

# Macroeconomic effects of the population ageing phenomenon

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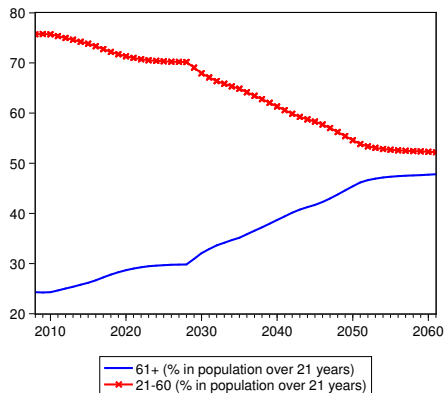
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# Ageing population phenomenon

- Indicator: share of individuals aged 61 years or over in total population aged 21 years or over. (Source: *Eurostat*)



# Theoretical impact of ageing

- Labor force shrinkage
- Modification of the capital to labor ratio:
  - $\implies$  the wage is expected to rise
  - $\implies$  the interest rate is expected to drop
- Fiscal sector:
  - taxation base decreases as the share of active agents declines.
  - growing health care expenditures as the share of old agents rises.
  - main challenge: **social security system under pressure.**

# Auerbach-Kotlikoff Overlapping Generations Model

- Deterministic model, perfect foresight.
- The total population consists of individuals of different ages (from 1 to  $T + T^R$ ). The model age of 1 year corresponds to real life age of 21 years.
- All agents of the same age form a generation (or cohort). The agents within a generation are identical.
- There are  $T + T^R$  unequal generations living simultaneously. Every period a generation dies and a new one is born.
- The agent works for the first  $T$  periods of his life, and is retired for the next  $T^R$  periods.
- Time is dedicated to labor and leisure:  $n_t^s = 1 - l_t^s$  for  $s = 1, 2, \dots, T$
- ...and retirement is mandatory:  $n_t^s = 0$  and  $l_t^s = 1$  for  $s = T + 1, 2, \dots, T + T^R$

# Households optimization problem

- The agent born in period  $t$  maximizes:

$$\sum_{s=1}^{T+T^R} \beta^{s-1} U(c_{t+s-1}^s, l_{t+s-1}^s) \quad (1)$$

- ...subject to the budget constraint for worker ( $s = 1, 2, \dots, T$ ):

$$k_{t+s}^{s+1} = (1 + r_{t+s-1})k_{t+s-1}^s + (1 - \tau_{t+s-1})w_{t+s-1}\xi^s n_{t+s-1}^s - c_{t+s-1}^s \quad (2)$$

- ...and to the budget constraint for retired ( $s = T + 1, \dots, T + T^R$ ):

$$k_{t+s}^{s+1} = (1 + r_{t+s-1})k_{t+s-1}^s + b_{t+s-1} - c_{t+s-1}^s \quad (3)$$

- The agent enters life without wealth and doesn't leave bequests.
- The terms capital and wealth are used interchangeably for  $k_{t+s-1}^s$ .

# Firms sector

- A single representative firm produces a homogeneous output according to a labor-augmenting Cobb-Douglas production function:

$$Y_t = (A_t N_t)^{1-\alpha} K_t^\alpha \quad (4)$$

- The technological progress  $A_t$  grows exogenously with  $g_A$
- The production factors are paid with their marginal product:

$$r_t = \frac{\partial Y_t}{\partial K_t} - \delta = \alpha (A_t N_t)^{1-\alpha} K_t^{\alpha-1} - \delta \quad (5)$$

$$w_t = \frac{\partial Y_t}{\partial N_t} = (1 - \alpha) A_t (A_t N_t)^{-\alpha} K_t^\alpha \quad (6)$$

## Government sector

- The government collects contributions, distributes pensions and maintains its budget balanced every period:

$$\tau_t w_t N_t = \sum_{s=T+1}^{T+T^R} b_t P_t^s \quad (7)$$

- The balancing is ensured by adjusting the revenues through the contribution rate  $\tau_t$  and/or the expenditures through the pension to net average wage ratio  $\zeta_t$ , known as replacement ratio

$$\zeta_t = \frac{b_t}{(1 - \tau_t) w_t \bar{n}_t} \quad (8)$$

- where the average effective labor supply  $\bar{n}$  is given by:

$$\bar{n}_t = \frac{\sum_{s=1}^T n_t^s \xi^s \frac{P_t^s}{T}}{\sum_{s=1}^T P_t^s} \quad (9)$$



