CreditMetrics™ – Technical Document

The benchmark for understanding credit risk

- A value-at-risk (VaR) framework applicable to *all institutions worldwide that carry credit risk in the course of their business.*
- A full portfolio view addressing credit event correlations which can identify the costs of over concentration and benefits of diversification in a mark-to-market framework.
- Results that drive: *investment decisions, risk-mitigating actions,* consistent risk-based *credit limits,* and rational *risk-based capital allocations.*

New York
April 2, 1997

J.P. Morgan

Co-sponsors:
Bank of America
BZW
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KMV Corporation
Swiss Bank Corporation
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This *Technical Document* describes CreditMetrics™, a framework for quantifying credit risk in portfolios of traditional credit products (loans, commitments to lend, financial letters of credit), fixed income instruments, and market-driven instruments subject to counterparty default (swaps, forwards, etc.). This is the first edition of what we intend will be an ongoing refinement of credit risk methodologies.

Just as we have done with RiskMetrics™, we are making our methodology and data available for three reasons:

1. We are interested in promoting greater transparency of credit risk. Transparency is the key to effective management.

2. Our aim is to establish a benchmark for credit risk measurement. The absence of a common point of reference for credit risk makes it difficult to compare different approaches to and measures of credit risk. Risks are comparable only when they are measured with the same yardstick.

3. We intend to provide our clients with sound advice, including advice on managing their credit risk. We describe the CreditMetrics™ methodology as an aid to clients in understanding and evaluating that advice.

Both J.P. Morgan and our co-sponsors are committed to further the development of CreditMetrics™ as a fully transparent set of risk measurement methods. This broad sponsorship should be interpreted as a signal of our joint commitment to an open and evolving standard for credit risk measurement. We invite the participation of all parties in this continuing enterprise and look forward to receiving feedback to enhance CreditMetrics™ as a benchmark for measuring credit risk.

CreditMetrics™ is based on, but differs significantly from, the risk measurement methodology developed by J.P. Morgan for the measurement, management, and control of credit risk in its trading, arbitrage, and investment account activities. **We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks.** CreditMetrics™ is nothing more than a high-quality tool for the professional risk manager in the financial markets and is not a guarantee of specific results.

CreditMetrics™—Technical Document

This book

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This is the reference document for CreditMetrics™. It is meant to serve as an introduction to the methodology and mathematics behind statistical credit risk estimation, as well as a detailed documentation of the analytics that generate the data set we provide.

This document reviews:

- the conceptual framework of our methodologies for estimating credit risk;
- the description of the obligors' credit quality characteristics, their statistical description and associated statistical models;
- the description of credit exposure types across "market-driven" instruments and the more traditional corporate finance credit products; and
- the data set that we update periodically and provide to the market for free.

In the interest of establishing a benchmark in a field with as little standardization and precise data as credit risk measurement, we have invited five leading banks, Bank of America, BZW, Deutsche Morgan Grenfell, Swiss Bank Corporation, and Union Bank of Switzerland, and a leading credit risk analytics firm, KMV Corporation, to be co-sponsors of CreditMetrics. All these firms have spent a significant amount of time working on their own credit risk management issues, and we are pleased to have received their input and support in the development of CreditMetrics. With their sponsorship we hope to send one clear and consistent message to the marketplace in an area with little clarity to date.

We have also had many fruitful dialogues with professionals from Central Banks, regulators, competitors, and academics. We are grateful for their insights, help, and encouragement. Of course, all remaining errors and omissions are solely our responsibility.

How is this related to RiskMetrics™?

We developed CreditMetrics to be as good a methodology for capturing counterparty default risk as the available data quality would allow. Although we never mandated during this development that CreditMetrics must resemble RiskMetrics, the outcome has yielded philosophically similar models. One major difference in the models was driven by the difference in the available data. In RiskMetrics, we have an abundance of daily liquid pricing data on which to construct a model of conditional volatility. In CreditMetrics, we have relatively sparse and infrequently priced data on which to construct a model of unconditional volatility.

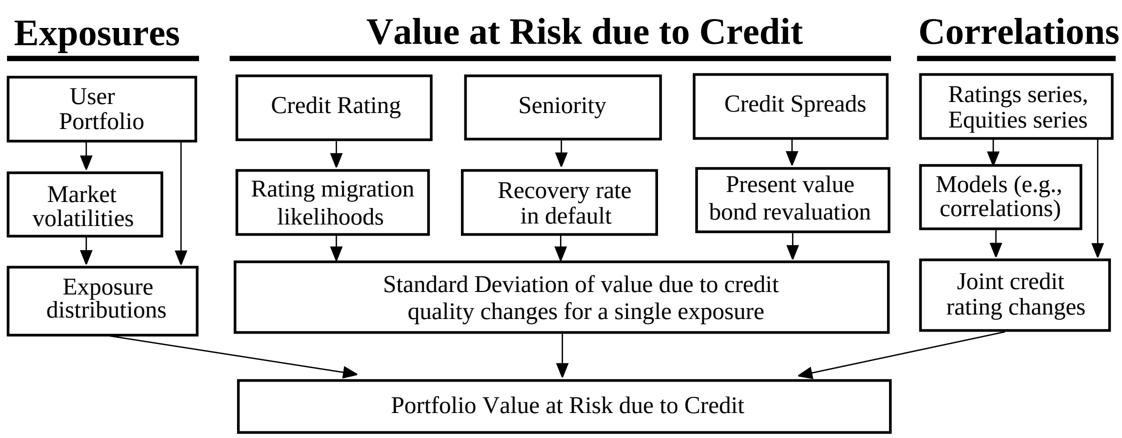
What is different about CreditMetrics?

Unlike market risks where daily liquid price observations allow a direct calculation of value-at-risk (VaR), CreditMetrics seeks to *construct* what it cannot directly *observe*: the volatility of value due to credit quality changes. This constructive approach makes CreditMetrics less an exercise in fitting distributions to observed price data, and more an exercise in proposing models which explain the changes in credit related instruments.

And as we will mention many times in this document, the models which best describe credit risk do not rely on the assumption that returns are normally distributed, marking a significant departure from the RiskMetrics framework.

In the end, we seek to balance the best of all sources of information in a model which looks across broad historical data rather than only recent market moves and across the full range of credit quality migration — upgrades and downgrades — rather than just default.

Our framework can be described in the diagram below. The many sources of information may give an impression of complexity. However, we give a step-by-step introduction in the first four chapters of this book which should be accessible to all readers.



One of our fundamental techniques is *migration analysis*, that is, the study of changes in the credit quality of names through time. Morgan developed transition matrices for this purpose as early as 1987. We have since built upon a broad literature of work which applies migration analysis to credit risk evaluation. The first publication of transition matrices was in 1991 by both Professor Edward Altman of New York University and separately by Lucas & Lonski of Moody's Investors Service. They have since been published regularly (see Moody's Carty & Lieberman [96a][1] and Standard & Poor's *Creditweek* [15-Apr-96]) and are also calculated by firms such as KMV.

Are RiskMetrics and CreditMetrics comparable?

Yes, in brief, RiskMetrics looks to a horizon and estimates the *value-at-risk* across a distribution of historically estimated realizations. Likewise, CreditMetrics looks to a horizon and constructs a distribution of historically estimated credit outcomes (rating migrations including potentially default). Each credit quality migration is weighted by its likelihood (transition matrix analysis). Each outcome has an estimate of change in value (given by either credit spreads or studies of recovery rates in default). We then aggregate volatilities across the portfolio, applying estimates of correlation. Thus, although the relevant time horizon is usually longer for credit risk, with CreditMetrics we compute credit risk on a comparable basis with market risk.

[1] Bracketed numbers refer to year of publication.

What CreditMetrics is not

We have sought to add value to the market's understanding of credit risk estimation, not by replicating what others have done before, but rather by filling in what we believe is lacking. Most prior work has been on the estimation of the relative likelihoods of default for individual firms; Moody's and S&P have long done this and many others have started to do so. We have designed CreditMetrics to accept as an input any assessment of default probability[2] which results in firms being classified into discrete groups (such as rating categories), each with a defined default probability. It is important to realize, however, that these assessments are only inputs to CreditMetrics, and not the final output.

We wish to estimate the *volatility of value* due to changes in credit quality, not just the *expected loss*. In our view, as important as default likelihood estimation is, it is only one link in the long chain of modeling and estimation that is necessary to fully assess credit risk (volatility) within a portfolio. Just as a chain is only as strong as its weakest link, it is also important to diligently address: (i) uncertainty of exposure such as is found in swaps and forwards, (ii) residual value estimates and their *uncertainties*, and (iii) credit quality *correlations* across the portfolio.

How is this document organized?

One need not read and fully understand the details of this entire document to understand CreditMetrics. This document is organized into three parts that address subjects of particular interest to our diverse readers.

Part I **Risk Measurement Framework**
This section is for the general practitioner. We provide a practicable framework of how to think about credit risk, how to apply that thinking in practice, and how to interpret the results. We begin with an example of a single bond and then add more variation and detail. By example, we apply our framework across different exposures and across a portfolio.

Part II **Model Parameters**
Although this section occasionally refers to advanced statistical analysis, there is content accessible to all readers. We first review the current academic context within which we developed our credit risk framework. We review the statistical assumptions needed to describe discrete credit events; their mean expectations, volatilities, and correlations. We then look at how these credit statistics can be estimated to describe what happened in the past and what can be projected in the future.

Part III **Applications**
We discuss two implementations of our portfolio framework for estimating the *volatility of value due to credit quality changes*. The first is an analytic calculation of the mean and standard deviation of value changes. The second is a simulation approach which estimates the full distribution of value changes. These both embody the same modeling framework and

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[2] These assessments may be agency debt ratings, a user's internal ratings, the output of a statistical default prediction model, or any other approach.

produce comparable results. We also discuss how the results can be used in portfolio management, limit setting, and economic capital allocation.

Future plans

We expect to update this *Technical Document* regularly. We intend to further develop our methodology, data and software implementation as we receive client and academic comments.

CreditMetrics has been developed by the Risk Management Research Group at J.P. Morgan. Special mention must go to Greg M. Gupton who conceived of this project and has been working on developing the CreditMetrics approach at JPMorgan for the last four years. We welcome any suggestions to enhance the methodology and adapt it further to the changing needs of the market. We encourage academic studies and are prepared to supply data for well-structured projects.

Acknowledgments

We would like to thank our co-sponsors for their input and support in the writing and editing of this document. In particular, we thank the KMV Corporation, which has been a pioneer in developing portfolio approaches to credit risk, and whose work has influenced many of the methods presented here.

We thank numerous individuals at J.P. Morgan who participated in this project, as well as professionals at other banks and academic institutions who offered input at various levels. Also, this document could not have been produced without the contributions of our consulting editor, Margaret Dunkle. We apologize for any omissions.

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Overview of Part III

To this point, we have detailed an analytic approach to compute the mean and standard deviation of portfolio value change, presented calculations for one- and two-asset portfolios, and discussed the inputs to these calculations. In this section we discuss approaches to computing risk measures other than standard deviation and apply the CreditMetrics methodology to a larger portfolio.

Both issues – alternative measures of risk and computations for a larger portfolio – point us to a central theme of this section: simulation. By this we mean the generation of future portfolio scenarios according to the models already discussed.

Implementation of a simulation approach involves a tradeoff. On the one hand, we are able to describe in much more detail the distribution of portfolio value changes; on the other, we introduce noise into what has been an exact solution for the risk estimates. We will continue to discuss this tradeoff as we go.

Part III is composed of four chapters which describe the methods and discuss the outputs of the CreditMetrics methodology for larger portfolios. The chapters dealing with simulation focus on computing advanced (beyond the mean and standard deviation) risk estimates. This section is organized as follows:

- **Chapter 9: Analytic portfolio calculation.** We extend the methods discussed in *Chapter 3* for computing the standard deviation and marginal standard deviation to a large (more than two instruments) portfolio.

- **Chapter 10: Simulation.** We address the assumptions necessary to specify the portfolio distribution completely, describe the Monte Carlo approach to this distribution, and discuss how to produce percentile levels as well as marginal statistics. We focus on computing advanced (beyond the mean and standard deviation) risk estimates for larger portfolios.

- **Chapter 11: Portfolio example.** We choose a portfolio of 20 instruments of varying maturities and rating and specify the asset correlations between their issuers. We then utilize the simulation approach of the previous section to estimate certain risk statistics and interpret these results in the context of the portfolio.

- **Chapter 12: Application of model outputs.** We consider how the analysis in *Chapter 11* might lead to risk management actions such as prioritizing risk reduction, setting credit risk limits, and assessing economic capital.

Chapter 9. Analytic portfolio calculation

In *Chapter 3*, we discussed the computation of the standard deviation of value change for a portfolio of two instruments. We refrained from extending this computation to larger portfolios, stating that the standard deviation of value for larger portfolios involves no different calculations than the standard deviation for two-asset portfolios. In this chapter, we illustrate this point for a three-asset portfolio, and discuss as well the calculation of marginal standard deviations for this portfolio. The generalization of these calculations to portfolios of arbitrary size is straightforward, and is detailed in *Appendix A*.

9.1 Three-asset portfolio

Our example is a portfolio consisting of three assets, all annual coupon bonds. We take the first two of these bonds to be issued by the BBB and A rated firms of *Chapter 3* and the third to be a two-year bond paying a 10% coupon and issued by a CCC rated firm. We will refer to the firms respectively as Firms 1, 2, and 3. Suppose that the Firm 1 issue has a notional amount of 4mm, the Firm 2 issue an amount of 2mm, and the Firm 3 issue an amount of 1mm. Denote by V_1, V_2, and V_3, the values at the end of the risk horizon of the three respective issues.

We present transition probabilities for the three firms in *Table 9.1* below, and revaluations in *Table 9.2*.

Table 9.1
Transition probabilities (%)

	Transition probability (%)		
Rating	**Firm 1**	**Firm 2**	**Firm 3**
AAA	0.02	0.09	0.22
AA	0.33	2.27	0.00
A	5.95	91.05	0.22
BBB	86.93	5.52	1.30
BB	5.30	0.74	2.38
B	1.17	0.26	11.24
CCC	0.12	0.01	64.86
Default	0.18	0.06	19.79

Table 9.2
Instrument values in future ratings ($mm)

	Value of issue ($mm)		
Future rating	**Firm 1**	**Firm 2**	**Firm 3**
AAA	4.375	2.132	1.162
AA	4.368	2.130	1.161
A	4.346	2.126	1.161
BBB	4.302	2.113	1.157
BB	4.081	2.063	1.142
B	3.924	2.028	1.137
CCC	3.346	1.774	1.056
Default	2.125	1.023	0551

Utilizing the methods of *Chapter 2* and the information in the tables above, we may compute the mean value for each issue:

[9.1] $$\mu_1 = \$4.28\text{mm}, \mu_2 = \$2.12\text{mm}, \text{ and } \mu_3 = \$0.97\text{mm},$$

giving a portfolio mean of $\mu_p = \$7.38\text{mm}$. We may also compute the variance of value for each of the three assets, obtaining

[9.2] $$\sigma^2(V_1) = 0.014, \sigma^2(V_2) = 0.001, \text{ and } \sigma^2(V_3) = 0.044.$$

Note that since the standard deviations are in units of ($mm), the units for $\sigma^2(V_1)$, $\sigma^2(V_2)$, and $\sigma^2(V_3)$ are ($mm)^2.

Now to compute σ_p, the standard deviation of value for the portfolio, we could use the standard formula

[9.3] $$\sigma_p^2 = \sigma^2(V_1) + \sigma^2(V_2) + \sigma^2(V_3) + 2 \cdot COV(V_1, V_2) + 2 \cdot COV(V_1, V_3) + 2 \cdot COV(V_2, V_3).$$

This would require the calculation of the various covariance terms. Alternatively, noting that

[9.4] $$\sigma^2(V_1 + V_2) = \sigma^2(V_1) + 2 \cdot COV(V_1, V_2) + \sigma^2(V_2),$$

we may express σ_p by

[9.5] $$\sigma_p^2 = \sigma^2(V_1 + V_2) + \sigma^2(V_1 + V_3) + \sigma^2(V_2 + V_3) - \sigma^2(V_1) - \sigma^2(V_2) - \sigma^2(V_3).$$

The above formula has the attractive feature of expressing the portfolio standard deviation in terms of the standard deviations of single assets (e.g. $\sigma(V_1)$) and the standard deviations of two-asset subportfolios (e.g. $\sigma(V_1 + V_2)$). Thus, to complete our computation of σ_p, it only remains to identify each two-asset subportfolio, compute the standard deviations of each, and apply Eq. [9.5].

The standard deviation for two-asset portfolios was covered in *Chapter 3*, and so in principle, we have described all of the portfolio calculations. We present the two-asset case again as a review. Consider the first pair of assets, the BBB and A rated bonds. In order to compute the variance for the portfolio containing only these assets, we utilize the joint transition probabilities in *Table 3.2*, which are an output of the asset value model of the previous chapter, with an assumed asset correlation of 30%. Along with these probabilities we need the values of this two-asset portfolio in each of the 64 joint rating states; we present these values in *Table 9.3*. Note that the values in *Table 9.3* differ from those in *Table 3.2* since the notional amounts of the issues in these two cases are different.

Table 9.3
Values of a two-asset portfolio in future ratings ($mm)

New rating for Firm 1 (currently BBB)	**New rating for Firm 2 (currently A)**							
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	6.51	6.51	6.50	6.49	6.44	6.40	6.15	5.40
AA	6.50	6.50	6.49	6.48	6.43	6.40	6.14	5.39
A	6.48	6.48	6.47	6.46	6.41	6.37	6.12	5.37
BBB	6.43	6.43	6.43	6.42	6.37	6.33	6.08	5.33
BB	6.21	6.21	6.21	6.19	6.14	6.11	5.86	5.10
B	6.06	6.05	6.05	6.04	5.99	5.95	5.70	4.95
CCC	5.48	5.48	5.47	5.46	5.41	5.37	5.12	4.37
Default	4.26	4.26	4.25	4.24	4.19	4.15	3.90	3.15

Applying Eq. [3.1] to the probabilities in *Table 3.2* and the values in *Table 9.3*, we then compute $\sigma^2(V_1 + V_2) = 0.018$. In a similar fashion, we specify that the asset correlations between the first and third and between the second and third obligors are also 30%, and then create analogs to *Table 3.2* and *Table 9.3*. This allows us to compute $\sigma^2(V_1 + V_3) = 0.083$ and $\sigma^2(V_2 + V_3) = 0.051$. Finally, we apply Eq. [9.6] to obtain $\sigma_p^2 = 0.093$, and thus $\sigma_p = \$0.305\text{mm}$.

