58th INTERNATIONAL ATLANTIC ECONOMIC CONFERENCE
Chicago, Illinois,U.S.A., October 7-10, 2004

Fiscal Policy, Public Capital and Economic Growth

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JEL Clasification : C 61, D 99,E 31, E 41, E 62, E 63, H 62, H 63, O 41.

Macro focus

1. INTRODUCTION

Government productive activities play unambiguously a prominent role in the new generation of endogeneous growth models. Both theoretically (after the work of Barro, 1990, Barro and Sala-i-Martin, 1992, and others) and empirically (see Aschauer, 1989, and the voluminous empirical literature that followed) government productive activities are widely recognized as key determinants of lung-run growth and productivity. Their positive spillovers are transmitted in the economy via private firms' production function either as a flow (government productive services) or as a stock (public or infrastructural capital).

The effect of public investment on private capital formation is a crucial public policy issue. Empirical research into this question was stimulated by Aschauer (1989a, b) who suggested that public capital has powerful impact on the productivity of private capital. Aschauer's results were controversial and have generated substantial empirical research directed at determining the robustness of his position. While the evidence is mixed, there seems to be a consensus generally supporting the productivity of public investment, although suggesting that its impact is somewhat weaker than that originally proposed by Aschauer.

The theoretical analysis of the productivity of public investment proceeds by introducing government expenditure as an argument in the production function, to reflect, among other reasons, an externality in production. Two formulations can be identified. Most of the existing literature treats the current flow of government expenditure as the source of contribution to productive capacity. For example, Aschauer and Greenwood (1985), Aschauer (1988), Barro (1989), and Turnovsky and Fisher (1995) do so in neoclassical Ramsey framework. Barro (1990) and Turnovski (1995) employ a simple 'A-K' endogenous growth model. While the flow specification has the virtue of tractability, it is open to criticism that insofar as productive government expenditures are intended to represent public infrastructure, such as roads and education, it is the accumulated stock, rather than the current flow, that is relevant.

Despite this criticism, few authors have adopted the alternative approach of specifying productive government expenditure as stock. Arrow and Kurz (1970) were the first authors to formulate government expenditure as a form of investment. More recently, Baxter and King (1995) study the macroeconomic implications of increases in the stocks of public goods. They derive the transitional dynamic responses of output, investment, consumption, employment, and interest rates to such policies by calibrating a real business cycle model. Futagami et al. (1993) extend the Barro (1990) A-K growth model to include government capital.

2. THE ASSUMPTIONS OF THE MODEL

We assume that the model consists of N identical agents. The individual consumer-producer agent chooses his consumption, C, and stock of private capital, *k*, to maximize the following concave intertemporal utility function:

$$\max \sum_{t=1}^{\infty} \delta^t U(c_t) \qquad (1),$$

where:

$$U'(\cdot) > 0, \; U''(\cdot) < 0 \qquad (2)$$

Within the model we assume that the utility function is of the Bernoulli form:

$$U(c) = \frac{1}{1-\sigma} c^{1-\sigma} \qquad (3).$$

The production function of the individual agent is:

$$\begin{aligned} &f : \mathbf{R}_+ \times \mathbf{R}_+ \to \mathbf{R}_+ \qquad (4) \\ &y = f(k, S) \end{aligned}$$

where

$$f_1'(\cdot) > 0; \qquad f_2'(\cdot) > 0 \qquad (5)$$

and the Hessian matrix is negative definite.

We denoted by S the productive services desired by the representative agent from government capital. These are represented by an Edwards (1990) type function:

$$S = K^g \left(\frac{k}{K}\right)^{1-\alpha}, \qquad 0 \le \alpha \le 1 \qquad (6)$$

where K^g denotes the aggregate public capital, K denotes the aggregate private capital and α represents the degree of congestion associated with the public goods.

The case $\alpha = 1$ corresponds to a non-rival, non-excludable public capital good, that is available equally to each firm, independent of the size of the economy; there is no congestion .

The case $\alpha = 0$ corresponds to a phenomenon of proportional congestion, meaning that the congestion grows in direct proportion to the size of the economy. The cases $0 < \alpha < 1$, describe partial congestion.

With all agents being identical, capital stocks are related by:

$$K = N \cdot k$$

where N is the number of representative agents (firms).

In this case, the function S regarding the services derived by the representative agent from the government capital, becomes:

$$S = K^g \left(\frac{k}{N \cdot k}\right)^{1-\alpha} = K^g \cdot h \qquad (6')$$

where
$$h = \left(\frac{1}{N}\right)^{1-\alpha} \qquad (7).$$

So, function (4) becomes:

$$y = f(k, h \cdot K^g) \qquad (8)$$

With the above notations, the model of the representative agent is:

$$\begin{cases} \max \sum_{t=1}^{\infty} \delta^t U(c_t) \\ k_{t+1} = (1-\mu_1)k_t + (1-\tau) f(k_t, h \cdot K_t^g) - c_t - M \\ k_0 = k^0 - given \end{cases} \qquad (9)$$

where :

δ - constant rate of consumer time preference;
μ_1 - constant rate of physical depreciation of private capital;
τ - distortionary rate of income tax;
M – lump sum taxation;

For the beginning, we will assume that the variables $\tau, M, h \text{ and } K_t^g$ are exogenous. To deduce the optimal solution of the model (9) we apply the Maximum Principle for systems with discrete variables .

