

Fiscal and Monetary Policies and Economic Growth

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Abstract

The paper analyses the way in which monetary and fiscal policy influences the performances of economic growth. The analysis is made on the basis of a dynamic model with discrete variables of the Sidrauski- Brock type, with infinite-lived households and money in the utility function. The model is with a representative private agent and a government sector consisting of a consolidated fiscal authority and central bank. Households receive an exogenous perishable endowment each period, decide about consumption and pay net real lump-sum tax.

The state variable of the model is government debt, and the decision variables are: consumption and the amount of money detained by the agent.

The optimality conditions are obtained by using the Maximum Principle for discrete dynamic systems. A qualitative analysis of the optimal trajectories is performed, on the basis of the information provided by the Maximum Principle, concerning the dynamics of the dual variable and the properties of the Lagrange multipliers.

Finally, we analyze the influence of several monetary and fiscal decisions on the optimal trajectories and on the performance-function of the model.

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1. Introduction

In recent years relative effectiveness of monetary and fiscal policy action on economic growth has been debated by both economists and policy makers.

The traditional optimal currency areas literature pointed out long ago that, in a monetary union, fiscal policy has to play a more important role in cyclical stabilization given the loss of national monetary independence. This is particularly the case if shocks are not perfectly correlated across frontiers. Fiscal flexibility, together with budgetary discipline and co-ordination, has come to be seen as a central pillar of fiscal policy in a currency area (Commission, 1990). The Stability and Growth Pact (SGP) has been the operational response of EU countries to the quest for budgetary discipline in EMU.

Recent theoretical and empirical developments have shed new light on the ‘old’ issue of the interaction between monetary and fiscal authorities.

There are numerous studies, both theoretical and empirical, analyzing the relation between inflation and long-run growth.

In the past decade the development of the endogenous growth literature pioneered by Romer (1986), Lucas (1988), and Rebelo (1991) has enhanced our understanding about how an economy’s growth performance can be affected by public policies. For instance, Barro (1990) and King and Rebelo (1990) study the effects of fiscal policies, such as government spending and taxation, on economic growth. The general conclusion is that taxation adversely affects long-run growth performance and that the quantitative impacts are much larger than those found in exogenous growth models. Chari, Jones, and Manuelli (1995) and van der Ploeg and Alogoskoufis (1994) examine the effects of monetary policies, such as changes in the growth rates of nominal money supply, on long-run real activity. These authors find support for the conventional wisdom that inflation and long-run growth are inversely related. These studies also represent an advance in our understanding of the impact of alternative policies on inflation and growth.

Recently economists have been paying increasing attention to a dynamic general equilibrium approach to the theory of price level that is often called the fiscal theory of the price level, or FTPL. This way of thinking emphasizes the role of fiscal and monetary policy in determining the risk and return properties of government

liabilities. It is particularly useful in analyzing proposals for large-scale institutional changes that imply shifts in monetary and fiscal policies. When dollarization is considered from this perspective, some disadvantages are brought to light that may not be so apparent from other points of view.

The so-called fiscal theory of the price level (FTPL) has highlighted that, if government solvency is not guaranteed, monetary policy will not be able to control the price level. In order to ensure stability, fiscal policy has to react sufficiently strongly to a rise in the interest rate in the event of inflationary pressures by increasing the primary surplus. In other words, monetary policy aiming at keeping inflation in check – as the ECD is mandated to behave – has to go hand-in-hand with a fiscal policy obeying a solvency constraint. Once the FTPL is applied to the EMU institutional set-up, however, seemingly different conclusions are drawn. While Sims (1999) considers the Maastricht *cum* SGP rules insufficient to rule out FTPL's doom scenario, Canzoneri and Diba (2001) conclude that the SGP appears far too strict from the point of view of guaranteeing fiscal solvency. The latter authors, in particular, call for shifting attention from actual to cyclically-adjusted budget balances in assessing the compliance of EMU members with budgetary prudence so as not to hamper fiscal stabilization.

