials & Merchants Index
	Chemicals	FT-SE-A 350 Chemicals Index
	Electronics	FT-SE-A 350 Electronic & Electrical Equipment Index
	Food	FT-SE-A 350 Food Producers Index
	Health care and pharmaceuticals	FT-SE-A 350 Health Care Index
	Insurance	FT-SE-A 350 Insurance Index
	Hotels	FT-SE-A 350 Leisure & Hotels Index
	Metals mining	FT-SE-A 350 Extractive Industries Index
	Oil and gas -- refining and marketing	FT-SE-A 350 Gas Distribution Index
	Energy	FT-SE-A 350 Oil Integrated Index
	Paper and forest products	FT-SE-A 350 Paper, Packaging & Printing Index
	Telecommunications	FT-SE-A 350 Telecommunications Index
	Textiles	FT-SE-A 350 Textiles & Apparel Index
	Transportation	FT-SE-A 350 Transport Index
United States	General	MSCI United States Of America Index
	Automobiles	S&P Automobiles Index
	Banking and finance	S&P Financial Index
	Broadcasting and media	S&P Broadcasting (Television, Radio & Cable)
	Construction and building materials	S&P Building Materials Index
	Chemicals	S&P Chemicals Index
	Electronics	S&P Electronics (Instrumentation)
	Energy	S&P Energy Index
	Entertainment	S&P Entertainment Index
	Food	S&P Foods Index
	Health care and pharmaceuticals	S&P Health Care Index
	Insurance	S&P Insurance Composite Index
	Hotels	S&P Lodging-Hotels Index
	Machinery	S&P Machinery (Diversified)
	Manufacturing	S&P Manufacturing (Diversified)
	Metals mining	S&P Metals Mining Index
	Oil and gas -- refining and marketing	S&P Oil & Gas (Refining & Marketing)
	Paper and forest products	S&P Paper & Forest Products Index
	Publishing	S&P Publishing Index
	Technology	S&P Technology Index
	Telecommunications	S&P Telecommunications (Long Distance)
	Textiles	S&P Textiles (Apparel)
	Transportation	S&P Transport Index
	Utilities	S&P Utilities Index
South Africa	General	MSCI South Africa (Gross Dividends Reinvested)
	Banking and finance	Johannesburg SE Financial Index
	Metals mining	Johannesburg SE Mining Holding Index

	Asset Category	Index
MSCI Worldwide	Automobiles	Automobiles Price Index
	Banking and finance	Banking Price Index
	Broadcasting and media	Broadcasting & Pubs Price Index
	Construction and building materials	Construction & Housing (US$) Price Index
	Chemicals	Chemicals Price Index
	Electronics	Electronic Comps/Inst. Price Index
	Energy	Energy Sources Price Index
	Entertainment	Recreation & Other Goods Price Index
	Food	Food & Household Products Price Index
	Health care and pharmaceuticals	Health & Personal Care Price Index
	Insurance	Insurance Price Index
	Hotels	Leisure & Tourism Price Index
	Machinery	Machinery & Engineering Price Index
	Metals mining	Metals Nonferrous Price Index
	Paper and forest products	Forest Products/Paper Price Index
	Telecommunications	Recreation & Telecommunications Price Index
	Textiles	Textiles & Apparel Price Index
	Transportation	Transport Shipping Price Index
	Utilities	Utilities Electric & Gas Price Index
MSCI Regional	EMF Latin America	
	Europe 14	
	Nordic Countries	
	North America	
	Pacific	
	Pacific ex Japan	

Reference

Glossary of terms

This glossary defines important terms in CreditMetrics.

accounting analytic. The use of financial ratios and fundamental analysis to estimate firm specific credit quality examining items such as leverage and coverage measures, with an evaluation of the level and stability of earnings and cash flows. *(*See *page 58.)*

allowance for loan and lease losses. An accounting reserve set aside to equate expected (mean) losses from credit defaults. It is common to consider this reserve as the buffer for expected losses and some risk-based economic capital as the buffer for unexpected losses. *(See page 60.)*

autocorrelation (serial correlation). When time series observations have a non-zero correlation over time. Two empirical examples of autocorrelation are:

- Interest rates exhibit mean reversion behavior and are often negatively autocorrelated (i.e., an up move one day will suggest a down move the next). But note that mean reversion does not technically necessitate negative autocorrelation.

- Agency credit ratings typically exhibit move persistence behavior and are positively autocorrelated during downgrades (i.e., a downgrade will suggest another downgrade soon). But, for completeness, note that upgrades do not better predict future upgrades – we find, they predict a "quiet" period; see also Altman & Kao [92].