The calculation of portfolio variance in terms of the variance of two-asset subportfolios may seem unusual to those accustomed to the standard covariance approach. We remark that we have all of the information necessary to compute the covariances and correlations between our three assets. Thus, since

$$[9.6] \qquad COV(V_1, V_2) = \frac{\sigma^2(V_1 + V_2) - \sigma^2(V_1) - \sigma^2(V_2)}{2},$$

we have $COV(V_1, V_2) = 0.0015$. Similarly, we obtain $COV(V_1, V_3) = 0.0125$ and $COV(V_2, V_3) = 0.0030$. This allows us to then compute correlations between the asset values using

$$[9.7] \qquad CORR(V_1, V_2) = \frac{COV(V_1, V_2)}{\sqrt{\sigma^2(V_1) \times \sigma^2(V_2)}}.$$

We then have $CORR(V_1, V_2) = 40.1\%$, $CORR(V_1, V_3) = 50.4\%$, and $CORR(V_2, V_3) = 45.2\%$. It is a simple matter then to check that the standard formula Eq. [9.1] yields the same value for σ_p as we computed above.

We refer to σ_p as the *absolute* measure of the portfolio standard deviation. Alternatively, we may express this risk in percentage terms; we thus refer to σ_p/μ_p (which is equal to 4.1% in our example) as the *percent* portfolio standard deviation. These notions of absolute and percent measures will be used for other portfolio statistics, with the percent statistic always representing the absolute statistic as a fraction of the mean portfolio value.

To extend this calculation to larger portfolios is straightforward. We present the details of this in *Appendix A*.

9.2 Marginal standard deviation

As defined in *Section 3.3*, the marginal standard deviation for a given instrument in a portfolio is the difference between the standard deviation for the entire portfolio and the standard deviation for the portfolio not including the instrument in question. Thus, since we now are able to compute the standard deviation for a portfolio of arbitrary size, the calculation of marginal standard deviations is clear.

Consider the Firm 1 issue in our portfolio above. We have seen that the standard deviation for the entire portfolio is \$0.46mm. If we remove the Firm 1 issue, then the new portfolio variance is given by $\hat{\sigma}_p^2 = \sigma^2(V_2 + V_3) = 0.051$, making the new portfolio standard deviation $\hat{\sigma}_p = \$0.225\text{mm}$. The *marginal standard deviation* of the Firm 1 issue is then the difference between the absolute portfolio standard deviation and this figure, or $\sigma_p - \hat{\sigma}_p = \$0.080\text{mm}$. Thus, we see that we can reduce the total portfolio standard deviation by \$0.080mm if we liquidate the Firm 1 issue While this is a measure of the absolute risk contributed by the Firm 1 issue, we might also wish to characterize the riskiness of this instrument independently of its size. To this end, we may express the marginal standard deviation as a percentage of μ_1, the mean value of the Firm 1 issue. We refer to this figure, 1.9% in this case, as the *percent marginal standard deviation* of this issue.

The difference between marginal and stand-alone statistics gives us an idea of the effect of diversification on the portfolio. Note that if we consider the Firm 1 issue alone, its standard deviation of value is \$0.117mm. If this asset were perfectly correlated with the other assets in the portfolio, its marginal impact on the portfolio standard deviation would be exactly this amount. However, we have seen that the marginal impact of the Firm 1 issue is only \$0.080mm, and thus that we benefit from the fact that this issue is not in fact perfectly correlated with the others.

The risk measures produced in this section may strike the reader as a bit small, particularly in light of the riskiness of the CCC rated issue in our example. This might be explained by the fact that the size of this issue is quite small in comparison with the other assets in the portfolio. However, since we have only considered the standard deviation to this point, it may be that to adequately describe the riskiness of the portfolio, we need more detailed information about the portfolio distribution. In order to obtain this higher order information, it will be necessary to perform a simulation based analysis, which is the subject of the following two chapters.

Chapter 10. Simulation

Our methodology up to this point has focused on analytic estimates of risk, that is, estimates which are computed directly from formulas implied by the models we assume. This analytical approach has two advantages:

1. **Speed**. Particularly for smaller portfolios, the direct calculations require fewer operations, and thus can be computed more quickly.
2. **Precision.** No random noise is introduced in the calculations and, therefore, no error in the risk estimates.

However, it has also two principal disadvantages. One is that for large portfolios, number 1 above is no longer true. The other is that by restricting ourselves to analytical approaches, we limit the available of statistics that can be estimated.

Throughout this document, we have discussed methods to compute the standard deviation of portfolio value; yet we have also stressed that this may not be a meaningful measure of the credit risk of the portfolio. To provide a methodology that better describes the distribution of portfolio values, we present in this chapter a simulation approach known as “Monte Carlo.”

The three sections of this chapter treat the three steps to a Monte Carlo simulation:

1. **Generate scenarios.** Each scenario corresponds to a possible “state of the world” at the end of our risk horizon. For our purposes, the “state of the world” is just the credit rating of each of the obligors in our portfolio.
2. **Value portfolio.** For each scenario, we revalue the portfolio to reflect the new credit ratings. This step gives us a large number of possible future portfolio values.
3. **Summarize results.** Given the value scenarios generated in the previous steps, we have an estimate for the distribution of portfolio values. We may then choose to report any number of descriptive statistics for this distribution.

We will continue to consider the example portfolio of the previous chapter: three two-year par bonds issued by BBB, A, and CCC rated firms. The notional values of these bonds are \$4mm, \$2mm, and \$1mm.

10.1 Scenario generation

In this section, we will discuss how to generate scenarios of future credit ratings for the obligors in our portfolio. We will rely heavily on the asset value model discussed in *Section 8.4*. The steps to scenario generation are as follows:

1. Establish asset return thresholds for the obligors in the portfolio.
2. Generate scenarios of asset returns according to the normal distribution.
3. Map the asset return scenarios to credit rating scenarios.

In *Table 10.1* below, we restate the transition probabilities for the three issues.

Table 10.1
Transition probabilities (%)

	Transition Probability (%)		
Rating	**Firm 1**	**Firm 2**	**Firm 3**
AAA	0.02	0.09	0.22
AA	0.33	2.27	0.00
A	5.95	91.05	0.22
BBB	86.93	5.52	1.30
BB	5.30	0.74	2.38
B	1.17	0.26	11.24
CCC	0.12	0.01	64.86
Default	0.18	0.06	19.79

We then present in *Table 10.2*[1] the asset return thresholds for the three firms, which are obtained using the methods of *Section 8.4.*

Table 10.2
Asset return thresholds

Threshold	**Firm 1**	**Firm 2**	**Firm 3**
Z_{AA}	3.54	3.12	2.86
Z_{A}	2.78	1.98	2.86
Z_{BBB}	1.53	-1.51	2.63
Z_{BB}	-1.49	-2.30	2.11
Z_{B}	-2.18	-2.72	1.74
Z_{CCC}	-2.75	-3.19	1.02
Z_{Def}	-2.91	-3.24	-0.85

Recall that the thresholds are labeled such that a return falling just below a given thresholds corresponds to the rating in the threshold's subscript. That is, a return less than Z_{BB} (but greater than Z_B) corresponds to a rating of BB.

In order to describe how the asset values of the three firms move jointly, we state that the asset returns in for each firm are normally distributed, and specify the correlations for each pair of firms[2]. For our example, we assume the correlations in *Table 10.3*.

1 Recall the comment at the end of *Chapter 8* that asset return volatility does not affect the joint probabilities of rating changes. For this reason, we may consider standardized asset returns, and report the thresholds for these.

2 Technically, the assumption is that the joint distribution of the asset returns of any collection of firms is multivariate normal.

Table 10.3
Correlation matrix for example portfolio

	Firm 1	**Firm 2**	**Firm 3**
Firm 1	1.0	0.3	0.1
Firm 2	0.3	1.0	0.2
Firm 3	0.1	0.2	1.0

Generating scenarios for the asset returns of our three obligors is a simple matter of generating correlated, normally distributed variates. There are a number of methods for doing this – Cholesky factorization, singular value decomposition, etc. – for discussions of which see, for example, Strang [88]. In *Table 10.4*, we list ten scenarios which might be produced by such a procedure. In each scenario, the three numbers represent the standardized asset return for each of the three firms.

Table 10.4
Scenarios for standardized asset returns

Scenario	**Firm 1**	**Firm 2**	**Firm 3**
1	-0.7769	-0.8750	-0.6874
2	-2.1060	-2.0646	0.2996
3	-0.9276	0.0606	2.7068
4	0.6454	-0.1532	-1.1510
5	0.4690	-0.5639	0.2832
6	-0.1252	-0.5570	-1.9479
7	0.6994	1.5191	-1.6503
8	1.1778	-0.6342	-1.7759
9	1.8480	2.1202	1.1631
10	0.0249	-0.4642	0.3533

To fully specify our scenarios, it is only necessary to assign ratings to the asset return scenarios. For example, consider scenario 2 of *Table 10.4*. The standardized return for Firm 1 is –2.1060, which falls between Z_B (–2.18 from *Table 10.2*) and Z_{BB} (–1.49 from *Table 10.2*) for this name. This corresponds to a new rating of BB. For Firm 2, the return is –2.0646, which falls between Z_{BB} and Z_{BBB} for this name, corresponding to a new rating of BBB. Continuing this process, we may fill in *Table 10.5*, which completes the process of scenario generation

Table 10.5
Mapping return scenarios to rating scenarios

	Asset Return			New Rating		
Scenario	**Firm 1**	**Firm 2**	**Firm 3**	**Firm 1**	**Firm 2**	**Firm 3**
1	-0.7769	-0.8750	-0.6874	BBB	A	CCC
2	-2.1060	-2.0646	0.2996	BB	BBB	CCC
3	-0.9276	0.0606	2.7068	BBB	A	A
4	0.6454	-0.1532	-1.1510	BBB	A	Default
5	0.4690	-0.5639	0.2832	BBB	A	CCC
6	-0.1252	-0.5570	-1.9479	BBB	A	Default
7	0.6994	1.5191	-1.6503	BBB	A	Default
8	1.1778	-0.6342	-1.7759	BBB	A	Default
9	1.8480	2.1202	1.1631	A	AA	B
10	0.0249	-0.4642	0.3533	BBB	A	CCC

Notice that for this small number of trials, the scenarios do not correspond precisely to the transition probabilities in *Table 10.1*. (For example, in four of the ten scenarios, Firm 3 defaults, while the probability that this occurs is just 20%.) These random fluctuations are the source of the lack of precision in Monte Carlo estimation. As we generate more scenarios, these fluctuations become less prominent, but it is important to quantify how large we can expect the fluctuations to be. This is the topic of *Appendix B*.

10.2 Portfolio valuation

For non-default scenarios, this step is no different here than in the previous chapters. For each scenario and each issue, the new rating maps directly to a new value. To recall the specifics of valuation, refer back to *Chapter 4*.

For default scenarios, the situation is slightly different. We discussed in *Chapter 7* that recovery rates are not deterministic quantities but rather display a large amount of variation. This variation of value in the case of default is a significant contributor to risk. To model this variation, we obtain the mean and standard deviation of recovery rate for each issue in our portfolio according to the issue's seniority. For example, in our BBB rated senior unsecured issue, the recovery mean is 53% and the recovery standard deviation is 33%. For each default scenario, we generate a random recovery rate according to a beta distribution[3] with these parameters[4]. These recovery rates then allow us to obtain the value in each default scenario.

In the end, we obtain a portfolio value for each scenario. The results for the first ten scenarios for our example are presented in *Table 10.6*.

[3] Recall that the beta distribution only produces numbers between zero and one, so that we are assured of obtaining meaningful recovery rates.

[4] Note that we assume here that the recovery rate for a given obligor is independent of the value of all other instruments in the portfolio.

Table 10.6
Valuation of portfolio scenarios ($mm)

	Rating			Value			
Scenario	**Firm 1**	**Firm 2**	**Firm 3**	**Firm 1**	**Firm 2**	**Firm 3**	**Portfolio**
1	BBB	A	CCC	4.302	2.126	1.056	7.484
2	BB	BBB	CCC	4.081	2.063	1.056	7.200
3	BBB	A	A	4.302	2.126	1.161	7.589
4	BBB	A	Default	4.302	2.126	0.657	7.085
5	BBB	A	CCC	4.302	2.126	1.056	7.484
6	BBB	A	Default	4.302	2.126	0.754	7.182
7	BBB	A	Default	4.302	2.126	0.269	6.697
8	BBB	A	Default	4.302	2.126	0.151	6.579
9	A	AA	B	4.346	2.130	1.137	7.613
10	BBB	A	CCC	4.302	2.126	1.056	7.484

Note that for a given issue, the value is the same in scenarios with the same (non-default) credit rating. For defaults, this is not the case – the values of the Firm 3 issue in the default scenarios are different – since recovery rates are themselves uncertain. Thus, each default scenario requires an independently generated recovery rate.

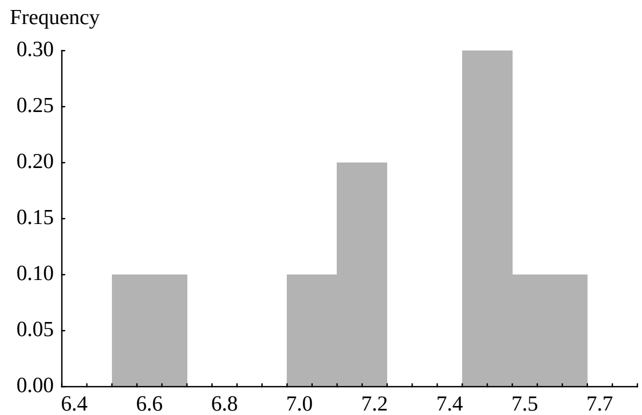
10.3 Summarizing the results

At this point, we have created a number of possible future portfolio values. The final task is then to synthesize this information into meaningful risk estimates.

In this section, we will examine a number of descriptive statistics for the scenarios we have created. In the section to follow, we will examine the same statistics, but for an example in which we consider a larger portfolio and a larger number of scenarios, so as to obtain more significant results.

In order to gain some intuition about the distribution of values, we first examine a plot of the ten scenarios for our example. This plot is presented in *Chart 10.1*. For a larger number of scenarios, we would expect this plot to become more smooth, and approach something like the histogram we will see in *Chart 11.1*.

Chart 10.1
Frequency plot of portfolio scenarios



Even for small number of scenarios, we begin to see the heavy downside tail typical of credit portfolio distributions.

The first statistics we examine are those which we are able to compute analytically: the mean and standard deviation of future portfolio value. Let $V^{(1)}, V^{(2)}, V^{(3)}, \ldots$ indicate the portfolio value in the respective scenarios. Then we may compute the sample mean (μ) and standard deviation (σ) of the scenarios as follows:

[10.1] $$\mu_p = \frac{1}{N}\sum_{i=1}^{N} V^{(i)} = \$7.24\text{mm and } \sigma_p = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(V^{(i)} - \mu)^2} = \$0.37\text{mm}$$

where N is the number of scenarios (in our case, N=10).

As we have mentioned before, the mean and standard deviation may not be the best measures of risk in that, since the distribution of values is not normal, we cannot infer percentile levels from the standard deviation. We are thus motivated to perform simulations in order to capture more information about the distribution of values. Estimates of percentile levels are straightforward. For example, to compute the tenth percentile given our scenarios, we choose a level (x) at which one of the ten scenarios is less than x and the other nine scenarios are greater than x. For our scenarios, this level is between \$6.58mm and \$6.70mm. This imprecision is due to simulation noise, but we will see in the next chapter that as we consider more scenarios, our estimates of percentiles become more precise.

To this point, we have considered only statistics which describe the portfolio distribution. We would also like to consider individual assets and to ascertain how much risk each asset contributes to the portfolio. To this end, we will describe marginal statistics.

We have discussed marginal standard deviations previously. This concept may be generalized, and we may compute a marginal analog of any of the statistics (standard deviation, percentile) discussed above. In general, the marginal statistic for a particular asset is the difference between that statistic for the entire portfolio and that statistic for the portfolio not including the asset in question. Thus, if we wish to compute the marginal tenth percentile of the third asset in our portfolio (the CCC rated bond), we take

[10.2] $$\theta_{10}(V_1 + V_2 + V_3) - \theta_{10}(V_1 + V_2)$$

where V_1, V_2, and V_3 represent the future values of the first, second, and third assets, respectively, and θ_{10} represents the tenth percentile of the values in question. For the scenarios above, the tenth percentile for the entire portfolio is \$6.64mm, while that for just the first two assets is \$6.29mm; and thus the marginal standard deviation for the third asset is \$0.35mm. This marginal figure may be interpreted as the amount by which we could decrease the risk on our portfolio by removing the CCC rated bond.

As we have mentioned a number of times, the statistics obtained through Monte Carlo simulation are subject to fluctuations; any set of scenarios may not produce a sample mean or sample 5^{th} percentile which is equal to the true mean or 5^{th} percentile for the portfolio. Thus, it is important to quantify, given the number of scenarios which are generated, how close we expect our estimates of various portfolio statistics to be to their true value. In fact, a reasonable way to choose the number of scenarios to be generated is to specify some desired level of precision for a particular statistic, and generate enough scenarios to achieve this. Quantifying the precision of simulation based statistics is the subject of *Appendix B*.

Chapter 11. Portfolio example

In this chapter, we examine a more realistic example portfolio and discuss the results of a simulation-based analysis of this portfolio. The risk estimates are no different than those in the previous chapter, but should take on more meaning here in the context of a larger portfolio.

