Dynamic Adverse Selection and Liquidity

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Traditional View

- More informed traders $\implies$ more adverse selection for market makers $\implies$ larger bid-ask spreads (less liquidity)
- Bagehot (1971)

  > *"The essence of market making, viewed as a business, is that in order for the market maker to survive and prosper, his gains from liquidity-motivated transactors must exceed his losses to information motivated transactors. [...] The spread he sets between his bid and asked price affects both: the larger the spread, the less money he loses to information-motivated, transactors and the more he makes from liquidity-motivated transactors."*

Glosten and Milgrom (1985)

- Risky asset has liquidation value $v \in \{0, 1\}$
- Competitive risk-neutral dealer $\implies$ sets $p_t = \mathrm{E}_t(v)$
- Trading at $t = 0, 1, 2, \ldots$
  - At most one unit
  - Buy order executes at ask a_t, sell order executes at bid b_t
- At each t, trader selected at random:
  - Informed (fraction ρ): observes v, buys if $v > a_t$ or sells if $v < b_t$
  - Uninformed (fraction ρ): buys or sells with equal probability

Glosten and Milgrom (1985)

- Equilibrium is non-stationary
  - Eventually the dealer learns v and spread becomes zero
- Evolution in time: when ρ is large (many informed traders)
  - Initially: the bid-ask spread is larger
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- What happens in a stationary equilibrium?
- We need time-varying v to make it interesting

Model

- Same as GM85, except v_t follows a random walk
  - $v_{t+1} - v_t$ is normal $\mathcal{N}(0, \sigma_v^2)$
  - σ_v is called *fundamental volatility*
- **Simplifying assumption 1:** an informed trader observing $v_t \in [b_t, a_t])$ is immediately replaced by an uninformed trader
  - Otherwise there may be no trade at t
- **Simplifying assumption 2:** dealer is approximately Bayesian
  - Regards v_t as normally distributed $\mathcal{N}(\mu_t, \sigma_t^2)$
  - Correctly computes posterior first and second moments

Results

- Equilibrium converges to a stationary equilibrium
  - Non-stationary behavior is similar to GM85
- Stationary equilibrium
  - Spread is constant, equal to $2\sigma_v$
  - Spread does not depend on informed share ρ
- Positive shock to informed share $\implies$ spread initially jumps, then gradually reverts to stationary value
  - Liquidity is resilient, for purely informational reasons

Efficient Density

- The dealer regards v_t as normally distributed: $\mathcal{N}(\mu_t, \sigma_t^2)$
  - This is called the *efficient density*
  - μ_t is the *efficient mean*
  - σ_t is the *efficient volatility*
- Efficient volatility measures dealer's uncertainty about v_t

Equilibrium: Evolution of Efficient Density

- Efficient mean evolves according to:

$$\mu_{t+1} = \mu_t \pm \delta\sigma_t$$

where δ is an increasing function of ρ

- Efficient volatility evolves according to

$$\sigma_{t+1}^2 = (1-\delta^2)\sigma_t^2 + \sigma_v^2$$

Therefore

$$\sigma_t^2 - \sigma_*^2 = (\sigma_0^2 - \sigma_*^2)(1-\delta^2)^t$$

where

$$\sigma_* = \frac{\sigma_v}{\delta}$$

Equilibrium: Evolution of Bid-Ask Spread

- Spread is always proportional to efficient volatility:

$$s_t = 2\delta\sigma_t$$

- Spread converges to

$$s_* = 2\sigma_v$$

Stationary Equilibrium

- Efficient mean evolves according to:

$$\mu_{t+1} = \mu_t \pm \sigma_v$$

- Efficient volatility is constant

$$\sigma_* = \frac{\sigma_v}{\delta}$$

- Spread is constant

$$s_* = 2\sigma_v$$

Note: Spread does not depend on informed share ρ

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    - Spread $\nearrow$: **Dynamic efficiency effect**
- Dynamic efficiency: many informed trades (ρ is high) $\implies$ dealer learns fast $\implies$ uncertainty $\searrow$, spread $\searrow$

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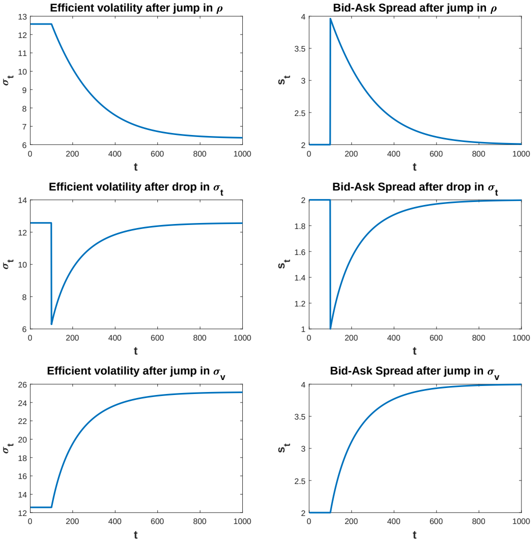
- So $\Delta = \sigma_v \implies$ spread independent of the informed share

Evolution after Shocks



Collin-Dufresne and Fos (2015,2016)

- In those papers, more informed trading is associated with more liquidity
- Intuition similar to Admati and Pfleiderer (1988):
  - Discretionary liquidity traders cluster in time
  - Discretionary informed traders prefer to trade in more liquid times
  - Despite the increase in informed trading in liquid times, market remains more liquid in those times
- Current mechanism: **dynamic efficiency**
  - Even when liquidity trading is constant over time

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