*(*See *page 32.)*

average exposure. Credit exposure arising from market-driven instruments will have an ever-changing mark-to-market exposure amount. The amount of exposure relevant to our credit analysis is the time-bucketed average exposure in each forward period across the life of the transaction across all – probability weighted – market rate paths. *(*See *page 49.)*

average shortfall. The expected loss given that a loss occurs, or as the expected loss given that losses exceed a given level. *(*See *page 137.)*

credit exposure. The amount subject to changes in value upon a change in credit quality through either a market based revaluation in the event of an up(down)grade or the application of a recovery fraction in the event of default. (See *page 42*).

commitment. A legally binding obligation (subject usually both to conditions precedent and to continuing conditions) to make available loans or other financial accommodation for a specified period; this includes revolving facilities. Even during publicly known credit distress, a commit can be legally binding if drawndown before it is formally withdraw for cause.

concentration risk. Portfolio risk resulting from increased exposure to one obligor or groups of correlated (e.g., by industry or location) obligors. *(*See *page 6.)*

correlation. A linear statistical measure of the co-movement between two random variables. A correlation (Greek letter "ρ", pronounced "rho") will range from +1.0 to -1.0. Observing "clumps" of firms defaulting together by industry or geographically is an example of positive correlation of default events. *(*See *page 35.)*

$$\rho_{X,Y} = \frac{COV_{X,Y}}{\sigma_X \cdot \sigma_Y} = \frac{\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{N}(X_i - \bar{X})^2} \cdot \sqrt{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}}$$

counterparty. The partner in a credit facility or transaction in which each side takes broadly comparable credit risk to the other. When a bank lends a company money, the borrower (not Counterparty) has no meaningful credit risk to the bank. When the same two agree on an at-the-money forward exchange contract or swap, the company is at risk if the bank fails just as much as the bank is at risk if the counterparty fails (although for the opposite movement in exchange or interest rates). After inception, swap positions often move in/out-of-the-money and the relative credit risk changes accordingly. *(*See *page 47.)*

covenants. The terms under which a credit facility will be monitored. Covenants are most effective when they are specific measures that state the acceptable limits for change in the obligor's financial and overall condition. They clearly define what is meant by "significant" deterioration in the obligor's credit quality. Financial covenants are more explicit (and therefore more desirable) than a "material adverse change" clause. Cross default provisions are common: allowing acceleration of debt repayment. *(*See *page 43.)*

credit distress. A firm can have many types of credit obligations outstanding. These may be of all manner of seniority, security and instrument type. In bankruptcy proceedings, it is not uncommon for different obligations to realize different recovery rates including perhaps 100% recovery – zero loss. In our terminology, it is the obligor that encounters credit distress carrying all of his obligations with him even though some of these may not realize a true *default* (i.e., some may have zero loss). *(*See *page 65.)*

credit exposure. The amount subject to either changes in value upon credit quality up(down)grade or loss in the event of default. *(*See *page 42.)*

credit quality. Generally meant to refer to an obligor's relative chance of default, usually expressed in alphabetic terms (e.g., Aaa, Aa, A, etc.). CreditMetrics makes use of an extended definition that includes also the volatility of up(down)grades.

credit scoring. Generically, credit scoring refers to the estimation of the relative likelihood of default of an individual firm. More specifically, this is a reference to the application of linear discriminant analysis to combine financial rations to quantitatively predict the relative chance of default. *(*See *page 57.)*

current exposure. For market-driven instruments, the amount it would cost to replace a transaction today should a counterparty default. If there is an enforceable netting agreement with the counterparty, then the current exposure would be the net replacement cost; otherwise, it would be the gross amount.

default probability. The likelihood that an obligor or counterparty will encounter credit distress within a given time period. "Credit distress" usually leads to either an omitted delayed payment or distressed exchange which would impair the value to senior unsecured debt holders. Note that this leaves open the possibilities that:

- Subordinated debt might default without impairing senior debt value, and
- Transfers and clearing might continue even with a senior debt impairment.