2. The model

2.1 Household and government behavior

In this section we study a dynamic model, originally due to Obstfeld and Rogoff (1983), that shows how standard monetary models have a continuum of equilibrium time paths for the inflation rate. The model is a Sidrauski – Brock type model with infinite-lived households and money in the utility function. Such models are ones in which it is common for Ricardian equivalence results to be obtained, which makes the fiscalist theory's claim more remarkable than if developed in a model of the overlapping generations type. This type of models was used by several

authors, such as Woodford (1994, 1995, 2000), Sims (1994, 1996), McCallum (1998), Buiter (1999, 2001), Kocherlakota and Phelan (1999), Chanda and Nolan (2002).

The model is with a representative private agent and a government sector, consisting of a consolidated fiscal authority and central bank.

There is no uncertainty and markets are complete. Time indexed by t is measured in discrete intervals of equal length, normalized to unity.

Households receive an exogenous perishable endowment, $y_t > 0$, each period, consume $c_t \geq 0$ and pay net real lump-sum tax h_t .

The money price of output in period t is P_t . The quantities of money and nominal bonds outstanding at the beginning of period t (and the end of period $t-1$) are denoted M_t , respectively B_t . The i_t is one-period risk-free nominal interest rate in period t , and r_t the one-period risk-free real interest rate in period t .

The Fisher equation is:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_t} \quad (1)$$

where

$$1 + \pi_t = \frac{P_{t+1}}{P_t}.$$

The single-period household budget constraint is:

$$P_t c_t + M_{t+1} + \frac{B_{t+1}}{1 + i_t} \leq M_t + B_t + P_t y_t - P_t h_t, \forall t \geq 0 \quad (2)$$

We denote:

$$W_t = B_t + M_t \quad (3)$$

In order to transform inequality (2) into an equation, we introduce the slack variable x_t :

$$\frac{W_{t+1}}{1 + i_t} = W_t + P_t (y_t - h_t - c_t) - \frac{i_t}{1 + i_t} M_{t+1} - \frac{x_t}{1 + i_t}, x_t \geq 0, \forall t \geq 0 \quad (4)$$

In equilibrium, when planned expenditure equals supply, it follows that the economy-wide resource constraint is given by:

$$c_t + g_t = y_t, \forall t \geq 0 \quad (5)$$

where we denoted by g_t real government expenditure in period t . Equation (4)

becomes:

$$\frac{W_{t+1}}{1+i_t} = W_t + P_t(g_t - h_t) - \frac{i_t}{1+i_t} M_{t+1} - \frac{x_t}{1+i_t}, x_t \geq 0, \forall t \geq 0 \quad (6)$$

or,

$$W_t = \frac{W_{t+1}}{1+i_t} + P_t(h_t - g_t) + \frac{i_t}{1+i_t} M_{t+1} + \frac{x_t}{1+i_t} \quad (6')$$

where $h_t - g_t$ is primary surplus.

The difference equation can be solved forward to yield:

$$W_t = B_t + M_t = \prod_{j=0}^{T-1} \left(\frac{1}{1+i_{t+j}} \right) W_{t+j} + \sum_{k=0}^{T-1} \left(\prod_{j=0}^{k-1} \frac{1}{1+i_{t+j}} \right) \left[P_{t+k}(h_{t+k} - g_{t+k}) + \frac{i_{t+k}}{1+i_{t+k}} M_{t+k+1} + x_{t+k} \right] \quad (7)$$

If the transversality condition:

$$\lim_{T \rightarrow \infty} \left(\prod_{j=0}^{T-1} \frac{1}{1+i_{t+j}} \right) W_{t+j} = 0 \quad (8)$$

is satisfied, then, from (7), it follows:

$$B_t + M_t = \sum_{k=0}^{\infty} \left(\prod_{j=0}^{k-1} \frac{1}{1+i_{t+j}} \right) \left[P_{t+k}(h_{t+k} - g_{t+k}) + \frac{i_{t+k}}{1+i_{t+k}} M_{t+k+1} + x_{t+k} \right] \quad (9)$$

Relationship (9) represents a solvency condition for the government, or a non-Ponzi game condition (Artis and Marcellino – 1998).