# Equilibrium

- In the equilibrium, the households and the firm are on their maximizing path, the government has a balanced budget and the labor, capital and goods markets clear
- Labor market clearing:

$$N_t = \sum_{s=1}^T n_t^s \xi^s P_t^s \quad (10)$$

- Capital market clearing:

$$K_t = \sum_{s=1}^{T+T^R} k_t^s P_t^s \quad (11)$$

- Goods market clearing:

$$Y_t = C_t + I_t = \sum_{s=1}^T c_t^s P_t^s + K_{t+1} - (1 - \delta)K_t \quad (12)$$

# Balanced growth path

- The balanced growth path is an equilibrium in which all variables grow at constant, but possibly different rates.
- The balanced growth path is defined so that the growth rates of the variables are consistent with the "stylized facts" of growth:
  - output per capita, aggregate capital per capita and aggregate consumption per capita grow at  $g_A$ .
  - the wage  $w$ , the individual wealth  $k$ , the pension  $b$  and individual consumption  $c$  grow at  $g_A$ .

# Stationary transformation

- The model is expressed in stationary form and in per capita terms.
- Aggregate variables  $Y_t$ ,  $K_t$ ,  $C_t$ , are divided by  $A_t P_t$  and aggregate labor supply  $N_t$  is divided by  $P_t$ .
- Individual variables  $c_t$ ,  $k_t$ ,  $w_t$ ,  $b_t$  are divided by  $A_t$ .  $\implies$  productivity detrended variables.
- In the stationary-transformed model, the balanced growth path is a steady state in detrended variables.

## Simulation description (1)

- Following Börsch-Supan et al. (2006), two extreme cases are considered, depending on the method of balancing the budget. First, the replacement ratio  $\zeta$  is constant and the contribution rate  $\tau_t$  adjusts. Second, the contribution rate  $\tau$  is constant and the replacement ratio  $\zeta$  adjusts.
- From the balanced budget condition, the expressions for the contribution rate  $\tau_t$  and the replacement ratio  $\zeta$  are:

$$\tau_t = \frac{\zeta \rho_t}{\zeta \rho_t + 1 - \rho_t} \quad \text{and} \quad \zeta_t = \frac{\tau}{1 - \tau} \frac{1 - \rho_t}{\rho_t} \quad (13)$$

where  $\rho_t$  denotes the ratio of retired agents to total population.

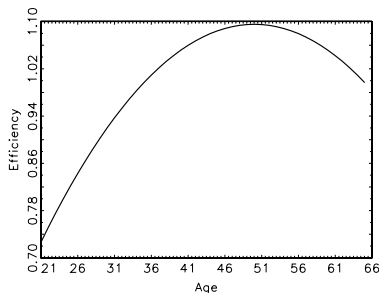
## Simulation description (2)

- In the first scenario, the relative benefit after retirement is guaranteed, while in the second the agents are aware that the pension will decline and have an incentive to save more.
- The first scenario bears resemblance to the old pay-as-you-go system, while the second has similarities with transition to a partially funded system.
- Also, a case for extending the retirement age was considered, in line with the current proposed legislation.

# Calibration

- The parameters were set mainly following Heer and Maussner (2009) and standard economic literature.

- $U(c, l) = \frac{(cl^\gamma)^{1-\eta}-1}{1-\eta}$
- $\eta = 2$  ;  $\gamma = 2$
- $\beta = 0.99$  ;  $\alpha = 0.3$  ;  
 $\delta = 0.06$
- $\zeta = 0.5$  or  $\tau = 0.105$
- $T = 40$  ;  $T^R = 20$
- $\xi^s = 0.03s + 0.0006s^2$