*(*See *page 65.)*

dirty price. Inclusion of the accrued value of the coupon in the quoted price of a bond For instance, a 6% annual coupon bond trading at par would have a dirty price of $106 just prior to coupon payment. CreditMetrics estimates dirty prices since the coupon is paid in non-default states but assumed not paid in default. *(*See *page 10.)*

distressed exchange. During a time of credit distress, debt holders may be effectively forced to accepted securities in exchange for their debt claim – such securities being of a lower value than the nominal present value of their original claim. They may have a lower coupon, delayed sinking funds, and/or lengthened maturity. For historical estimation of default probabilities, this would count as a default event since it can significantly impair value. In the U.S., exchange offers on traded bonds may be either registered with the SEC or unregistered if they meet requirements under Section 3(a)(9) of the Securities Act of 1933. Refer to Asquith, Mullins & Wolff [89]. *(*See *page 65.)*

duration. The weighted average term of a security's cash flows. The longer the duration, the larger the price movement given a 1bp change in the yield.

expected excession of a percentile level. For a specified percentile level, the expected loss given that the loss is more extreme than that percentile level. *(*See *page 137.)*

exposure. The amount which would be lost in a default given the worst possible assumptions about recovery in the liquidation or bankruptcy of an obligor. For a loan or fully drawn facility, this is the full amount plus accrued interest; for an unused or partly used facility it is the full amount of the facility, since the worst assumption is that the borrower draws the full amount and then goes bankrupt.

- Exposure is not usually a statistical concept; it does not make any attempt to assess the probability of loss, it only states the amount at risk.
- For market-driven instruments, (e.g., foreign exchange, swaps, options and derivatives) a proxy for exposure is estimated given the volatility of underlying market rates/prices. See Loan Equivalent Exposure.

facility. A generic term which includes loans, commitments, lines, letters, etc. Any arrangement by which a bank accepts credit exposure to an obligor. *(*See *page 79.)*

fallen angels. Obligors having both relatively high percentage risk and relatively large exposure, whose large exposures were created when their credit ratings were better, but who now have much higher percentage risk due to recent downgrades.

ISDA. (Institutional Swap Dealers Association) A committee sponsored by this organization was instrumental in drafting an industry standard under which securities dealers would trade swaps. Included in this was a draft of a master agreement by which institutions outlined their rights to net multiple offsetting exposures which they might have to a counterparty at the time of a default.

issuer exposure. The credit risk to the issuer of traded instruments (typically a bond, but also swaps, foreign exchange, etc.). Labeling credit spread volatility as either market or credit risk is a question of semantics. CreditMetrics addresses market price volatility as it is caused by changes in credit quality.

joint probabilities. Stand-alone obligors have some likelihood of each possible credit quality migration. Between two obligors there is some likelihood of each possible joint credit quality migration. The probabilities are commonly influences by the correlation between the two obligors. *(*See *page 36.)*

kurtosis. Characterizes relative peakedness or flatness of a given distribution compared to a normal distribution. It is the fourth moment of a distribution.

$$K_X = \left\{ \frac{N^2 - 2N + 3}{(N-1)(N-2)(N-3)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}}{\sigma_x} \right)^4 \right\} - 3 \frac{(N-1)(N-3)}{N(N-2)(N-3)}$$

Since the unconditional normal distribution has a kurtosis of 3, excess kurtosis is defined as *Kx-3*. See *leptokurtosis.*

leptokurtosis (fat tails). The property of a statistical distribution to have more occurrences far away from the mean than would be predicted by a Normal distribution. Since a normal distribution has a kurtosis measure of 3, excess kurtosis is defined as *Kx-3 > 0*.

A credit portfolio loss distribution will typically be leptokurtotic given positive obligor correlations or coarse granularity in the size / number of exposures. This means that a downside confidence interval will be further away from the mean than would be expected given the standard deviation and skewness.

letter of credit. A promise to lend issued by a bank which agrees to pay the addressee, the “beneficiary”, under specified conditions on behalf of a third party, also known as the “account party”. *(*See *page 46)*.

There are different types of letters of credit. A *financial* letter of credit (also termed a stand-by letter of credit) is used to assure access to funding without the immediate need for funds and is triggered at the obligor’s discretion. A *project* letter of credit is secured by a specific asset or project income. A *trade* letter of credit is typically triggered by a non credit related (and infrequent) event.

liquidity. There are two separate meanings:

- At the enterprise level, the ability to meet current liabilities as they fall due; often measures as the ratio of current assets to current liabilities.
- At the security level, the ability to trade in volume without directly moving the market price; often measured as bid/ask spread and daily turnover.

loan exposure. The face amount of any loan outstanding plus accrued interest plus. See *dirty price*.

marginal standard deviation. Impact of a given asset on the total portfolio standard deviation. *(*See *page 129.)*

marginal statistic. A statistic for a particular asset which is the difference between that statistic for the entire portfolio and that for the portfolio not including the asset.

market-driven instruments. Derivative instruments that are subject to counterparty default (e.g., swaps, forwards, options, etc.). The distinguishing feature of these types of credit exposures is that their amount is only the net replacement cost – the amount the position is in-the-money – rather than a full notional amount. *(*See *page 47)*.

market exposure. For market-driven instruments, there is an amount at risk to default only when the contract is in-the-money (i.e., when the replacement cost of the contract exceeds the original value). This exposure/uncertainty is captured by calculating the netted mean and standard deviation of exposure(s).