From (9), substituting $g_t = y_t - c_t$, we obtain the solvency condition for households:

$$B_t + M_t = \sum_{k=0}^{\infty} \left(\prod_{j=0}^{k-1} \frac{1}{1+i_{t+j}} \right) \left[\frac{i_{t+k}}{1+i_{t+k}} M_{t+k+1} - P_{t+k}(y_{t+k} - h_{t+k} - c_{t+k}) + x_{t+k} \right] \quad (10)$$

2.2 The Optimization Problem

The representative consumer maximizes the functional J given by (11):

$$J = \sum_{t=0}^{\infty} \gamma^t U\left(c_t, \frac{Z_t}{P_t}\right) \quad (11)$$

where $U(\cdot, \cdot)$ denotes the utility, increasing in both arguments, strictly concave and twice differentiable, i.e.:

$$U'_c(\cdot, \cdot) > 0, U'_z(\cdot, \cdot) > 0 \quad (12)$$

and the Hessian:

$$\begin{pmatrix} U''_{cc} & U''_{cz} \\ U''_{zc} & U''_{zz} \end{pmatrix} \quad (13)$$

is negative definite.

Moreover,

$$\lim_{c \rightarrow 0} U'_c\left(c, \frac{Z}{P}\right) = \lim_{z \rightarrow 0} U'_z\left(c, \frac{Z}{P}\right) = \infty \quad (14)$$

$$\lim_{c \rightarrow \infty} U'_c\left(c, \frac{Z}{P}\right) = 0 \quad (15)$$

We denoted by:

$$Z_t = M_{t+1} \quad (16)$$

and γ is the discount factor which equals:

$$\gamma = \frac{1}{1 + \delta}; \delta > 0 \quad (17)$$

where $\delta > 0$ is the subjective rate of time preference.

The representative agent maximizes (11) subject to the following dynamic equation:

$$W_{t+1} = (1 + i_t)[W_t + P_t(y_t - h_t - c_t)] - i_t Z_t - x_t \quad (18)$$

W_0 given and:

$$W_{t+1} \geq Z_t \quad (19)$$

$$x_t \geq 0 \quad (20)$$

$$\lim_{T \rightarrow \infty} \left(\prod_{j=0}^{T-1} \frac{1}{1 + i_{t+j}} \right) W_{T+j} = 0 \quad (21)$$

Condition (19) can be written as:

$$B_{t+1} + M_{t+1} \geq M_{t+1} \text{ or } B_{t+1} \geq 0 \quad (19'')$$

In order to obtain the optimality conditions, we shall apply the Maximum Principle for dynamic systems with discrete variables (Altar, 1976).

The Hamiltonian is:

$$\begin{aligned} H_t = \gamma^t U\left(c_t, \frac{Z_t}{P_t}\right) + \psi_t \{ (1+i_t)[W_t + P_t(y_t - h_t - c_t)] - i_t Z_t - x_t \} + \\ + \lambda_t \{ (1+i_t)[W_t + P_t(y_t - h_t - c_t)] - (1+i_t)Z_t - x_t \} + \mu_t x_t \end{aligned} \quad (22)$$

where we denoted by ψ_t the dual variable, λ_t and μ_t are the Lagrange multipliers corresponding to constraints (19) and (20).