11.1 The example portfolio

In this chapter, we consider a portfolio of 20 corporate bonds (each with a different issuer) of varying rating and maturity. The bonds are listed in *Table 11.1*. The total market value of the portfolio is $68mm.

Table 11.1.
Example portfolio

Asset	Credit rating	Principal amount	Maturity (years)	Market value
1	AAA	7,000,000	3	7,821,049
2	AA	1,000,000	4	1,177,268
3	A	1,000,000	3	1,120,831
4	BBB	1,000,000	4	1,189,432
5	BB	1,000,000	3	1,154,641
6	B	1,000,000	4	1,263,523
7	CCC	1,000,000	2	1,127,628
8	A	10,000,000	8	14,229,071
9	BB	5,000,000	2	5,386,603
10	A	3,000,000	2	3,181,246
11	A	1,000,000	4	1,181,246
12	A	2,000,000	5	2,483,322
13	B	600,000	3	705,409
14	B	1,000,000	2	1,087,841
15	B	3,000,000	2	3,263,523
16	B	2,000,000	4	2,527,046
17	BBB	1,000,000	6	1,315,720
18	BBB	8,000,000	5	10,020,611
19	BBB	1,000,000	3	1,118,178
20	AA	5,000,000	5	6,181,784

Recall that for each asset, the credit rating determines the distribution of future credit rating, and thus also the distribution of future value. For the portfolio, however, we must also specify the asset correlations in order to describe the distribution of future ratings and values. For this example, we assume the correlations in *Table 11.2*.

Table 11.2
Asset correlations for example portfolio

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	1	0.45	0.45	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.45	1	0.45	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
3	0.45	0.45	1	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
4	0.45	0.45	0.45	1	0.15	0.15	0.15	0.15	0.15	0.15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
5	0.15	0.15	0.15	0.15	1	0.35	0.35	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
6	0.15	0.15	0.15	0.15	0.35	1	0.35	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
7	0.15	0.15	0.15	0.15	0.35	0.35	1	0.35	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
8	0.15	0.15	0.15	0.15	0.35	0.35	0.35	1	0.35	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
9	0.15	0.15	0.15	0.15	0.35	0.35	0.35	0.35	1	0.35	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
10	0.15	0.15	0.15	0.15	0.35	0.35	0.35	0.35	0.35	1	0.2	0.2	0.2	0.2	0.2	0.15	0.15	0.15	0.1	0.1
11	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	1	0.45	0.45	0.45	0.45	0.2	0.2	0.2	0.1	0.1
12	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	1	0.45	0.45	0.45	0.2	0.2	0.2	0.1	0.1
13	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	1	0.45	0.45	0.2	0.2	0.2	0.1	0.1
14	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	0.45	1	0.45	0.2	0.2	0.2	0.1	0.1
15	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.45	0.45	0.45	0.45	1	0.2	0.2	0.2	0.1	0.1
16	0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	1	0.55	0.55	0.25	0.25
17	0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	0.55	1	0.55	0.25	0.25
18	0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	0.15	0.15	0.2	0.2	0.2	0.2	0.2	0.55	0.55	1	0.25	0.25
19	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.25	0.25	0.25	1	0.65
20	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.25	0.25	0.25	0.65	1

Observe that there are five groups of issuers (those for assets 1-4, 6-10, 11-15, 16-18, and 19-20, in the shaded areas of the table) within which the asset correlations are relatively high, while the correlations between these groups are lower. This might be the case for a portfolio containing issues from firms in five different industries; the correlations between firms in a given industry are high, while correlations across industries are lower.

11.2 Simulation results

Using the methodology of the previous chapter, we generate 20,000 portfolio scenarios, that is, 20,000 possible future occurrences in one year's time of the credit ratings for each of our issues. For each scenario, we then obtain a portfolio value for one year into the future. In Charts *11.1* through *11.3*, we present histograms of the portfolio value scenarios. Note the axes on each chart carefully. The first chart illustrates the distribution of the most common scenarios, the second moves a bit further into the left tail of the distribution, and the third shows the distribution of the most extreme 5% of all cases. The vertical axis, which represents relative frequency, is ten times smaller in the second chart than in the first, and twenty times smaller in the third chart than in the second.

Chart 11.1
Histogram of future portfolio values – upper 85% of scenarios

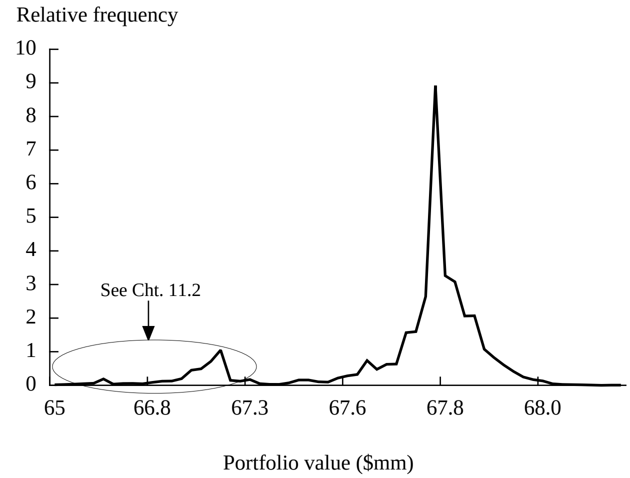


Chart 11.2
Histogram of future portfolio values – scenarios between 95th and 65th percentiles

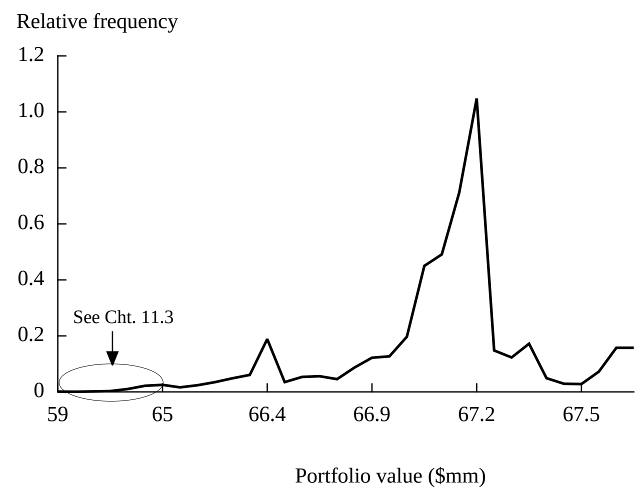
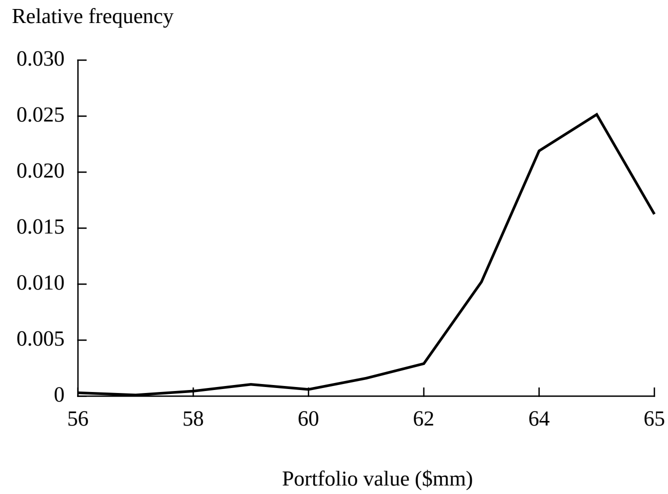


Chart 11.3
Histogram of future portfolio values – lower 5% of scenarios



We may make several interesting observations of these charts. First, by far the most common occurrence (almost 9% of all scenarios, exhibited by the spike near $67.8mm in *Table 11.1*) is that none of the issuers undergoes a rating change. Further, in well over half of the scenarios, there are no significant credit events, and the portfolio appreciates.

The second observation is the odd bimodal structure of the distribution. This is due to the fact that default events produce much more significant value changes than any other rating migrations. Thus, the distribution of portfolio value is driven primarily by the number of issues which default. The second hump in the distribution (the one between $67mm and $67.2mm) represents scenarios in which one issue defaults.

The two other humps further to the left in the distribution represent scenarios with two and three defaults, respectively. For larger portfolios, these humps become even more smoothed out, while for smaller ones, the humps are generally more prominent.

Regardless of the particulars of the shape of the value distribution, one feature persists: the heavy downward skew. Our example distribution is no different, displaying a large probability of a marginal increase in value along with a small probability of a more significant drop in value.

As in the previous chapter, the first two statistics we present are the mean and standard deviation of the portfolio value. For our case, we have:

- Mean portfolio value (μ) = $67,284,888.

- Standard deviation of portfolio value (σ) = $1,136,077.

As we have mentioned before, the mean and standard deviation may not be the best measures of risk in that, since the loss distribution is not normal, we cannot infer confidence levels from these parameters. We can however estimate percentiles directly from our scenarios.

For example, if we wish to compute the 5th percentile (the level below which we estimate that 5% of portfolio values fall), we sort our 20,000 scenarios in ascending order and take the 1000th of these sorted scenarios (that is, $64.98mm) as our estimate. (Our assumption is then that since 5% of the simulated changes in value were less than -$5.69mm, there is a 5% chance that the actual portfolio value change will be less than this level.) Here we see the advantage of the simulation approach, in that we can estimate arbitrary percentile levels, where in the analytic approach, because the portfolio distribution is not normal, we are only able to compute two statistics.

In *Table 11.3* below, we present various percentiles of our scenarios of future portfolio values. For comparison and in order to illustrate the non-normality of the portfolio distribution, we also give the percentiles which we would have estimated had we utilized the sample mean and standard deviation, and assumed that the distribution was normal.

Table 11.3
Percentiles of future portfolio values ($mm)

	Actual scenarios	**Normal distribution**	
Percentile	**Portfolio value ($mm)**	**Formula**	**Portfolio value ($mm)**
95%	67.93	$\mu+1.65\sigma$	69.15
50%	67.80	μ	67.28
5%	64.98	$\mu-1.65\sigma$	65.42
2.5%	63.97	$\mu-1.96\sigma$	65.06
1%	62.85	$\mu-2.33\sigma$	64.64
0.5%	61.84	$\mu-2.58\sigma$	64.36
0.1%	57.97	$\mu-3.09\sigma$	63.77

Using the scenarios, we estimate that 2.5% of the time (or one year in forty), our portfolio in one year will drop in value to $63.97mm or less. If we had used a normal assumption, we would have estimated that this percentile would correspond to only a drop to $65.06mm, a much more optimistic risk estimate.

On the other hand, if we examine the median value change (the 50% level), the normal assumption leads to a more pessimistic forecast: there is a 50% chance that the portfolio is less valuable than the mean value of $67.28mm. By contrast, the scenarios point to a higher mean, and thus to a greater than 50% chance that the portfolio value will exceed its mean.

Another interesting observation is that the 5th and 1st percentiles of the scenarios are 2 and 2.9 standard deviations, respectively, below the mean. This is further evidence that it is best not to use the standard deviation to infer percentile levels for a credit portfolio.

11.3 Assessing precision

In this section, we utilize the methods of *Appendix B* to give confidence bands around our estimated statistics, and examine how these confidence bands evolve as we increase the number of scenarios which we consider.

For the 20,000 scenarios in our example, we have the results shown in *Table 11.4*.

Table 11.4
Portfolio value statistics with 90% confidence levels ($mm)

Statistic	Lower bound	Estimate	Upper bound
Mean portfolio value	67.27	67.28	67.30
Standard deviation	1.10	1.14	1.17
5th percentile	64.94	64.98	65.02
1st percentile	62.66	62.85	62.97
0.5 percentile[1]	61.26	61.84	62.08
0.1 percentile[2]	56.11	57.97	58.73

[1] 1 in 200 chance of shortfall
[2] 1 in 1,000 chance of shortfall

For both the mean and standard deviation, and for the 5th and 1st percentiles, the confidence bands are reasonably tight, and we feel assured of making decisions based on our estimates of these quantities. For the more extreme percentiles, we see that the true loss level could well be at least 10% greater than our estimate. If we desire estimates for these levels, we would be best off generating more scenarios.

With regard to the question of how many scenarios we need to obtain precise estimates, we may examine the evolution of our confidence bands for each estimate as we consider more and more scenarios. We present this information for the six statistics above in the following charts.

Chart 11.4
Evolution of confidence bands for portfolio mean ($mm)

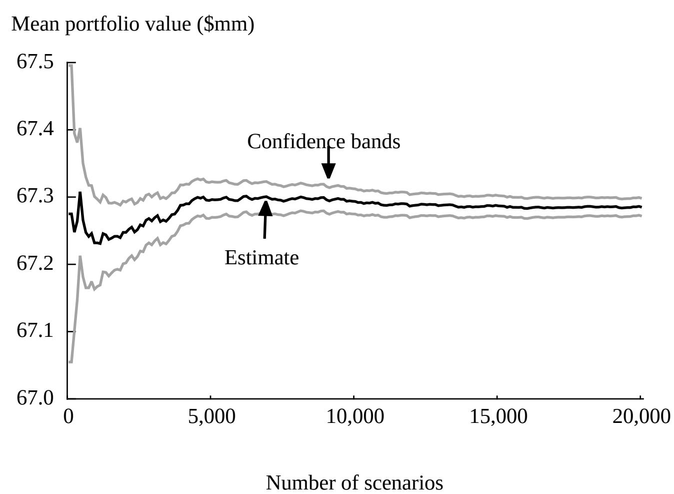


Chart 11.5
Evolution of confidence bands for standard deviation ($mm)

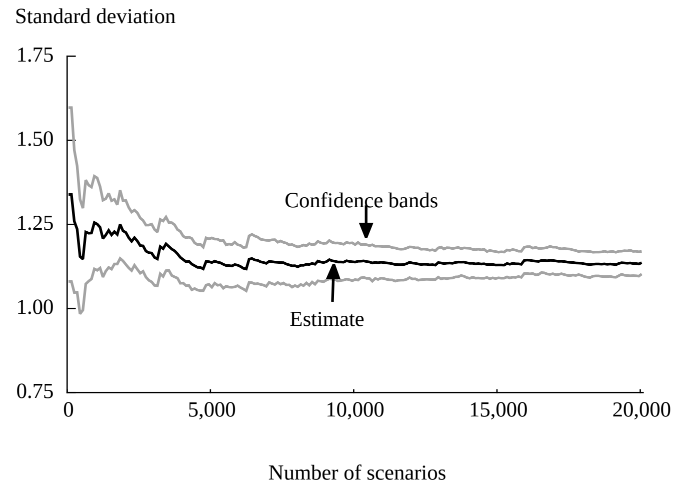


Chart 11.6
Evolution of confidence bands for 5th percentile ($mm)

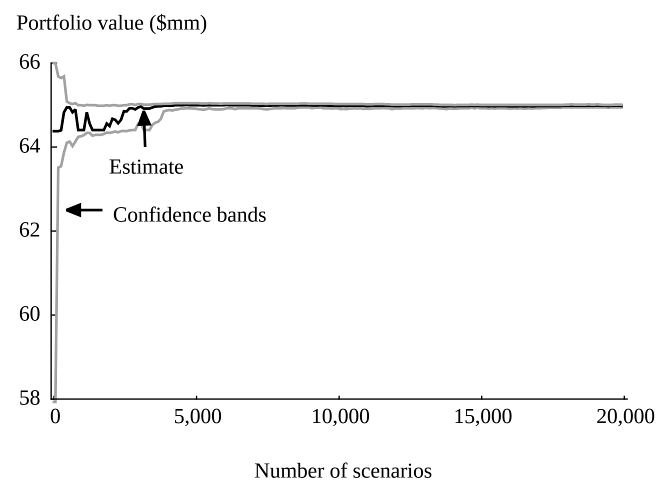


Chart 11.7
Evolution of confidence bands for 1st percentile ($mm)

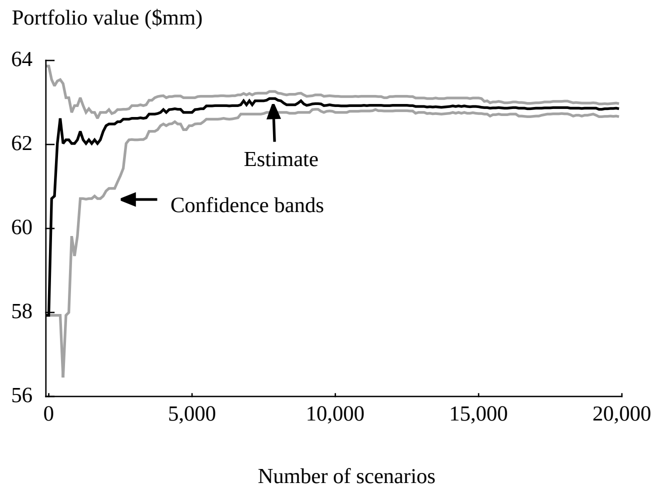


Chart 11.8
Evolution of confidence bands for 0.5 percentile ($mm)

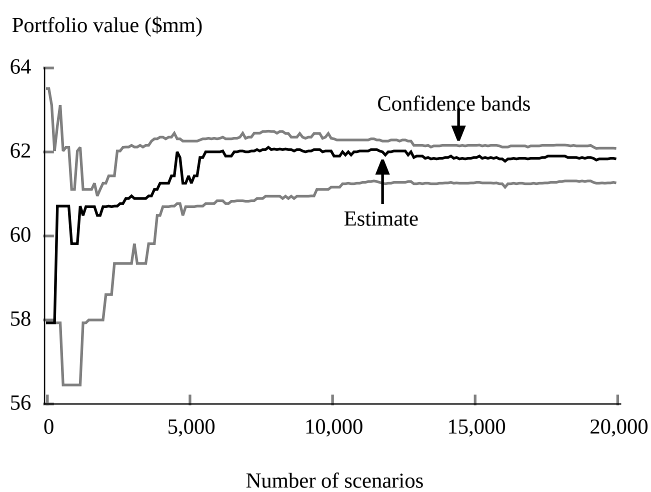
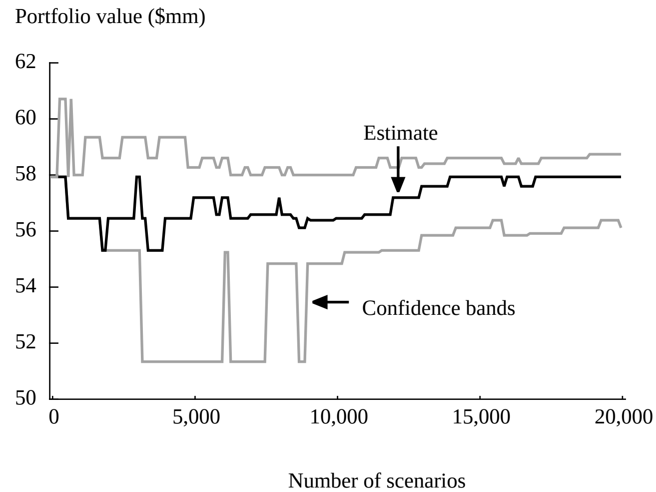


Chart 11.9
Evolution of confidence bands for 0.1 percentile ($mm)



It is interesting to note here that few of the plots change beyond about 10,000 scenarios; we could have obtained similar estimates and similar confidence bands with only half the effort. In fact, if we had been most concerned with the 5^{th} percentile, we might have been satisfied with the precision of our estimate after only 5000 trials, and could have stopped our calculations then. For the most extreme percentile level, note that the estimates and confidence bands do not change frequently. This is due to the fact that on average only one in one thousand scenarios produces a value which truly influences our estimate. This suggests that to meaningfully improve our estimate will require a large number of additional scenarios.