Markov process. A model which defines a finite set of "states" and whose next progression is determinable solely by the current state. A transition matrix model is an example of a Markov process. *(*See *page 71.)*

mean. A statistical measure of central tendency. Sum of observation values divided by the number of observations. It is the first moment of a distribution. There are two types of means. A mean calculated across a sample from a population is referred to as $\bar{X}$, while means calculated across the entire population – or means given exogenously – are referred to as μ, pronounced "mu." *(*See *page 15.)*

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

mean reversion. The statistical tendency in a time series to gravitate back towards a long term historical level. This is on a much longer scale than another similar measure, called autocorrelation; and these two behaviors are mathematically independent of one another.

migration. Credit quality migration describes the possibility that a firm or obligor with some credit rating today may move to (or "migrate") to potentially any other credit rating – or perhaps default – by the risk horizon. *(*See *page 24.)*

migration analysis. The technique of estimating the likelihood of credit quality migrations. See *transition matrix.*

moments (of a statistical distribution). Statistical distributions show the frequency at which events might occur across a range of values. The most familiar distribution is a Normal "Bell Shaped" curve. In general though, the shape of any distribution can be described by its (infinitely many) moments.

1. The **first** moment is the ***mean*** which indicates the central tendency.
2. The **second** moment is the ***variance*** which indicates the width.
3. The **third** moment is the ***skewness*** which indicates any asymmetric "leaning" either left or right.
4. The **fourth** moment is the ***kurtosis*** which indicates the degree of central "peakedness" or, equivalently, the "fatness" of the outer tails.

monotinicity. See *rank order.*

move persistence. The statistical tendency in a time series to move on the next step in the same direction as the previous step (see also, positive autocorrelation).

netting. There are at least three types of netting:

close-out netting: In the event of counterparty bankruptcy, all transactions or all of a given type are netted at market value. The alternative would allow the liquidator to choose which contracts to enforce and which not to (and thus potentially "cherry pick"). There are international jurisdictions where the enforceability of netting in bankruptcy has not been legally tested.

netting by novation: The legal obligation of the parties to make required payments under one or more series of related transactions are canceled and a new obligation to make only the net payment is created.

settlement or payment netting: For cash settled trades, this can be applied either bilaterally or multilaterally and on related or unrelated transactions.

notional amount. The face amount of a transaction typically used as the basis for interest payment calculations. For swaps, this amount is not itself a cash flow. Credit exposure arises – not against the notional – but against the present value (market replacement cost) of in-the-money future terminal payment(s).

obligor. A party who is in debt to another: (i) a loan borrower; (ii) a bond issuer; (iii) a trader who has not yet settled; (iv) a trade partner with accounts payable; (v) a contractor with unfinished performance, etc.; see *Counterparty.* *(*See *page 5.)*

option theoretic. An approach to estimating the expected default frequency of a particular firm. It applies Robert Merton's model-of-the-firm which states that debt can be valued as a put option of the underlying asset value of the firm. *(*See *page 36.)*

originator. The financial institution that extends credit on a facility which may later be held by another institution through, for instance, a loan sale.

peak exposure. For market-driven instruments, the maximum (perhaps netted) exposure expected with 95% confidence for the remaining life of a transaction. CreditMetrics does not utilize this figure because it is not possible to aggregate tail statistics across a portfolio, since it is not the case that these "peaks" will all occur at the same time.

percent marginal standard deviation. Expression in percent terms of the impact of a given asset on the total portfolio standard deviation. *(*See *page 129.)*

percentile level. A measure of risk based on the specified confidence level of the portfolio value distribution: e.g., the likelihood that the portfolio market falls below the 99th percentile number is 1%. *(*See *page 16.)*

pricing grid. A schedule of credit spreads listed by credit rating that are applied to either a loan or Credit-Sensitive Note (CSN) upon an up(down)grade of the obligor or issuer. If the spreads are specified at market levels, then such terms reduce the volatility of value across all non-default credit quality migrations by keeping the instrument close to par. *(*See *page 67.)*

rank order. A quality of data often found across credit rating categories where values consistently progress in one direction – never reversing direction. Mathematicians term this property of data, *monotonicity.* *(*See *page 66.)*

receivables. Non interest bearing short term extensions of credit in the normal course of business, *"trade credit,"* that are at risk to the extent that the customer may not pay its obligation in full. *(*See *page 42)*.