The optimality conditions are:

$$\begin{cases} \frac{\partial H_t}{\partial c} = 0 \\ \frac{\partial H_t}{\partial Z} = 0 \\ \frac{\partial H_t}{\partial x} = 0 \end{cases} \quad (23)$$

or

$$\begin{cases} \gamma^t U'_c\left(c_t, \frac{Z_t}{P_t}\right) = (1+i_t)P_t(\psi_t + \lambda_t) \\ \gamma^t U'_z\left(c_t, \frac{Z_t}{P_t}\right) \cdot \frac{1}{P_t} = i_t \psi_t + (1+i_t)\lambda_t \\ \psi_t = \mu_t \end{cases} \quad (24)$$

The dynamic equation of the dual variable ψ_t is:

$$\Psi_{t-1} = \frac{\partial H_t}{\partial W_t} \quad (25)$$

or

$$\Psi_{t-1} = (1 + i_t) \Psi_t + (1 + i_t) \lambda_t \quad (26)$$

Concerning the Lagrange multipliers, the Maximum Principle provides the following information:

$$\begin{aligned} \lambda_t &\geq 0 ; & \lambda_t [W_{t+1} - Z_t] &= 0 \\ \mu_t &\geq 0 ; & \mu_t \cdot x_t &= 0 \end{aligned} \quad (27)$$

From the second condition (29) we can see that $x_t > 0$ implies $\mu_t = 0$.

In this case, from the third relation (24) it would follow $\Psi_t = \mu_t = 0$, which would make the problem senseless. It follows that $x_t = 0, \forall t \geq 0$, hence the consumer budget constraint (2) on the optimal trajectory is satisfied as an equality.

From the first relation (27) it follows that $\lambda_t > 0$ implies $W_{t+1} - Z_t = 0$, i.e. $B_{t+1} = 0$. In other words, if $\lambda_t > 0$, then the agent will not buy bonds in the given period.

We perform the substitutions:

$$q_t = \frac{\Psi_t}{\gamma^t}; \alpha_t = \frac{\lambda_t}{\gamma^t} \quad (28)$$

In this case, the optimality conditions (24) become:

$$\begin{cases} U'_c \left(c_t, \frac{Z_t}{P_t} \right) = (1 + i_t) P_t (q_t + \alpha_t) \\ U'_c \left(c_t, \frac{Z_t}{P_t} \right) = P_t i_t q_t + (1 + i_t) \alpha_t \end{cases} \quad (29)$$

and the dynamic equation becomes:

$$q_{t-1} = \gamma(1 + i_t)(q_t + \alpha_t)$$

or

$$q_t = \frac{1}{\gamma(1+i_t)} q_{t-1} - \alpha_t \quad (30)$$

2.3 Qualitative analysis of optimal trajectories

In what follows, to facilitate the analysis, we assume that the utility function is separable in both arguments:

$$U(c_t, \frac{Z_t}{P_t}) = V(c_t) + \varphi(\frac{Z_t}{P_t}) \quad (31)$$

In this case, the optimality conditions (29) become:

$$V'(c_t) = (1+i_t)P_t(q_t + \alpha_t) \quad (32)$$

$$\varphi'(\frac{Z_t}{P_t}) = i_t P_t q_t + P_t(1+i_t)\alpha_t \quad (33)$$

If $B_{t+1} > 0$, in other words, if the agent buys bonds the n^{th} multiplier α_t vanishes and the relations (30) – (33) become:

$$q_t = \frac{1}{\gamma(1+i_t)} q_{t-1} \quad (34)$$

$$V'(c_t) = (1+i_t)P_t q_t \quad (35)$$

$$\varphi'(\frac{Z_t}{P_t}) = P_t i_t q_t \quad (36)$$

Writing the optimality condition (35) for two consecutive periods, we have:

$$V'(c_t) = (1+i_t)P_t q_t$$

$$V'(c_{t-1}) = (1+i_{t-1})P_{t-1} q_{t-1}$$

and, taking into account relation (34), it follows:

$$\frac{V'(c_t)}{V'(c_{t-1})} = \frac{1}{\gamma} \frac{1}{1+i_{t-1}} \frac{P_t}{P_{t-1}} \quad (37)$$

Denoting

$$1 + \pi_{t-1} = \frac{P_t}{P_{t-1}} \quad (38)$$

where π_{t-1} stands for inflation rate, relation (37) becomes:

$$\frac{V'(c_t)}{V'(c_{t-1})} = \frac{1 + \delta}{1 + r_{t-1}} \quad (39)$$

where δ is the subjective rate of preference and r is the risk-free real interest rate.