11.4 Marginal risk measures

To examine the contribution of each individual asset to the risk of the portfolio, we compute marginal statistics. Recall that for any risk measure, the marginal risk of a given asset is the difference between the risk for the entire portfolio and the risk of the portfolio without the given asset.

As an example, let us consider the standard deviation. For each asset in the portfolio, we will compute four numbers. First, we compute each asset's *stand-alone standard deviation* of value, that is the standard deviation of value for the asset computed without regard for the other instruments in the portfolio. Second, we compute the *stand-alone percent standard deviation*, which is just the stand-alone standard deviation expressed as a percentage of the mean value for the given asset. Third, we compute each asset's *marginal standard deviation*, the impact of the given asset on the total portfolio standard deviation. Last, we express this figure in percent terms, giving the *percent marginal standard deviation*. These four statistics are presented for each of the 20 assets in *Table 11.5*.

Table 11.5
Standard deviation of value change

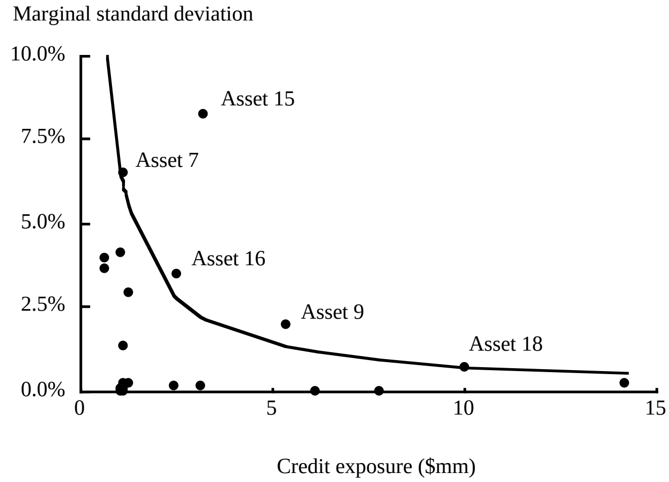
		Stand-alone		Marginal	
Asset	**Credit rating**	**Absolute ($)**	**Percent**	**Absolute ($)**	**Percent**
1	AAA	4,905	0.06	239	0.00
2	AA	2,007	0.17	114	0.01
3	A	17,523	1.56	693	0.06
4	BBB	40,043	3.37	2,934	0.25
5	BB	99,607	8.63	16,046	1.39
6	B	162,251	12.84	37,664	2.98
7	CCC	255,680	22.67	73,079	6.48
8	A	197,152	1.39	35,104	0.25
9	BB	380,141	7.06	105,949	1.97
10	A	63,207	1.99	5,068	0.16
11	A	15,360	1.30	1,232	0.10
12	A	43,085	1.73	4,531	0.18
13	B	107,314	15.21	25,684	3.64
14	B	167,511	15.40	44,827	4.12
15	B	610,900	18.72	270,000	8.27
16	B	322,720	12.77	89,190	3.53
17	BBB	28,051	2.13	2,775	0.21
18	BBB	306,892	3.06	69,624	0.69
19	BBB	1,837	0.16	120	0.01
20	AA	9,916	0.16	389	0.01

The difference between the stand-alone and marginal risk for a given asset is an indication of the effect of diversification. We see in general that for the higher rated assets, there is a greater reduction from the stand-alone to marginal risk than for the lower rated assets. This is in line with our intuition that a much larger portfolio is required to diversify the effects of riskier credit instruments.

An interesting way to visualize these outputs is to plot the percent marginal standard deviations against the market value of each asset, as in *Chart 11.10*. Points in the upper left of the chart represent assets which are risky in percent terms, but whose exposure sizes are small, while points in the lower right represent large exposures which have relatively small chances of undergoing credit losses. Note that the product of the two coordinates (that is, the percent risk multiplied by the market value) gives the absolute marginal risk. The curve in *Chart 11.10* represents points with the same absolute risk; points which fall above the curve have greater absolute risk, while points which fall below have less.

Chart 11.10

Marginal risk versus current value for example portfolio



Based on the discussion above, we may identify with the aid of the curve the five greatest contributors to portfolio risk. Some of these “culprits” are obvious: Asset 7 is the CCC rated issue, and has a much larger likelihood of default, whereas Asset 18 is BBB rated, but is a rather large exposure.

On the other hand, the other “culprits” seem to owe their riskiness as much to their correlation with other instruments as to their individual characteristics. For instance, Asset 9 has a reasonably secure BB rating, but has a correlation of 35% with Asset 7, the CCC rated issue, while Asset 16 is rated B, but has a 55% correlation with Asset 18. Finally, the appearance of Asset 15 as the riskiest in absolute terms seems to be due as much to its 45% correlation with two other B issues as to its own B rating.

With this, we conclude the chapter. The reader should now an understanding of the various descriptors of the future portfolio distribution which can be used to assess risk. In the following chapter, we step away from the technical, and discuss what policy implications the assessment of credit risk might have, as well as how the use of a risk measure should influence the decision on precisely which measure to use.

Chapter 12. Application of model outputs

The measures of credit risk outlined in the preceding sections can have a variety of applications; we will highlight just a few:

- to set priorities for actions to reduce the portfolio risk;
- to measure and compare credit risks so that an institution can best apportion scarce risk-taking resources by limiting over-concentrations; and
- to estimate *economic capital* required to support risk-taking.

The objective of all of the above is to utilize risk-taking capacity more efficiently. Whether this is achieved by setting limits and insisting on being adequately compensated for risk, or by allocating capital to functions which have proven to take risk most effectively, is a policy issue. The bottom line is that in order to optimize the return we receive for the risk we take, it is necessary to measure the risk we take; and this is the contribution of CreditMetrics.

Note that we do not address the issue of credit pricing. Although credit risk can be an important input into a credit pricing decision, we believe that there are significant other determinants for pricing which are beyond the scope of CreditMetrics. These additional factors are non-trivial and so we have chosen to focus this current version on the already challenging task of risk estimation.[1]

12.1 Prioritizing risk reduction actions

The primary purpose of any risk management system is to direct *actions*. But there are many actions that may be taken towards addressing risk – so they must be prioritized. For this discussion, we will make reference to *Chart 12.1*, which is exactly like *Chart 11.10*, but for a hypothetical portfolio with a very large number of exposures.

There are at least two features of risk which are worth reducing, but the trade-off between them is judgmental: (i) absolute exposure size, and (ii) statistical risk level. Thus approaches include:

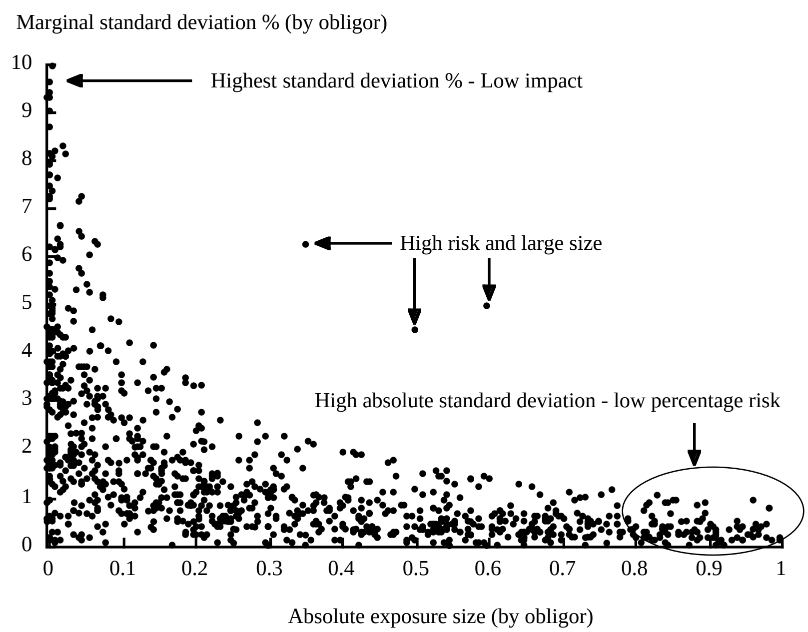
- reevaluate obligors having the largest *absolute size* (the lower right corner of the chart) arguing that a single default among these would have the greatest impact.
- reevaluate obligors having the highest *percentage level of risk* (the upper left corner of the chart) arguing that these are the most likely to contribute to portfolio losses.

[1] Researchers interested in valuation and pricing models may refer to the following: Das & Tufano [96], Foss [95], Jarrow & Turnbull [95], Merton [74], Shimko, Tejima & Van Deventer [93], Skinner [94], and Sorensen & Bollier [94]. Other research on historical credit price levels and relationships includes: Altman & Haldeman [92], Eberhart, Moore & Roenfeldt [90], Fridson & Gao [96], Hurley & Johnson [96], Madan & Unal [96], Neilsen & Ronn [96], and Sarig & Warga [89].

- reevaluate obligors contributing the largest *absolute amount of risk* (points towards the upper right corner of the chart) arguing that these are the single largest contributors to portfolio risk.

Although all three approaches are perfectly valid, we advocate the last one, setting as the highest priority to address those obligors which are both relatively high percentage risk and relatively large exposure. These are the parties which contribute the greatest volatility to the portfolio. In practice, these are often "fallen angels," whose large exposures were created when their credit ratings were better, but who now have much higher percentage risk due to recent downgrades.

Chart 12.1
Risk versus size of exposures within a typical credit portfolio



Like *Chart 11.10*, this chart illustrates a risk versus size profile for a credit portfolio. Obligors with high percentage risk – and presumably high anticipated return – can be tolerated if they are small in size. Large exposures are typically allowed only if they have relatively small percentage risk levels. Unfortunately, the quality of a credit can change over time and a large exposure may have its credit rating downgraded (i.e., its point will move straight up in this chart). The portfolio will then have a large exposure with also a relatively large absolute level of risk. It is this type of obligor which we advocate addressing first.

Chart 12.1 does not completely describe the portfolio in question, however, as it does not address the issue of returns. Thus, there is another issue to consider when considering which exposures should be addressed: whether the returns on the exposures in question adequately compensate their risk. This is where the power of a portfolio analysis becomes evident. In general, it can be assumed that assets will be priced according to their risk on a stand-alone basis, or otherwise, in a CAPM (capital asset pricing model) framework, according to their correlation with a broad universe of assets. What this means is that a given asset may contribute differently to the risk of distinct portfolios, and yet yield the same returns in either case.

Consequently, we can imagine the following situation. Two managers identify a risky asset in their portfolios. It turns out that the two assets are of the same maturity, credit rating, and price, and are expected to yield equivalent returns. However, because of the structure of the two portfolios, if the managers swap these assets, the risk of both portfolios will be reduced without the expected return on either being affected. This might be the case if two managers are heavily concentrated in two different industries. By swapping similar risky assets, the managers reduce their concentration, and thus their risk, without reducing their expected profits.

We see then not only the importance of evaluating the contribution of each asset to the risk of the portfolio, but also the identification of how each asset makes its contribution. When the risk of an asset is due largely to concentrations particular to the portfolio, as in the example above, an opportunity could well exist to restructure the portfolio in such a way as to reduce its risk without altering its profitability.

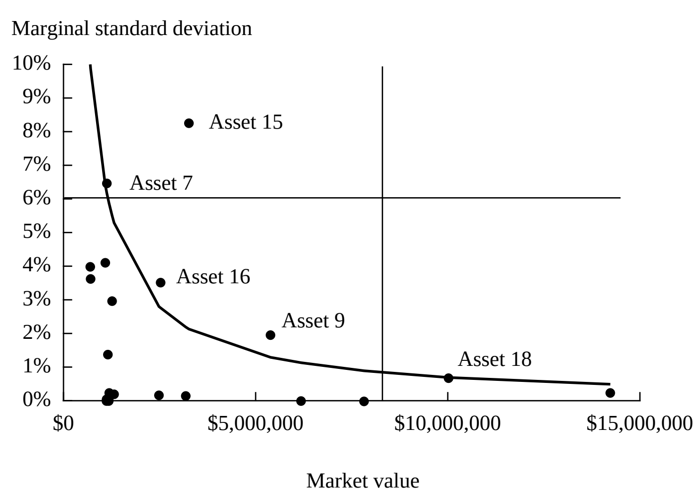
12.2 Credit risk limits

In terms of policy rigor, the next step beyond using risk statistics for prioritization is to use them for limit setting. Of course, what type of risk measure to use for limits, as well as what type of policy to take with regard to the limits, are management decisions. In this section, we discuss three aspects a user might consider with regard to using CreditMetrics for limit purposes: what type of limit to set, which risk measure to use for the limits, and what policy to employ with regard to the limits.

12.2.1 Types of credit risk limits

This section's discussion will make reference to *Chart 12.2*, which the reader might recognize as exactly the same as *Chart 11.10*, but with two additional barriers included.

Chart 12.2
Possible risk limits for an example portfolio



We might consider each of the three possibilities mentioned in the previous section as candidates for credit risk limits. We treat each in turn:

- *Set limits based on percentage risk.* This would correspond to a limit like the horizontal line in *Chart 12.2.* If we measured risk in absolute terms, this would correspond exactly to a limit on credit quality, that is, a limit restricting the portfolio to contain only exposures rated, say, B or higher. Since we measure risk in marginal terms, this limit would be slightly different in that it would also restrict exposures that are more correlated to the portfolio, since these contribute more to portfolio risk.

- *Set limits based on exposure size.* This would correspond to a limit like the vertical bar in *Chart 12.2.* Such a limit would restrict the portfolio to have no exposures, regardless of credit quality, above a given size.

- *Set limits based on absolute risk.* This would correspond to a limit like the curve in *Chart 12.2.* Such a limit would prevent the addition to the portfolio of any exposure which increased the portfolio risk by more than a given amount. In effect, this would cap the total risk of the portfolio at a certain amount above the current risk.

In the previous section, we argued that it is best to address exposures with the highest level of absolute risk first, since these have the greatest impact on the total portfolio risk. By the same token, it is most sensible to set limits in terms of absolute (rather than percent) risk. Moreover, limiting absolute risk is consistent with the natural tendencies of portfolio managers; in other words, it is perfectly intuitive to require that exposures which pose a greater chance of decreases in value due to credit be smaller, while allowing those with less chance of depreciating to be greater.

We see the natural tendency to structure portfolios in this way in both *Charts 12.1* and *12.2;* in both cases, the risk profiles tend to align themselves with the curve rather than with either the vertical or horizontal line. Thus, setting limits based on absolute risk would take the qualitative intuition that currently drives decisions and make it quantitative.

It is worth mentioning here that the risk limits we have discussed are not meant to replace existing limits to individual names. Limits based on the notion that there is a maximum amount of exposure we desire to a given counterparty, regardless of this counterparty's credit standing, are certainly appropriate. Such limits may be thought of as conditional, in that they reflect the amount we are willing to lose conditioned on a given counterparty's defaulting, and do not depend on the probability that the counterparty actually defaults. The limits proposed in this section are meant to supplement, but not replace, these conditional limits.

12.2.2 Choice of risk measure

Given a choice of what type of limit to implement, it is necessary next to choose the specific risk measure to be used. Essentially, there are two choices to make: first, whether to use a marginal or stand-alone statistic, and second, whether to use standard deviation, percentile level, or another statistic.

The arguments for using marginal statistics have been made before. These statistics allow the user to examine an exposure with regard to its effect on the actual portfolio, taking into accounts the effects of correlation and diversification. Thus, marginal statistics provide a better picture of the true concentration risk with respect to a given counterparty.

On the other hand, certain circumstances suggest the use of absolute risk measures for limits. For instance, suppose a portfolio contains a large percentage of a bond issue of a given name. Even if the name has a very low correlation with the remainder of the portfolio (meaning that the bond has low marginal risk), the position should be considered risky because of the liquidity implications of holding a large portion of the issue. Thus, it is important in this case to know the stand-alone riskiness of the position.

As to what statistic to use, we describe four statistics below, and discuss the applicability of each to credit risk limits.

As always, the easiest statistic to compute is the *standard deviation*. However, as a measure of credit risk, it has a number of deficiencies. First, the standard deviation is a "two-sided" measure, measuring the portfolio value's likely fluctuations to the upside or downside of the mean. Since we are essentially concerned with only the downside, this makes the standard deviation somewhat misleading. In addition, since distributions of credit portfolios are mostly non-normal, there is no way to infer concrete information about the distribution from just the standard deviation.

We have also discussed the use of *percentile levels* at some length. The advantages of this statistic are that it is easy to define and has a very concrete meaning. When we state the first percentile level of a portfolio, we know that this is precisely the level below which we can expect losses only one percent of the time. There is a price for this precision, however, as we cannot derive such a measure analytically, and must resort to simulations. Thus, our measure is subject to the random errors inherent in Monte Carlo approaches.