The Hamiltonian of the system is:

$$H_t = \delta^t U(c_t) + \psi_t \left[(1-\mu_1)k_t + (1-\tau) f(k_t, h \cdot K_t^g) - c_t - M \right] \qquad (10)$$

We denoted by ψ_t the dual variable of the system:

The optimality conditions are:

$$\frac{\partial H_t}{\partial c} = 0 \qquad (11)$$

and

$$\delta^t U'(c_t) = \psi_t ,$$

respectively.

The dynamic equation of the dual variable ψ_t are:

$$\psi_{t-1} = \frac{\partial H_t}{\partial K} \qquad (12)$$

and

$$\psi_{t-1} = \psi_t \left[1 - \mu_1 + (1-\tau) f_1'(k_t, h \cdot K_t^g) \right] \qquad (12')$$

respectively.

We denoted by $f_1'(\cdot)$ the derivative of the production function with respect to the first variable.

The transversality condition is:

$$\lim_{t \to \infty} \psi_t \cdot k_t = 0$$

Denoting

$$q_t = \psi_t \cdot \delta^{-t} ,$$

equations $(11')$ and $(12')$ become:

$$U'(c_t) = q_t \qquad (13)$$

$$q_t = \frac{1}{\delta \left[1 - \mu_1 + (1-\tau) f_1'(k_t, h \cdot K_t^g) \right]} q_{t-1} \qquad (14)$$

and the transversality condition becomes

$$\lim_{t \to \infty} q_t \delta^{-t} \cdot k_t = 0 \qquad (15)$$

In the case in which the utility function is of the Bernoulli form (3), the dynamic equation (14) becomes:

$$c_t = \left[\delta(1-\mu_1 + (1-\tau) f_1'(k_t, h\cdot K_t^g)\right]^{1/\sigma} c_{t-1} \qquad (16)$$

From now on, we will denote by k^g the stock of public capital per capita , namely

$$k^g = \frac{K^g}{N} \qquad (17)$$

The dynamic equation of the variable k^g is the following:

$$k_{t+1}^g = (1-\mu_2)k_t^g + g \qquad (18)$$

where g represents the level of public investment in infrastructure, and μ_2 represents the rate of government capital depreciation.

The government finances its investment activities by using either distortionary or lump-sum taxation, in accordance with its' flow budget constraint:

$$\tau\ f(k, h\cdot Nk^g) + M = g \qquad (19)$$

3. THE MACROECONOMIC DYNAMIC EQUILIBRIUM

The macroeconomic dynamic equilibrium is summarized by the following three equations:

$$\begin{cases} k_{t+1} = (1-\mu_1)k_t + f(k_t, h\cdot N\cdot k_t^g) - c_t - g \\ k_{t+1}^g = (1-\mu_2)k_t + g \\ c_t = \left[\delta(1-\mu_1 + (1-\tau) f_1'(k_t, h\cdot N\cdot k_t^g)\right]^{1/\sigma} c_{t-1} \end{cases} \qquad (20)$$

Given the degree of congestion, as parameterized by *h*, and the scale of the economy *N*, equations (20) represent an autonomous dynamic system in the two private variables, *k* and *c*, and the stock of government capital k^g . The precise nature of the steady-state equilibrium occurs when:

$$k_{t+1} = k_t,\ k_{t+1}^g = k_t^g,\ c_t = c_{t-1} \qquad (21)$$

and consists of the following relationships:

$$
\begin{cases}
c^* = f(k^*, h \cdot N(k^g)^*) - \mu \cdot k^* - g \\
\mu_2 (k^g)^* = g \\
(1-\tau) f_1'(k^*, h \cdot N(k^g)^*) = \dfrac{1}{\sigma} - 1 + \mu_1
\end{cases}
\tag{22}
$$

The steady state government budget constraint is:

$$
\tau \cdot f(k^*, h \cdot N(k^g)^*) + M = g = \mu_2 (k^g)^* \tag{23}
$$

Taking into account the second relation in the system (22), we obtain:

$$
\begin{cases}
c^* = f(k^*, h \cdot N \cdot g \cdot \mu_2^{-1}) - \mu_1 k^* - g & (24a) \\
f_1'(k^*, h \cdot N \cdot g \cdot \mu_2^{-1}) = \dfrac{1}{1-\tau}\left[\dfrac{1}{\delta} - 1 + \mu_1\right] & (24b)
\end{cases}
$$

Assuming that all the parameters of the model are fixed, except the investments in the public capital, *g*, from equations (24a) and (24b) we deduce:

$$
\begin{aligned}
k^* &= k^*(g) \\
c^* &= c^*(g)
\end{aligned}
\tag{25}
$$

From equation (24b) we deduce:

$$
f_1'(k^*(g), h \cdot N \cdot g \cdot \mu_2^{-1}) = \frac{1}{1-\tau}\left[\frac{1}{\delta} - 1 + \mu_2\right] \tag{26}
$$

and

$$
f_{11}''(\cdot,\cdot)\frac{\partial k^*(g)}{\partial g} + f_{12}''(\cdot,\cdot) \cdot h \cdot N \cdot \mu_2^{-1} = 0 \tag{27}
$$

respectively.