Relation (39) allows to identify the following situations:

- a) if $r_{t-1} = \delta$, then $c_t = c_{t-1}$
- b) if $r_{t-1} > \delta$, then $c_t > c_{t-1}$
- c) if $r_{t-1} < \delta$, then $c_t < c_{t-1}$

Therefore, depending on the evolution of inflation and of the risk-free nominal interest rate, consumption can be constant, increasing or decreasing.

To identify the evolution of the demand for money, we use optimality condition (36). We obtain:

$$\frac{\varphi'\left(\frac{Z_t}{P_t}\right)}{\varphi'\left(\frac{Z_{t-1}}{P_{t-1}}\right)} = \frac{P_t}{P_{t-1}} \cdot \frac{i_t}{i_{t-1}} \cdot \frac{1}{\gamma(1 + i_t)} \quad (40)$$

or

$$\frac{\varphi'\left(\frac{Z_t}{P_t}\right)}{\varphi'\left(\frac{Z_{t-1}}{P_{t-1}}\right)} = \frac{P_t}{P_{t-1}} \cdot \frac{i_t}{i_{t-1}} \cdot \frac{1 + \delta}{1 + i_t} \quad (41)$$

If we admit that:

- a) inflation is constant: $\frac{P_t}{P_{t-1}} = 1 + \pi$
- b) $i_t = i_{t-1}$

$$\text{c) } \delta = r = \frac{1+i}{1+\pi}$$

then, from (41), we obtain:

$$\frac{\varphi'\left(\frac{Z_t}{P_t}\right)}{\varphi'\left(\frac{Z_{t-1}}{P_{t-1}}\right)} = 1$$

or

$$\frac{Z_t}{P_t} = \frac{Z_{t-1}}{P_{t-1}} \quad (42)$$

and

$$\frac{M_{t+1}}{M_t} = \frac{P_t}{P_{t-1}} = 1 + \pi.$$

Thus, under assumptions (a), (b), (c), the demand for money grows with the same rate as inflation:

$$M_{t+1} = (1 + \pi) \cdot M_t \quad (43)$$

2.4. Transversality conditions

For the optimal trajectories, the Maximum Principle provides the transversality condition:

$$\lim_{T \rightarrow \infty} \psi_T W_T = 0 \quad (44)$$

Taking into account the substitution (28), relation (44) becomes:

$$\lim_{T \rightarrow \infty} \gamma^T q_T W_T = 0 \quad (45)$$

Since $W_T = B_T + M_T$ with $B_T \geq 0$ and $M_T \geq 0$, the transversality condition (45) becomes:

$$\lim_{T \rightarrow \infty} \gamma^T q_T B_T = 0 \quad (46)$$

$$\lim_{T \rightarrow \infty} \gamma^T q_T M_T = 0 \quad (47)$$

From the dynamic equation (34) for the dual variable, we obtain:

$$q_T = \frac{1}{\gamma^T} \prod_{k=1}^T \left(\frac{1}{1+i_k} \right) q_0 \quad (48)$$

Taking into account relation (48), the transversality conditions can be written as:

$$\lim_{T \rightarrow \infty} \prod_{k=1}^T \left(\frac{1}{1+i_k} \right) B_T = 0 \quad (46')$$

$$\lim_{T \rightarrow \infty} \prod_{k=1}^T \left(\frac{1}{1+i_k} \right) M_T = 0 \quad (47')$$

Remark: Relation (20), given earlier, follows from (46') and (47').