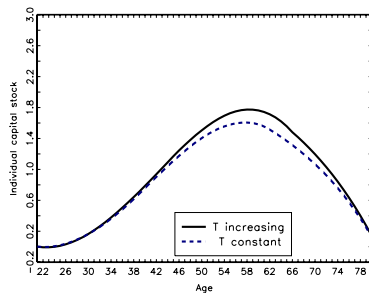


Efficiency profile

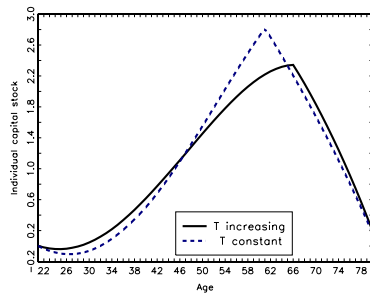
# Data

- The demographic evolution is taken exogenous.
- Most important: the ratio of retired agents to total population  $\rho_t$  .
- Historical demographic data from 1950 were collected from Eurostat. The population projection *Europop2008* was used for data until 2061.
- The share of individuals aged 61 years or over in total population aged 21 year or over appears to stabilize after 2050 (shown in the figure in Introduction). It was assumed to remains constant at 48%, the projected value corresponding to year 2061, for the next 88 years, until 2149. The sizes of the generations are also assumed constant.