Another statistic which is often mentioned for characterizing risk is *average shortfall*. This statistic is defined as the expected loss given that a loss occurs, or as the expected loss given that losses exceed a given level. While this does give some intuition about a portfolio's riskiness, it does not have quite as concrete an interpretation as a percentile level. For instance, if we say that given a loss of over $3mm occurs, we expect that loss to be $6mm, we still do not have any notion of how likely a $6mm loss is. Along the same lines, we might consider using the *expected excession of a percentile level*. For the 1st percentile level, this statistic is defined as the expected loss given that the loss is more extreme than the 1st percentile level. If this statistic were $12mm, then the interpretation would be that in the worst 1 percent of all possible cases, we would expect our losses to be $12mm. This is a very reasonable characterization of risk, but like percentile level and average shortfall, requires a simulation approach.

When choosing a risk statistic, it is important to keep in mind its application. For limits, and particularly for prioritization, it is not absolutely necessary that we be able to infer great amounts of information about the portfolio distribution from the risk statistics that we use. What is most important is that the risk estimates give us an idea of the *relative* riskiness of the various exposures in our portfolio. It is reasonable to claim that the standard deviation does this. Thus, for the purpose of prioritization or limit setting, it would be sensible to sacrifice the intuition we obtain from percentile levels or expected excessions if using the standard deviation provides us with significant improvements in computational speed.

12.2.3 Policy issues

The fundamental point of a limit is that it triggers action. There can be many levels of limits which we classify according to the severity of action taking in the case the limit is exceeded.

For informational limits, an excession of the limit might require more in-depth reporting, additional authorization to increase exposure size, or even supplemental covenant protection or collateral. The common thread is that exposures which exceed the limits are permitted, but trigger other actions which are not normally necessary.

Alternatively, one might set hard limits, which would preclude any further exposure to an individual name, industry, geographical region, or instrument type. In practice, one might implement both types of limits – an informational limit at some low level of risk or exposure and a hard limit at a higher level. And these limits might even be based on two different risk measures – a marginal measure at one level and an absolute measure at the other.

The assumption for both types of limits above is that the limits are in place before the exposures, and each exposure we add to the portfolio satisfies the limits. However, for the aforementioned fallen angels, this will not be the case. These exposures satisfy the risk limits when they are added to the portfolio, but subsequently exceed the limits due to a change in market rates or to a credit rating downgrade. Excessions of this type are essentially uncontrollable, although a portfolio manager might seek to reduce the risk in these cases by curtailing additional exposure, reducing existing exposure, or hedging with a credit derivative.

It is not uncommon to set limits at different levels of aggregation since different levels of oversight may occur at higher and higher levels. For instance, there might be limits on individual names, plus industry limits, plus sector limits, plus even an overall credit portfolio limit.

It should always be the case that a limit will be less than or equal to the sum of limits one level lower in the hierarchy. Thus, the financial sector limit should not be greater than the sum of limits to industries underneath it such as banks, insurers, brokers, etc. This will be true whether limits are set according to exposures (which can be aggregated by simply summing them) or according to risk (which can be aggregated only after accounting for diversification).

12.3 Economic capital assessment

For the purposes of prioritization and limit setting, the subjects of the first two sections, we examined risk measures in order to evaluate and manage individual exposures. The total risk of the portfolio might guide the limit-setting process, but it was the relative riskiness of individual exposures which most concerned us.

In this section, we examine a different application of credit risk measures, that of assessing the capital which a firm puts at risk by holding a credit portfolio. We are no longer trying to compare different exposures and decide which contribute most to the riskiness of the portfolio, but rather are seeking to understand the risk of the entire portfolio with regard to what this risk implies about the stability of our organization.

To consider risk in this way, we look at risk in terms of capital; but rather than considering the standard regulator or accounting view of capital, we examine capital from a risk management informational view. The general idea is that if a firm's liabilities are constant, then it is taking risk by holding assets that are volatile, to the extent that the asset volatility could result in such a drop in asset value that the firm is unable to meet its liability obligations.

This risk-taking capability is not unlimited, as there is a level beyond which no manager would feel comfortable. For example, if a manager found that given his asset portfolio, there was a ten percent chance for such a depreciation to occur in the next year as to cause organization-wide insolvency, then he would likely seek to decrease the risk of the asset portfolio. For a portfolio with a more reasonable level of risk, the manager cannot add new exposures indiscriminately, since eventually the portfolio risk will surpass the "comfort level." Thus, each additional exposure utilizes some of a scarce resource, which might be thought of as risk-taking capability, or alternately, as economic capital.

To measure or assess the economic capital utilized by an asset portfolio, we may utilize the distribution of future portfolio values which we describe elsewhere in this document. This involves a choice, then, of what statistic to use to describe this distribution. The choice is in some ways similar to the choice of risk statistic for limits which we discussed in the previous section; however, the distinct use of risk measures here make the decision different. For limits, we were concerned with individual exposures and relative measures; for economic capital, we are interested in a portfolio measure and have more need for a more concrete meaning for our risk estimate. These issues should become clear as we consider the risk statistics below.

For limits we could argue that the standard deviation was an adequate statistic in that it could capture the relative risks of various instruments. In this case, however, it is difficult to argue that a standard deviation represents a good measure of capital since we are unable to attach a concrete interpretation to this statistic. Yet this statistic is practical to compute and for this reason alone may be the logical choice.

As an indicator of economic capital, a percentile level seems quite appropriate. Using for example the 1st percentile level, we could define economic capital as the level of losses on our portfolio which we are 99% certain (or in the words of Jacob Bernoulli, "morally certain"[2]) that we will not experience in the next year. This fits nicely with our discussion of capital above. If it is our desire to be 99% certain of meeting our financial obligations in the next year, then we may think of the 1st percentile level as the risk we are taking, or as the economic capital which we are allocating to our asset portfolio. If this level ever reaches the point at which such a loss will prevent us from meeting obligations, then we will have surpassed the maximum amount of economic capital we are willing to utilize.

As with limits, we may consider average shortfall as a potential statistic. Yet just as in the case of limits, it is difficult to consider an expected shortfall of $6mm as a usage of capital since we do not know how likely such a loss actually is. On the other hand, the expected excession of a percentile level does seem worth consideration. Recall that if this statistic were $12mm at the first percentile level, then the interpretation would be

[2] As quoted in Bernstein [96].

that in the worst 1 percent of all possible cases, we would expect our losses to be $12mm. So like the percentile level above, this seems to coincide with our notion of economic capital, and thus seems a very appropriate measure.

All of the above measures of economic capital differ fundamentally from the capital measures mandated for bank regulation by the Bank for International Settlements (BIS). For a portfolio of positions not considered to be trading positions, the BIS risk-based capital accord of 1988 requires capital that is a simple summation of the capital required on each of the portfolio's individual transactions, where each transaction's capital requirement depends on a broad categorization (rather than the credit quality) of the obligor; on the transaction's exposure type (e.g., drawn loans versus undrawn commitments); and, for off-balance-sheet exposures, on whether the transaction's maturity is under one-year or over one year. The weaknesses of this risk-based structure – such as its one-size-fits-all risk weight for all corporate loans and its inability to distinguish diversified and undiversified portfolios – are increasingly apparent to regulators and market participants, with particular concern paid to the uneconomic incentives created by the regulatory regime and the inability of regulatory capital adequacy ratios to accurately portray actual bank risk levels. In response to these concerns, bank regulators are increasingly looking for insights in internal credit risk models that generate expected losses and a probability distribution of unexpected losses.[3]

12.4 Summary

In summary, the CreditMetrics methodology gives the user a variety of options to use for measuring economic capital which may in turn lead to further uses of CreditMetrics. We briefly touch on three applications of an economic capital measure: *exposure reduction*, *limit setting*, and *performance evaluation*.

An assessment of economic capital may guide the user to actions which will alter the characteristics of his portfolio. For example, if the use of economic capital is too high, it will be necessary to take actions on one or more exposures, possibly by prohibiting additional exposure, or else by reducing existing exposures by unwinding a position or hedging with a credit derivative. How to choose which exposures to treat could then be guided by the discussions in *Section 12.1*.

On the other hand, one might wish to use the measure of economic capital in order to aid the limit-setting process, assuring that if individual or industry level exposures are within the limits, then the level of capital utilization will be at an acceptable level.

A third use is performance evaluation. The traditional practice has been to evaluate portfolio managers based on return, leading to an incentive structure which encourages these managers to take on lower rated exposures in order to boost performance. Adding a measure of economic capital utilization allows for a more comprehensive measure of performance; when managers' returns are paired with such a risk measure, it can be seen which managers make the most efficient use of the firm's economic capital. Examining performance in this way retains the incentive to seek high returns, but penalizes for taking undue risks to obtain these returns.

3 See Remarks by Alan Greenspan, Board of Governors of the Federal Reserve System, before the 32nd Annual Conference on Bank Structure and Competition, FRB of Chicago, May 2, 1996.

By examining rates of return on economic capital and setting targets for these returns, a manager or firm goes a step beyond the traditional practice of requiring one rate of return on its most creditworthy assets and a higher rate on more speculative ones; the new approach is to consider a hurdle rate of return on risk, which is more clear and more uniform than the traditional practice. Identifying portfolios or businesses that achieve higher returns on economic capital essentially tells a manager which areas are providing the most value to the firm. And just as it is possible to allocate any other type of capital, areas where the return on risk is higher may be allocated more economic capital, or more risk-taking ability. By focusing capital on the most efficient parts of a firm or portfolio, profits are maximized, but within transparent, responsible risk guidelines.

Appendices

Appendices

In CreditMetrics we use certain general statistical formulas, data, and indices in several different capacities. We have chosen to address each of them in detail here in an appendix so that we may give them the depth they deserve without cluttering the main body of this *Technical Document*. These appendices include:

Appendix A: Analytic standard deviation calculation.

A generalization of the methods presented in *Chapter 9* to compute the standard deviation for a portfolio of arbitrary size.

Appendix B: Precision of simulation-based estimates.

Techniques to assess the precision of portfolio statistics obtained through simulation.

Appendix C: Derivation of the product of N random variables.

Used to: (i) combine the uncertainty of spread and exposure risk and (ii) for the derivation of risk across mutually exclusive outcomes.

Appendix D: Derivation of risk across mutually exclusive outcomes.

Used for both: the value variance of a position across *N*-states and the covariance between positions across *N*-states.

Appendix E: Derivation of the correlation of two binomials.

Used to link correlation between firms' value to their default correlations.

Appendix F: Inferring default correlations from default volatilities.

Used as alternative method to estimate default correlations which corroborates our equity correlation approach.

Appendix G: International bankruptcy code summary.

Contains this information in tabular format.

Appendix H: Model inputs.

Describes the CreditMetrics data files and required inputs.

Appendix I: Indices used for asset correlations.

Contains this information in tabular format.

Appendix A. Analytic standard deviation calculation

In *Chapter 9*, we presented the calculation of the standard deviation for an example three asset portfolio, and stated that the generalization of this calculation to a portfolio of arbitrary size was straightforward. In this appendix, we present this generalization in detail.

Consider a portfolio of n assets. Denote the value of these assets at the end of the horizon by V_1, V_2, ..., V_n; let these values' means be $\mu_1,\mu_2,...,\mu_n$ and their variances be $\sigma^2(V_1), \sigma^2(V_2), ..., \sigma^2(V_n)$. The calculation of these individual means and variances is detailed in *Chapter 2*.

The value of the portfolio at the end of the forecast horizon is just $V_1+V_2+...+V_n$, and the mean value is $\mu_p=\mu_1+\mu_2+...+\mu_n$. To compute the portfolio standard deviation (σ_p), we may use the standard formula:

[A.1] $$\sigma_p^2 = \sum_{i=1}^{n} \sigma^2(V_i) + 2 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} COV(V_i, V_j).$$

Alternatively, we may relate the covariance terms to the variances of pairs of assets,

[A.2] $$\sigma^2(V_i + V_j) = \sigma^2(V_i) + 2 \cdot COV(V_i, V_j) + \sigma^2(V_j),$$

and using this fact, express the portfolio standard deviation in terms of the standard deviations of subportfolios containing two assets:

[A.3] $$\sigma_p^2 = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma^2(V_i + V_j) - (n-2) \cdot \sum_{i=1}^{n} \sigma^2(V_i).$$

As in *Chapter 9*, we see that the portfolio standard deviation depends only on the variances for pairs of assets and the variances of individual assets. This makes the computation of the portfolio standard deviation straightforward. We begin by computing the variances of each individual asset; we then identify all pairs of assets among the n assets in the portfolio[1] and compute the variances for each of these pairs using the methods in *Chapter 3*; finally, we apply Eq. [A.3].

[1] There will be $n \cdot (n-1)/2$ pairs.

Appendix B. Precision of simulation-based estimates

In *Chapter 10*, we presented a methodology to compute portfolio statistics using Monte Carlo simulation and mentioned that statistics which are estimated in this way are subject to random errors. In this appendix, we discuss how we may quantify the sizes of these errors, and thus discover how confident we may be of the risk estimates we compute. We devote one subsection each to the treatment of the sample mean, sample standard deviation, and sample percentile levels.

Throughout this section, we will use $V^{(1)}, V^{(2)}, V^{(3)}, \ldots, V^{(N)}$ to indicate the portfolio values across scenarios and $V^{[1]}, V^{[2]}, V^{[3]}, \ldots, V^{[N]}$ to indicate the same values sorted into ascending order (so that, for example $V^{[2]}$ is the second smallest value). Further, let μ_n denote the sample mean and σ_n the sample standard deviation of the first n scenarios.

B.1 Sample mean

Quantifying the error about our estimate of the mean portfolio value is straightforward. For large n, μ_n will be approximately normally distributed with standard deviation $\sigma_n / \sqrt{n}$. Thus, after generating n scenarios, we may say that we are 68%[2] confident that the true mean portfolio value lies between $\mu_n - \sigma_n / \sqrt{n}$ and $\mu_n + \sigma_n / \sqrt{n}$ and 90% confident the true mean lies between $\mu_n - (1.65 \cdot \sigma_n / \sqrt{n})$ and $\mu_n + 1.65 \cdot \sigma_n / \sqrt{n}$. Note that these bands will tighten as n increases.

B.2 Sample standard deviation

Our confidence in the estimate σ_n is more difficult to quantify since the distribution of the estimate is less well approximated by a normal distribution, and the standard deviation of the estimate is much harder to estimate.

The simplest approach here is to break the full set of scenarios into several subsets, compute the sample standard deviation for each subset, and examine how much fluctuation there is in these estimates. For example, if we have generated 20,000 portfolio scenarios, then we might divide these scenarios into fifty separate groups of 400. We could then compute the sample standard deviation within each group, obtaining fifty different estimates $\sigma^{(1)}, \sigma^{(2)}, \ldots, \sigma^{(50)}$. The sample standard deviation of these estimates, which we denote by s, is then an estimate for the standard error of σ_{400}. In order to extrapolate to an estimate for the standard error of σ_{20000}, we assume that the same scaling holds as with the sample mean, and take $s / \sqrt{50}$. Then we can say that we are approximately 90% confident that the true value of our portfolio standard deviation lies between $\mu_{2000} - (1.65 \cdot s / \sqrt{50})$ and $\mu_{2000} + (1.65 \cdot s / \sqrt{50})$ [3]. This procedure is commonly referred to as "jackknifing."

For the sample mean and standard deviation, our approach to assessing precision was the same. Motivated by the fact that the estimates we compute are sums over a large number

[2] Since the probability that a normally distributed random variable falls within one standard deviation of its mean is 68%.

[3] This methodology is somewhat sensitive to the choice of how many separate groups to divide the sample into. We choose 50 here, but in practice suggest that the user experiment with various numbers in order to get a feel for the sensitivity of the confidence estimates to this choice.

of independent trials, we approximated the distributions of the estimates as normal. The rest of the analysis then focused on computing the standard errors for the estimates. Moreover, in some sense, the assessment of precision for estimates of these two statistics is somewhat redundant, as it is possible to obtain exact values in both cases.

In the next section, we treat estimates of percentile levels, for which neither of these points applies. Estimates are not just sums over the scenarios, and thus we cannot expect the distributions of the estimates to be normal; further, we have no way of computing percentile levels directly, and thus are much more concerned with the precision of our estimates.

B.3 Sample percentile levels

As an example, say we are trying to estimate the 5^{th} percentile level, and let θ_5 be the true value of this level. Each scenario which we generate then (by definition) has a 5% chance of producing a portfolio value less than θ_5. Now consider 1000 independent scenarios, and let N_5 be the number of these scenarios which fall below θ_5. Note that N_5 follows the binomial distribution. Clearly, the expected value of N_5 is $1000 \cdot 5\% = 50$, while the standard deviation is $\sqrt{1000 \cdot 5\% \cdot (100\% - 5\%)} = 6.9$. For this many trials, it is reasonable to approximate the distribution of N_5 by the normal. Thus, we estimate that there is a 68% chance that N_5 will be between 50-6.9=43.1 and 50+6.9=56.9, and a slightly higher chance that N_5 will be between 43 and 57. Further, there is a 90% chance that N_5 falls between $50 - 1.65 \cdot 6.9 = 38.6$ and $50 + 1.65 \cdot 6.9 = 61.2$.

At this point we have characterized N_5. This may not seem particularly useful, however, since N_5 is not actually observable. In other words, since we do not actually know the level θ_5 (this is what we are trying to estimate), we have no way of knowing how many of our scenarios fell below θ_5. We assert that it is not necessary to know N_5 exactly, since we can gain a large amount of information from its distribution.