If the nominal risk-free interest rate is constant:

$$i_k = i_{k+1} = i, \forall k \geq 0 \quad (49)$$

then the transversality conditions become:

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+i} \right)^T B_T = 0 \quad (46'')$$

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+i} \right)^T M_T = 0 \quad (47'')$$

This means that the sequences $\{B_t\}_{t \in \mathbb{N}}$ and $\{M_t\}_{t \in \mathbb{N}}$ should increase slower than the sequence $\left\{ \left(\frac{1}{1+i} \right)^t \right\}_{t \in \mathbb{N}}$

2.5. The case of Bernoulli type utility functions

In order to have a convenient parametric example, we assume that the utility functions $V(\cdot)$ and $\varphi(\cdot)$ are of Bernoulli type, i.e. their elasticity of marginal utility is constant.

Let us suppose that :

$$\begin{aligned} V(c) &= \frac{1}{1-\sigma} c^{1-\sigma} \\ \varphi\left(\frac{Z_t}{P_t}\right) &= \frac{1}{1-\theta} \left(\frac{Z_t}{P_t}\right)^{1-\theta} \end{aligned} \quad (50)$$

where $\sigma \in (0,1)$.

In this case, condition (39) deduced from the optimality conditions becomes:

$$c_t = \left(\frac{1+r_{t-1}}{1+\delta} \right)^{\frac{1}{\sigma}} c_{t-1} \quad (51)$$

Relation (51) represents a dynamic equation for the control variable c_t .

As a matter of fact, it is known that the Bernoulli type utility functions are the only utility functions allowing the inference of a dynamic equation for the control variable.

Relation (51) shows that, in order to know the dynamics of optimal consumption, it is sufficient to know the evolution of the risk-free real interest rate, as well as the initial value c_0 of consumption.

From (51) it follows:

$$c_t = \prod_{k=0}^{t-1} \left(\frac{1+r_k}{1+\delta} \right)^{\frac{1}{\sigma}} c_0 \quad (51')$$

or

$$c_{t+T} = \prod_{k=0}^T \left(\frac{1+r_{t+k}}{1+\delta} \right)^{\frac{1}{\sigma}} c_t \quad (52)$$

As concerns the demand for money, from relations (35) and (36) we obtain:

$$\frac{\varphi'\left(\frac{Z_t}{P_t}\right)}{V'(c_t)} = \frac{i_t}{1+i_t} \quad (53)$$

Substituting the expressions of the derivatives, we obtain

$$\frac{Z_t}{P_t} = \left(\frac{1+i_t}{i_t} \right)^{\frac{1}{\theta}} c_t^{\frac{\sigma}{\theta}} \quad (54)$$

Taking into account that $Z_t = M_{t+1}$, relation (54) can be written as:

$$M_{t+1} = P_t \left(\frac{1+i_t}{i_t} \right)^{\frac{1}{\theta}} c_t^{\frac{\sigma}{\theta}} \quad (55)$$

It can be seen that the demand for money M_{t+1} increases as P_t and C_t increase and decreases as i_t increases, which is consistent with the properties that a function of money demand should have.

We denote:

$$m_{t+1} = \frac{M_{t+1}}{P_{t+1}} \quad (56)$$

From (55), it follows:

$$m_{t+1} = \frac{1}{1+\pi_t} \left(\frac{1+i_t}{i_t} \right)^{\frac{1}{\theta}} c_t^{\frac{\sigma}{\theta}} \quad (57)$$

It can be seen that m_{t+1} decreases as the inflation rate π_t increases.