# Wealth profile for 2010 labor force entrant



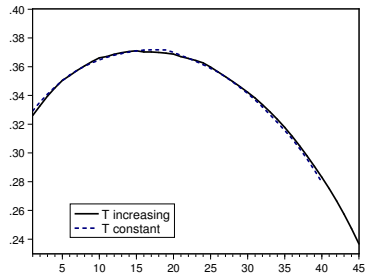
Constant replacement ratio scenario



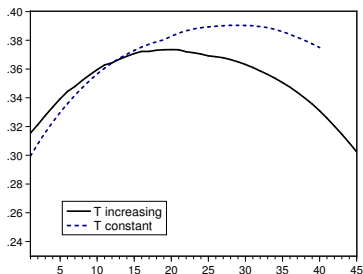
Constant contribution rate scenario



# Optimal labor supply for 2010 labor force entrant

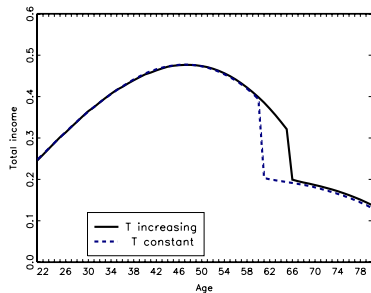


Constant replacement ratio scenario

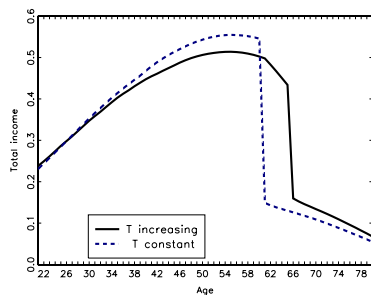


Constant contribution rate scenario

# Total income for 2010 labor force entrant

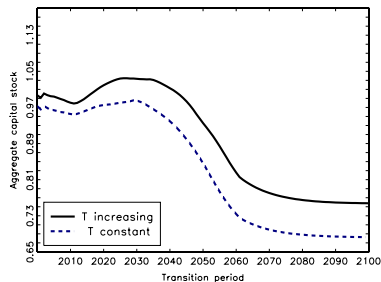


Constant replacement ratio scenario

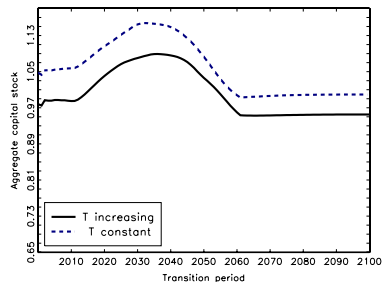


Constant contribution rate scenario

# Productivity detrended aggregate capital stock per capita

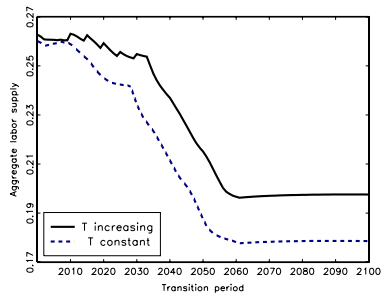


Constant replacement ratio scenario

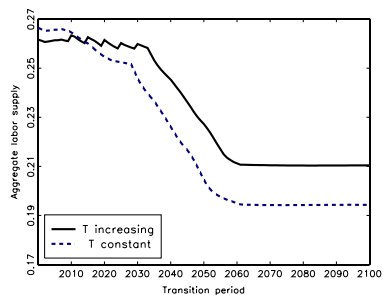


Constant contribution rate scenario

# Aggregate labor supply per capita

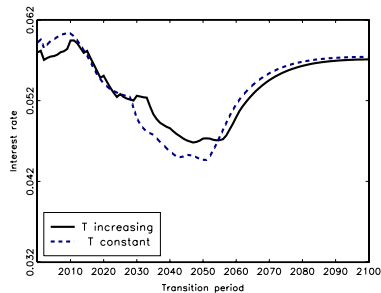


Constant replacement ratio scenario

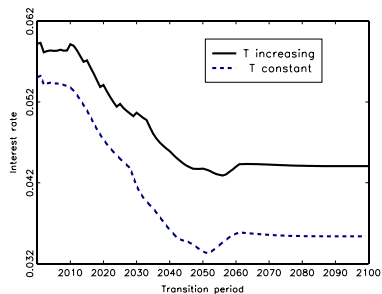


Constant contribution rate scenario

# Interest rate

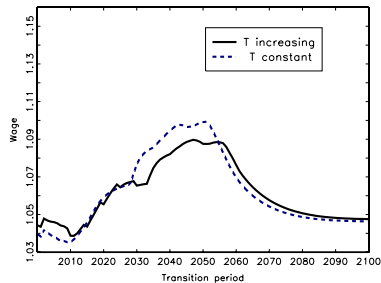


Constant replacement ratio scenario

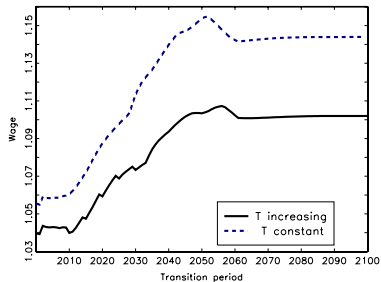


Constant contribution rate scenario

# Productivity detrended wage

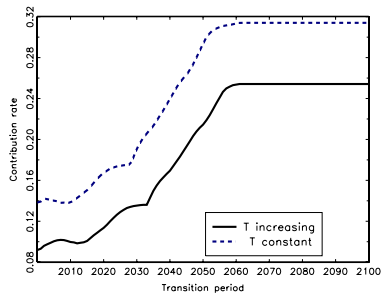


Constant replacement ratio scenario

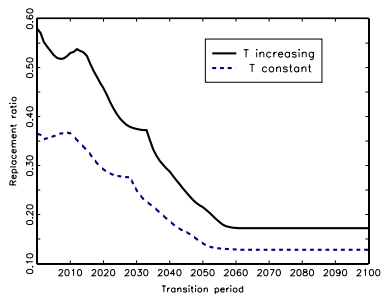


Constant contribution rate scenario

# The contribution rate and the replacement ratio

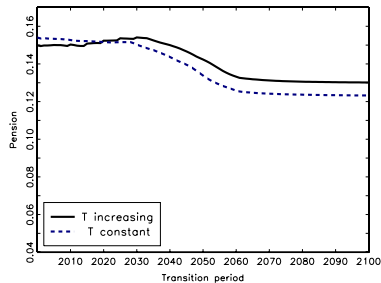


Constant replacement ratio scenario

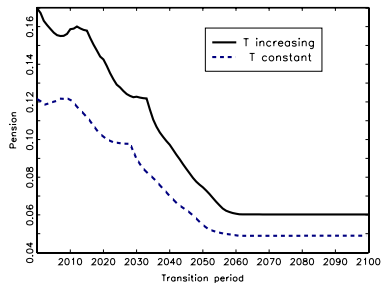


Constant contribution rate scenario

# Pension



Constant replacement ratio scenario



Constant contribution rate scenario



# Conclusions

- The wealth profile confirms the of the life-cycle theory.
- In the constant replacement ratio scenario, because of the guaranteed relative benefit after retirement and the high contribution rate, agents work and save less.
- Agents are encouraged to accumulate wealth and supply labor in the constant contribution rate scenario.
- As the ageing process determines a smaller capital stock and aggregate labor supply, the impact of ageing is higher in the first scenario as compared to the second.
- Extending the retirement age mitigates only to a small extent the pressure on the social security system.
- The retirement of the numerous generations after 2028 significantly impacts the evolution of the variables.

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