Observe that if N_5 is greater than or equal to 43, then at least 43 of our scenarios are less than θ_5. This implies that θ_5 is at least as large as the 43rd smallest of our portfolio values. (Recall that in our notation, this scenario is denoted by $V^{[43]}$.) On the other hand, if N_5 is less than or equal to 57, then it must be true that θ_5 is no larger than the 57th smallest of the portfolio values (that is, $V^{[57]}$). Thus, we have argued that the event

[B.1] $$43 \le N_5 \le 57$$

is exactly the same as the event

[B.2] $$V^{[43]} < \theta_5 < V^{[57]}.$$

Now since these two events are the same, they must have the same probability, and thus

[B.3] $$\Pr\{V^{[45]} < \theta_5 < V^{[57]}\} = \Pr\{43 \le N_5 \le 57\} = 68\%$$

and so we have a confidence bound for our estimate of θ_5. To recap, using 1000 scenarios, we estimate the 5^{th} percentile portfolio value by the 50^{th} smallest scenario, and state that we are 68% confident that the true percentile lies somewhere between the 43rd and 57th smallest scenarios.

In general, if we wish to estimate the p^{th} percentile using N scenarios, we first consider the number of scenarios that fall below the true value of this percentile. We characterize this number via the following:

[B.4]

$$\text{lower bound: } \quad l = N \cdot p - \alpha \cdot \sqrt{N \cdot p \cdot (1-p)}$$

$$\text{mean: } \quad m = N \cdot p, \; s = \sqrt{N \cdot p \cdot (1-p)}$$

and

$$\text{upper bound: } \quad u = N \cdot p + \alpha \cdot \sqrt{N \cdot p \cdot (1-p)}$$

where α depends on the level of confidence which we desire. (That is, if we desire 68% confidence, then α=1, if we desire 90%, then α=1.65, etc.) If either l or m are not whole numbers, we round them downwards, while if u is not a whole number, we round upwards. We then estimate our percentile by $V^{[m]}$ and state with our desired level of confidence that the true percentile lies between $V^{[l]}$ and $V^{[u]}$.

For further discussion of these methods, see DeGroot [86], p. 563. Note that the only assumption we make in this analysis is that the binomial distribution is well approximated by the normal. In general, this will be the case as long as the expected number of scenarios falling below the desired percentile (that is, $N{\cdot}p$) is at least 20 or so. In cases where this approximation is not accurate, we may take the same approach as in this section, but characterize the distribution precisely rather than using the approximation. The result will be similar, in that we will obtain confidence bands on the number of scenarios falling below the threshold, and then proceed to infer confidence intervals on the estimated percentile.

Appendix C. Derivation of the product of *N* random variables

First we examine in detail the volatility of the product of two random variables. Let X and Y be any independent and uncorrelated distributions defined as follows:

[C.1] $$X \sim \mu_x + \sigma_x \cdot Z_x \quad Y \sim \mu_y + \sigma_y \cdot Z_y \ (where\ \sim denotes\ \text{distributed as})$$

where all distributions, Z, are independent and standardized but can otherwise have any desired shape: normal, highly skewed, binomial, etc.

[C.2] $$\sigma^2_{X \cdot Y} = E(X^2 \cdot Y^2) - E(X \cdot Y)^2 \quad \text{(Textbook formula)}$$

First, we will multiply out x · y.

[C.3] $$X \cdot Y = \mu_x\mu_y + \mu_x\sigma_y Z_Y + \mu_y\sigma_x Z_X + \sigma_x Z_X \sigma_y Z_Y$$

Since the expected value of Z is zero, the *E( )*'s simplify greatly.

[C.4]
$$\begin{aligned} E(X \cdot Y) &= \mu_x\mu_y \\ E(X \cdot Y)^2 &= \mu_x^2\mu_y^2 && \text{(Since: E(Z) = 0)} \\ E(X^2 \cdot Y^2) &= \mu_x^2\mu_y^2 + \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2 && \text{(Since: } E(Z)^2 = 1\text{)} \end{aligned}$$

Now $\sigma_{X \cdot Y}$ is only a matter of algebra.

[C.5]
$$\begin{aligned} \sigma^2_{X \cdot Y} &= (\mu_x^2\mu_y^2 + \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2) - (\mu_x^2\mu_y^2) \\ &= \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2 \end{aligned}$$
$$\boxed{\sigma^2_{X \cdot Y} = \sqrt{\mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2}}$$

By induction, we can we can extend the volatility estimation for the product of arbitrarily many independent events. First, the expectation of this product is simply the product of its expectations:

[C.6] $$E\left(\prod_i^N \Phi_i\right) = \prod_i^N \mu_i \text{ where all } \Phi_i \sim \mu_i + \sigma_i \cdot Z_i$$
and all Z_i are standardized (0,1)

The variance of the product of N distributions will in general have, $2^N - 1$, terms. For the case of the product of three distributions, the result is:

[C.7]
$$VAR(\Phi_X \cdot \Phi_Y \cdot \Phi_Z) = \left(\begin{array}{l} + \mu_x^2\mu_y^2\sigma_z^2 + \mu_x^2\sigma_y^2\sigma_z^2 \\ + \mu_x^2\sigma_y^2\mu_z^2 + \sigma_x^2\mu_y^2\sigma_z^2 + \sigma_x^2\sigma_y^2\sigma_z^2 \\ + \sigma_x^2\mu_y^2\mu_z^2 + \sigma_x^2\sigma_y^2\mu_z^2 \end{array} \right)$$

In general, the pattern continues and can be denoted as follows for N distributions. In this notation, j and m denote sets whose elements comprise the product sums:

[C.8]
$$VAR\left(\prod_i^N \Phi_i\right) = \sum_{k=1}^{N}\left[\prod_{J \in s(N,k)}\left(\sigma_j^2 \cdot \prod_{m = S(N) - j} \mu_m^2\right)\right] - \prod_i^N \mu_i^2$$

where the sets $S(N) = \{1, 2, 3, \ldots, N\}$

and $s(N, k) = \{j_i, \ldots j_k \mid 1 \le j_1 < \ldots < j_k \le N, k \le N\}$.

Appendix D. Derivation of risk across mutually exclusive outcomes

Imagine that there were two alternative outcomes (subscripts *1* and *2*) that might occur in the event of default with probabilities of p_1 and p_2 which sum to the total probability of default. For completeness, subscript ω is the case of no default. Each of these three cases has some distribution of losses denoted, $\Phi_i(x)$, with statistics, μ_i and σ_i.

Definitions: $1 = p_1 + p_2 + p_\omega$ and $\Phi_T(x) = p_1\Phi_1(x) + p_2\Phi_2(x) + p_\omega\Phi_\omega(x)$.

[D.1] Expected Total Loss

$$\begin{aligned}\mu_T &= \int x\Phi_T(x)dx \\ &= \int x(p_1\Phi_1(x) + p_2\Phi_2(x) + p_\omega\Phi_\omega(x))dx \\ &= p_1\mu_1 + p_2\mu_2 + p_\omega\mu_\omega\end{aligned}$$

[D.2] Variance of Total Loss

$$\begin{aligned}\sigma_T^2 &= \int(x-\mu_T)^2\Phi_T(x)dx \\ &= \int(x^2 - 2x\mu_T + \mu_T^2)(p_1\Phi_1(x) + p_2\Phi_2(x) + p_\omega\Phi_\omega(x))dx \\ &= \left(\begin{array}{c} \underbrace{p_1\int x^2\Phi_1(x) \quad + \quad p_2\int x^2\Phi_2(x) \quad + \quad p_\omega\int x^2\Phi_\omega(x)}_{\text{These simplify}} \\ -2\mu_T\underbrace{(p_1\mu_1 + p_2\mu_2 + p_\omega\mu_\varpi)}_{\text{Note that this equals } \mu_T \text{ see above}} \\ +\underbrace{\mu_T^2(p_1 + p_2 + p_\omega)}_{\text{Note that this sums to 1}} \end{array}\right) \\ &= \left(\begin{array}{c} p_1(\mu_1^2 + \sigma_1^2) + p_2(\mu_2^2 + \sigma_2^2) + p_\omega(\mu_\omega^2 + \sigma_\omega^2) \\ -2\mu_T^2 \\ +\mu_T^2 \end{array}\right) \\ &= p_1(\mu_1^2 + \sigma_1^2) + p_2(\mu_2^2 + \sigma_2^2) + p_\omega(\mu_\omega^2 + \sigma_\omega^2) - \mu_T^2\end{aligned}$$

The above derivation requires a substitution for an integral that merits further discussion. The problem of multiplying a random variable by itself was addressed in the prior appendix note (see *Appendix C)*. If the two are the same distribution, then the correlation is simply 1.0.

[D.3] Mean of Product of Two Random Variables

$\mu_{(i \cdot j)}$	$= \mu_i\mu_j + \rho\sigma_i\sigma_j$	See prior appendix note.
	$= \mu_i^2 + \sigma_i^2$	Since i = j and $\rho = 1.0$
	$= \int x_2\Phi_i(x)dx$	Substitution made above.

For completeness, we have included terms describing the losses in the case of no default: μ_ω and σ_ω. But these are both zero since there will be no losses in the case of no default. Thus the overall total mean and standard deviation of losses in this process simplifies as follows:

[D.4] $$\sigma_T = \sqrt{\sum_{i=1}^{S} p_i(\mu_i^2 + \sigma_i^2) - \mu_T^2} \qquad \text{where } \mu_T = \sum_{i=1}^{S} p_i \mu_i$$

Appendix E. Derivation of the correlation of two binomials

The traditional textbook formula for covariance is shown below.

$$cov_{x,y} = \sum_{i=1}^{n} W_i(x_i - \mu_x)(y_i - \mu_y)$$

The expected probabilities, *p'*s, of the two binomials, *x* and *y*, are termed μ_x and μ_y respectively. Normally all the *n* observations would be equally weighted ($^1/_n$), but here the probability weights W_i will equal the likelihood of each possible outcome. For the joint occurrence of two binomials, there will be exactly four possible outcomes. We can simply list them explicitly. The probability weights W_i are easily calculated for the case of independence, but we will leave them as variables to allow for any degree of possible correlation. As shown below, defaults will have value 1 and non-defaults will have value 0.

[E.1]

Obligor Y		**Obligor X**	
Default	**No Default**	**Default**	**No Default**
1: X& Y default	3: Only X defaults	1: X& Y default	2: Only Y defaults
2: Only Y defaults	4: Neither defaults	3: Only X defaults	4: Neither defaults

$$cov_{x,y} = \begin{array}{l} W_1(1-\mu_x)(1-\mu_y) \\ + W_2(0-\mu_x)(1-\mu_y) \\ + W_3(1-\mu_x)(0-\mu_y) \\ + W_4(0-\mu_x)(0-\mu_y) \end{array}$$

The difficult problem in defining the probability weights *W'*s is knowing the correlated joint probability of default (cell #1 above). We will label this joint probability as α. Multiplying and simplifying the resulting formula, see below, yields an intuitive result for our covariance. If the joint default probability, α, is greater than the independent probability, (that is μ_x times μ_y), then the covariance is positive; otherwise it is negative.

$$\text{[E.2]} \qquad cov_{x,y} = \begin{pmatrix} W_1(1-\mu_x)(1-\mu_y) \\ +W_2(0-\mu_x)(1-\mu_y) \\ +W_3(1-\mu_x)(0-\mu_y) \\ +W_4(0-\mu_x)(0-\mu_y) \end{pmatrix}$$

$$= \begin{pmatrix} [\quad \alpha \quad](1-\mu_x)(1-\mu_y) \\ +[\mu_y - \alpha](0-\mu_x)(1-\mu_y) \\ +[\mu_x - \alpha](1-\mu_x)(0-\mu_y) \\ +[1-\mu_x-\mu_y+\alpha](0-\mu_x)(0-\mu_y) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha - \alpha\mu_y - \alpha\mu_x + \alpha\mu_x\mu_y \\ -\mu_x\mu_y + \mu_x\mu_y^2 + \alpha\mu_x - \alpha\mu_x\mu_y \\ -\mu_x\mu_y + \mu_x^2\mu_y + \alpha\mu_y - \alpha\mu_x\mu_y \\ +\mu_x\mu_y - \mu_x^2\mu_y - \mu_x\mu_y^2 + \alpha\mu_x\mu_y \end{pmatrix}$$

$$= \alpha - \mu_x\mu_y$$

Now that we have derived the covariance as a function of the joint default probability, α, we can redefine α in terms of the correlation of our two binomials. Again, we can start with a textbook formula for the covariance:

$$\text{[E.3]} \qquad \begin{array}{ccc} cov_{x,y} = \rho_{x,y}\sigma_x\sigma_y & & \alpha = \mu_x\mu_u + \rho_{x,y}\sigma_x\sigma_y \\ so & \text{thus} & and \\ \alpha - \mu_x\mu_y = \rho_{x,y}\sigma_x\sigma_y & & \rho_{x,y} = (\alpha - \mu_x\mu_y)/\sigma_x\sigma_y \end{array}$$

Interestingly, the above definition of α and ρ is identical the formula for the mean of the product of two correlated random variables as shown above (see *Appendix A)*. Importantly, this correlation ρ_{xy} is the resulting correlation of the joint binomials[4]. It does not represent some underlying firm-asset correlation that (via a bivariate normal assumption) might lead to correlated binomials. The σ's here are the usual binomial standard deviations, $\sqrt{\mu(1-\mu)}$. This formula for ρ_{xy} implies that there are bounds on ρ_{xy} since α is at least max(*0,* μ_x+μ_y-1) and at most min (μ_x, μ_y). Thus:

$$\text{[E.4]} \qquad \frac{(max(0, \mu_x + \mu_y - 1) - \mu_x\mu_y)}{\sigma_x\sigma_y} \le \rho_{x,y} \le \frac{(min(\mu_x, \mu_y) - \mu_x\mu_y)}{\sigma_x\sigma_y}$$

[4] Other researchers have used this same binomial correlation, see Lucas [95a].

Appendix F. Inferring default correlations from default volatilities

For N firms in a grouping with identical default rate (i.e., within a single credit rating category), let X_1 be a random variable which is either 1 or 0 according to each firm's default event realization with mean default rate, $\mu(X_1)$, and binomial default standard deviation, $\sigma(X_1)$, defined as follows:

[F.1]
$$\left(X_i = \begin{cases} 1 \text{ if company } i \text{ defaults} \\ 0 \text{ otherwise} \end{cases} \right)$$

$$\mu_{CrRt} = \mu(X_i) = \frac{1}{N}\sum_i^N X_i$$

$$\sigma(X_i) = \sqrt{\mu_{CrRt}(1-\mu_{CrRt})}$$

Let D represent the number of defaults, $D = \sum_i^N X_i$. So the variance of D is as follows:

[F.2]
$$\begin{aligned} VAR(D) &= \sum_i^N\sum_j^N \rho_{ij}\sigma(X_i)\sigma(X_j) \\ &= \sum_i^N\sum_j^N \rho_{ij}\sigma(X_i)^2 \qquad \text{Since all } i \text{ and } j \text{ have the same default rate.} \\ &= \sum_i^N\sum_j^N \rho_{ij}(\mu_{CrRt}-\mu_{CrRt}^2) \\ &= (\mu_{CrRt}-\mu_{CrRt}^2)\left[N + \sum_i^N\sum_{j\le i}^N \rho_{ij}\right] \end{aligned}$$

Rather than each ρ_{ij}, we are interested in the average correlation, $\bar{\rho}_{CrRt}$, and define this as follows

[F.3]
$$\bar{\rho}_{CrRt} = \left[\sum_i^N\sum_{j\le i}^N \rho_{ij}\right] / (N^2 - N)$$

and so we can now define

[F.4]
$$VAR(D) = (\mu_{CrRt}-\mu_{CrRt}^2)[N + (N^2-N)\bar{\rho}_{CrRt}].$$

Across many firms we can observe the volatility of defaults, $\sigma_{CrRt}^2 = VAR(D/N)$, thus:

$$\text{[F.5]} \qquad \sigma^2_{CrRt} = VAR\left(\frac{D}{N}\right) = \frac{VAR(D)}{N^2}$$

$$= (\mu_{CrRt} - \mu^2_{CrRt}) \cdot \frac{1 + (N-1)\bar{\rho}_{CrRt}}{N} \quad \therefore \quad \boxed{\bar{\rho}_{CrRt} = \frac{N\left(\frac{\sigma^2_{CrRt}}{\mu_{CrRt} - \mu^2_{CrRt}}\right) - 1}{N-1}}$$

This can be applied with good result in a simplified form if N is "large":

$$\text{[F.6]} \qquad \bar{\rho}_{CrRt} = \frac{N\left(\frac{\sigma^2_{CrRt}}{\mu_{CrRt} - \mu^2_{CrRt}}\right) - 1}{N-1} = \frac{8,500\left(\frac{1.4\%^2_{Ba}}{1.42\%_{Ba} - 1.42\%^2_{Ba}}\right) - 1}{8,500 - 1} = 1.3886\%$$

$$= \frac{\sigma^2_{CrRt}}{\mu_{CrRt} - \mu^2_{CrRt}} = \frac{1.4\%^2_{Ba}}{1.42\%_{Ba} - 1.42\%^2_{Ba}} = 1.4002\%$$

The estimate of 8,5000 firm-years above stems from Moody's reporting of 120 firms being rated Ba one calendar year prior to default (8,500 ≅ 120/1.42%), see Carty & Lieberman [96a].

Appendix G. International bankruptcy code summary

The practical result of the seniority standing of debt will vary across countries according to local bankruptcy law. Of course, this will affect the likely recovery rate distributions. Major differences will apply to secured versus unsecured debt. The following summary table is reproduced from Rajan & Zingales [95] – who in turn reference Keiser [94], Lo Pucki & Triantis [94], and White [93].