If the nominal interest rate and inflation rate are constant and, moreover,

$$1+r = \frac{1+i}{1+\pi} = 1+\delta \quad (58)$$

then, from (51), it follows that consumption is constant, i.e.

$$c_t = c_{t-1}, \forall t \quad (59)$$

In this case, the real demand for money will also be constant:

$$m = \frac{1}{1+\pi} \left(\frac{1+i}{i} \right)^{\frac{1}{\theta}} c^{\frac{\sigma}{\theta}} \quad (60)$$

From (58) we obtain the nominal interest rate:

$$i = (1+\delta)(1+\pi) - 1 \quad (61)$$

and formula (60) becomes

$$m = \frac{1}{1 + \pi} \left(\frac{1}{1 - \frac{1}{(1 + \delta)(1 + \pi)}} \right)^{\frac{1}{\theta}} c^{\frac{\sigma}{\theta}} \quad (62)$$

Therefore, under the above assumption, if the Central Bank decides about the magnitude of the inflation rate π , the magnitude of the nominal interest rate will be given by (61), and the real demand for money will be given by (62).

3. Compatibility between Monetary and Fiscal Policy. Fiscal Solvency.

The interaction between monetary and fiscal policy remains a topic of intense interest to macroeconomists.

In this section, we adopt several simple assumptions concerning monetary and fiscal policy, in order to see how the Fiscal Solvency conditions are satisfied (these are, in fact, transversality conditions (46') and (47')).

As it concerns the **monetary policy** of the Central Bank, we assume that the following conditions are satisfied:

- a). constant inflation rate;
- b). constant nominal interest rate.

Moreover, we assume that the nominal interest rate is that given by the formula (61), which implies constant consumption.

For the sake of simplicity, we assume that the exogenous endowment y is the same in each period:

$$y_{t-1} = y_t = y \quad \forall t \geq 0 \quad (63)$$

Since the real government expenditure in each period is

$$g_t = y_t - c_t$$

it follows that this is also constant, hence

$$g_{t-1} = g_t = g \quad \forall t \geq 0 \quad (64)$$

Dividing by P_t the budget equation (2) of the household, we obtain

$$c_t + \frac{M_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} + \frac{B_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} \frac{1}{1+i} = \frac{M_t}{P_t} + \frac{B_t}{P_t} + y - h_t \quad (65)$$

Denoting

$$b_t = \frac{B_t}{P_t} ; m_t = \frac{M_t}{P_t} \quad (66)$$

and taking into account that $y-c=g$, relation (66) can be written as

$$\frac{b_{t+1}}{1+r} = b_t + (g - h_t) - [m_{t+1}(1+\pi) - m_t] \quad (67)$$

Taking into account the assumptions concerning inflation rate and nominal interest rate, it follows that the real demand for money is also constant:

$$m_{t+1} = m_t = m \quad \forall t \geq 0 \quad (68)$$

its magnitude being given by relation (62).

Formula (67) can be written as:

$$\frac{b_{t+1}}{1+r} = b_t + (g - h_t) - \pi m \quad (69)$$

or

$$\frac{b_{t+1}}{1+r} = b_t - S_t \quad (69')$$

where we denoted by S_t primary surplus inclusive of seignorage;

$$S_t = h_t + \pi m - g \quad (70)$$

As it concerns fiscal policy, we assume a constant lump-sum tax:

$$h_t = h \quad \forall t \geq 0 \quad (71)$$

In this case, the primary surplus inclusive of seignorage is constant :

$$S_t = S \quad \forall t \geq 0 \quad (72)$$

and equation (69') becomes:

$$\frac{b_{t+1}}{1+r} = b_t - S \quad (73)$$

or

$$b_{t+1} = (1+r)b_t - (1+r)S \quad (73')$$

The solution of the difference equation (73') is

$$b_T = (1+r)^T b_0 - \frac{1+r}{r} [(1+r)^T - 1] S \quad (74)$$

where from:

$$\frac{b_T}{(1+r)^T} = b_0 - \frac{1+r}{r} \left[1 - \frac{1}{(1+r)^T} \right] S \quad (75)$$