revolving commitment (revolver). A generic term referring to some facility which a client can use – or refrain from using – without canceling the facility.

sector loadings. For correlation analysis, a firm or industry group is said to be dependent upon underlying economic factors or "sectors" such as: (i) the market as a whole, (ii) interest rates, (iii) oil prices, etc. As two industries "load" – are influenced by – common factors, they will have a higher correlation between them.

serial correlation. See *autocorrelation.*

skewness. A statistical measure which characterizes the asymmetry of a distribution around its mean. Positive skews indicate asymmetric tail extending toward positive values (right-hand side). Negative skewness implies asymmetry toward negative values (left-hand side). It is the third moment of a distribution.

$$S_x = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}}{\sigma_X} \right)^3$$

The distribution of losses across a credit portfolio will be positively skewed if there is positive correlation between obligors or the size / number of exposures is coarsely granular. This means that the confidence interval out on the downside tail will be further away from the mean than would be expected given the portfolio's standard deviation alone.

stand-alone standard deviation. Standard deviation of value for an asset computed without regard for the other instruments in the portfolio. *(*See *page 129.)*

standard deviation. A statistical measure which indicates the width of a distribution around the mean. A standard deviation (Greek letter "σ," pronounced "sigma") is the square root of the second moment of a distribution.

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{x})^2}$$

The distribution of losses across a credit portfolio will (typically) have a standard deviation which is much larger than its mean and yet negative losses are not possible. Thus, it is not meaningful to think of a standard deviation as being a +/- range within which will lie X% of the distribution – as one would naturally do for a normal distribution. *(*See *page 15.)*

stand-alone percent standard deviation. Stand-alone standard deviation expressed as a percentage of the mean value for the given asset. *(*See *page 129.)*

stand-by letter of credit. See *letter of credit.*

state of the world. A credit rating migration outcome; a new credit rating arrived at the risk horizon. This can be either for a single obligor on a stand-alone basis or jointly between two obligors. *(See page 24.)*

stochastic. Following a process which includes a random element. *(*See *page 70.)*

trade credit. See *"receivables."*

transition matrix. A square table of probabilities which summarize the likelihood that a credit will migrate from its current credit rating today to any possible credit rating – or perhaps default – in one period. *(*See *page 25.)*

unexpected losses. A popular term for the volatility of losses but also used when referring to the *realization* of a large loss which, in retrospect, was unexpected. (See *page 60*.)

value-at-risk (VaR). A measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a preset horizon. *(*See *page 5.)*

variance. A statistical measure which indicates the width of a distribution around the mean. It is the second moment of a distribution. A related measure is the standard deviation, which is the square root of the variance. *(*See *page 16.)*

$$VAR_x = \sigma_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{x})^2$$

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Part III: Applications

Look to the J.P. Morgan site on the Internet located at http://jpmorgan.com for updates of examples illustrating points from this technical document or useful new tools. Numerical examples are implemented in Excel spreadsheets.

These Excel spreadsheets are intended as a demonstration of the CreditMetrics credit risk management methodology. They have been designed as an educational tool and should not be used for the risk estimation of actual portfolio positions. Separately, J.P. Morgan sells software which embodies the CreditMetrics methodology. If you have any questions about the use of these spreadsheets contact your local J.P. Morgan representative or:

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CreditMetrics™ Products

Introduction to CreditMetrics™: An abbreviated document which broadly describes the CreditMetrics™ methodology for measuring portfolio credit risk.

CreditMetrics™ – Technical Document: A manual describing the CreditMetrics™ methodology for estimating credit risks. It fully specifies how we construct the volatility of value due to credit quality changes for both stand-alone exposures and portfolios of exposures. It also discloses our approach to estimating credit exposures by instrument type and a method of estimating correlations of credit quality co-movements. Finally, the manual describes the format of the data set.

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Trouble accessing the Internet? If you encounter any difficulties in either accessing the J.P. Morgan home page on http://www.jpmorgan.com or downloading the CreditMetrics™ data files, you can call (1-800) JPM-INET in the United States.

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