Table G.1
Summary of international bankruptcy codes

Country	Forms of Liquidation	Forms of Reorganization	Management Control in Bankruptcy	Automatic Stay	Rights of Secured Creditors
United States	Chapter 7: Can be voluntary (management files) or involuntary (creditors file).	Chapter 11: Can be voluntary (management files) or involuntary (creditors file).	Trustee appointed in Chapter 7. Management stays in control in Chapter 11.	Automatic stay on any attempts to collect debt once filing takes place.	Secured creditors get highest priority in any attempts to collect payment are also stayed unless court or trustee approves
Japan	Court Supervised Liquidation (*Hasan*) and Special Liquidation (*Tokubetsu Seisan*). The latter is less costly and a broader set of firms are eligible to file.	Composition (*Wagi-ho*), Corporate Arrangement (*Kaisha Seiri*) and Reorganization (*Kaisha Kosei-ho*). The list in order of increasing eligibility. Only debtors file.	Third party is appointed except in composition and corporate arrangement.	All creditors are stayed except in court supervised liquidation and composition where only unsecured creditors are stayed.	Secured Creditors have highest priority and greater voting rights in renegotiation. However, can be subject on the petition that is filed.
Germany	Liquidation (*Konkursordnung*) can be requested by creditors or debtor. Management required to file as soon as it learns it is insolvent.	Composition (*Vergleich* or *Zwangvergleich*) can be filed for only by debtor.	Receiver appointed to manage firm.	Only unsecured creditors are stayed.	Secured creditors can recover their claims even after a bankruptcy filing. No stay for secured creditors.
France	Liquidation (*Liquidation Judicaire*)	Negotiated Settlement (*Reglement Amiable*) where a court appointed conciliator attempts a settlement with creditors and Judicial Arrangement (*Redressement Judiciare*).	Debtor loses control in liquidation. Debtor remains in control otherwise but submits to court appointed administrator's decisions in a judicial arrangement.	Stay on all creditors in judicial arrangement.	Secured creditors may lose status if court determines the security is necessary for continuation of the business, or if the securing asset is sold as part of settlement.
Italy	Bankruptcy (*Fallimento*)	Preventive Composition (*Concordato Preventino*)	Debtor is removed from control over the firm.	Stay on all creditors.	Secured creditors stayed in bankruptcy, through composition allowed only if enough value exists to pay secured creditors in full and 40% of unsecured creditor claims. Secured creditors follow administrative claims in priority.
United Kingdom	Members' voluntary winding up, Creditors' voluntary winding up, Compulsory winding up.	Administration, Administrative Receivership (usually ends in sale of business), and Voluntary Arrangement.	Debtor is removed from control except in members' voluntary winding up.	Stay on all creditors in administration, on unsecured only in liquidation, and no stay in a voluntary arrangement until a proposal is approved.	Secured creditor may prevent administration order by appointing his own receiver. A creditor with a fixed or floating charge can appoint an administrative receiver to realize the security and pay the creditor.
Canada	Liquidation proceedings much like Chapter 7 in the United States	Firms can file for automatic stay under the Companies Creditors Arrangement Act or the Bankruptcy and Insolvency Act.	Firm is in control in reorganizations while trustee is appointed for liquidation. Trustee may be appointed to oversee management in some reorganizations at the discretion of the court.	Stay on all creditors in reorganization.	Secured creditors have to give 10 days notice to debtor of intent to repossess collateral. Repossession even close to bankruptcy filing is permitted, but stayed after filing.

Appendix H. Model inputs

Available for free download from the Internet http://jpmorgan.com/ is a data set of all the elements described in this technical document and necessary to implement the CreditMetrics methodology. Here, we briefly list what is provided and the format in which it is available.

CreditMetrics data files include:

- country/industry index volatilities and correlations,
- yield curves,
- spread curves, and
- transition matrices.

H.1 Common CreditMetrics data format characteristics

In general, CreditMetrics data files are text (ASCII) files which use tab characters (ASCII code 9) as column delimiters, and carriage returns/line feeds as row delimiters.

Every CreditMetrics data file begins with a header, for example:

CDFVersion	v1.0
Date	02/15/1997
DataType	CountryIndustryVolCorrs

The header is followed by a row of column headers, followed by the data.

Cells in the data rows must contain data. If the value is unavailable or not applicable, the cell should contain the keyword NULL.

H.2 Country/industry index volatilities and correlations

This file is named **indxvcor.cdf**. The data represent the weekly volatilities and correlations discussed in *Chapter 8*.

CDFVersion	v1.0					
Date	02/15/1997					
DataType	CountryIndustryVolCorrs					
IndexName	Volatility					
MSCI Australia Index (.CIAU)	0.0171	1.0000	0.6840	0.6911	0.7343	0.6377
ASX Banks & Finance Index (.ABII)	0.0219	0.6840	1.0000	0.4360	0.4580	0.4436
ASX Media Index (.AMEI)	0.0257	0.6911	0.4360	1.0000	0.5528	0.3525

H.3 Yield curves

This file is named **yldcrv.cdf**. A yield curve is defined by currency . Allowable currencies are the standard three-letter ISO currency codes (e.g., CHF, DEM, GBP, JPY, USD).

CDFVersion	v1.0		
Date	02/15/1997		
DataType	YieldCurves		
Currency	CompoundingFrequency	Maturity	YieldToMaturity
CHF	1	1.0	0.055
CHF	1	2.0	0.05707

H.4 Spread curves

Bridge will be the initial data provider for credit spreads. Their contact number is (1-800) 828 - 8010.

Bridge credit spread data is derived through a compilation of information provided by major dealers including Citibank, CS First Boston, Goldman Sachs, Liberty Brokerage, Lehman Brothers, Morgan Stanley, Salomon Brothers and J.P. Morgan. A team of evaluators reviews the contributed information on a daily bvasis to ensure accuracy and consistency.

This file is named **sprdcrv.cdf**. A spread curve is defined by a combination of rating system, rating, and a yield curve (a yield curve being defined as a combination of currency and asset type). Allowable currencies are the standard 3-letter ISO currency codes (e.g. CHF, DEM, GBP, JPY, USD). Allowable asset types are BOND, LOAN, COMMITMENT, RECEIVABLE, and MDI.

Initial data is available only for USD and BOND

CDFVersion	v1.0					
Date	02/15/1997					
DataType	SpreadCurves					
RatingSystem	Rating	Currency	AssetType	CompoundingFrequency	Maturity	Spread
Moody8	Aaa	CHF	BOND	1	5.0	0.01118
Moody8	Aaa	CHF	BOND	1	3.0	0.00866
Moody8	Aaa	CHF	BOND	1	10.0	0.015811
Moody8	Aaa	CHF	BOND	1	15.0	0.019365
Moody8	Aaa	CHF	BOND	1	2.0	0.007071
Moody8	Aaa	CHF	BOND	1	20.0	0.022361

H.5 Transition matrices

This file is named **trnsprb.cdf**. This contains transition probabilities for both Moody's major and modified ratings, S&P major rating transition matrix, and J.P. Morgan derived matrices estimating long-term ratings behavior. Initially, they will have data for a one year risk horizon. However, the format supports other horizons.

The FromRating and ToRating columns of descriptive rating labels are included for readability. CreditMetrics only utilizes the numerical FromRating and ToRating columns.

CDFVersion	v1.0					
Date	02/15/97					
DataType	TransitionProbabilities					
RatingSystem	FromRank	ToRank	FromRating	ToRating	HorizonInMonths	Probability
Moody18	0	0	Aaa	Aaa	12	0.880784
Moody18	0	1	Aaa	Aa1	12	0.050303
Moody18	0	2	Aaa	Aa2	12	0.029015

H.6 Data Input Requirements to the Software Implementation of CreditMetrics

Table H.1
Required inputs for each issuer

Data Type	Description
Issuer name	Must be unique.
Credit Rating/Agency	Long term rating that applies to the issuer's senior unsecured debt regardless of the particular seniority class(es) listed as its exposure. Each rating has an agency (Moody's, S&P, etcetera)
Market Capitalization	Stock price times number of shares outstanding
Country & Industry	Proportion of sales assigned to specified countries and industries.
Issuer-specific risk	Volatility of issuer asset returns not explained by industry/country group(s).

Table H.2
Required inputs for each exposure type

Property	Bond	Loan	Commitment	MDI	Receivable
Issuer Name	x	x	x	x	x
Portfolio	x	x	x	x	x
Currency	x	x	x	x	x
Asset type	x	x	x	x	x
Par value	x	x			x
Maturity	x	x	x		x
Seniority class	x				
Recovery rate	x	x	x	x	x
Recovery rate std	x	x	x	x	x
Fixed or floating	x	x	x		
Coupon or spread	x	x	x		
Coupon frequency	x	x	x		
Current line			x	x	
Current drawdown			x		
Expected drawdown			x		
Duration				x	
Expected exposure				x	
Average exposure				x	
Forward value				x	

Appendix I. Indices used for asset correlations

	Asset Category	Index
Australia	General	MSCI Australia Index
	Banking and finance	ASX Banks & Finance Index
	Broadcasting and media	ASX Media Index
	Construction and building materials	ASX Building Materials Index
	Chemicals	ASX Chemicals Index
	Energy	ASX Energy Index
	Food	ASX Food & Household Goods Index
	Insurance	ASX Insurance Index
	Paper and forest products	ASX Paper & Packaging Index
	Transportation	ASX Transport Index
Austria	General	MSCI Austria Index
Belgium	General	MSCI Belgium Index
Canada	General	MSCI Canada Index
	Automobiles	Toronto SE Automobiles & Parts Index
	Banking and finance	Toronto SE Financial Services Index
	Broadcasting and media	Toronto SE Broadcasting Index
	Construction and building materials	Toronto SE Cement & Concrete Index
	Chemicals	Toronto SE Chemicals Index
	Hotels	Toronto SE Lodging, Food & Health Index
	Insurance	Toronto SE Insurance Index
	Food	Toronto SE Food Stores Index
	Electronics	Toronto SE Electrical & Electronics Index
	Metals mining	Toronto SE Metals Mines Index
	Energy	Toronto SE Integrated Oils Index
	Health care and pharmaceuticals	Toronto SE Biotechnology & Pharmaceuticals Index
	Publishing	Toronto SE Publishing & Printing Index
	Transportation	Toronto SE Transportation Index
Germany	General	MSCI Germany Index
	Automobiles	CDAX Automobiles Index
	Banking and finance	CDAX Investment Company Index
	Chemicals	CDAX Chemicals Index
	Construction and building materials	CDAX Construction Index
	Insurance	CDAX Insurance Index
	Machinery	CDAX Machinery Index
	Paper and forest products	CDAX Paper Index
	Textiles	CDAX Textiles Index
	Transportation	CDAX Transport Index
	Utilities	CDAX Utilities Index
Greece	General	MSCI Greece Index
	Banking and finance	Athens SE Banks Index
	Insurance	Athens SE Insurance Index
Finland	General	MSCI Finland Index
	Banking and finance	Helsinki SE Banks & Finance Index
	Metals mining	Helsinki SE Metal Index
	Paper and forest products	Helsinki SE Forest & Wood Index
	Insurance	Helsinki SE Insurance & Investment Index

	Asset Category	Index
France	General	MSCI France Index
	Automobiles	SBF Automotive Index
	Banking and finance	SBF Finance Index
	Construction and building materials	SBF Construction Index
	Energy	SBF Energy Index
	Food	SBF Food Index
Hong Kong	General	MSCI Hong Kong Index
	Banking and finance	Hang Seng Finance Index
	Utilities	Hang Seng Utilities Index
Indonesia	General	MSCI Indonesia Index
Italy	General	MSCI Italy Index
	Chemicals	Milan SE Chemical Current Index
	Banking and finance	Milan SE Financial Current Index
	Food	Milan SE Food & Groceries Current Index
	Paper and forest products	Milan SE Paper & Print Current Index
	Metals mining	Milan SE Mine & Metal Current Index
Japan	General	MSCI Japan Index
	Banking and finance	Topix Banking Index
	Broadcasting and media	Topix Communications Index
	Construction and building materials	Topix Construction Index
	Chemicals	Topix Chemical Index
	Electronics	Topix Electrical Appliances Index
	Food	Topix Foods Index
	Insurance	Topix Insurances Index
	Machinery	Topix Machinery Index
	Metals mining	Topix Mining Index
	Health care and pharmaceuticals	Topix Pharmaceuticals Index
	Paper and forest products	Topix Pulp and Paper Index
	Energy	Topix Electric Power and Gas Index
	Oil and gas -- refining and marketing	Topix Oil and Coal Products Index
	Textiles	Topix Textile Products Index
	Transportation	Topix Transportation Equipment Index
Korea	General	MSCI Korea Index
	Banking and finance	Korea SE Finance Major Index
	Construction and building materials	Korea SE Construction Major Index
	Chemicals	Korea SE Chemical Company Major Index
	Food	Korea SE Food & Beverage Major Index
	Insurance	Korea SE Insurance Major Index
	Machinery	Korea SE Fabricated Metal & Machinery Major Index
	Metals mining	Korea SE Mining Major Index
	Paper and forest products	Korea SE Paper Product Major Index
	Textiles	Korea SE Textile & Wear Major Index
	Transportation	Korea SE Transport & Storage Major Index
Malaysia	General	MSCI Malaysia Index
	Banking and finance	KLSE Financial Index
	Metals mining	KLSE Mining Index

	Asset Category	Index
Mexico	General	MSCI Mexico Index
	Transportation	Mexican SE Commercial & Transport Index
	Metals mining	Mexican SE Mining Index
	Construction and building materials	Mexican SE Construction Index
New Zealand	General	MSCI New Zealand Index
Norway	General	MSCI Norway Index
	Banking and finance	Oslo SE Bank Index
	Insurance	Oslo SE Insurance Index
Philippines	General	MSCI Philippines Index
	Metals mining	Philippine SE Mining Index
	Oil and gas -- refining and marketing	Philippine SE Oil Index
Poland	General	MSCI Poland Index
Portugal	General	MSCI Portugal Index
Singapore	General	MSCI Singapore Index
	Hotels	All-Singapore Hotel Index
	Banking and finance	All-Singapore Finance Index
Spain	General	MSCI Spain Index
Sweden	General	MSCI Sweden Index
	Banking and finance	Stockholm SE Banking Sector Index
	Construction and building materials	Stockholm SE Real Estate & Construction Index
	Chemicals	Stockholm SE Pharmaceutical & Chemical Index
	Paper and forest products	Stockholm SE Forest Industry Sector Index
Switzerland	General	MSCI Switzerland Index
	Banking and finance	SPI Banks Cum Dividend Index
	Construction and building materials	SPI Building Cum Dividend Index
	Chemicals	SPI Chemical Cum Dividend Index
	Electronics	SPI Electronic Cum Dividend Index
Thailand	General	MSCI Thailand Index
	Banking and finance	SET Finance Index
	Chemicals	SET Chemicals & Plastics Index
	Electronics	SET Electrical Components Index
	Technology	SET Electrical Products &Computers Index
	Construction and building materials	SET Building & Furnishing Materials Index
	Energy	SET Energy Index
	Food	SET Food & Beverages Index
	Health care and pharmaceuticals	SET Health Care Services Index
	Insurance	SET Insurance Index
	Hotels	SET Hotel & Travel Index
	Machinery	SET Machinery & Equipment Index
	Metals mining	SET Mining Index
	Paper and forest products	SET Pulp & Paper Index
	Publishing	SET Printing & Publishing Index
	Textiles	SET Textile Index
	Transportation	SET Transportation Index

	Asset Category	Index
United Kingdom	General	MSCI United Kingdom Index
	Banking and finance	FT-SE-A 350 Banks Retail Index
	Broadcasting and media	FT-SE-A 350 Media Index
	Construction and building materials	FT-SE-A 350 Building Materials & Merchants Index
	Chemicals	FT-SE-A 350 Chemicals Index
	Electronics	FT-SE-A 350 Electronic & Electrical Equipment Index
	Food	FT-SE-A 350 Food Producers Index
	Health care and pharmaceuticals	FT-SE-A 350 Health Care Index
	Insurance	FT-SE-A 350 Insurance Index
	Hotels	FT-SE-A 350 Leisure & Hotels Index
	Metals mining	FT-SE-A 350 Extractive Industries Index
	Oil and gas -- refining and marketing	FT-SE-A 350 Gas Distribution Index
	Energy	FT-SE-A 350 Oil Integrated Index
	Paper and forest products	FT-SE-A 350 Paper, Packaging & Printing Index
	Telecommunications	FT-SE-A 350 Telecommunications Index
	Textiles	FT-SE-A 350 Textiles & Apparel Index
	Transportation	FT-SE-A 350 Transport Index
United States	General	MSCI United States Of America Index
	Automobiles	S&P Automobiles Index
	Banking and finance	S&P Financial Index
	Broadcasting and media	S&P Broadcasting (Television, Radio & Cable)
	Construction and building materials	S&P Building Materials Index
	Chemicals	S&P Chemicals Index
	Electronics	S&P Electronics (Instrumentation)
	Energy	S&P Energy Index
	Entertainment	S&P Entertainment Index
	Food	S&P Foods Index
	Health care and pharmaceuticals	S&P Health Care Index
	Insurance	S&P Insurance Composite Index
	Hotels	S&P Lodging-Hotels Index
	Machinery	S&P Machinery (Diversified)
	Manufacturing	S&P Manufacturing (Diversified)
	Metals mining	S&P Metals Mining Index
	Oil and gas -- refining and marketing	S&P Oil & Gas (Refining & Marketing)
	Paper and forest products	S&P Paper & Forest Products Index
	Publishing	S&P Publishing Index
	Technology	S&P Technology Index
	Telecommunications	S&P Telecommunications (Long Distance)
	Textiles	S&P Textiles (Apparel)
	Transportation	S&P Transport Index
	Utilities	S&P Utilities Index
South Africa	General	MSCI South Africa (Gross Dividends Reinvested)
	Banking and finance	Johannesburg SE Financial Index
	Metals mining	Johannesburg SE Mining Holding Index

	Asset Category	Index
MSCI Worldwide	Automobiles	Automobiles Price Index
	Banking and finance	Banking Price Index
	Broadcasting and media	Broadcasting & Pubs Price Index
	Construction and building materials	Construction & Housing (US$) Price Index
	Chemicals	Chemicals Price Index
	Electronics	Electronic Comps/Inst. Price Index
	Energy	Energy Sources Price Index
	Entertainment	Recreation & Other Goods Price Index
	Food	Food & Household Products Price Index
	Health care and pharmaceuticals	Health & Personal Care Price Index
	Insurance	Insurance Price Index
	Hotels	Leisure & Tourism Price Index
	Machinery	Machinery & Engineering Price Index
	Metals mining	Metals Nonferrous Price Index
	Paper and forest products	Forest Products/Paper Price Index
	Telecommunications	Recreation & Telecommunications Price Index
	Textiles	Textiles & Apparel Price Index
	Transportation	Transport Shipping Price Index
	Utilities	Utilities Electric & Gas Price Index
MSCI Regional	EMF Latin America	
	Europe 14	
	Nordic Countries	
	North America	
	Pacific	
	Pacific ex Japan	

Reference

Glossary of terms

This glossary defines important terms in CreditMetrics.

accounting analytic. The use of financial ratios and fundamental analysis to estimate firm specific credit quality examining items such as leverage and coverage measures, with an evaluation of the level and stability of earnings and cash flows. (See *page 58.)*

allowance for loan and lease losses. An accounting reserve set aside to equate expected (mean) losses from credit defaults. It is common to consider this reserve as the buffer for expected losses and some risk-based economic capital as the buffer for unexpected losses. *(See page 60.)*

autocorrelation (serial correlation). When time series observations have a non-zero correlation over time. Two empirical examples of autocorrelation are:

- Interest rates exhibit mean reversion behavior and are often negatively autocorrelated (i.e., an up move one day will suggest a down move the next). But note that mean reversion does not technically necessitate negative autocorrelation.