Taking into account that

$$b_T = \frac{B_T}{P_T} \quad (76)$$

$$P_T = (1+\pi)^T P_0$$

we obtain from (75)

$$\frac{B_T}{(1+i)^T} = \frac{B_0}{P_0} - \frac{1+r}{r} \left[1 - \left(\frac{1}{1+r} \right)^T \right] S \quad (77)$$

and

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+i)^T} = b_0 - \frac{1+r}{r} S \quad (78)$$

The transversality conditions will be satisfied only if

$$S = \frac{r}{1+r} b_0 \quad (79)$$

i.e if the magnitude of the lump-sum tax is

$$h = g - \pi m + \frac{r}{1+r} b_0 \quad (80)$$

We remark that if the value of the tax is given by (80), then from the dynamic equation (73), we obtain

$$b_0 = b_1 = b_2 = \dots = b_t, \forall t \in \mathbb{N} \quad (81)$$

i.e. the real debt is constant, and the nominal debt increases at the inflation rate:

$$B_T = (1+\pi)^T B_0 \quad (82)$$

From the budget constraint equation for $t = 0$, we have

$$m = m_0 \quad (83)$$

where $m_0 = \frac{M_0}{P_0}$ is given.

Taking into account the relation (62) it follows that:

$$c = (1 + \pi)^{\frac{\theta}{\sigma}} \left[\frac{(1 + \delta)(1 + \pi) - 1}{(1 + \delta)(1 + \pi)} \right]^{\frac{1}{\sigma}} m_0^{\frac{\theta}{\sigma}} \quad (84)$$

For the considered hypotheses the optimal solution is as follows:

$$c_0 = c_1 = \dots = c_t = c, \forall t \geq 0 \quad (85)$$

where c is given in (84)

$$m_0 = m_1 = \dots = m_t = m, \forall t \geq 0 \quad (86)$$

It follows that the nominal money demand is given by:

$$M_t = (1 + \pi)^t M_0, \forall t \geq 0 \quad (87)$$

Taking into account (11), (31) and (50) the objective function becomes:

$$J = \sum_{t=0}^{\infty} \gamma^t \left[\frac{1}{1 - \sigma} c_t^{1 - \sigma} + \frac{1}{1 - \theta} \left(\frac{M_{t+1}}{P_t} \right)^{1 - \theta} \right] \quad (88)$$

Using the optimal solution given by (85) and (86) we have:

$$J = \frac{1 + \delta}{\delta} c + \frac{1 + \delta}{\delta - \pi} \left(\frac{M_0}{P_0} \right) \quad (89)$$

Considering (84) it follows

$$J(\pi) = \frac{1 + \delta}{\delta} (1 + \pi)^{\frac{\theta}{\sigma}} \left[\frac{(1 + \delta)(1 + \pi) - 1}{(1 + \delta)(1 + \pi)} \right]^{\frac{1}{\sigma}} m_0^{\frac{\theta}{\sigma}} + \frac{1 + \delta}{\delta - \pi} m_0 \quad (90)$$

One can observe that the objective function depends on the inflation rate π , set by the Central Bank, as well as on the initial real money balances.

4. Conclusions

We studied in this paper a discrete time economic growth model, having elements of monetary and fiscal policy. For the case when the utility function of the household is of Bernoulli type, we obtained the dynamic equation of consumption. The dynamics depend on the nominal interest rate and on the inflation rate. If the real interest rate coincides with the subjective discount rate, then optimal consumption is constant on the whole horizon. The real demand for money is also constant.

Assuming a monetary policy rule with constant nominal interest rate and constant inflation rate, we inferred the fiscal policy rules compatible with this one.

Fiscal solvency is an important concern in the design of macroeconomic policy in EMU. Within the model, fiscal solvency conditions practically coincides with the transversality conditions provided by the Maximum Principle.

There are a number of directions in which our analysis can be developed. Obvious extensions include the incorporation of distortionary taxation and of “useful” government expenditure and the way in which these contribute to the “welfare” of the agent.

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