From (27) we deduce:

$$
\frac{\partial k^*(g)}{\partial g} = -\frac{f_{12}''(\cdot,\cdot)}{f_{11}''(\cdot,\cdot)} h \cdot N \cdot \mu_2^{-1} \tag{28}
$$

Because $f''_{12}(\cdot,\cdot) < 0$, if $f''_{12}(\cdot,\cdot) > 0$ then $\frac{\partial k^*(g)}{\partial g} > 0$. This means that the size of the private capital increases with the growth of public investment *g*. As concerns the behavior of private consumption *c** with respect to public investment, *g*, we deduce from (24a):

$$\frac{\partial c^*}{\partial g} = f'_1(\cdot,\cdot)\frac{\partial k^*}{\partial g} + f'_2(\cdot,\cdot)\cdot h\cdot N\cdot \mu_2^{-1} - \mu_1\frac{\partial k^*}{\partial g} - 1 \qquad (29)$$

and:

$$\frac{\partial c^*}{\partial g} = \left[f'_1(\cdot,\cdot) - \mu_1\right]\frac{\partial k^*}{\partial g} + \left[f'_2(\cdot,\cdot)\cdot h\cdot N\cdot \mu_2^{-1} - 1\right] \qquad (29')$$

respectively.

In general, the first term in brackets is positive:

$$f'_1(\cdot,\cdot) > 0$$

If

$$f'_2(\cdot,\cdot)\cdot h\cdot N\cdot \mu_2^{-1} \geq 1 \qquad (30)$$

we deduce that

$$\frac{\partial c^*}{\partial g} > 0 \qquad (31)$$

which means that private consumption increases along with the increase of public investment.

4. THE OPTIMAL LEVEL OF PUBLIC INVESTMENT

In the model analyzed in the previous section the level of investment ,g, was fixed exogenously.

In this section, we try to deduce the optimal level for public investment.

We consider the following dynamic model:

$$\begin{cases} \max \sum_{t=0}^{\infty} \delta^t U(c_t) \\ k_{t+1} = (1-\mu_1)k_t + f(k_t, h\cdot k\cdot N\cdot k_t^g) - c_t - g_t \\ k_{t+1}^g = (1-\mu_2)k_t^g + g_t \end{cases} \qquad (32)$$

Model (32) contains two state variables, k_t, k_t^g, as well as two decision variables, namely: c_t, g_t.

The optimality conditions are:

$$\frac{\partial H_t}{\partial c} = 0 \text{ or, } U'(c_t) = \psi_t \qquad (34)$$

$$\frac{\partial H_t}{\partial g} = 0 \text{ or, } \psi_t = \psi_t^g \quad \text{, respectively} \qquad (35)$$

and the dynamic equations of the dual variables are:

$$\psi_{t-1} = \psi_t \left[1 - \mu_1 + f_1'(k_t, h \cdot N \cdot k_t^g)\right] \qquad (36)$$

$$\psi_{t-1}^g = \psi_t \cdot f_2'(k_t, h \cdot N \cdot k_t^g) \cdot h \cdot N + \psi_t^g (1 - \mu_2) \qquad (37)$$

Taking into account the optimality condition (35), we deduce

$$1 - \mu_1 + f_1'(k_t \cdot h \cdot N \cdot k_t^g) = 1 - \mu_2 + f_2'(k_t \cdot h \cdot N \cdot k_t^g) \cdot h \cdot N \qquad (38)$$

To simplify, we assume that

$$\mu_1 = \mu_2 \qquad (39)$$

In this case, from (38) it follows

$$f_1'(k_t \cdot h \cdot N \cdot k_t^g) = f_2'(k_t \cdot h \cdot N \cdot k_t^g) \cdot h \cdot N \qquad (40)$$

Relation (40) shows that the marginal productivity of private capital must be equal to the marginal productivity of public capital, multiplied by the congestion quotient *h* and with the scale quotient *N*.

From (40) will follow the relationship between public and private capital is:

$$k_t^g = k_t^g(k_t, N, h) \qquad (41)$$

On the basis of the optimality condition (34), relationship (41) and the dynamic equations (32), we can calculate the optimal values of private consumption, as well as the optimal level of public investment g_t.

5. CONCLUDING REMARKS

The impact of public investment on private capital formation remains one of the important issues in macroeconomics. In this paper we have analysed the subject in an intertemporal optimizing market-clearing framework. The key feature characterizing our analysis is that we have treated the public good as taking the form of a durable capital good, subject to congestion. This view of government expenditure provides a more realistic approach to analyzing the intertemporal tradeoffs that public expenditure policy imposes on the private sector.

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