- Agency credit ratings typically exhibit move persistence behavior and are positively autocorrelated during downgrades (i.e., a downgrade will suggest another downgrade soon). But, for completeness, note that upgrades do not better predict future upgrades – we find, they predict a "quiet" period; see also Altman & Kao [92].

(See *page 32.)*

average exposure. Credit exposure arising from market-driven instruments will have an ever-changing mark-to-market exposure amount. The amount of exposure relevant to our credit analysis is the time-bucketed average exposure in each forward period across the life of the transaction across all – probability weighted – market rate paths. (See *page 49.)*

average shortfall. The expected loss given that a loss occurs, or as the expected loss given that losses exceed a given level. (See *page 137.)*

credit exposure. The amount subject to changes in value upon a change in credit quality through either a market based revaluation in the event of an up(down)grade or the application of a recovery fraction in the event of default. (See *page 42*).

commitment. A legally binding obligation (subject usually both to conditions precedent and to continuing conditions) to make available loans or other financial accommodation for a specified period; this includes revolving facilities. Even during publicly known credit distress, a commit can be legally binding if drawndown before it is formally withdraw for cause.

concentration risk. Portfolio risk resulting from increased exposure to one obligor or groups of correlated (e.g., by industry or location) obligors. (See *page 6.)*

correlation. A linear statistical measure of the co-movement between two random variables. A correlation (Greek letter "ρ", pronounced "rho") will range from +1.0 to -1.0. Observing "clumps" of firms defaulting together by industry or geographically is an example of positive correlation of default events. *(*See *page 35.)*

$$\rho_{X,Y} = \frac{COV_{X,Y}}{\sigma_X \cdot \sigma_Y} = \frac{\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{N} (X_i - \bar{X})^2} \cdot \sqrt{\sum_{i=1}^{N} (Y_i - \bar{Y})^2}}$$

counterparty. The partner in a credit facility or transaction in which each side takes broadly comparable credit risk to the other. When a bank lends a company money, the borrower (not Counterparty) has no meaningful credit risk to the bank. When the same two agree on an at-the-money forward exchange contract or swap, the company is at risk if the bank fails just as much as the bank is at risk if the counterparty fails (although for the opposite movement in exchange or interest rates). After inception, swap positions often move in/out-of-the-money and the relative credit risk changes accordingly. *(*See *page 47.)*

covenants. The terms under which a credit facility will be monitored. Covenants are most effective when they are specific measures that state the acceptable limits for change in the obligor's financial and overall condition. They clearly define what is meant by "significant" deterioration in the obligor's credit quality. Financial covenants are more explicit (and therefore more desirable) than a "material adverse change" clause. Cross default provisions are common: allowing acceleration of debt repayment. *(*See *page 43.)*

credit distress. A firm can have many types of credit obligations outstanding. These may be of all manner of seniority, security and instrument type. In bankruptcy proceedings, it is not uncommon for different obligations to realize different recovery rates including perhaps 100% recovery – zero loss. In our terminology, it is the obligor that encounters credit distress carrying all of his obligations with him even though some of these may not realize a true *default* (i.e., some may have zero loss). *(*See *page 65.)*

credit exposure. The amount subject to either changes in value upon credit quality up(down)grade or loss in the event of default. *(*See *page 42.)*

credit quality. Generally meant to refer to an obligor's relative chance of default, usually expressed in alphabetic terms (e.g., Aaa, Aa, A, etc.). CreditMetrics makes use of an extended definition that includes also the volatility of up(down)grades.

credit scoring. Generically, credit scoring refers to the estimation of the relative likelihood of default of an individual firm. More specifically, this is a reference to the application of linear discriminant analysis to combine financial rations to quantitatively predict the relative chance of default. *(*See *page 57.)*

current exposure. For market-driven instruments, the amount it would cost to replace a transaction today should a counterparty default. If there is an enforceable netting agreement with the counterparty, then the current exposure would be the net replacement cost; otherwise, it would be the gross amount.

default probability. The likelihood that an obligor or counterparty will encounter credit distress within a given time period. "Credit distress" usually leads to either an omitted delayed payment or distressed exchange which would impair the value to senior unsecured debt holders. Note that this leaves open the possibilities that:

- Subordinated debt might default without impairing senior debt value, and
- Transfers and clearing might continue even with a senior debt impairment.

*(*See *page 65.)*

dirty price. Inclusion of the accrued value of the coupon in the quoted price of a bond For instance, a 6% annual coupon bond trading at par would have a dirty price of $106 just prior to coupon payment. CreditMetrics estimates dirty prices since the coupon is paid in non-default states but assumed not paid in default. *(*See *page 10.)*

distressed exchange. During a time of credit distress, debt holders may be effectively forced to accepted securities in exchange for their debt claim – such securities being of a lower value than the nominal present value of their original claim. They may have a lower coupon, delayed sinking funds, and/or lengthened maturity. For historical estimation of default probabilities, this would count as a default event since it can significantly impair value. In the U.S., exchange offers on traded bonds may be either registered with the SEC or unregistered if they meet requirements under Section 3(a)(9) of the Securities Act of 1933. Refer to Asquith, Mullins & Wolff [89]. *(*See *page 65.)*

duration. The weighted average term of a security's cash flows. The longer the duration, the larger the price movement given a 1bp change in the yield.

expected excession of a percentile level. For a specified percentile level, the expected loss given that the loss is more extreme than that percentile level. *(*See *page 137.)*

exposure. The amount which would be lost in a default given the worst possible assumptions about recovery in the liquidation or bankruptcy of an obligor. For a loan or fully drawn facility, this is the full amount plus accrued interest; for an unused or partly used facility it is the full amount of the facility, since the worst assumption is that the borrower draws the full amount and then goes bankrupt.

- Exposure is not usually a statistical concept; it does not make any attempt to assess the probability of loss, it only states the amount at risk.
- For market-driven instruments, (e.g., foreign exchange, swaps, options and derivatives) a proxy for exposure is estimated given the volatility of underlying market rates/prices. See Loan Equivalent Exposure.

facility. A generic term which includes loans, commitments, lines, letters, etc. Any arrangement by which a bank accepts credit exposure to an obligor. *(*See *page 79.)*

fallen angels. Obligors having both relatively high percentage risk and relatively large exposure, whose large exposures were created when their credit ratings were better, but who now have much higher percentage risk due to recent downgrades.

ISDA. (Institutional Swap Dealers Association) A committee sponsored by this organization was instrumental in drafting an industry standard under which securities dealers would trade swaps. Included in this was a draft of a master agreement by which institutions outlined their rights to net multiple offsetting exposures which they might have to a counterparty at the time of a default.

issuer exposure. The credit risk to the issuer of traded instruments (typically a bond, but also swaps, foreign exchange, etc.). Labeling credit spread volatility as either market or credit risk is a question of semantics. CreditMetrics addresses market price volatility as it is caused by changes in credit quality.

joint probabilities. Stand-alone obligors have some likelihood of each possible credit quality migration. Between two obligors there is some likelihood of each possible joint credit quality migration. The probabilities are commonly influences by the correlation between the two obligors. *(*See *page 36.)*

kurtosis. Characterizes relative peakedness or flatness of a given distribution compared to a normal distribution. It is the fourth moment of a distribution.

$$K_X = \left\{\frac{N^2 - 2N + 3}{(N-1)(N-2)(N-3)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}}{\sigma_x}\right)^4\right\} - 3\frac{(N-1)(N-3)}{N(N-2)(N-3)}$$

Since the unconditional normal distribution has a kurtosis of 3, excess kurtosis is defined as *Kx-3*. See *leptokurtosis.*

leptokurtosis (fat tails). The property of a statistical distribution to have more occurrences far away from the mean than would be predicted by a Normal distribution. Since a normal distribution has a kurtosis measure of 3, excess kurtosis is defined as $Kx-3 > 0$.

A credit portfolio loss distribution will typically be leptokurtotic given positive obligor correlations or coarse granularity in the size / number of exposures. This means that a downside confidence interval will be further away from the mean than would be expected given the standard deviation and skewness.

letter of credit. A promise to lend issued by a bank which agrees to pay the addressee, the “beneficiary”, under specified conditions on behalf of a third party, also known as the “account party”. *(*See *page 46)*.

There are different types of letters of credit. A *financial* letter of credit (also termed a stand-by letter of credit) is used to assure access to funding without the immediate need for funds and is triggered at the obligor’s discretion. A *project* letter of credit is secured by a specific asset or project income. A *trade* letter of credit is typically triggered by a non credit related (and infrequent) event.

liquidity. There are two separate meanings:

- At the enterprise level, the ability to meet current liabilities as they fall due; often measures as the ratio of current assets to current liabilities.
- At the security level, the ability to trade in volume without directly moving the market price; often measured as bid/ask spread and daily turnover.

loan exposure. The face amount of any loan outstanding plus accrued interest plus. See *dirty price*.

marginal standard deviation. Impact of a given asset on the total portfolio standard deviation. *(*See *page 129.)*

marginal statistic. A statistic for a particular asset which is the difference between that statistic for the entire portfolio and that for the portfolio not including the asset.

market-driven instruments. Derivative instruments that are subject to counterparty default (e.g., swaps, forwards, options, etc.). The distinguishing feature of these types of credit exposures is that their amount is only the net replacement cost – the amount the position is in-the-money – rather than a full notional amount. *(*See *page 47)*.

market exposure. For market-driven instruments, there is an amount at risk to default only when the contract is in-the-money (i.e., when the replacement cost of the contract exceeds the original value). This exposure/uncertainty is captured by calculating the netted mean and standard deviation of exposure(s).

Markov process. A model which defines a finite set of "states" and whose next progression is determinable solely by the current state. A transition matrix model is an example of a Markov process. *(*See *page 71.)*

mean. A statistical measure of central tendency. Sum of observation values divided by the number of observations. It is the first moment of a distribution. There are two types of means. A mean calculated across a sample from a population is referred to as $\bar{X}$, while means calculated across the entire population – or means given exogenously – are referred to as μ, pronounced "mu." *(*See *page 15.)*

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

mean reversion. The statistical tendency in a time series to gravitate back towards a long term historical level. This is on a much longer scale than another similar measure, called autocorrelation; and these two behaviors are mathematically independent of one another.

migration. Credit quality migration describes the possibility that a firm or obligor with some credit rating today may move to (or "migrate") to potentially any other credit rating – or perhaps default – by the risk horizon. *(*See *page 24.)*

migration analysis. The technique of estimating the likelihood of credit quality migrations. See *transition matrix.*

moments (of a statistical distribution). Statistical distributions show the frequency at which events might occur across a range of values. The most familiar distribution is a Normal "Bell Shaped" curve. In general though, the shape of any distribution can be described by its (infinitely many) moments.

1. The **first** moment is the ***mean*** which indicates the central tendency.
2. The **second** moment is the ***variance*** which indicates the width.
3. The **third** moment is the ***skewness*** which indicates any asymmetric "leaning" either left or right.
4. The **fourth** moment is the ***kurtosis*** which indicates the degree of central "peakedness" or, equivalently, the "fatness" of the outer tails.

monotinicity. See *rank order.*

move persistence. The statistical tendency in a time series to move on the next step in the same direction as the previous step (see also, positive autocorrelation).

netting. There are at least three types of netting:

close-out netting: In the event of counterparty bankruptcy, all transactions or all of a given type are netted at market value. The alternative would allow the liquidator to choose which contracts to enforce and which not to (and thus potentially "cherry pick"). There are international jurisdictions where the enforceability of netting in bankruptcy has not been legally tested.

netting by novation: The legal obligation of the parties to make required payments under one or more series of related transactions are canceled and a new obligation to make only the net payment is created.

settlement or payment netting: For cash settled trades, this can be applied either bilaterally or multilaterally and on related or unrelated transactions.

notional amount. The face amount of a transaction typically used as the basis for interest payment calculations. For swaps, this amount is not itself a cash flow. Credit exposure arises – not against the notional – but against the present value (market replacement cost) of in-the-money future terminal payment(s).

obligor. A party who is in debt to another: (i) a loan borrower; (ii) a bond issuer; (iii) a trader who has not yet settled; (iv) a trade partner with accounts payable; (v) a contractor with unfinished performance, etc.; see *Counterparty.* *(*See *page 5.)*

option theoretic. An approach to estimating the expected default frequency of a particular firm. It applies Robert Merton's model-of-the-firm which states that debt can be valued as a put option of the underlying asset value of the firm. *(*See *page 36.)*

originator. The financial institution that extends credit on a facility which may later be held by another institution through, for instance, a loan sale.

peak exposure. For market-driven instruments, the maximum (perhaps netted) exposure expected with 95% confidence for the remaining life of a transaction. CreditMetrics does not utilize this figure because it is not possible to aggregate tail statistics across a portfolio, since it is not the case that these "peaks" will all occur at the same time.

percent marginal standard deviation. Expression in percent terms of the impact of a given asset on the total portfolio standard deviation. *(*See *page 129.)*

percentile level. A measure of risk based on the specified confidence level of the portfolio value distribution: e.g., the likelihood that the portfolio market falls below the 99th percentile number is 1%. *(*See *page 16.)*

pricing grid. A schedule of credit spreads listed by credit rating that are applied to either a loan or Credit-Sensitive Note (CSN) upon an up(down)grade of the obligor or issuer. If the spreads are specified at market levels, then such terms reduce the volatility of value across all non-default credit quality migrations by keeping the instrument close to par. *(*See *page 67.)*

rank order. A quality of data often found across credit rating categories where values consistently progress in one direction – never reversing direction. Mathematicians term this property of data, *monotonicity.* *(*See *page 66.)*

receivables. Non interest bearing short term extensions of credit in the normal course of business, *"trade credit,"* that are at risk to the extent that the customer may not pay its obligation in full. *(*See *page 42)*.

revolving commitment (revolver). A generic term referring to some facility which a client can use – or refrain from using – without canceling the facility.

sector loadings. For correlation analysis, a firm or industry group is said to be dependent upon underlying economic factors or "sectors" such as: (i) the market as a whole, (ii) interest rates, (iii) oil prices, etc. As two industries "load" – are influenced by – common factors, they will have a higher correlation between them.

serial correlation. See *autocorrelation.*

skewness. A statistical measure which characterizes the asymmetry of a distribution around its mean. Positive skews indicate asymmetric tail extending toward positive values (right-hand side). Negative skewness implies asymmetry toward negative values (left-hand side). It is the third moment of a distribution.

$$S_x = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} \left(\frac{X_i - \bar{x}}{\sigma_X} \right)^3$$

The distribution of losses across a credit portfolio will be positively skewed if there is positive correlation between obligors or the size / number of exposures is coarsely granular. This means that the confidence interval out on the downside tail will be further away from the mean than would be expected given the portfolio's standard deviation alone.

stand-alone standard deviation. Standard deviation of value for an asset computed without regard for the other instruments in the portfolio. *(*See *page 129.)*

standard deviation. A statistical measure which indicates the width of a distribution around the mean. A standard deviation (Greek letter "σ," pronounced "sigma") is the square root of the second moment of a distribution.

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{x})^2}$$

The distribution of losses across a credit portfolio will (typically) have a standard deviation which is much larger than its mean and yet negative losses are not possible. Thus, it is not meaningful to think of a standard deviation as being a +/- range within which will lie X% of the distribution – as one would naturally do for a normal distribution. *(*See *page 15.)*

stand-alone percent standard deviation. Stand-alone standard deviation expressed as a percentage of the mean value for the given asset. *(*See *page 129.)*

stand-by letter of credit. See *letter of credit.*

state of the world. A credit rating migration outcome; a new credit rating arrived at the risk horizon. This can be either for a single obligor on a stand-alone basis or jointly between two obligors. *(See page 24.)*

stochastic. Following a process which includes a random element. *(*See *page 70.)*

trade credit. See *"receivables."*

transition matrix. A square table of probabilities which summarize the likelihood that a credit will migrate from its current credit rating today to any possible credit rating – or perhaps default – in one period. *(*See *page 25.)*

unexpected losses. A popular term for the volatility of losses but also used when referring to the *realization* of a large loss which, in retrospect, was unexpected. (See *page 60*.)

value-at-risk (VaR). A measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a preset horizon. *(*See *page 5.)*

variance. A statistical measure which indicates the width of a distribution around the mean. It is the second moment of a distribution. A related measure is the standard deviation, which is the square root of the variance. *(*See *page 16.)*

$$VAR_x = \sigma_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{x})^2$$

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Part III: Applications

Look to the J.P. Morgan site on the Internet located at http://jpmorgan.com for updates of examples illustrating points from this technical document or useful new tools. Numerical examples are implemented in Excel spreadsheets.

These Excel spreadsheets are intended as a demonstration of the CreditMetrics credit risk management methodology. They have been designed as an educational tool and should not be used for the risk estimation of actual portfolio positions. Separately, J.P. Morgan sells software which embodies the CreditMetrics methodology. If you have any questions about the use of these spreadsheets contact your local J.P. Morgan representative or:

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